Some Mathematical Formulas

These are a few mathematical formulas/tools which may prove useful to statisticians (and indeed anyone in a STEM-related field). It is by no means exhaustive.

1 Sums

1.1 Basic Sums

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

1.2 Geometric Sums

- $\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$
- $\sum_{k=0}^{\infty} r^k = \frac{1}{1-r};\ \ |r| < 1$
- $\sum_{k=a}^{\infty} r^k = \frac{r^a}{1-r};\ \ |r| < 1$

1.3 Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

1.4 Taylor Series Expansion about *a*

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \\ &= f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{6}f'''(a)(x-a)^3 + \dots \end{aligned}$$

1.5 Maclaurin Expansions

- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{4}x^2 + \frac{1}{6}x^3 + \dots$
- $e^{-x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!} = 1 - x + \frac{1}{4}x^2 - \frac{1}{6}x^3 + \dots$
- $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2k+1} = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \dots$
- $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2k} = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \dots$

2 Limits

2.1 Limits involving e

- $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$
- $\lim_{n\to\infty}\left(1-\frac{x}{n}\right)^n = e^{-x}$

2.2 L'Hôspital's Rule

$$\lim_n \frac{f(n)}{g(n)} = \lim_n \frac{f'(n)}{g'(n)},\ \text{provided we have an indeterminate form } 0/0 \text{ or } \infty/\infty$$

3 Derivatives

3.1 Product Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

3.2 Quotient Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

3.3 Chain Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[f(g(x))\right] = f'(g(x))g'(x)$$

3.4 Derivative of an Inverse

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}$$

3.5 Useful Derivatives

- $\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$
- $\frac{\mathrm{d}}{\mathrm{d}x}\ln(x) = \frac{1}{x}$
- $\frac{\mathrm{d}}{\mathrm{d}x}\cos(x) = -\sin(x)$
- $\frac{\mathrm{d}}{\mathrm{d}x}a^x = a^x\ln(a)$
- $\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x)$
- $\frac{\mathrm{d}}{\mathrm{d}x}\tan(x) = \sec^2(x)$

4 Integrals

4.1 First Fundamental Theorem of Calculus

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_{g(x)}^{h(x)} f(t)\,\mathrm{d}t\right) = f(h(x))h'(x) - f(g(x))g'(x)$$

4.2 Integration By Substitution

$$\int_a^b f(g(x))g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\,\mathrm{d}u$$

4.3 Integration By Parts

$$\int_a^b u\,\mathrm{d}v = uv\Big]_a^b - \int_a^b v\,\mathrm{d}u$$

4.4 Useful Integrals

- $\displaystyle\int x^n\,\mathrm{d}x = \begin{cases} \frac{1}{n+1}x^{n+1} & n \neq -1 \\ \ln|x| + C & n = -1 \end{cases}$
- $\displaystyle\int e^{-x}\,\mathrm{d}x = -e^{-x} + C$
- $\displaystyle\int xe^{-x}\,\mathrm{d}x = -e^{-x}(x+1) + C$
- $\displaystyle\int \frac{1}{(ax)^2 + b^2}\,\mathrm{d}x = \frac{1}{ab}\arctan\left(\frac{ax}{b}\right) + C$
- $\displaystyle\int \frac{1}{\sqrt{1-(ax)^2}}\,\mathrm{d}x = \frac{1}{a}\arcsin(ax) + C$
- $\displaystyle\int_0^\infty x^{r-1}e^{-x}\,\mathrm{d}x =: \Gamma(r)$ (called the **Gamma function**)
- $\displaystyle\int_0^1 x^{a-1}(1-x)^{b-1}\,\mathrm{d}x =: \mathrm{B}(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ (sometimes called the **Beta function**)