HW01: THEORETICAL QUESTIONS

Spring 2024, with Ethan P. Marzban

PSTAT 100: Data Science Concepts and Analysis

Welcome to the first PSTAT 100 Homework! This quarter, homework will consist of a mix of programming/coding questions, as well as some conceptual/theoretical questions.

> **!** Important
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> - This PDF contains only the theoretical questions to Homework 01. Computing questions appear in a separate document.
> - Do **not** try to write your solutions to the theoretical questions on this PDF- please write your solutions on a separate sheet of paper.
> - When submitting, merge your PDFs (containing your computing and theoretical question answers), and upload this merged PDF to Gradescope.
>   - After submitting, make sure to **match pages**.

Probability Review

1) **Does Rounding Preserve Uniformity?** Let $U \sim \text{Unif}[0, 10]$ and let $Y := \text{round}(U)$, where $\text{round}(t)$ denotes the value of t rounded to the nearest integer. (For example, $\text{round}(2.5) = 3$ and $\text{round}(2.4) = 2$.)

   (a) Write down the support (state space) S_Y of Y. Is Y a discrete or a continuous random variable?

   (b) Define $A_3 := \{Y = 3\}$; i.e. A_3 is the event that Y equals 3. Find $\mathbb{P}(A_3)$.

   (c) Define $A_0 := \{Y = 0\}$; i.e. A_0 is the event that Y equals 0. Find $\mathbb{P}(A_0)$.

   (d) Extend your logic from parts (a) and (b) to find an expression for $\mathbb{P}(Y = y)$, for appropriate values of y. (This amounts to finding the PMF/PDF of Y.)

   (e) Is Y uniformly distributed (discrete or continuous)?

   (f) Compute $\mathbb{E}[Y]$ and $\text{Var}(Y)$. Some useful facts to remember:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$