Homework 8.

In class, we talked about Lobatto IIIC 4th order. In this homework you are assigned to solve 2nd order ODE with Lobatto IIIC 3rd order (easier that what we did in class) and other easier methods (Scipy dopri5 & explicit Euler). To do this homework, I suggest you to review the class material on Lobatto IIIC 4th order (multi-variable), first.

From http://homepage.math.uiowa.edu/~ljay/publications.dir/Lobatto.pdf page 10, Lobatto IIIC 3rd order has the Butcher Tableau of

$$\begin{array}{c|ccc} 0 & \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{5}{12} & \frac{-1}{12} \\ 1 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ \hline & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{array}$$

(Notice that summation of each row equal to the value of the first column.)

Solve

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6xy = x + sin(y)$$

for x in the range of 0.1 to 1.9.
Boundary conditions are
dy/dx = 1.8 at x = 1.5
y = 1.2 at x = 0.5
Note that input for the above sine function must be in radiant.
By using
8.1) Explicit Euler, (1 point). Show two cases of the step-size magnitude (abs(h) value)
8.2) **Scipy** Dormand-Prince 5(4), (2 points). Show two cases of the step-size magnitude (abs(h) value). Show the intermediate point values (plot as dots, not just continuous line). If no intermediate value is obtained, you get zero. The unique feature of uneven discretization of Dormand-Prince must be shown. You may need to use Scipy Dormand-Prince together with other scipy function such as scipy.optimize.fsolve, or you may just use scipy.integration.solve_bvp if you can.
8.3) Lobatto IIIC 3rd order, (6 points). Show two cases of the step-size magnitude (abs(h) value)

Then,
8.4) Plot graph to compare result from 8.1 to 8.3 for two different step-size magnitude cases. (1 point) Small and large step-size value used in 8.1 to 8.3 must be the same. Note that the boundary conditions are at 0.5 and 1.5, but your answer must cover the range of **0.1** to **1.9**. Submit one ipynb file. Solve every questions with Python programming language, only (no Excel). If you use any code outside lecture, give a citation properly. If your code is from the lecture / modification of lecture code, no need to cite (for this homework).