Proof. From the definition of virtual queue in (8), we can get $Q_k(t+1)^2 \leq (Q_k(t)+\beta_k - x_k(t))^2$, which can be combined with (11).

$$\begin{aligned}
\triangle(Q(t)) &= \mathbb{E}[L(Q(t+1)) - L(Q(t))|Q(t)] \\
&= \mathbb{E}[\frac{1}{2}\sum_{k\in\mathcal{K}} Q_k(t+1)^2 - \frac{1}{2}\sum_{k\in\mathcal{K}} Q_k(t)^2|Q(t)] \\
&\leq \mathbb{E}[\frac{1}{2}\sum_{k\in\mathcal{K}}(\beta_k - x_k(t))^2 + \sum_{k\in\mathcal{K}} Q_k(t)(\beta_k - x_k(t))|Q(t)] \\
&\leq B + \mathbb{E}[\sum_{k\in\mathcal{K}} Q_k(t)(\beta_k - x_k(t))|Q(t)]
\end{aligned}$$

Here, we have $B = \operatorname{argmax}\{\frac{1}{2}(\beta_k - x_k(t))\}$, which shows that:

$$\begin{aligned}
\sum_{t\in T} Vu_k(t) - \triangle(Q(t)) &= \sum_{t\in T}\sum_{k\in\mathcal{K}}\{V\mathbb{I}_{\{\tau_k(t)\leq\tau^t_{max}\}}u_k(t) - \mathbb{E}[L(Q(t+1)) - L(Q(t))|Q(t)]\} \\
&\geq \sum_{t\in T}\sum_{k\in\mathcal{K}}\{V\mathbb{I}_{\{\tau_k(t)\leq\tau^t_{max}\}}u_k(t) - Q_k(t)(\beta_k - x_k(t)) - B\}
\end{aligned}$$