MF and ACE Target Detection on the Cooke City Data

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ACE Derivation

$$L(x;\mu,\Sigma) = f(\mu,\Sigma;x) = \frac{1}{\sqrt{(2\pi)^p|\Sigma|}} e^{-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)}$$

$$\ln\left(\frac{L(x;t,\Sigma)}{L(x;\mu,\Sigma)}\right) = \ln\left(\frac{\frac{1}{\sqrt{(2\pi)^p|\Sigma|}}e^{-1/2(x-t)^T\Sigma^{-1}(x-t)}}{\frac{1}{\sqrt{(2\pi)^p|\Sigma|}}e^{-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)}}\right)$$

$$\boxed{\ln\left(\frac{a}{b}\right) = \ln(ab^{-1}) = \ln(a) + \ln(b^{-1}) = \ln(a) - \ln(b)}$$

$$\ln\left(\frac{1}{\sqrt{(2\pi)^p|\Sigma|}}e^{-1/2(x-t)^T\Sigma^{-1}(x-t)}\right) - \ln\left(\frac{1}{\sqrt{(2\pi)^p|\Sigma|}}e^{-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)}\right)$$

$$\cancel{\ln\left(\frac{1}{\sqrt{2\pi^p|\Sigma|}}\right)} + \ln\left(e^{-1/2(x-t)^T\Sigma^{-1}(x-t)}\right) - \cancel{\ln\left(\frac{1}{\sqrt{2\pi^p|\Sigma|}}\right)} - \ln\left(e^{-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)}\right)$$

$$\ln\left(e^{-1/2(x-t)^T\Sigma^{-1}(x-t)}\right) - \ln\left(e^{-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)}\right)$$

$\ln e^x = x$

$$-1/2(x-t)^T\Sigma^{-1}(x-t) - \left(-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

$$\cancel{-1/2}(x-t)^T\Sigma^{-1}(x-t) + \cancel{1/2}(x-\mu)^T\Sigma^{-1}(x-\mu)$$

$$-1/2\underbrace{\left((x-t)^T\Sigma^{-1}(x-t) - (x-\mu)^T\Sigma^{-1}(x-\mu)\right)}_{\text{Paper only includes this part - where does the -1/2 go?}}$$

Drop constant

$$(x-t)^T\Sigma^{-1}(x-t) - (x-\mu)^T\Sigma^{-1}(x-\mu)$$

$t = at + (1-a)\mu$

$$(x-(at+(1-a)\mu))^T\Sigma^{-1}(x-(at+(1-a)\mu)) - (x-\mu)^T\Sigma^{-1}(x-\mu)$$

$$(x-at-\mu+a\mu)^T\Sigma^{-1}(x-at-\mu+a\mu) - \overbrace{(x-\mu)^T\Sigma^{-1}(x-\mu)}^{X^TX}$$

$$((x-\mu)-a(t-\mu))^T\Sigma^{-1}((x-\mu)-a(t-\mu))$$

$(A+B)^T = A^T + B^T$

$$\underbrace{(x-\mu)^T - a(t-\mu)^T}_{X^T-aT^T}\underbrace{(\Sigma^{-1}(x-\mu) - \Sigma^{-1}a(t-\mu))}_{X-aT}$$

Whiten

$$(X^T - aT^T)(X - aT) - X^TX$$

$NF = \frac{X^TT}{T^TT}$

$$(X^TX - aX^TT - aT^TX + a^2T^TT) - X^TX$$

$$\left(\cancel{X^TX} - \frac{(X^TT)^2}{T^TT} - \cancel{\frac{(X^TT)^2}{T^TT}} + \frac{(X^TT)^2}{(T^TT)^2}\cancel{(T^TT)}\right) - \cancel{X^TX}$$

$$+\cancel{\frac{(X^TT)^2}{T^TT}}$$

$$\boxed{= \frac{(X^TT)^2}{T^TT}}$$

IOU

$$ACE = \frac{(X^TT)^2}{(X^TX)(T^TT)}$$

ACE Code

```python
numerator = np.matmul(WimList,Wspec.T)

XX = np.reshape(np.sqrt(np.sum(WimList**2,axis=1)), (224000,1))
TT = np.reshape(np.sqrt(np.sum(Wspec**2,axis=1)),(18,1))
denominator = np.matmul(XX,TT.T)

ACE = np.squeeze(numerator / denominator)

# Lets reshape this result so we can compare it to the image
ACE = np.reshape(ACE, (im.nrows, im.ncols, ntargets))
```

Plots

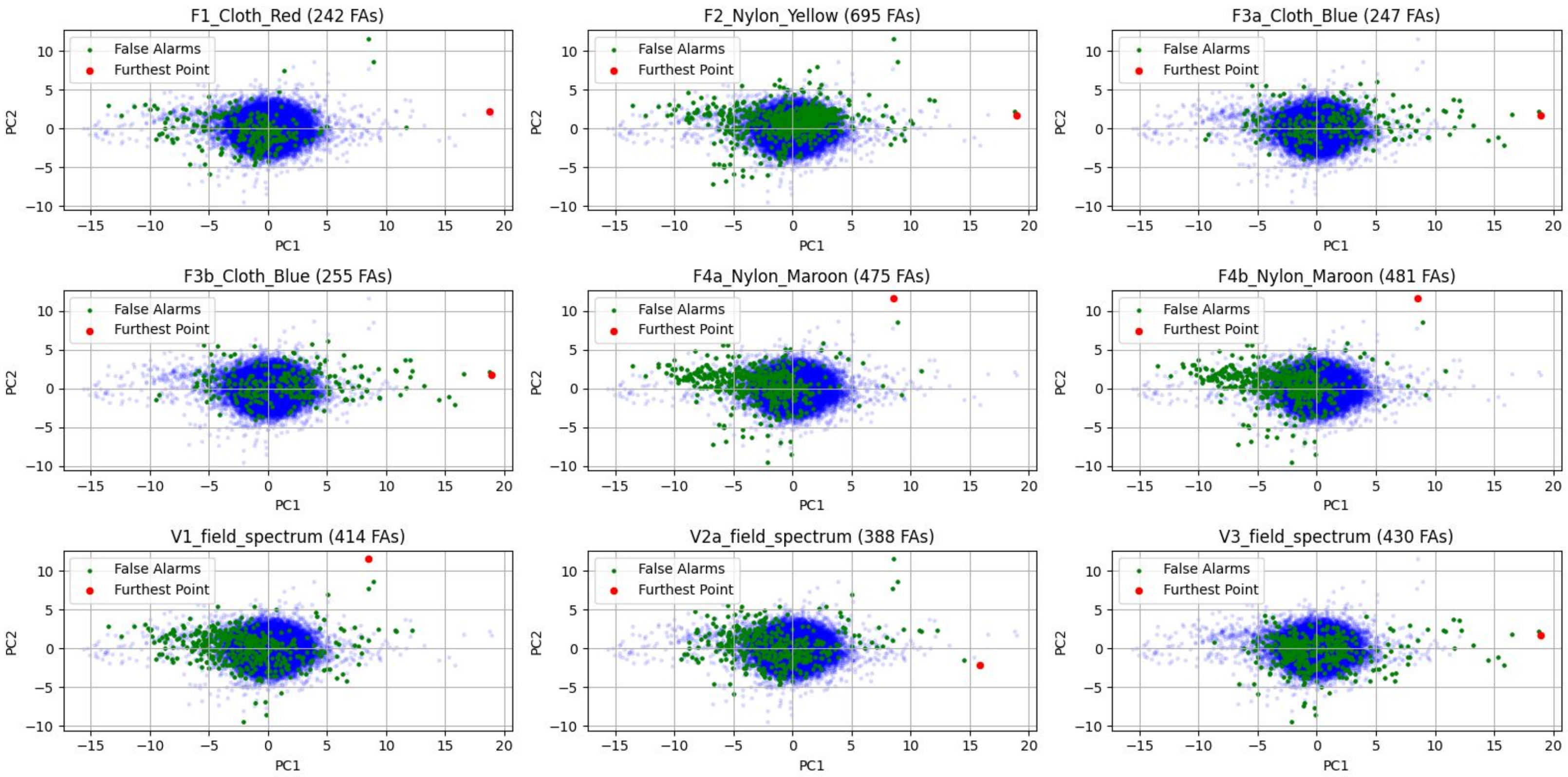


Fig. 5 Plots of the image data projected from whitened space with false alarms for MF shown in green. The vertical axis is adjacent side and the horizontal axis is opposite side from Eq. (8). The highest scoring target pixel from the ground truth is shown in red. The false alarms—pixels scoring greater than the highest scoring target pixel from the ground truth—are shown in green. For the fabric panels additional ground truth target pixels are shown in teal. The number of false alarms is provided in the title for each plot.

MF

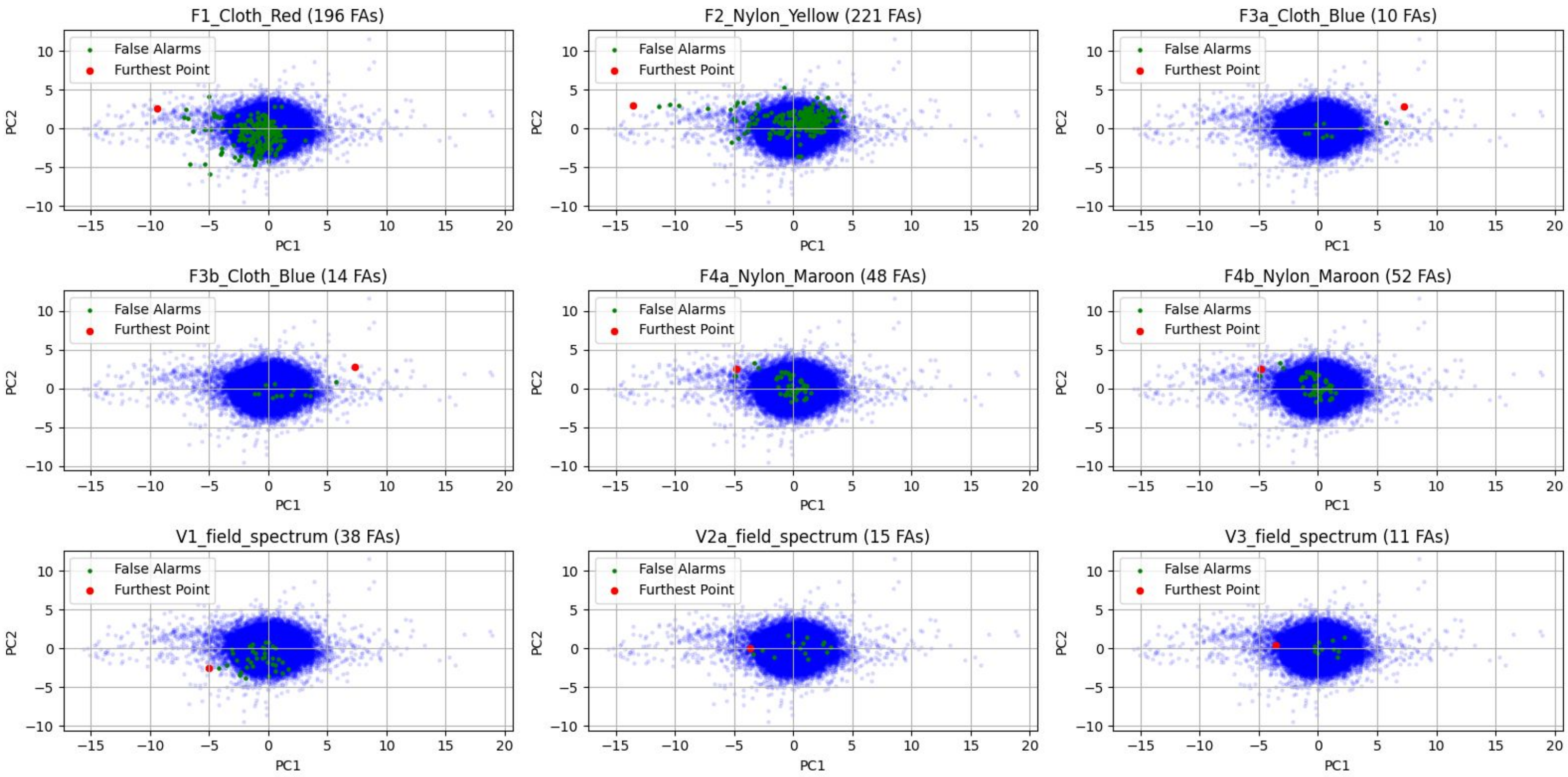


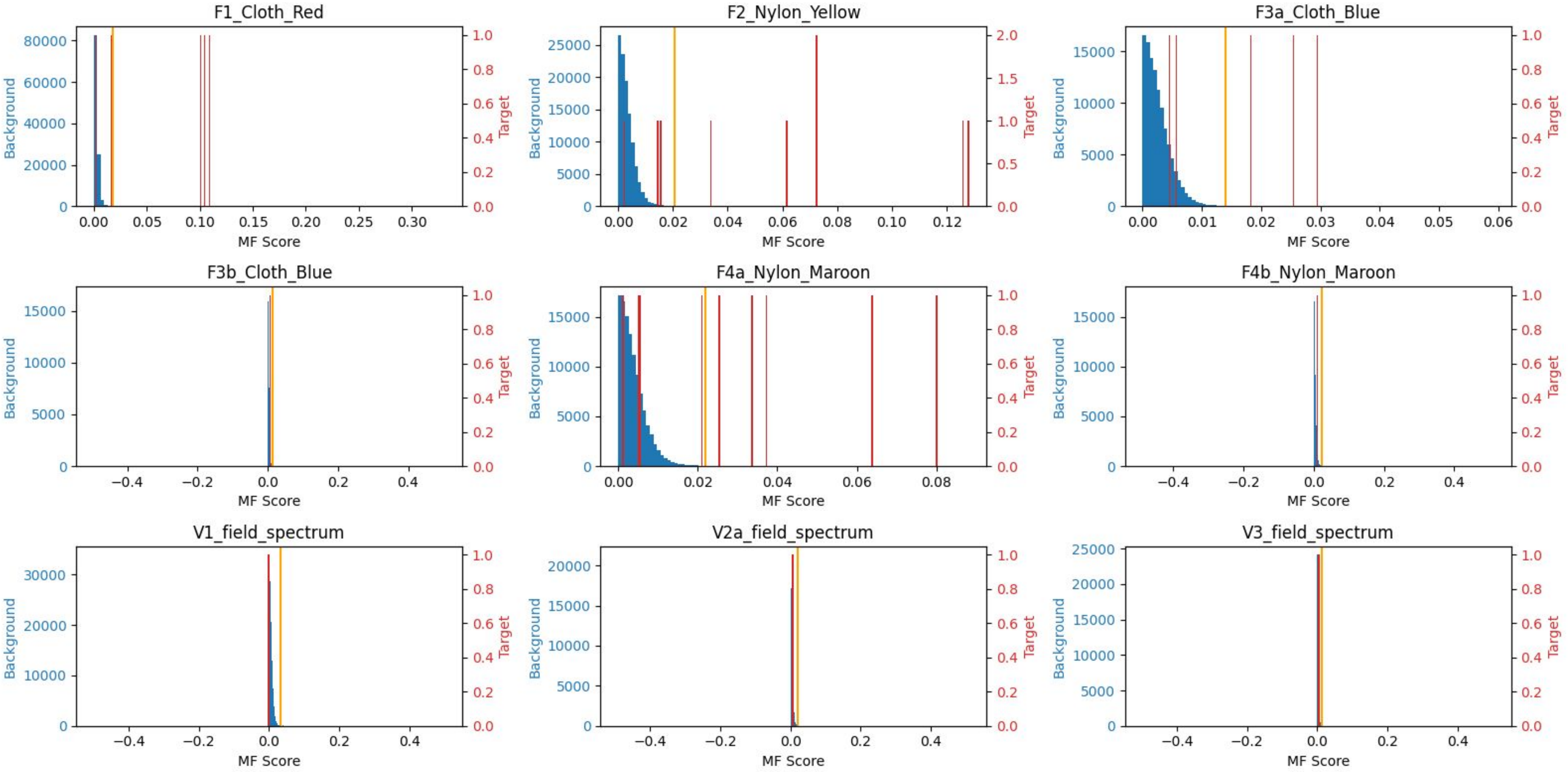
Fig. 6 Plots of the image data projected from “nonmean-centered whitened space” space with false alarms for ACE-NM shown in green. The vertical axis is adjacent side and the horizontal axis is opposite side from Eq. (8). The highest scoring target pixel from the ground truth is shown in red. The false alarms—pixels scoring greater than the highest scoring target pixel from the ground truth—are shown in green. For the fabric panels additional ground truth target pixels are shown in teal. The number of false alarms is provided in the title for each plot.

False Alarm Rates

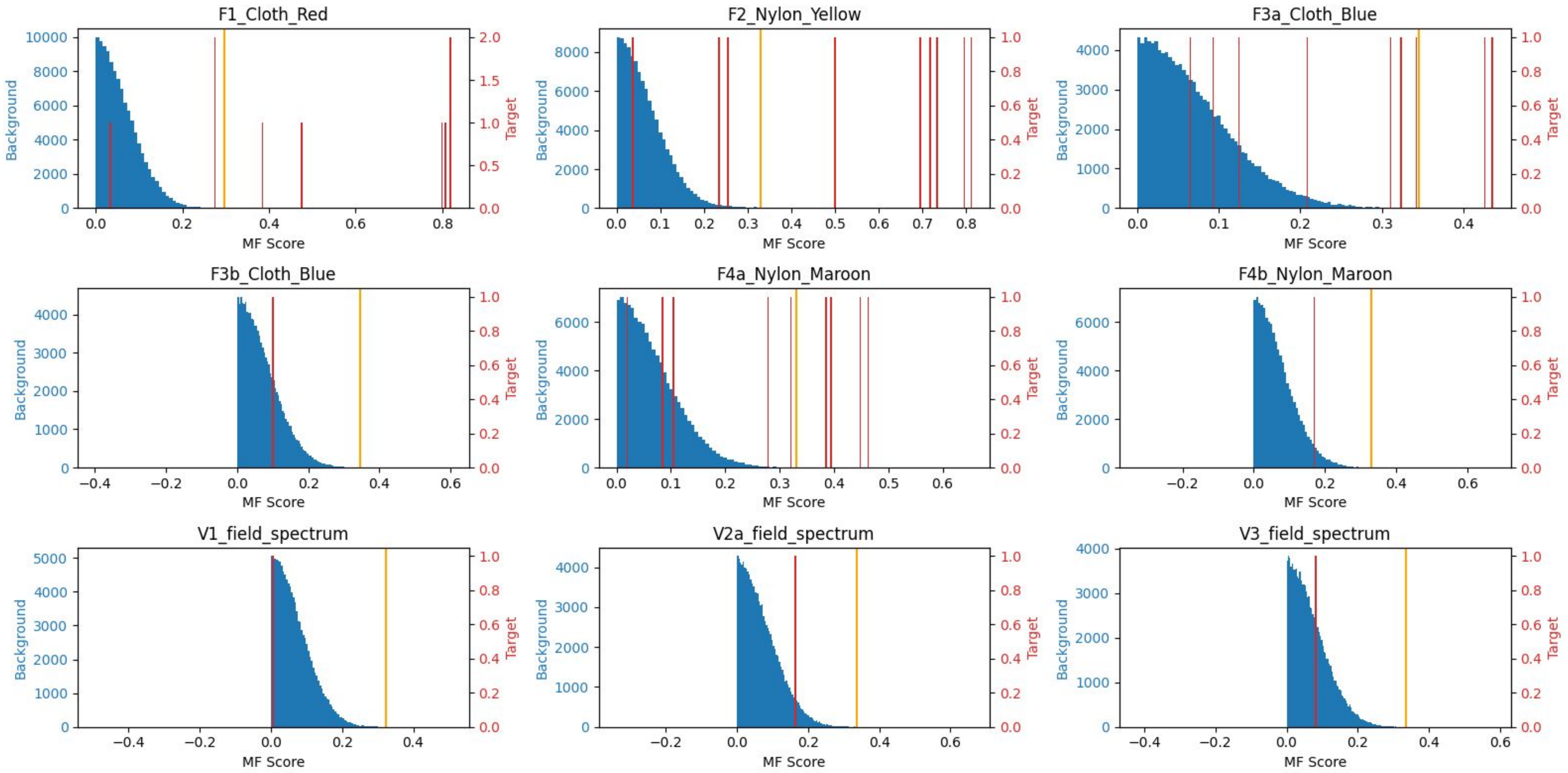
	F1_Cloth_Red	F2_Nylon_Yellow	F3a_Cloth_Blue	F3b_Cloth_Blue	F4a_Nylon_Maroon	F4b_Nylon_Maroon	V1_field_spectrum	V2a_field_spectrum	V3_field_spectrum
MF	30.375	87.00	31.00	255.0	59.500	481.0	414.0	388.0	430.0
ACE	24.625	27.75	1.25	14.0	6.125	52.0	38.0	15.0	11.0

Target/Non-Target Histogram Plots

MF


F1_Cloth_Red
F2_Nylon_Yellow
F3a_Cloth_Blue
F3b_Cloth_Blue
F4a_Nylon_Maroon
F4b_Nylon_Maroon
V1_field_spectrum
V2a_field_spectrum
V3_field_spectrum
Background
Target
MF Score

ACE



F1_Cloth_Red
F2_Nylon_Yellow
F3a_Cloth_Blue
F3b_Cloth_Blue
F4a_Nylon_Maroon
F4b_Nylon_Maroon
V1_field_spectrum
V2a_field_spectrum
V3_field_spectrum
Background
Target
MF Score

The histograms show that the thresholding is doing a pretty good job at minimizing both the false alarms and missed targets

Conclusions

Read the paper [1]. What are the primary conclusions of that paper? What is meant by a ‘Quadratic Detector’?

Primary Conclusions: Rooted in the geometry of each detection algorithm, it was found that quadratic detectors were the best detection algorithms specifically after anomaly removal from the background. The paper aimed to provide insights into why different target detection algorithms do what they do in order to inform future research as opposed to determining a generalizable "best" method.

Quadratic Detector: A detector with a quadratic-shape or cone-shape like ACE as opposed to a linear detector with a linear-shaped detection surface like MF. Given the shape of spectral data, I would assume that quadratic methods would show improvements over linear ones.

Do you see trends that either MF or ACE seems more effective? If one seems more effective that the other, can you see any reasons one might use the less-effective detector?

ACE had improved performance compared to MF specifically within false alarm rates. One reason why ACE had lower false alarm rates might be that high reflectance pixels (such as bright red) are a potential false alarm for linear detectors, creating artificially larger projections whereas that is not an issue for quadratic detectors. However, the true positive rates were very similar between MF and ACE.