Search problems

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ULiege - INFO0049

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Search problem

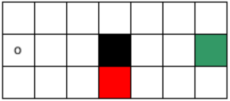
A search problem is defined by the following components :

- A set of state, which also includes :
  - An initial state,
  - A subset of forbidden states.
- A set of available actions per state,
- A transition model which maps a state-action pair to another state,
- A goal test.

Definitions

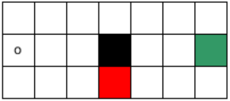
- A state space is the set of all states reachable from the initial state using the transition model,
- A path is a sequence of state generated by the transition model with an available action for each state,
- A solution is a path which satisfies the goal test, which generally accept paths ending with a (set of) particular state(s) (alias *final state(s)*).
  - A numeric value can be assigned to a solution to represent its cost,
  - If the solution achieves the lowest possible cost, then it is *optimal*.
  - Note that a path which contains a cycle is generally not a solution. A possible solution is to forbid to revisit any state integrated in the path.

Example



- State space : $S = \{(i,j) \in \mathbb{N}_1^2 \mid \forall i \leqslant 3, j \leqslant 7\} \setminus (2,4)$,
- Initial state : $(2,1)$,
- Available actions :
  - $U = \{(0,1),(1,0),(-1,0),(0,-1)\}$ for all states except for $(2,3)$, $(2,5)$, $(1,4)$ and $(3,4)$,
  - $U \setminus (0,1)$ for state $(2,3)$, $U \setminus (0,-1)$ for state $(2,5)$.
- Final state : $(2,7)$,
- Forbidden state : $(3,4)$.

Example (cont'd)



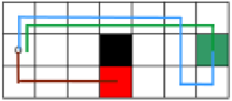
- Transition model :

$$t((i,j),(x,y)) = \begin{cases} (i+x, j+y) & \text{if} \quad (i+x, j+y) \in S \setminus (2,4) \\ (i,j) & \text{otherwise} \end{cases}$$

- Transition cost is 1 everywhere.

Paths

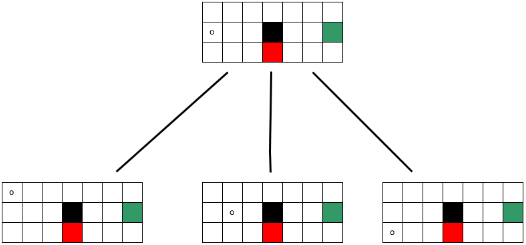
(not exhaustive)



- Optimal path solution
- Path solution
- Not a path solution

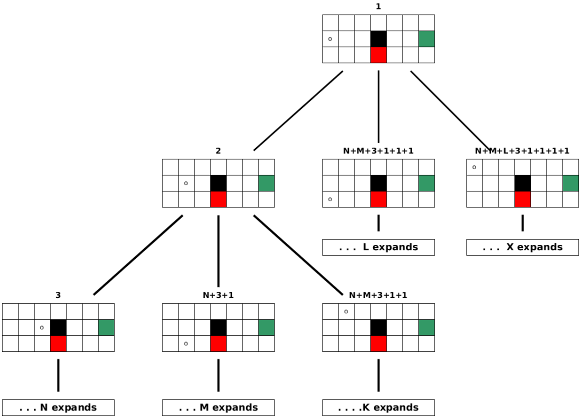
Searching over a tree

⚠ do not confuse with search tree in prolog !



- *Node expansion* leads to a successor state per action.
- Return the first path found which satisfies the goal test
- Which node should be expanded first ?

Naive approach (1) - Depth First Search

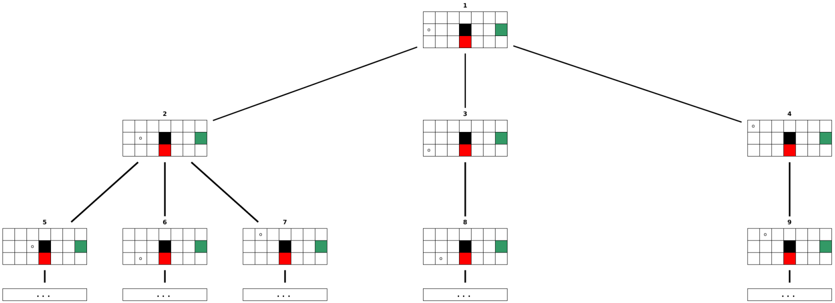


Node labels define expansion order

Naive approach (1) - DFS (cont'd)

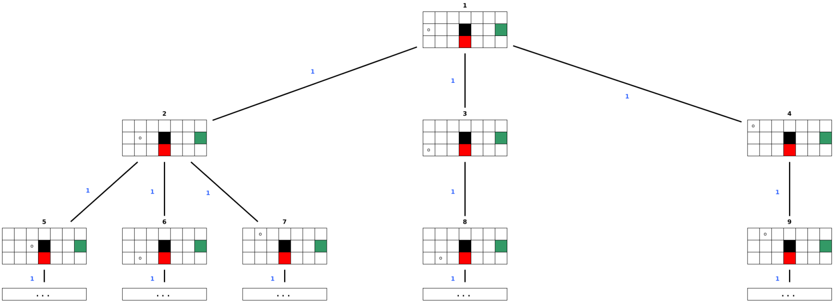
- Node depth defines expansion order.
- Properties ? Issues ?

Naive approach (2) - Breadth First Search



- Node depth defines expansion order.
- Properties ? Issues ?

Naive approach (3) - Uniform Cost Search (UCS)



- Node path cost defines expansion order.
- Properties ? Issues ?

What is missing ?

- Previous methods are *uninformed*.
- How to provide a generic guidance for the expansion order ?

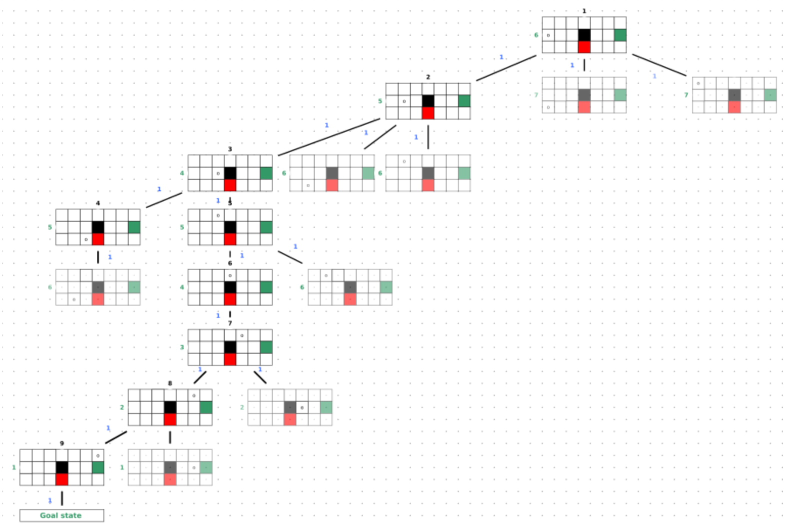
State evaluation

- Say we define a heuristic which is a state evaluation function $h : S \rightarrow \mathbb{R}_+$
- Ideally, we would like a function $h^*(x)$ which gives the lowest achievable cost by reaching the goal state from x.
- Unfortunately, we are unlikely to have this function. Are we done ? No !

A*

- A* = UCS + heuristic
- We can have an approximation of h^*, as long as it is admissible
  - Let h^0 be the 0-everywhere heuristic
  - An heuristic h is admissible if $h^0 \leq h \leq h^*$
  - Note that h^0 reduces A* to UCS
- Manhattan distance is a well known heuristic
  - $d(A,B) = |X_b - X_a| + |Y_a - Y_b|$
- Quality of the heuristic will directly influence the number of expanded nodes.

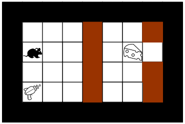
A* example



Remarks

- This kind of search problem can also be addressed through a Traveling Salesman Person solver-like.
  - In one sentence, looks for the shortest path that crosses each node (at most) once.
  - NP-Complete problem. Many greedy algorithms and integer programming can be used to approximate the optimal solution.
- What is something tries to prevent us to reach the objective ? This is *adversarial search*. We'll see it later.

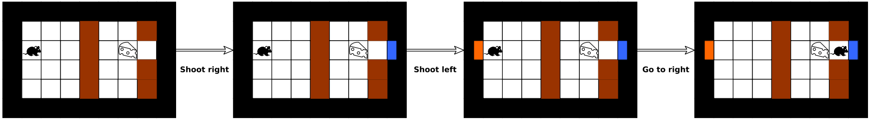
Homework : Aperture maze



Goal : collect the cheese(s) without crossing a red cell

- The mouse can move left,right, up or down.
- When equipped with a gun, the mouse can also shoot a bullet left,right, up or down.
- Once a bullet reaches a wall, it creates a portal oriented in the opposite direction.
- At most two portals can be created. If a third portal is created, the first one directly vanish.
- If the mouse reaches a portal, it lands directly to the adjacent non-wall cell.

Homework : Motion example



For simplicity here we assume that the mouse has already grabbed the gun.

Homework : Input/Output

- The input is a list of list of the following symbols :
  - '%' is a wall,
  - 'r' is a red cell,
  - 'm' is the mouse,
  - 'g' is the gun,
  - 'c' is the cheese,
  - ' ' is a white cell
- The output is a list of the actions below
  - 'l','r','u','d' respectively for left,right,up,down,
  - 'j','l','i','k' respectively for shooting at left, right, up, or down
- ⚠Ensure that your path meets the goal test.

Homework : Deliverable and deadline

Expected : a single Prolog file, to be sent via the Montefiore Submission Platform in the indicated deadline. This file needs to contains the following parts :

- A formalization of the Aperture Maze search problem.
- The source code of your implementation of an algorithm presented here
  - It needs to contains a main predicate *solve(M,O)* where *M* is the input and *O* is the output as specified in the previous slide.
  - You may consider other algorithms but be careful to explain them clearly and why you used them.
- Some executions examples with solution/computation time
  - Use the predicate *time(P)* with *P* your predicate call (it needs to be named *solve*).

Homework : Evaluation

- Formalization of the search problem.
- Structure and clarity of your code.
  - A clear and concise explanation of your solving strategy is expected.
  - There is no expected report so be really careful about the documentation.
- Performance (computation time and quality of the solution).
  - Your solution will be tested on big grids.
- Oral presentation.
  - Scheduled for 27/03/2018.

That's all folks

If you want to know more about AI :

Stuart J. Russell and Peter Norvig. 2003. Artificial Intelligence: A Modern Approach (2 ed.). Pearson Education.