

## Some Mathematical Formulas

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These are a few mathematical formulas/tools which may prove useful to statisticians (and indeed anyone in a STEM-related field). It is by no means exhaustive.

### 1 Sums

#### 1.1 Basic Sums

$$\bullet \sum_{i=1}^n i = \frac{n(n+1)}{2} \quad \bullet \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad \bullet \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

#### 1.2 Geometric Sums

$$\bullet \sum_{k=0}^n r^k = \frac{1-r^{n+1}}{1-r} \quad \bullet \sum_{k=0}^{\infty} r^k = \frac{1}{1-r}; \quad |r| < 1 \quad \bullet \sum_{k=a}^{\infty} r^k = \frac{r^a}{1-r}; \quad |r| < 1$$

#### 1.3 Binomial Theorem

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

#### 1.4 Taylor Series Expansion about $a$

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \\ &= f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{6}f'''(a)(x-a)^3 + \dots \end{aligned}$$

#### 1.5 Maclaurin Expansions

$$\begin{aligned} \bullet e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots \\ \bullet e^{-x} &= \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \dots \\ \bullet \sin(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \dots \\ \bullet \cos(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \dots \end{aligned}$$

## 2 Limits

### 2.1 Limits involving $e$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x \qquad \bullet \lim_{n \rightarrow \infty} \left(1 - \frac{x}{n}\right)^n = e^{-x}$$

### 2.2 L'Hôspital's Rule

$$\lim_n \frac{f(n)}{g(n)} = \lim_n \frac{f'(n)}{g'(n)}, \text{ provided we have an indeterminate form } 0/0 \text{ or } \infty/\infty$$

## 3 Derivatives

### 3.1 Product Rule

$$\frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

### 3.2 Quotient Rule

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

### 3.3 Chain Rule

$$\frac{d}{dx} [f(g(x))] = f'(g(x))g'(x)$$

### 3.4 Derivative of an Inverse

$$\frac{d}{dx} [f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$$

### 3.5 Useful Derivatives

$$\begin{array}{lll} \bullet \frac{d}{dx} x^n = nx^{n-1} & \bullet \frac{d}{dx} \ln(x) = \frac{1}{x} & \bullet \frac{d}{dx} \cos(x) = -\sin(x) \\ \bullet \frac{d}{dx} a^x = a^x \ln(a) & \bullet \frac{d}{dx} \sin(x) = \cos(x) & \bullet \frac{d}{dx} \tan(x) = \sec^2(x) \end{array}$$

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## 4 Integrals

### 4.1 Second Fundamental Theorem of Calculus

$$\frac{d}{dx} \left( \int_{g(x)}^{h(x)} f(t) dt \right) = f(h(x))h'(x) - f(g(x))g'(x)$$

### 4.2 Integration By Substitution

$$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

### 4.3 Integration By Parts

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

### 4.4 Useful Integrals

- $\int x^n dx = \begin{cases} \frac{1}{n+1}x^{n+1} & n \neq -1 \\ \ln|x| + C & n = -1 \end{cases}$
  - $\int e^{-x} dx = -e^{-x} + C$
  - $\int xe^{-x} dx = -e^{-x}(x+1) + C$
  - $\int \frac{1}{(ax)^2 + b^2} dx = \frac{1}{ab} \arctan\left(\frac{ax}{b}\right) + C$
  - $\int \frac{1}{\sqrt{1-(ax)^2}} dx = \frac{1}{a} \arcsin(ax) + C$
  - $\int_0^\infty x^{r-1}e^{-x} dx =: \Gamma(r)$  (called the **Gamma function**)
  - $\int_0^1 x^{a-1}(1-x)^{b-1} dx =: B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$  (sometimes called the **Beta function**)
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