Exercise 1

1. Euclidean Distance: $d(a,b) = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$

2. Correlation: $Cor(a,b) = \frac{1}{n} \frac{(a-\mu_a)\cdot(b-\mu_b)}{\sigma_a \sigma_b}$

3. Cosine: $\cos(\theta) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

Exercise 2

(where s = the distance between a and b)

1.
```
s = 0
for i in range(len(a)):
        s = s + (a[i] - b[i])**2
math.sqrt(s)
```

2.
```
mean_a = statistics.mean(a)
mean_b = statistics.mean(b)

s = 0
for i in range(len(a)):
     s = s + ((a[i] - mean_a) * (b[i] - mean_b)

s / (math.sqrt(sum((i - mean_a)**2 for i in a)) * \
    math.sqrt(sum((i - mean_b)**2 for i in b))
```

3.
```
numpy.matmul(a,b)/(math.sqrt(sum(i**2 for i in a)) \
 * math.sqrt(sum(i**2 for i in b)))
```

Exercise 3: $\det(\lambda I - B) = 0$

$$B = \begin{bmatrix} 0 & 2 \\ -4 & -6 \end{bmatrix},$$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -4 & -6 \end{bmatrix}\right) = 0$$

$$\det \begin{bmatrix} \lambda & -2 \\ 4 & \lambda+6 \end{bmatrix} = 0$$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\det(B) = ad - bc$

$$((\lambda)(\lambda+6)) - (\underbrace{4 \times -2}_{-8})$$

$$\lambda^2 + 6\lambda + 8 = 0$$

$$(\lambda+4)(\lambda+2) = 0$$

$$\boxed{\lambda_1 = -4 \quad \lambda_2 = -2}$$ eigenvalues

$(\lambda+4)(\lambda+2)$

$\lambda^2 + 4\lambda + 2\lambda + 8$

$\lambda + 6\lambda + 8$

$(-4I - B)X = 0$

$$-4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -4 & -6 \end{bmatrix}$$

$$\begin{bmatrix} -4 & -2 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 + R_1$

$$\begin{bmatrix} -4 & -2 \\ 0 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} -2 \\ -4 \end{bmatrix}}$$

eigenvector for $\lambda_1 = 4$

$(-2I - B)X = 0$

$$-2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -4 & -6 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 + R_1$

$$\begin{bmatrix} -2 & -2 \\ 0 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} -2 \\ -2 \end{bmatrix}}$$

eigenvector for $\lambda_2 = -2$