PETR 5313: CRN 38950, Fall 2017
Numerical Application in Petroleum Engineering,
Lesson 11: PDE Appendix,
Non-uniform material property

Ekarit Panacharoensawad, PhD
Terry Fuller Petroleum Engineering Research Building Room 236
ekarit.panacharoensawad@ttu.edu

Cite as: Panacharoensawad, E. (2017) "Numerical application in petroleum engineering, Lesson 11: PDE Appendix", presentation slide for Texas Tech PETR 5313 Fall 2017

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \frac{-\left(\alpha\frac{\partial T}{\partial x}\right)_{i-1/2} + \left(\alpha\frac{\partial T}{\partial x}\right)_{i+1/2}}{\Delta x}$$

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \frac{-\left(0.5(\alpha_{i-1}+\alpha_i)\frac{-T_{i-1}+T_i}{\Delta x}\right)}{\Delta x} \frac{+\left(0.5(\alpha_i+\alpha_{i+1})\frac{-T_i+T_{i+1}}{\Delta x}\right)}{\Delta x}$$

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \frac{-\left(0.5(\alpha_{i-1}+\alpha_i)\dfrac{-T_{i-1}+T_i}{\Delta x}\right)}{\Delta x} + \frac{\left(0.5(\alpha_i+\alpha_{i+1})\dfrac{-T_i+T_{i+1}}{\Delta x}\right)}{\Delta x}$$

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \left(\frac{0.5}{(\Delta x)^2}\right)\left[(\alpha_{i-1}+\alpha_i)T_{i-1} - (\alpha_{i-1}+\alpha_i)T_i - (\alpha_i+\alpha_{i+1})T_i + (\alpha_i+\alpha_{i+1})T_{i+1}\right]$$

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \left(\frac{0.5}{(\Delta x)^2}\right)[(\alpha_{i-1}+\alpha_i)T_{i-1} - (\alpha_{i-1}+\alpha_i)T_i - (\alpha_i+\alpha_{i+1})T_i + (\alpha_i+\alpha_{i+1})T_{i+1}]$$

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \left(\frac{0.5}{(\Delta x)^2}\right)[(\alpha_{i-1}+\alpha_i)T_{i-1} - (\alpha_{i-1}+2\alpha_i+\alpha_{i+1})T_i + (\alpha_i+\alpha_{i+1})T_{i+1}]$$

If α is the same everywhere, we get

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \left(\frac{\alpha}{(\Delta x)^2}\right)[T_{i-1} - 2T_i + T_{i+1}]$$ as usual

Non-Uniform Grid Case

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \frac{-\left(\alpha\frac{\partial T}{\partial x}\right)_{i-1/2} + \left(\alpha\frac{\partial T}{\partial x}\right)_{i+1/2}}{x_{i+1/2} - x_{i-1/2}}$$

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \frac{-\left(0.5(\alpha_{i-1} + \alpha_i)\frac{-T_{i-1} + T_i}{\Delta x_i}\right)}{x_{i+1/2} - x_{i-1/2}}$$

$$\frac{+\left(0.5(\alpha_i + \alpha_{i+1})\frac{-T_i + T_{i+1}}{\Delta x_{i+1}}\right)}{x_{i+1/2} - x_{i-1/2}}$$

Where $\Delta x_i = x_i - x_{i-1}$

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \frac{-\left(0.5(\alpha_{i-1}+\alpha_i)\dfrac{-T_{i-1}+T_i}{\Delta x_i}\right)}{0.5(\Delta x_i+\Delta x_{i+1})} + \frac{\left(0.5(\alpha_i+\alpha_{i+1})\dfrac{-T_i+T_{i+1}}{\Delta x_{i+1}}\right)}{0.5(\Delta x_i+\Delta x_{i+1})}$$

$$x_{i+1/2} - x_{i-1/2} = \frac{\Delta x_{i+1}}{2} + \frac{\Delta x_i}{2}$$

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \frac{1}{\Delta x_i + \Delta x_{i+1}}\left[\frac{(\alpha_{i-1}+\alpha_i)T_{i-1}}{\Delta x_i}\right.$$

$$\left. - \frac{(\alpha_{i-1}+\alpha_i)T_i}{\Delta x_i} - \frac{(\alpha_i+\alpha_{i+1})T_i}{\Delta x_{i+1}} + \frac{(\alpha_i+\alpha_{i+1})T_{i+1}}{\Delta x_{i+1}}\right]$$

For uniform grid and constant α , we get

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial T}{\partial x}\right) \approx \left(\frac{\alpha}{(\Delta x)^2}\right)[T_{i-1} - 2T_i + T_{i+1}]$$ as usual