Integrated Path Planning and Control for Lane Tracking with Moving Obstacle avoidance

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1. INTRODUCTION

In this paper, I explain the approach for "Integration of Path Planning and Control to achieve the objective of tracking given Lane Waypoints together with Collision avoidance of the surrounding Static & Moving Obstacles" using CASADI. I used the main ideas of fusing of Moving obstacle states, Vehicle Dynamic model, objective function, and vehicle dynamic rollover safety constraints in the NMPC formulation from A.B. Dudekula [1] where the objective was to determine optimum Integrated path planning and control for Unstructured Off-Road environments. The objective of my work is to apply similar ideas but which are relevant to determine optimum Integrated Path Planning for Moving obstacle avoidance and Tracking of the given Lane waypoints for Autonomous navigation in the Highway Structured environment. Due to altogether a different application, the code given in the thesis was almost completely modified, all the relevant parameters were retuned, and the various components of NMPC were reconstructed and implemented in a different manner to achieve the objective desired in this paper. I sincerely thank to the author of the thesis for open sourcing the code together with the thesis which helped me to understand how to formulate and implement various components of NMPC and therefore helped me to achieve my objectives in this paper.

2. PROBLEM FORMULATION: Moving Obstacle Avoidance & Tracking of the given Lane Waypoints

We have the below scenario for our control objective i.e. Tracking of the given Waypoints of the Lane in magenta which is the current preferred lane of the ego vehicle, avoiding collision with the surrounding stationary (Road boundaries in Black) & moving obstacles (in Blue) and reaching the given goal location. The basic sketch for our problem is shown below.

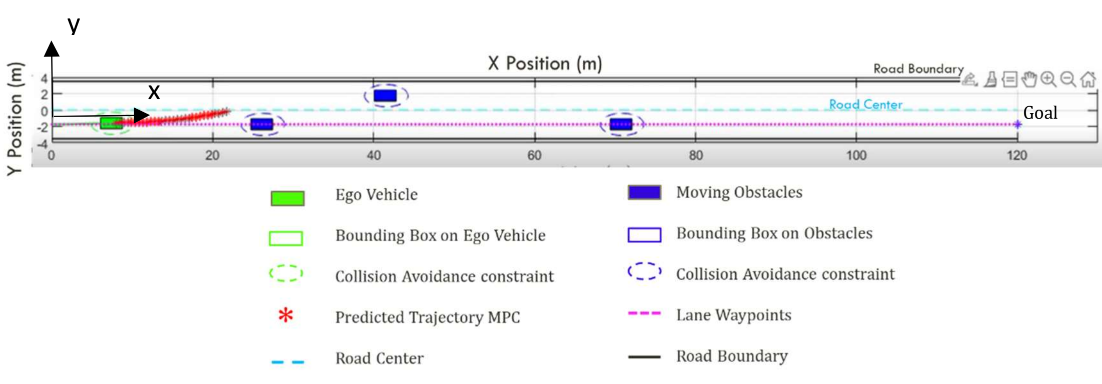

Fig. 1

Next, important aspects of the Vehicle Dynamic model of ego vehicle and moving obstacles, integration of Moving obstacle states with ego vehicle states, evaluation of vertical load on each wheel considering lateral and longitudinal weight transfer for wheel lift-off rollover safety constraints, objective function and constraints are explained.

3. Dynamic Model of Ego Vehicle:

The Dynamic model of the ego vehicle shown below in eq. (1) is defined like in [1] with 8 states as $[x\, y\, \psi\, v_x\, v_y\, \omega_z\, a_x\, \delta_f]^T$, which are defined as Position Vector x & y at the front axle of the vehicle along x and y direction of the Global Road frame as shown in the above figure, yaw angle ψ, longitudinal and lateral velocity along the vehicle body frame v_x & v_y, yaw rate ω_z, longitudinal acceleration a_x along the vehicle body frame and the front steering angle δ_f. The control inputs are Jerk J and Steering rate γ. We will also control the time step which will be explained later. The road is assumed to be straight in this example problem.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{v_x} \\ \dot{v_y} \\ \ddot{\psi} \\ \dot{a_x} \\ \dot{\delta_f} \end{bmatrix} = \begin{bmatrix} v_x \cos(\psi) - \left(v_y + l_f\dot{\psi}\right)\sin(\psi) \\ v_x \cos(\psi) - \left(v_y + l_f\dot{\psi}\right)\sin(\psi) \\ \omega_z \\ a_x \\ \frac{F_{yf}+F_{yr}}{M} - v_x\omega_z \\ \frac{F_{yf}l_f - F_{yr}l_r}{I_z} \\ J \\ \gamma \end{bmatrix} \tag{1}$$

Other parameters l_f, l_r, M, I_z are defined as distance from front and rear wheel axle to vehicle C.G. location in m; total mass of the vehicle in kg and the moment of inertia about z-axis in kg-m2 respectively. The tire model is assumed to be linear where the lateral forces F_{yf}, F_{yr} are defined using the well known equations of the bicycle model [2] as a function of cornering stiffness and side slip angle as in eq. (2) & eq. (3). The side slip angle for front and rear wheels is defined in eq. (4) & eq. (5) as the difference of the applied steering angle and the heading angle at the front & rear axle of the vehicle (defined as function of the ratio of lateral velocity and longitudinal velocity respectively at the front & rear axle of the vehicle).

$$F_{yf} = C_f\alpha_f \tag{2}$$

$$F_{yr} = C_r\alpha_r \tag{3}$$

$$\alpha_f = \delta_f - \operatorname{atan}\left(\frac{v_y + l_f\dot{\psi}}{v_x}\right) \tag{4}$$

$$\alpha_r = -\operatorname{atan}\left(\frac{v_y - l_r\dot{\psi}}{v_x}\right) \tag{5}$$

4. Dynamic Model of Moving Obstacles:

The Dynamic model of each of the moving obstacles shown below in eq. (6) is defined like in [1] with 2 states as $[x\ y\ ]^T$ which are defined as Position Vector at the C.G. of the moving obstacle along x and y direction of the Global Road frame as shown in the above figure. The dynamic model of each of the moving obstacles defined in eq. (6) is appended to the dynamic model of the ego vehicle defined in eq. (1)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} v_m \cos(\psi_m) \\ v_m \sin(\psi_m) \end{bmatrix} \tag{6}$$

5. Evaluation of Vertical Load for Vehicle Rollover safety constraints

The vertical load on each wheel needs to be evaluated as they will be used in the NMPC formulation to define their acceptable lower & upper bounds. This is to assure that any control action calculated by NMPC must respect these bounds so as to avoid the wheel lift off condition. The wheel lift off condition is assured by constraining the vertical load to be always greater than 1000N.

The vertical load has two components (i) Static: due to weight of vehicle's Sprung & Unsprung mass as defined in eq. (7) & eq. (8) for the entire front and rear axle (ii) Dynamic due to lateral and longitudinal accelerations as defined in eq. (9)-(12).

$$F_{zf0} = \left(\frac{M_s l_r}{l} + \frac{M_u}{2}\right) g \tag{7}$$

$$F_{zr0} = \left(\frac{M_s l_f}{l} + \frac{M_u}{2}\right) g \tag{8}$$

$$F_{zfl} = 0.5 * F_{zf0} - 0.5 * \mu_{zx}\, A_x - \mu_{zyf} A_y \tag{9}$$

$$F_{zfr} = 0.5 * F_{zf0} - 0.5 * \mu_{zx}\, A_x + \mu_{zyf} A_y \tag{10}$$

$$F_{zrl} = 0.5 * F_{zr0} + 0.5 * \mu_{zx}\, A_x - \mu_{zyf} A_y \tag{11}$$

$$F_{zrr} = 0.5 * F_{zr0} + 0.5 * \mu_{zx}\, A_x + \mu_{zyf} A_y \tag{12}$$

The equation of sprung and unsprung mass are defined as in eq. (13) & eq. (14) where the factor 0.142 Is assumed to be known.

$$M_u = 0.142 * M \tag{13}$$

$$M_s = M - M_u \tag{14}$$

The total lateral A_y and longitudinal A_x accelerations defined using the velocities in the local body frame are defined as in eq. (15) & (16). We can see that all the variables on which A_x and A_y depend on are known from the ego vehicle state vector and hence can be calculated at each time step.

$$A_x = (\dot{v_x} - v_y\omega_z) = a_x - v_y\omega_z \tag{15}$$

$$A_y = (\dot{v_y} + v_x\omega_z) = \frac{F_{yf}+F_{yr}}{M} \tag{16}$$

The longitudinal μ_{zx} and lateral μ_{zyf} & μ_{zyr} weight transfer coefficients can be either determined from experiments as in [1] or can be calculated for a Laterally and Longitudinally accelerating vehicle as derived in Reza N. Jazar [2] and written below as in eq. (17), (18) & (19).

$$\mu_{zx} = M_s \frac{h}{l} \tag{17}$$

$$\mu_{zyf} = M_s \frac{l_r}{l}\frac{h}{w} \tag{18}$$

$$\mu_{zyr} = M_s \frac{l_f}{l}\frac{h}{w} \tag{19}$$

6. Assumptions

In this section we highlight two main assumptions used to derive the above equations.

I. In eq. (2) and eq. (3) the slip angles for right and left tires is assumed as same.
II. The cornering stiffness is considered as same for right and left tire and is calculated as the slope of the curve (Lateral force Vs Side slip angle) corresponding to the vertical load which is determined by considering the longitudinal weight transfer but without considering the lateral weight transfer. If cornering stiffness curves are given for one tire then it must be multiplied by factor 2, as the two tires on an axle are assumed to be lumped together in the bicycle model, to get the total force for front/rear axle in the bicycle model.

Next we will describe all the important components of the NMPC controller i.e. System Model, Tracking Reference states from Lane Waypoints, Collision avoidance from road boundaries and Moving obstacles, Objective function and the equality constraints.

7. NMPC Controller: System Model

The system model of ego vehicle in eq. (1) and the moving obstacle in eq. (6) are integrated together into one system model as in eq. (20) with entire states vector (ego vehicle states and moving obstacles states) as z and control actions as u which are steering rate, jerk and time step in our problem.

$$\dot{z} = f(z, u) \tag{20}$$

$$[z(k+1)] = [z(k)] + \Delta T[f(z, u)] \qquad \forall\, k \in [1, N] \tag{21}$$

The above equation is integrated using Euler discretization as shown in the eq. (21) and is used to predict the future states of the ego vehicle and the moving obstacles over the entire prediction horizon length which is defined as the no of steps in future for which the states of the system are predicted. The predicted states of the system now available makes it possible for the ego vehicle to make decisions at the current time considering the predicted future states of itself and the moving obstacles which is the same way humans also drive and control the vehicle. Next we will define how we must choose the reference position from the given Lane waypoints which shall be achieved by the current and all the future states of the ego vehicle as per our primary objective of tracking the Lane waypoints.

8. NMPC Controller: Tracking Reference states from Lane Waypoints

The Waypoints on the preferred lane to be followed by the ego vehicle is assumed to be known from a high level planner algorithm like A*, D* or RRT*. The waypoints can be further interpolated as shown in Fig. 2 in magenta to achieve Lane following with higher accuracy. In NMPC controller, given the initial state of the ego vehicle, its future states are predicted for further N steps (prediction horizon length) resulting in the states available for total N+1 steps. N+1 points on the Lane are now required to be determined which will be given as reference to be achieved by the respective N+1 steps in the predicted trajectory of the ego vehicle. An example case is shown in Fig. 3 where it is illustrated how the reference point on the lane will be chosen for one particular state of the ego vehicle shown as red point. The current state shown as red point is projected on the Lane along the Global X & Y axis resulting in two points shown as cyan point. The cyan point which is closer to the goal (encircled in orange) will be chosen as the reference point to be achieved by that particular state of the ego vehicle which is at red point currently. Likewise, this is done for the all the N+1 states in the predicted trajectory of the ego vehicle. We will have a term in the objective function that will minimize the distance between each of these N+1 points of the Lane given as reference to be achieved by the respective N+1 steps in the predicted trajectory of the ego vehicle. This is done because a naïve approach of selecting reference points as per the closest distance on the lane sometimes lead to the vehicle tracking the lane but in opposite direction. This happens often when the predicted trajectories are laterally too far away from the reference lane to follow in order to avoid obstacles. Only the position states X & Y are given as the reference.

9. NMPC Controller: Collision avoidance from road boundaries

The linear obstacles for example the road boundaries highlighted in black in Fig. 2 are also assumed to be represented as points. In NMPC controller, given the initial state of the ego vehicle, its future states are predicted for further N steps (prediction horizon length) resulting in the states available for total N+1 steps. The points of the road boundaries most nearest to the respective N+1 steps in the predicted trajectory of the ego vehicle are determined resulting in the selection of total N+1 points from the road boundaries. Collision avoidance constraint will be defined between these set of N+1 points of the ego vehicle and the road boundaries. These constraints will ensure every step in the predicted trajectory of the ego vehicle to remain out of a certain elliptical region around the point of the road boundaries most nearest to it. Also we will have a term in the objective function that will maximize distance between every step in the predicted trajectory of the ego vehicle from the respective point of the road boundaries which is most nearest to it.

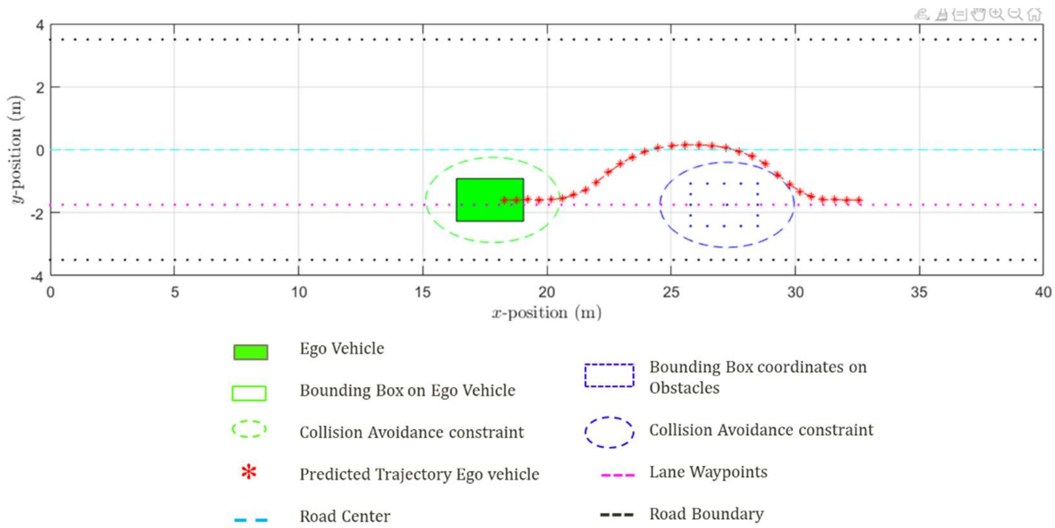

Fig. 2

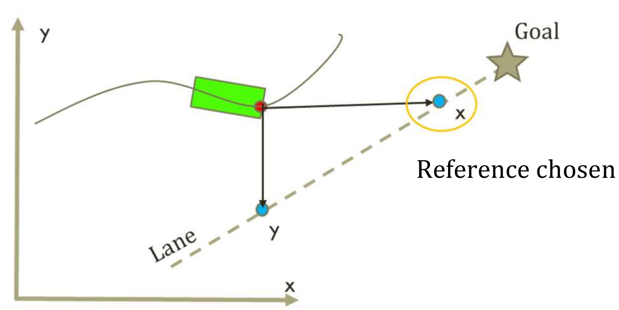

Fig. 3

10. NMPC Controller: Collision avoidance from Moving Obstacles

The Moving vehicles are highlighted in blue in Fig. 2. It is assume that the four coordinates of the bounding box around each of the moving obstacles is known from a prior perception algorithms. From these four coordinates, more points are interpolated in between to create more finely discretesized bounding box around moving obstacles as highlighted in blue in Fig. 2. With the assumption that the moving obstacles are rigid, same bounding box coordinates is also assumed around their C.G. during it's entire predicted trajectory. We find out the point from the finely discretesized bounding box points around the current state of the C.G. of the moving obstacle that is nearest to the current state of the ego vehicle. Likewise, this is done for all the N+1 steps in the predicted trajectory of the ego vehicle and the bounding boxes at the respective step in the entire trajectory of the moving obstacles. Collision avoidance constraint is defined for each step of the ego vehicle trajectory w.r.t the every point identified from the bounding boxes at each step in the entire trajectory of the moving obstacles. This will ensure that every step of the ego vehicle remain out of a certain elliptical region around all the N+1 identified points of the moving obstacles. Also we will have a term in the objective function that will maximize distance of the N+1th identified point of the moving obstacle which is most nearest to the last steps in the predicted trajectory of the ego vehicle.

11. NMPC Controller: Objective function

We will now define our objective function and constraints as below. In our NMPC, Only the position states x & y are required to track the given Lane waypoints as reference therefore Q∈R^{2x2}. The control variables are $[J\ \gamma\ \Delta T\ ]^T$ which are defined as Jerk rate, steering rate and the Time step used for every step, therefore R∈R^{3x3}. The next cost term is Wheel lift-off which is defined such that If greater than 1300N, -ve cost is applied and If less than 1300 positive cost is applied. To smoothen the transition tanh function is used. Next cost term is on large steering angle. Collision avoidance from road boundaries is the next cost term where distance between every point of the road boundaries which is most nearest to respective N+1 steps in the predicted trajectory of the ego vehicle is maximized. Next cost term is collision avoidance from the moving obstacles that will maximize distance of the point identified from the bounding box at the last step of the predicted trajectory of the surrounding moving obstacle which is most nearest to the last step in the predicted trajectory of the ego vehicle. Weights for the cost terms are defined as follows: Q is the weight matrix for reference trajectory tracking, R is the weight matrix for control actions Jerk Rate, steering rate and Time step, w_{fz} is the weight for Wheel lift off constraint, w_δ is the weight for the Steering Angle, $w_{obs_{road}}$ is the weight for collision avoidance from road boundaries and w_{obs_i} is the collision avoidance from moving obstacles.

$$\begin{aligned}\min_{x,u} J_N(x,u) = &\left(\sum_{k=0}^{N-1} \left\|x(k) - x_{ref}(k)\right\|_Q^2 + \left\|u(k) - u_{ref}\right\|_R^2\right.\\
&+ w_{fz}\left(\tanh\left(-\frac{F_{zfl}(k) - a}{b}\right) + \tanh\left(-\frac{F_{zfr}(k) - a}{b}\right) + \tanh\left(-\frac{F_{zrl}(k) - a}{b}\right)\right.\\
&\left.+ \tanh\left(-\frac{F_{zrr}(k) - a}{b}\right)\right) + w_\delta {\delta_f}^2\\
&\left.+ \frac{w_{ob\ \ road}}{(\|x(k) - x_{road}(k)\|^2 + \|y(k) - y_{road}(k)\|^2 + \xi)}\right)\\
&+ \left(\sum_{i=0}^{M} \frac{w_{obs_i}}{\left(\left\|x(N) - x_{obs_i}(N)\right\|^2 + \left\|y(N) - y_{ob\ \ i}(N)\right\|^2 + \xi\right)}\right)\end{aligned} \tag{22}$$

12. NMPC Controller: Equality & Inequality Constraints

System model Equality Constraints: We use Multiple shooting to convert NMPC problem in to NLP which will be solved by CASADI. In Multiple shooting both the System states and the control inputs for the entire prediction horizon are defined as Optimization variables. Therefore we use eq. (21) also to define equality constraints between states and control inputs at the two consecutive time steps. We rewrite eq. (21) below together with the constraint on the initial value of the states and control variables.

$$s.t.\ [z(k+1)] = [z(k)] + \Delta T[f(z,u)]\ \forall\, k \in [1, N] \tag{23}$$

$$z(0) = z_0 \tag{24}$$

Vehicle dynamic Rollover safety Constraints: Vehicle dynamic rollover safety constraints defined below ensures that vertical load on each of the wheels is higher than 1000N.

$$F_{zfl}, F_{zfr}, F_{zrl}, F_{zrr} \geq 1000N \tag{25}$$

*Prediction distance Constraints: M*inimum prediction distance of 15m is required at all the longitudinal speeds. As the prediction horizon is considered as fixed in our implementation, this is achieved by having time step as a control variable which is adjusted such that predicted distance is minimum 15m. Once the distance of the ego vehicle from the goal location becomes lesser than the 15m we will switch to constant time step from varying time step i.e. no constraint on the preview distance, which may now reduce with the reduction of the speed of the ego vehicle.

$$v_x * N * \Delta T \geq 15m \tag{26}$$

*State and Control Variable Constraints: C*ontrol bounds based upon the vehicle actuator limitations and the state bounds based on the spatial constraints specific to the type of scenario at hand are defined as below.

$$u(k) \in U\ \ \forall\, k \in [0, N-1] \tag{27}$$

$$z(k) \in Z\ \ \forall\, k \in [0, N] \tag{28}$$

Road boundaries Collision avoidance Constraint: Collision avoidance constraint w.r.t road boundaries is defined below in eq. (29)

$$\frac{\left\| x(k) - x_{obs_{road}}(k) \right\|^2}{a^2} + + \frac{\left\| y(k) - y_{obs_{road}}(k) \right\|^2}{b^2} \geq 2 \qquad \forall\, k \in [0, N] \tag{29}$$

Moving Obstacles Collision avoidance Constraint: Collision avoidance constraint w.r.t moving obstacles is defined below where O is the no. of obstacles. As can be observed below the constraint is defined for each step ($\forall\, k \in [0, N]$) of the ego vehicle trajectory w.r.t the every point identified from the bounding boxes at each step in the entire trajectory of the moving obstacles $\forall\, j \in [0, N]$.

$$\frac{\left\|x(k)-x_{obs_i}(j)\right\|^2}{a^2}+\frac{\left\|y(k)-y_{obs_i}(j)\right\|^2}{b^2} \geq 2 \quad \forall\, i \in [1 \ldots O] \;\; \forall\, k \in [0, N] \;\; \forall\, j \in [0, N] \quad (30)$$

13. NMPC Controller: Constraints

Below we see the results of our implementation. Fig. 4 shows the planned path by the ego vehicle in green while tracking the reference lane waypoints in magenta and avoiding both road boundaries and moving obstacle in black and blue respectively.

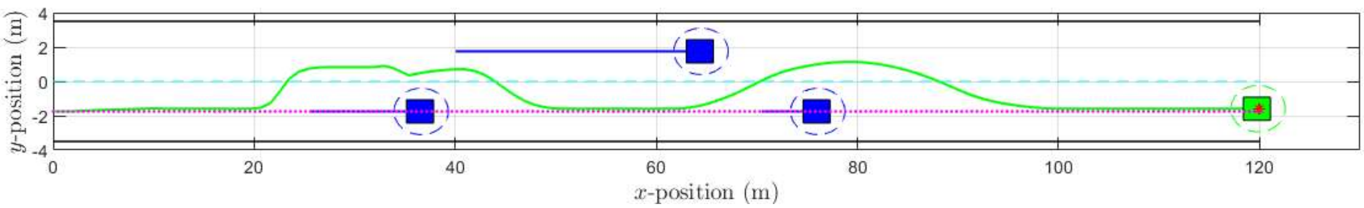

Fig. 4

In Fig. 5 we see the results of all the important states and the control variables. We can see that the constraints of all the states are control variables was well respected by NMPC.

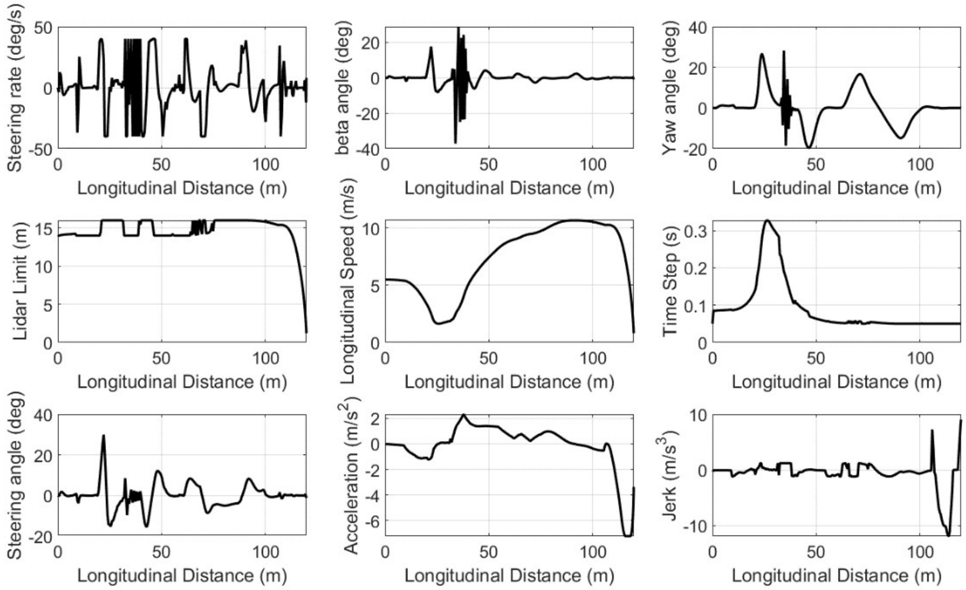

Fig. 5

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1. A.B. Dudekula, Sensor fusion and Non-linear NMPC controller development studies for Intelligent Autonomous vehicular systems, Thesis
2. Reza N. Jazar, Advanced Vehicle Dynamics, Springer Book.