Trajectory Planning for Overtaking of Autonomous Vehicles and Path Following using MPC

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(Video Link :https://youtu.be/QeKPRZ3XC3A)

***Abstract*— This paper presents a two-part solution to high-speed overtaking for autonomous vehicles. Finite-time optimization is used to generate a smooth and feasible "overtaking" trajectory around the obstacle vehicle. Model Predictive Control (MPC) is then used to control a vehicle model along the previously generated trajectory. The cost functions and constraints are designed such that the ego vehicle maintains a safe distance from the leading vehicle, stays within the two-lane workspace, and follows a smooth/natural trajectory. Moreover, the control method allows reliable overtaking maneuvers over a variety of initial conditions. In this study, the proposed solution is simulated, analyzed, and shown to be a robust method in achieving smooth high-speed overtaking.**

***Index Terms*— Trajectory Planning, MPC, Autonomous Cars**

I. INTRODUCTION

Recent advances in artificial intelligence, network infrastructure, and sensor technology have enabled increasingly complex levels of autonomy in self-driving vehicles. Furthermore, the field of autonomy and controls is rapidly evolving to meet the demands of autonomous-vehicle operations. The automotive industry is now looking to advance the scope of self-driving vehicles to accomplish end-to-end autonomy. This effort involves tackling challenging corner cases that arise in autonomy, such as avoiding unexpected obstacles or parking.

The "overtaking" maneuver is one such corner case which presents a unique challenge in the field of autonomy and self-driving vehicles. Unlike other rudimentary motions, such as turning or merging, the overtaking maneuver combines a variety of sub-maneuvers (lane-changing, lane-keeping, etc.) in a sequential and timely manner. Moreover, this complex motion must be completed smoothly, without sudden or abrupt movements, while maintaining a safe clearance as per state driving laws. Classical control techniques and novel path-search algorithms, such as A*, fall short in generating real-time solutions for overtaking problems. Optimization Techniques, however, provides a robust framework with which to generate trajectories and control vehicles through constrained maneuvers.

A. Problem Statement

In the context of this paper, "overtaking" is described as a complex maneuver which involves (i) lateral and longitudinal movement to an empty lane and back to the initial lane, and (ii) smooth/natural motion while maintaining a safe distance from the leading vehicle. As such, the objective of this paper is to design an optimal strategy (controller) to safely guide the ego vehicle around a slower leading vehicle in an overtaking maneuver. It is assumed that both vehicles will follow a two-lane freeway with both vehicles initialized in the right lane. The speeds of both cars are assumed to range anywhere from 15 m/s to 35 m/s; extreme speeds have been simulated to test for robustness. The ego vehicle and leading vehicles are initially separated by 30 meters – various distances are tested – and each lane is set as 10 meters wide. The ego vehicle is expected to stay within the parameters of the two lanes (20 meters total width), must hold a minimum safe distance of 10 meters from the leading vehicles, and must complete the maneuver smoothly while minimizing time.

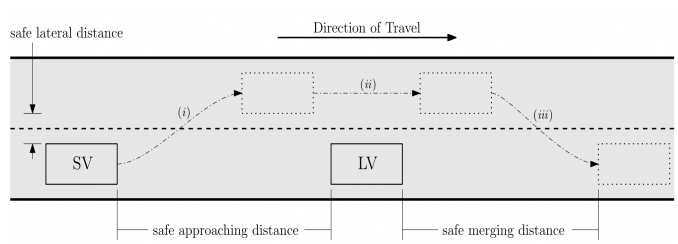

Fig. 1. Diagram of subject/ego vehicle overtaking leading vehicle [2]

B. Approach

To achieve safe overtaking, a two-layer control strategy (Fig. (**2**)) is formulated and validated. The control strategy is composed of two core elements: (i) Trajectory generation algorithm and a (ii) path following algorithm. The trajectory generator uses finite-time optimization to generate a smooth and feasible reference trajectory around the leading obstacle vehicle. This trajectory generator uses the Pyomo package for optimization and does not consider a vehicle model; rather, this algorithm applies environment, obstacle, and state constraints to generate an overtaking path. The reference trajectory (x, y, v, θ) then feeds into the path following algorithm. The path follower then generates control inputs to accurately guide a vehicle model along the reference trajectory. This algorithm uses model predictive control and assumes the kinematic bicycle model for vehicle dynamics. To validate the results, the control inputs are fed into a car model plant which will generate the true trajectory of the vehicle. The accuracy and quality of the actual trajectory is analyzed numerically and visually in the results section.

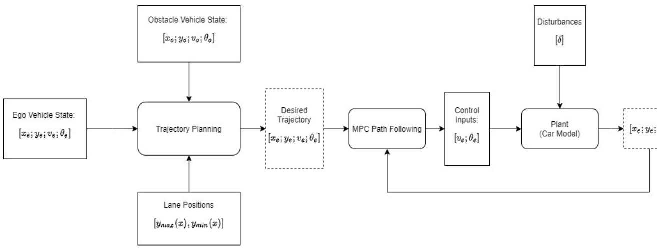

Fig. 2. Diagram of Approach [2]

C. Literature Survey

In recent years, a large body of work has been published regarding high-speed maneuvers for self-driving cars. Moreover, advances in control techniques have paved the way for truly novel approaches to vehicle autonomy. For instance, the "Model Predictive Control-based Lane Change Control System for An Autonomous Vehicle" by C. Huang, F. Naghdy, and H. Du, presents an MPC controller design used to generate vehicle control inputs for lane change maneuvers [4]. Model predictive control is also utilized heavily in research regarding trajectory generation for autonomous vehicles. "Path Planning for Autonomous Vehicles using Model Predictive Control" by C. Liu, S. Lee, S. Varnhagen, and E. Tseng is of particular relevance when considering high-speed maneuvers [3]. The paper proposes a two-layer control strategy composed of trajectory generation and path following. Model predictive control is used to generate and follow a reference trajectory which allows for maneuvers, such as merging into highways or intersection crossing. Drawing from the two-layer control strategy developed in Liu's et al. paper, a similar control method is designed and validated for high-speed overtaking in this paper.

II. Optimization Trajectory

The trajectory to be optimized is one of a ego vehicle starting behind an obstacle vehicle on a two lane highway. The obstacle and ego vehicle starts in the bottom lane traveling at a constant x velocity down the highway. The goal of the optimization problem is for the ego vehicle to overtake obstacle vehicle and return to the same lane without crashing into the ego vehicle, violating the constraints of the highway lanes, the speed limit of the highway, and the realistic steering angle of the ego vehicle. In addition, the trajectory should be able to handle both straight and curved roads.

A. Ego Vehicle and Obstacle Vehicle Notation/Simulation Parameters

In order to create an optimal trajectory for the ego vehicle around the obstacle vehicle, the states and the inputs of the ego and obstacle vehicles must be defined. These states and inputs are define below:

x_e = x coordinate of the ego vehicle
y_e = y coordinate of the ego vehicle
v_e = input forward velocity of the ego vehicle
θ_e = input steering angle of the ego vehicle
x_o = x coordinate of the obstacle vehicle
y_o = y coordinate of the obstacle vehicle
v_o = input forward velocity of the obstacle vehicle
θ_o = input steering angle of the obstacle vehicle
o = binary-like decision variable for overtaking vehicle

The coordinates of the ego vehicle and the obstacle vehicle are set into two states with the first being the x positions and the second being the y position of the ego and obstacle vehicles. The inputs of the ego and obstacle vehicle are the forward velocity of the vehicle and the steering angles of the vehicles. The binary decision variable o is used to alter the cost function depending on whether the vehicle has overtaken the obstacle vehicle or not.

The simulations parameters are set so that the sampling time is 0.1 seconds, with a horizon of 100 steps. All parameters were set to be in SI units. The highway constraints are set so that there are two lanes moving in the y direction the lane widths being 10 m wide. Finally the vehicles starting positions are in the middle of the bottom lane.

B. Road Formulation

For the highway, the vehicles move in the x direction of a two lane highway. The formulation of the y position of the middle divider is therefore formulated using (1) for each forward x coordinate of the road. This y position is also the path that the obstacle vehicle follows. This function shown is one of a curved lane, however the road can be formulated as any continuous function i.e. a straight line, a sine function etc. Using (2), the angle of the road and the horizontal can be computed for each horizontal x position. The highway boundaries are formulated using (2) and (3), to determine the y limits the ego vehicle can travel depending on it's x position, where $dx \ll 1$ and l_w is the lane width for the top offset is $y_{lw,top}$ and for the bottom offset, $y_{-lw,lower}$.

$$y_{middle}(x) = f(x) = 0.001x^2 \quad (1)$$

$$atan_x(x) = tan^{-1}\left(\frac{y_{middle}(x+dx) - y_{middle}(x-dx)}{2dx}\right) \quad (2)$$

$$y_{offset}(x, y) = y_{middle}(x) + \frac{y}{cos(tan^{-1}(atan_x(x))} \quad (3)$$

C. Ego Vehicle and Obstacle Vehicle Initial Conditions

The ego vehicle is expected to start at an x coordinate of 0 m and a y coordinate that is in the bottom lane or the negative half of the lane width with a starting velocity, which is set at 20 m/s, however, will be changed later and a steering angle of 0 radians. The obstacle vehicle on the other hand is expected to start at an x coordinate of 30 m ahead of the ego vehicle in the same lane, therefore its y coordinate is a function of its x coordinate and half of the lane width. The initial velocity of the obstacle vehicle is $20m/s$ with a 0 radians steering angle.

$x_{e,0} = 0; y_{e,0} = y_{offset(x_{e,0}, -l_{width}/2)}$; $v_{e,0} = 20m/s; \theta_{e,0} = 0$
$x_{o,0} = 30; y_{o,0} = y_{offset(x_{o,0}, -l_{width}/2)}$; $v_{o,0} = 20m/s; \theta_{o,0} = 0$

D. Obstacle Vehicle State and Inputs

The obstacle vehicle trajectory is set so that the obstacle vehicle continues at a constant velocity throughout the horizon. The obstacle vehicle's velocity at each time Ts is therefore equal to the initial velocity of 20 m/s and has an input steering angle of 0 radians. The x state coordinate is therefore the obstacle vehicle's initial position plus its constant velocity at each time Ts times the time step times the cosine of the angle between the road and the horizontal at each x position seen in (4). The y coordinate is the obstacle vehicle's initial y position plus the product of the velocity at each time Ts times the time step time the sin of the angle between the road and the horizontal seen in (5). $v_{o,0\to N} = v_{o,0} = 20; \theta_{o,0\to N} = \theta_{o,0} = 0$

$$x_{o,k\to N} = x_{o,k-1} + v_{o,k\to N} \times Ts \times cos(atan_x((x_{e,k-1})) \quad (4)$$

$$y_{o,k\to N} = y_{o,0} + v_{o,k\to N} \times Ts \times sin(atan_x((x_{e,k-1})) \quad (5)$$

E. Objective

$$\begin{aligned}\min_{z_0,\ldots,z_N,u_0,\ldots,u_{N-1}} \; & 10 * (y_{e,N} - y_{offset}(x_{e,N}, -l_w))^2 \\ & + 10 * (v_{e,N-1} - v_{e,0})^2 \\ & + 30 \sum_{k=t}^{N-1} (\theta_{e,t+1} - \theta_{e,t})^2 + 10 \sum_{k=t}^{N-1} \theta_{e,t}^2 \\ & - 50 \sum_{k=t}^{N} v_{e,t} * cos(atan_x(x_{e,t})) \\ & + 3 \sum_{k=t}^{N} (o_k)(y_{e,t} - y_{offset}(x_{e,t}, -l_w/2))^2 \\ & + 1.5 \sum_{k=t}^{N} (1 - o_k)(y_{e,t} - y_{offset}(x_{e,t}, l_w/2))^2\end{aligned} \quad (6)$$

The cost function seen in (6) is built so that the ego vehicle optimizes it's trajectory as it attempts to overtake the obstacle vehicle. The first sub-cost penalizes the final y position of the ego vehicle to make sure the terminal location of the ego vehicle is in the correct lane after overtaking and merging. The next part of the cost penalizes the velocity of the ego vehicle so that it minimizes its final speed to its initial speed. The first summation of the objective function is to penalize the cost so that the vehicle minimizes its change in steering angle, and therefore does not overturn. The next summation minimizes the steering angle of the car, so that it travels in more of a straight line, and the summation associated with the velocity of the ego vehicle minimizes the ego vehicle's velocity down its trajectory, so that it does not waste energy/gas. The last two summations deal with the overtaking cost of the ego vehicle. When the ego vehicle is behind the obstacle vehicle the first overtaking cost is active and the ego vehicle will try to change lanes. The opposite is true when the vehicle is in front of the obstacle vehicle and it will try to merge back into its original lane. Coefficients in front of each of the sub-costs of the overall cost are set in order to weigh each individual parts of the sub cost to optimize the trajectory. For example, the change in steering angle cost is of higher priority than the cost of overtaking in order to follow the dynamics of a vehicle.

F. State Constraints

$$x_{e,k+1} = x_{e,k} + v_{e,k} \times cos(\theta_{e,k} + atan_x(x_{e,k})) \times Ts \quad \text{for } k < N \quad (7)$$

$$y_{e,k+1} = y_{e,k} + v_{e,k} \times sin(\theta_{e,k} + atan_x(x_{e,k})) \times Ts \quad \text{for } k < N \quad (8)$$

As seen in (7) and (8), the ego vehicle's x and y coordinate follows a simple model where, the x coordinate of the ego vehicle is equal to the ego vehicle's initial x position plus the ego vehicles forward velocity times the cosine of it's steering angle times for each time interval Ts in the horizon plus the angle the road makes with the horizontal at that x position time the time step. The y coordinate of the ego vehicles position is equal to the ego vehicle's initial y position plus the ego vehicle's forward velocity time the sine of the steering angle of the ego vehicle and the angle the road makes with the horizontal at that x position times the time step of the horizon.

G. Speed and Acceleration Constraints

$$v_{e,k} \leq 35 \quad \text{for } k < N-1 \quad (9)$$

$$-3 \times Ts \leq v_{e,k+1} - v_{e,k} \leq 3 \times Ts \quad \text{for } k < N-1 \quad (10)$$

As the ego vehicle must obey the speed limit, the speed constraint is set that the ego vehicle's input forward velocity is never less than zero(the vehicle is always traveling forward), and does not go over 35 m/s for all time steps seen in (9). In addition, the ego vehicle can only accelerate between a certain range, which has been set to |4| m/s for all time steps seen in (10).

H. Angle and Angular Speed Constraints

$$-0.5 \leq \theta_{e,k} \leq 0.5 \quad \text{for } k < N \tag{11}$$

$$-0.175 * Ts \leq \theta_{e,k+1} - \theta_{e,k} \leq 0.175 * Ts \quad \text{for } k < N-1 \tag{12}$$

As with the speed and acceleration constraints, the angle and angular speed of the ego vehicle must have realistic constraints as well. Therefore, the ego vehicle steering angle is set to be $|\,0.5$ Radians$\,|$, and the angular speed is set to be $|\,0.175$ Radians$/s\,|$ to prevent the vehicle from turning at unrealistic speeds and angles, while traveling at it's relative speed seen in (11) and (12).

I. Obstacle Constraints

$$(x_{e,k} - x_{o,k})^2 + (y_{e,k} - y_{o,k})^2 \geq 100 \quad \text{for } k < N \tag{13}$$

In order to prevent the ego vehicle from crashing into the obstacle vehicle, a constraint is added so that the ego vehicle remains at least 10 meters away from the obstacle vehicle at all times steps throughout the horizon seen in (13). This ensures that the ego vehicle will never crash into the obstacle vehicle by using a Pythagorean theorem equation with the differences in distance of the ego and obstacle vehicle x and y coordinates.

J. Lane Constraints

$$y_{offset}(x_{e,k}, -l_w) \leq y_{e,k} \leq y_{offset}(x_{e,k}, l_w) \quad \text{for } k < N+1 \tag{14}$$

The lanes of the vehicle are set such that the vehicle remains in the highway boundaries of the bottom lane width offset and top lane width offset for each x position. Therefore the y position of the ego vehicle, during overtaking must remain within these boundaries in order to prevent the vehicle from driving off of the highway. These constraints can be seen in (14).

K. Overtaking Constraint

$$o_k = \frac{1}{1 + e^{x_{e,k} - x_{o,k} - 10}} \tag{15}$$

The overtaking constraint is used to determine a binary-like variable that edits the cost function. When the ego vehicle is behind the obstacle vehicle $o_k \approx 1$, therefore the binary-like variable is active, and therefore the sub cost associated with changing to another lane in the cost, (6) is active, while the other sub cost for going back into the same lane is not. When the ego vehicle is 10 m ahead of the obstacle vehicle $o_k \approx 0$, therefore the secondary sub cost is active, while the other one is not, and the vehicle will attempt to merge into it's original lane.

L. Final Constraints

$$\bar{v}_{e,N} = v_{e,0} \tag{16}$$

$$\bar{\theta}_{e,N} = \theta_{e,0} \tag{17}$$

The final constraint sets the final input velocity of the ego vehicle to its initial input velocity so that the vehicle stays below the speed limit seen in (16). It also sets the final input steering angle to zero seen in (17) so that the ego vehicle travels in a straight line in the same lane it started in.

III. MPC for Path Following

A. Car Model Formulation

To model our autonomous car, we model the dynamics of our car with a basic bicycle kinematic model with 4 states [x,y,v,θ]. The equations for the basic bicycle model for our car can be show in the equations below. A disturbance variable, δ was added to the dynamics of the model to introduce error in the model from the road and tires.

$$\begin{aligned}
\dot{x} &= v\cos(\psi + \beta) \\
\dot{y} &= v\sin(\psi + \beta) \\
\dot{v} &= a \\
\dot{\psi} &= \frac{v}{l_r}\sin(\beta) \\
\beta &= \tan^{-1}\left(\frac{l_r}{l_f + l_r}\tan(\delta_f)\right)
\end{aligned}$$

where

$x =$ global x CoG coordinate
$y =$ global y CoG coordinate
$v =$ speed of the vehicle
$\psi =$ global heading angle
$\beta =$ angle of the current velocity w/respect to the car
$a =$ acceleration of the center of mass into this direction
$l_r =$ distance from the center of the vehicle to the rear
$l_f =$ distance from the center of the vehicle to the front
$\delta_f =$ steering angle of the front wheels w/respect to the car

B. MPC Formulation

For the MPC, we set up the cost function and constraints based on the equations below. We perform the CFTOC calculation N times in as many for loops to cover our horizon. We also split the for loop into two parts to increase our accuracy and increase runtime to the original trajectory.

$$
\begin{aligned}
\min_{z_0,\ldots,z_N,u_0,\ldots,u_{N-1}} \quad & \sum_{k=N-1}^{N} (z_k - \bar{z}_k)^2 + 10 * \sum_{k=N-1}^{N} (z_k - \bar{z}_N)^2 \\
& + \sum_{k=N-1}^{N} u_k^T R u_k \\
& z_{k+1} = z_k + f(z_k, u_k)\Delta t \\
& z_{min} \leq z_k \leq z_{max} \\
& u_{min} \leq u_k \leq u_{max} \\
& |\beta_{k+1} - \beta_k| \leq \beta_d \\
& z_0 = \bar{z}_0 \\
& z_N = \bar{z}_N
\end{aligned}
$$

IV. Simulations & Results

The performance of the proposed two-layer control strategy is analyzed in detail below. Specifically, five tests have been performed to assess the effectiveness of the control method.

A. Straight Lane Overtaking

The first three experiments demonstrate trajectory generation and path following for high-speed overtaking along a *straight road*. To demonstrate robustness, the initial velocities of the ego vehicle is varied between 20, 25, and 30 m/s (45 mph to 67 mph). Note that the initial speed of the leading obstacle vehicle is held at 20 m/s in all three simulation. The results of the three simulations are shown in Figure 5. The figure presents the x position, y position, heading angle, and speed of the ego vehicle along the overtake for each test case.

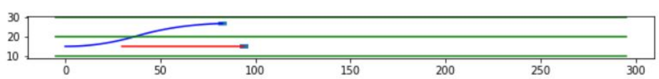

Fig. 3. Results from Start of Straight Lane Overtake

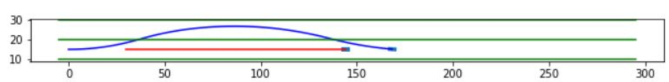

Fig. 4. Results from End of Straight Lane Overtake

Also note that the initial distance between the ego vehicle and the leading vehicle has been held constant in each simulations (30 meters). The trajectories shown in Fig. 3 and Fig. 4 demonstrate that the proposed control strategy enables high-speed overtaking along a linear road. Moreover, Fig. 5 shows that the heading angle varies linearly and stays strictly within the -0.2 to 0.2 radians constraint. Both the heading angle plots and the simulation snapshots (Fig. 4 and Fig. 5) show that the maneuver was performed naturally (without erratic motion). Similarly, the velocity for all three test cases varies linearly along the path allowing for a smooth overtake. However,

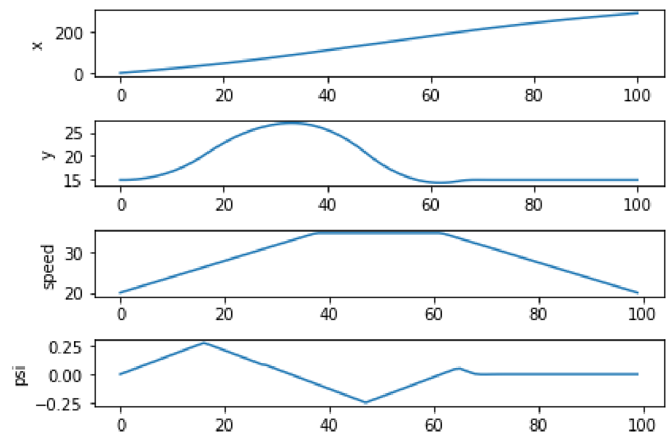

Fig. 5. Results from End of Straight Lane Overtake

the time required to perform the overtaking maneuver does vary with initial speed. Specifically, the 30 m/s test case required approximately 43 seconds to complete the overtaking maneuver while the 20 m/s ego vehicle required 57 seconds to complete the overtake. These results demonstrate that higher initial vehicle velocities allow for faster overtake maneuvers.

B. Curved Lane Overtaking

Simulation 4 and 5 (Fig. 6 and Fig. 7) demonstrate the controllers ability to plan and follow trajectories along curved roads. Figure 8 similarly shows the x position, y position, heading angle, and velocity of the ego vehicle along curved roads. The roads shown in figures 7 and 8 are modelled by the following equations respectively:

$$
\begin{aligned}
y &= x^2 \cdot 0.001 \\
y &= -x^2 \cdot 0.002
\end{aligned}
$$

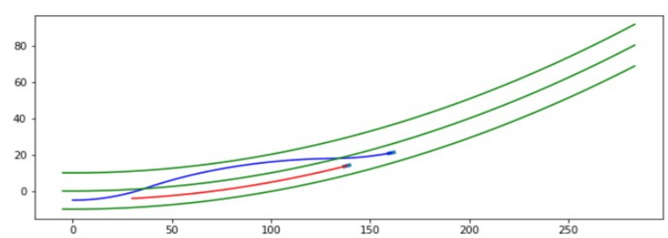

Fig. 6. Curved road trajectory generations (Sim. 4)

Note that the initial vehicle speeds are held constant in each curved road simulation – ego vehicle and leading vehicle initialized at 20 m/s. Similarly, the initial distance between the ego and leading vehicles are held constant (30 meters) in each case. The two simulations demonstrate that the control strategy allows smooth overtakes along both left curving and right curving roads. Also note that the road designed in simulation 4 (Fig. 6) features greater

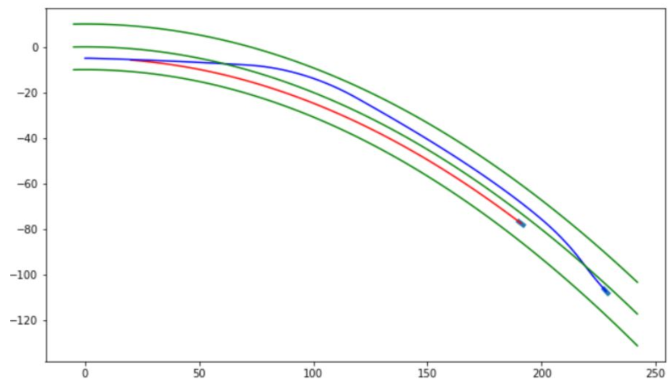

Fig. 7. Curved road trajectory generations (Sim. 5)

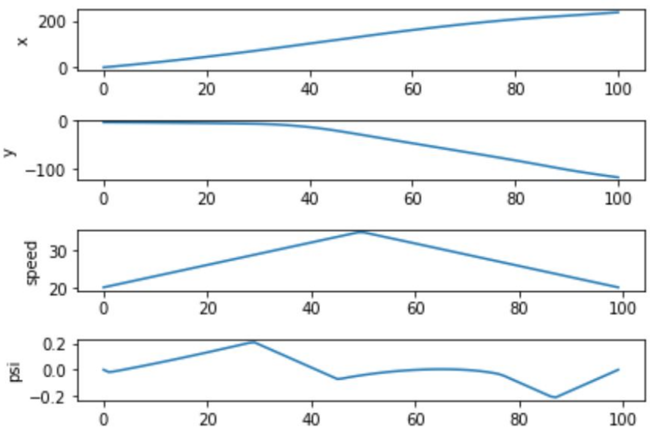

Fig. 8. States from curved trajectory

curvature than the road in simulation 5 (Fig. 7). While both experiments generate accurate maneuvers, the time required to overtake does vary with road curvature. For instance, the overtake was performed in 78 seconds on the least curved road (Fig. 6). However, the overtake maneuver required 97 seconds on the greater curved road (Fig. 7). Therefore, given constant initial speeds and initial positions, the overtake maneuver performs fastest on straight roads.

C. MPC Path Following

When implementing our MPC, we also added disturbances at the 20th time step using a random function to perturb the x and y location of the car and observe how the controller would direct the car back onto to its original trajectory. Note that the trajectory is generated offline for 100 timesteps and the MPC path follower only uses 10 future timesteps iteratively.

V. Conclusion and Future Work

This paper presented a two-layer control strategy to optimally perform high-speed overtaking. Optimization-based trajectory generation used with model predictive control path following provided a robust solution to the overtaking problem. This method was shown to generate solutions over a wide span of initial conditions and produced natural trajectories that conformed to United States driving regulations.

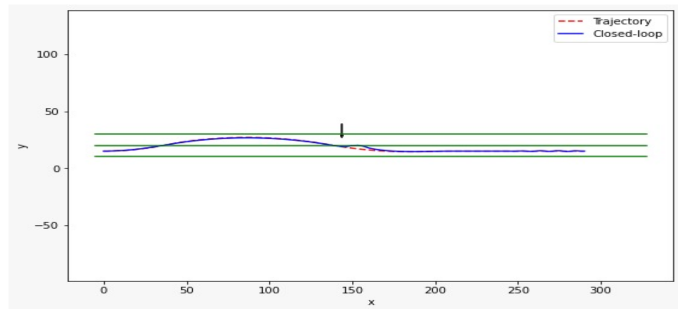

Fig. 9. MPC path following on linear road with disturbance (Sim. 1)

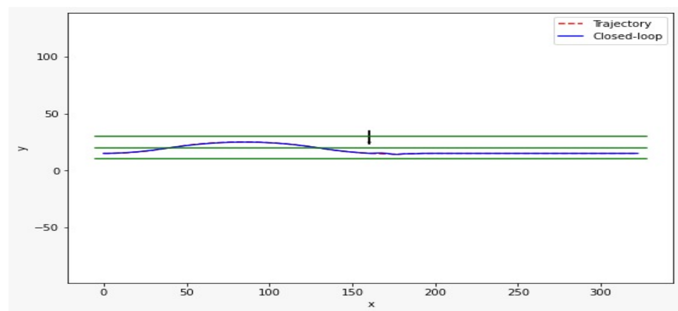

Fig. 10. MPC path following on linear road with disturbance (Sim. 3)

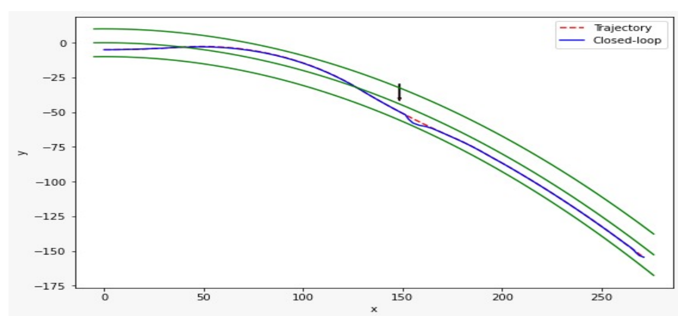

Fig. 11. MPC path following on curved road with disturbance (Sim. 5)

While the control strategy performed well under the given initial conditions, improvements can be made to the method in future works. For instance, this control method does not consider vehicles that may be present in the overtaking lane. Therefore, the strategy can be modified to make overtaking decisions based on incoming vehicles in other lanes. Additionally, the vehicle model utilized in this paper was a rudimentary kinematic bicycle model. For more realistic applications, high-fidelity vehicle models that consider tire dynamics, can be explored and implemented. Ultimately, the use of optimization trajectory generation and MPC path following allowed for a safe and feasible approach to high-speed overtaking.

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