Approximation of π with a Monte Carlo simulation

In the program `MonteCarloexample.m` we use a Monte Carlo method to obtain an approximated value of π. We draw points with random coordinates (x, y) inside a square. We consider a circle with a centre equal to the center of the square and radius equal to half the square size. When the number of draws tends to infinity, the value of π is given by the ratio between the number of points in the circle and the number of points in the square.

The function `approximation_pi` generates M points on a surface $[0, 1] \times [0, 1]$. The value of π is obtained from the ratio between the number of points in the disk with centre $(0.5, 0.5)$ and radius 0.5, and the total number of events M. This ratio is then divided by 4.

Figure 1 shows how the square and the circle get uniformly filled with points by increasing the value of M, to which we give the values of 10^2 ($\pi_{appr} = 3.08$), 10^3 ($\pi_{appr} = 3.184$), et 10^4 ($\pi_{appr} = 3.1304$). For $M = 10^8$ we obtain a value of $\pi_{appr} = 3.141654$, which approaches the exact value up to the third decimal digit.

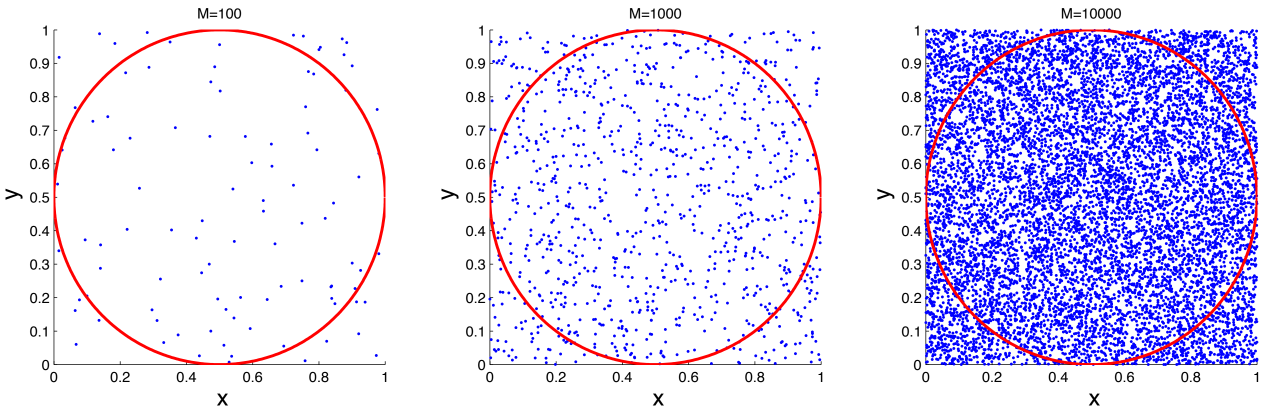

FIGURE 1 – Numerical calculation of the constant π via a Monte-Carlo method using a number of draws $M = 10^2, 10^3, 10^4$ (from left to right).

Figure 2 shows the convergence curve of the adopted Monte Carlo method, which is obtained by showing the error between the numerical solution and the exact one as a function of the number of M. The use of the logarithmic scales on both axes allows us to show that the convergence is of order $\mathcal{O}(\frac{1}{\sqrt{M}})$. In fact, the convergence curve of the numerical solution follows a line of slope 0.5. The curve is fitted with a line with slope -0.5467. In addition, it can be noted that after around 10^4 draws the error becomes smaller than 1%.

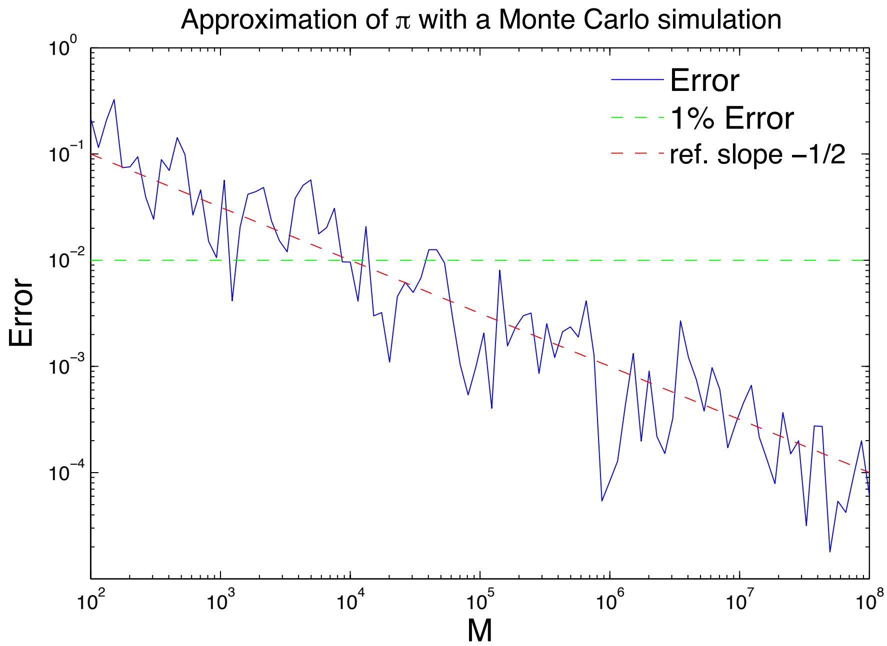

FIGURE 2 – Error of the numerical solution of the Monte-Carlo method for the approximation of π as a function of the number of draws M in double logarithmic scale. The convergence curve is compared with a line with -0.5 slope, as well as with a reference line corresponding to 1% error.