Basic electrical engineering concepts

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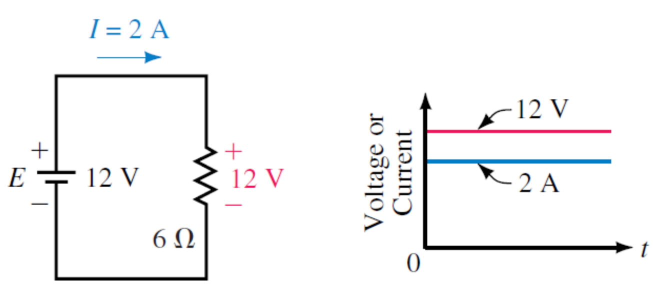
AC and DC current

Main characteristics

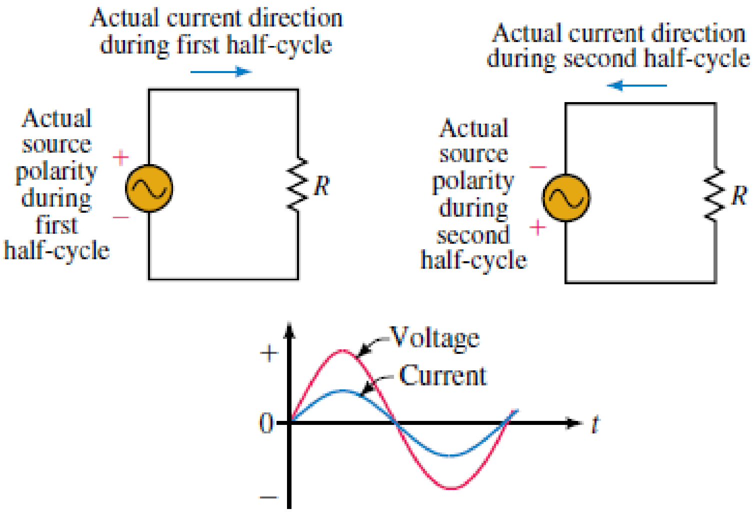
- DC (Direct Current)
  - Less losses
  - No distance limitation
  - No reactive power
  - More complex to change the voltage levels→ Power electronics required
  - More complex breakers (no zero crossing currents)



(a)

(b) Voltage and current versus time for dc

- AC (Alternating current)
  - Three-phase allows to generate a rotating magnetic field in electrical machines, enabling 'simple' generation.
  - Simple voltage change (step-up/step-down) - Transformers
  - 'Simple' breakers (zero crossing currents)



Robins & Miller "Circuit analysis theory and practice"

Circuit elements

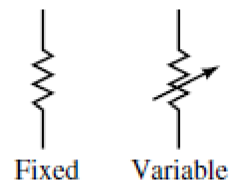
Basic elements

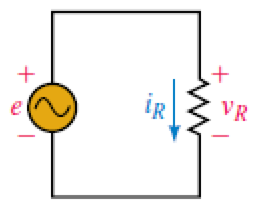
- Passive elements
  - Linear elements
    - Resistances (R), inductances (L), capacitors (C)
    - Lumped constants, passive elements connected using conductors.
    - Distributed constants (electrical lines)
  - Non-linear elements: semiconductors (power electronics), magnetic circuits (non-linear hystheresis)
- Active elements:
  - Energy sources
    - Voltage sources
    - Current sources

Passive elements

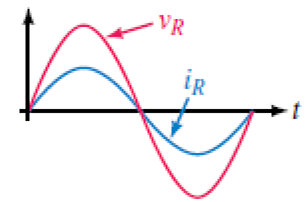
Resistances, inductances and capacitors

- Resistance R
  - $v_r(t) = R i_R(t)$



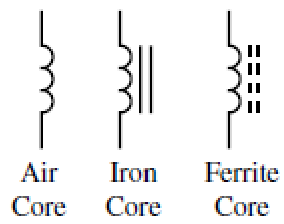


(a) Source voltage is a sine wave. Therefore, v_R is a sine wave.



(b) $i_R = v_R/R$. Therefore i_R is a sine wave.

- Inductance L
  - $v_L(t) = L\dfrac{di_L(t)}{dt}$
  - Equivalent to a conductor in steady state (DC)



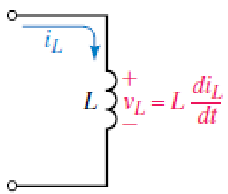
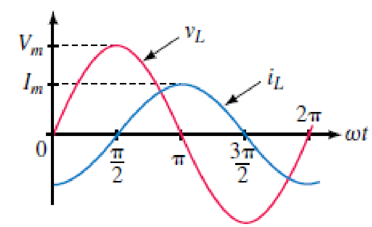
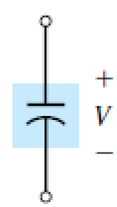
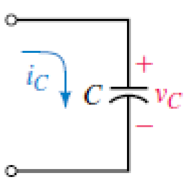


FIGURE 16–21 Voltage v_L is proportional to the rate of change of current i_L.

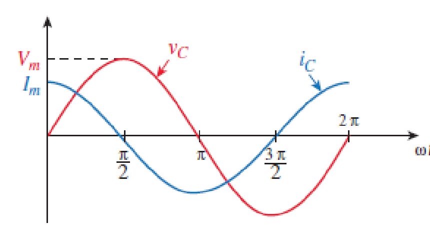


- Capacitor C
  - $i_C(t) = C\dfrac{dv_C(t)}{dt}$
  - Equivalent to an open circuit in steady state (DC)





(a) $i_C = C\dfrac{dv_C}{dt}$

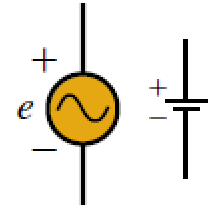


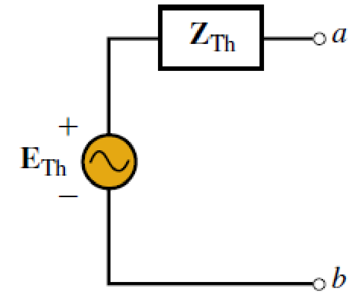
Robins & Miller "Circuit analysis theory and practice"

Main electrical sources

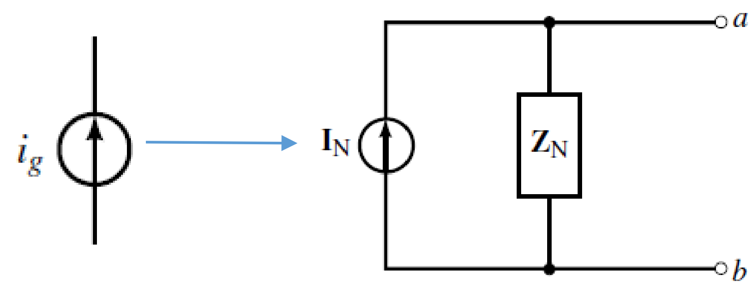
Voltage and current

- Voltage source
  - Ideal
  - Real (series impedance)
  - Critical situation: short-circuit → **Max current**





- Current source
  - Ideals
  - Reals (Parallel impedance)
  - Critical situation: open circuit → **Max voltage**



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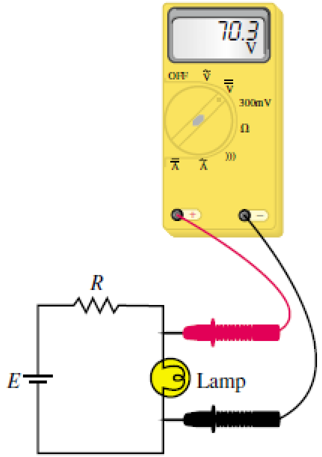
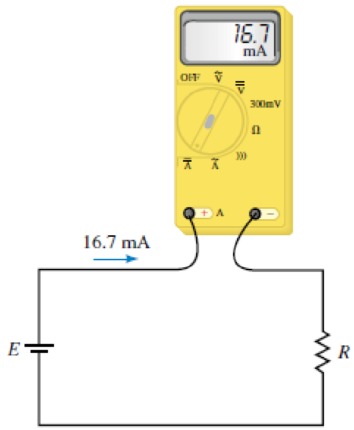
Equipment - Measurements

Voltage and current measurements

- Voltage measurement
  - Parallel connection
  - High impedance, ideally infinite.



- Current measurement
  - Series connection
  - Reduced impedance, ideally 0.



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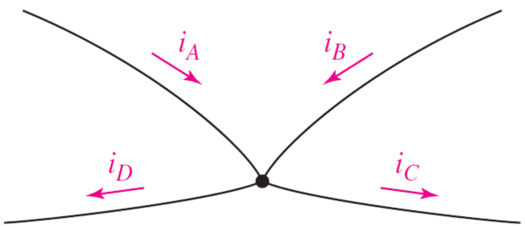
Kirchoff Law's

Kirchoff circuit and voltage laws

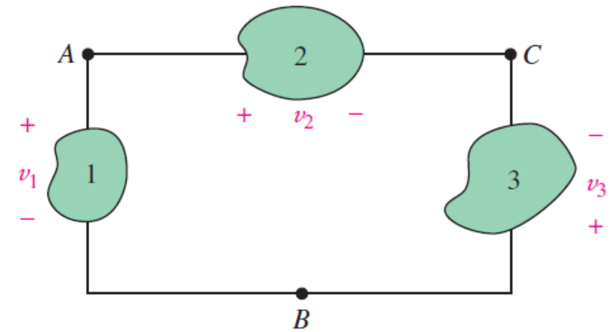
- Kirchoff Circuit Laws (KCL) – The sum of the currents entering a node has to be equal to the sum of the output currents

$$\sum \underline{I}_{input} = \sum \underline{I}_{output}$$



- Kirchoff voltage law (KVL) – The sum of the voltages in a closed circuit has to be equal to 0.

$$\sum \underline{V}_k = 0$$



- Then, the sum of all the powers in a circuit has to accomplish:

$$\sum \underline{P}_k = 0 \rightarrow \text{Active power}$$

$$\sum \underline{Q}_k = 0 \rightarrow \text{Reactive power}$$

Wait... we will explain later P and Q !!!

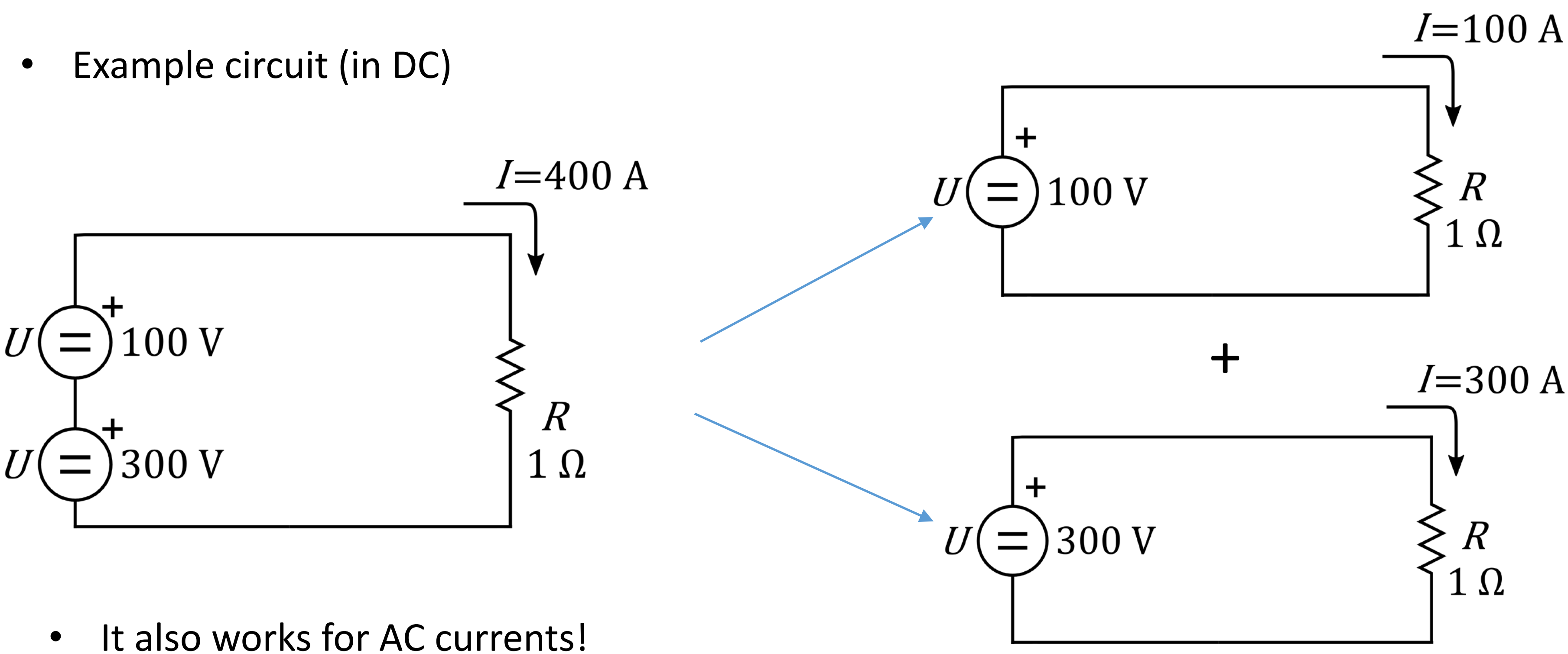
Hayt "Engineering circuit analysis"

Superposition principle

Main description

- If several simultaneous excitations are applied to the same network (either voltage or current sources), the global response is equal to the superposition of the partial results of each of the sources. (Voltage source → conductor | Current sources→ open circuit)

- Example circuit (in DC)



- It also works for AC currents!

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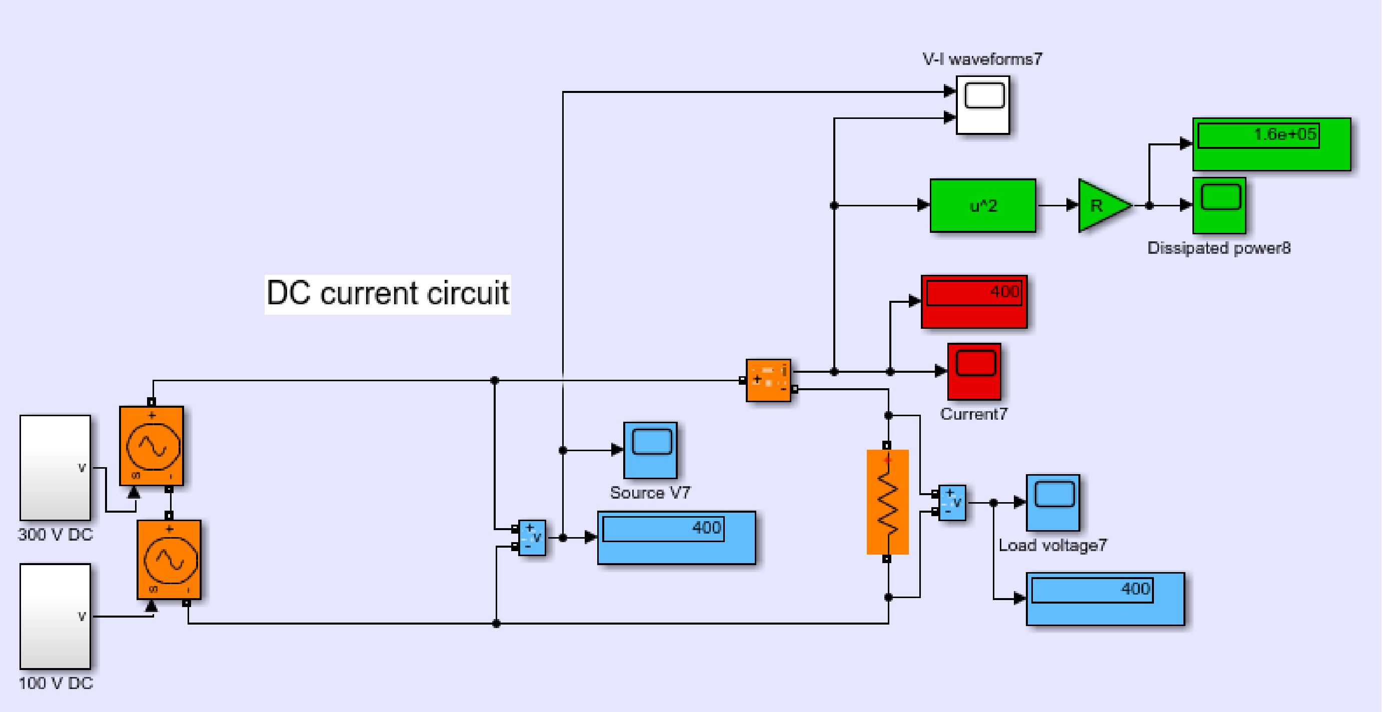
Activity 0

Illustrative example in DC

- Simple DC circuit (superposition of 2 voltage sources)



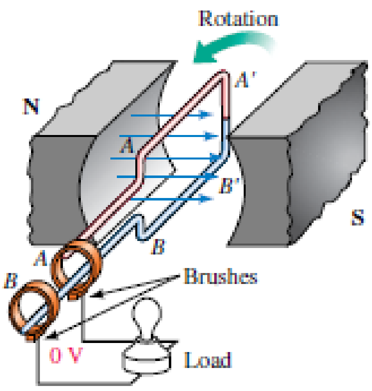
- Try it with AC!

Activity I

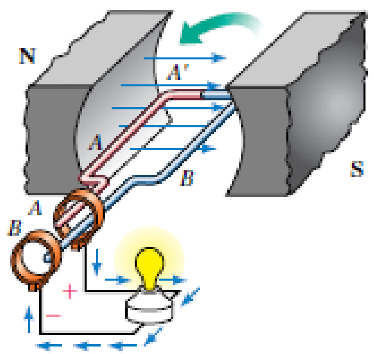
Illustrative example in DC

- Circuit 1 – Resistive circuit (DC)
- Circuit 2 – Capacitive circuit (DC)
- Circuit 3 – Inductive circuit (DC)
- Circuit 4 – Resistive-inductive circuit (DC) - Dynamics

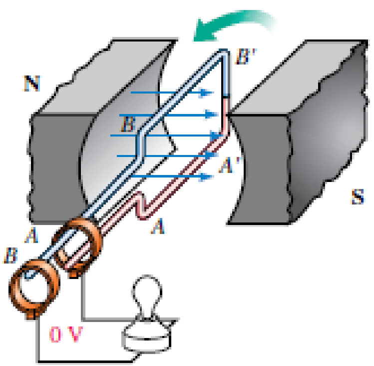
AC voltage generation



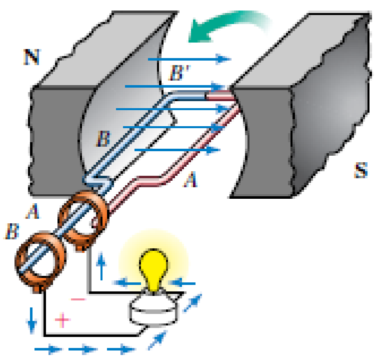
(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.



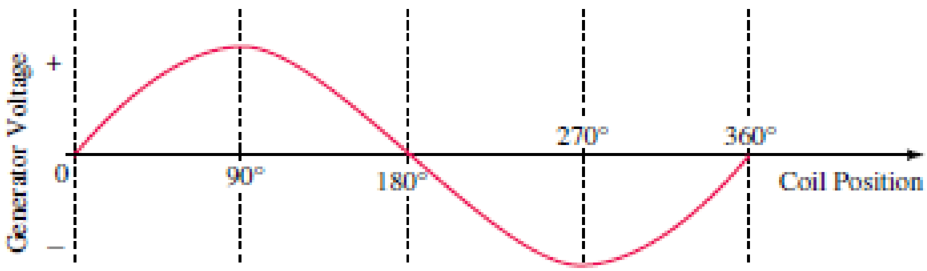
(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.



(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.



(d) 270° Position: Voltage polarity has reversed, therefore, current direction reverses.



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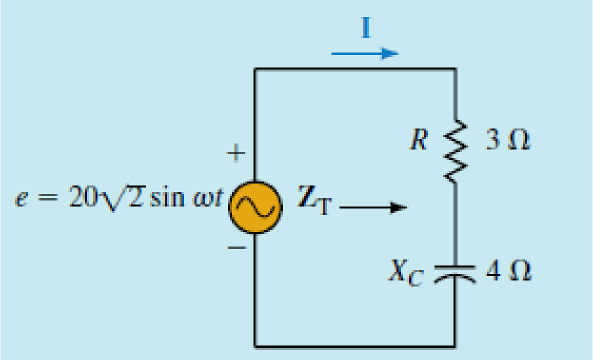
Analysis of AC circuits

How to operate?



- RC circuit equations:
  - $e(t) = 20\sqrt{2}\sin(\omega t) = Ri(t) + v_c(t)$
  - $i(t) = C\frac{dv_c(t)}{dt}$
  - $20\sqrt{2}\sin(\omega t) = Ri(t) + \frac{1}{C}\int i(t)dt$

How to calculate $\boldsymbol{i(t)}$???

- It is required to solve the differential equation and after observe the steady state.
- We could use simulation!
- See AC and DC examples

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Activity II

Illustrative example in AC – Simulations to solve differential equations

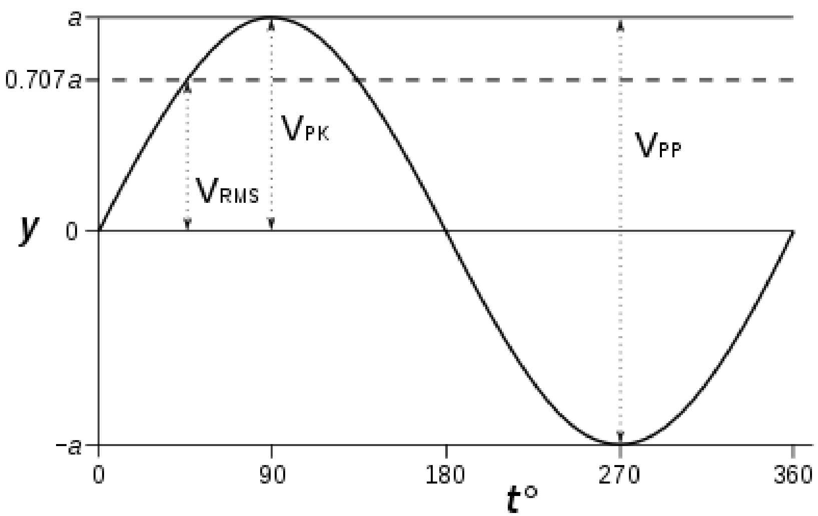
- Circuit 1 – Resistive circuit (AC)
- Circuit 2 – Capacitive circuit (AC)
- Circuit 3 – Inductive circuit (AC)
- Simulations are useful, but analytically, **phasors** will simplify the analysis!

Root Mean Square value (RMS)

Definition

- For alternating electric current, **RMS value is equal to the value of the direct current** that would produce the same average power dissipation in a resistive load – it is useful for power calculation in AC.
- If the waveform is a **pure sine wave** (as in AC), the **relationships between amplitudes** (peak-to-peak, peak) and **RMS are fixed and known**, as they are for any continuous periodic wave.
- For a sine wave, the relation between the peak and RMS value is:

$$V_{peak} = \sqrt{2}\, V_{RMS}$$

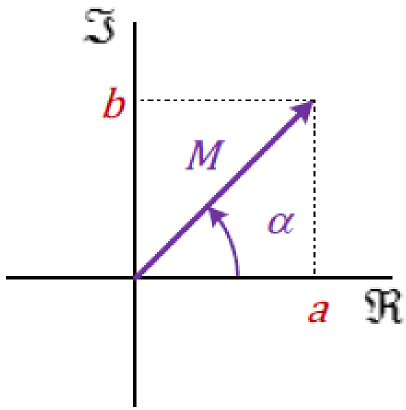


- Typically voltages are provided in RMS values
- Verify this in simulation!

Phasors

Concept definition

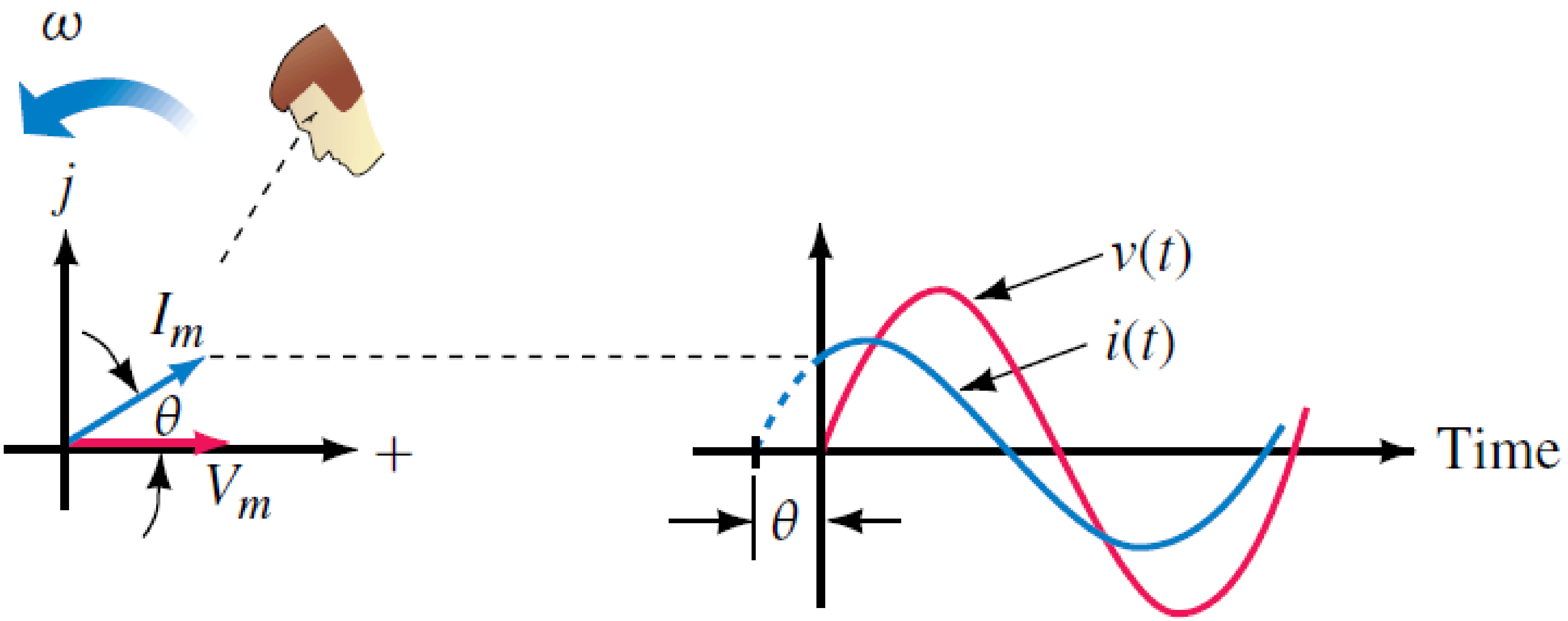
- Sine wave: $m(t) = \sqrt{2}M\cos(\omega t + \alpha)$
- Euler formula: $e^{jx} = \cos x + j\sin x$ $\qquad j = \sqrt{-1}$
- Combining both equations:
- $m(t) = \mathrm{Re}\left\{\sqrt{2}Me^{j(\omega t+\alpha)}\right\}= \mathrm{Re}\left\{\sqrt{2}Me^{(j\omega t+j\alpha)}\right\}= \mathrm{Re}\left\{\sqrt{2}Me^{j\omega t}e^{j\alpha}\right\}=\mathrm{Re}\left\{\sqrt{2}\underline{M}e^{j\omega t}\right\}$
- Then: $m(t) = \sqrt{2}M\cos(\omega t + \alpha) \Longleftrightarrow \underline{M} = Me^{j\alpha}$
- Graphic representation (counter-clockwise) $\longrightarrow$ $\underline{M} = M\,e^{j\alpha} = a + jb$
- It can also be expressed in a simplified manner (usually with the angle expressed in degrees:
  - $\underline{M} = Me^{j\alpha} \longrightarrow \underline{M} = M\angle\,(\alpha^o)$
  - $\underline{M} = M\angle\alpha = Me^{j\alpha} = a + jb = \underbrace{M\cos(\alpha)}_{a} + j\underbrace{M\sin(\alpha)}_{b}$



- You can use everything you know about complex numbers!!!

Equivalence phasors – time domain

Visual interpretation



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Phasor properties

Complex-number based operations

- Complex number-based operations
  - Sum/subtraction:

$$\underline{X}_1 \pm \underline{X}_2 = (a_1 + jb_1) \pm (a_2 + jb_2) = a_1 \pm a_2 + j(b_1 \pm b_2)$$

  - Product/division:

$$\underline{X}_1 \times \underline{X}_2 = X_1 X_{2\,\angle \alpha_1 + \alpha_2} \qquad \underline{X}_1 / \underline{X}_2 = \left(\frac{X_1}{X_2}\right)_{\angle \alpha_1 - \alpha_2}$$

  - Power:

$$(\underline{X}_1)^n = X_1{}^n{}_{\angle n\alpha_1}$$

  - Complex conjugate operations:

$$\underline{X}^*_1 = X_{1\angle -\alpha_1} = a_1 - jb_1 \qquad \underline{X}_1 \underline{X}^*_1 = X^2_1$$

  - Derivative

$$\frac{d(\underline{X})}{dt} = j\omega \underline{X}$$

Phasors for RLC circuits

Main equations (deduction)

- Resistance

$$v_R(t) = Ri_R(t) \rightarrow \text{if } v_R(t) = \sqrt{2}V_R \sin(\omega t + \alpha)$$

$$i_R(t) = \frac{1}{R}\boxed{\sqrt{2}V_R \sin(\omega t + \alpha)}$$

$$\underline{I}_R = \frac{\underline{V}_R}{R} \rightarrow \underline{V}_R = R\underline{I}_R$$

- Inductance

$$v_L(t) = L\frac{di_L(t)}{dt} \rightarrow \text{if } i_L(t) = \sqrt{2}I_L \sin(\omega t + \alpha)$$

$$v_L(t) = \sqrt{2}\omega L I_L \cos(\omega t + \alpha) = \omega L \boxed{\sqrt{2}I_L \sin(\omega t + \alpha + \pi/2)}$$

$$\underline{V}_L = \omega L \times I_{L\angle\alpha+\pi/2} = j\underbrace{\omega L}_{X_L}\underline{I}_L = jX_L\underline{I}_L$$

- Capacitor

$$i_C(t) = C\frac{dv_C(t)}{dt} \rightarrow \text{if } v_C(t) = \sqrt{2}V_C \sin(\omega t + \alpha)$$

$$i_C(t) = \sqrt{2}V_C C\omega \cos(\omega t + \alpha) = C\omega \boxed{\sqrt{2}V_C \sin(\omega t + \alpha + \pi/2)}$$

$$\underline{I}_C = \omega C V_{C\angle\alpha+\pi/2} = j\omega C\underline{V}_C \rightarrow \underline{V}_C = \frac{\underline{I}_C}{j\omega C} = -j\frac{1}{\underbrace{\omega C}_{X_C}}\underline{I}_C = -jX_C\underline{I}_C$$

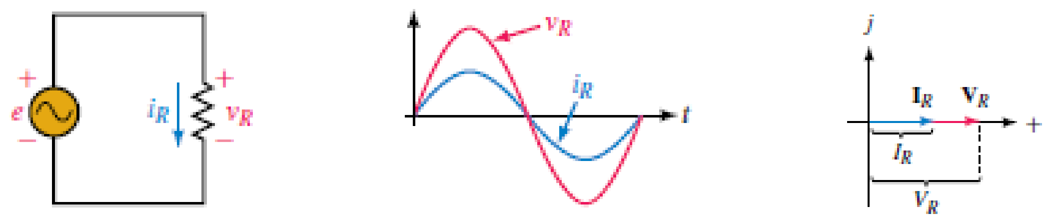
Phasors for RLC circuits

Phasors (summary)

- Resistance

$$v_R(t) = Ri_R(t)$$

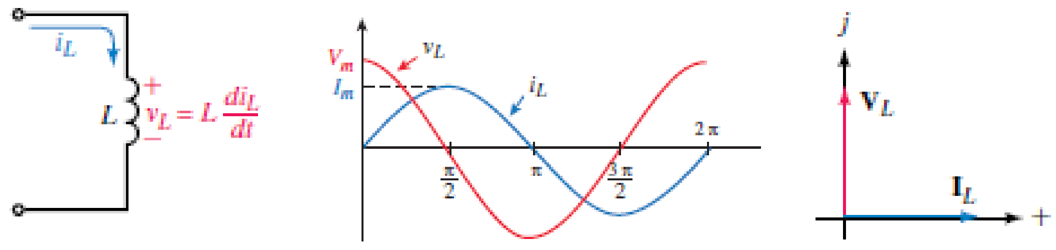
$$\underline{V}_R = R\underline{I}_R$$



- Inductance

$$v_L(t) = L\frac{di_L(t)}{dt}$$

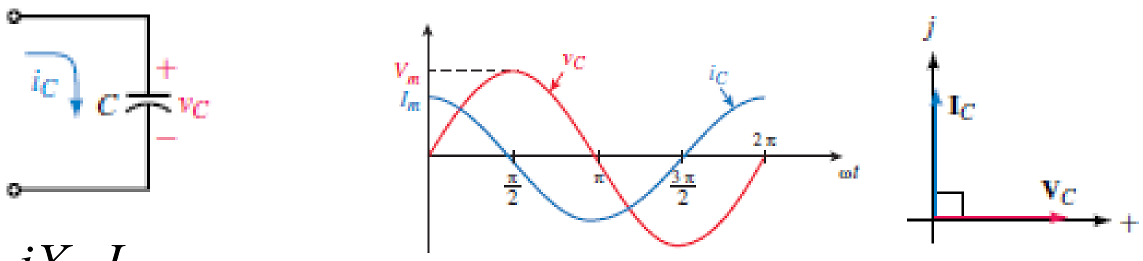
$$\underline{V}_L = j\underbrace{\omega L}_{X_L}\underline{I}_L = jX_L\underline{I}_L$$



- Capacitor

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

$$\underline{V}_C = \frac{\underline{I}_C}{j\omega C} = -j\frac{1}{\underbrace{\omega C}_{X_C}}\underline{I}_C = -jX_C\underline{I}_C$$



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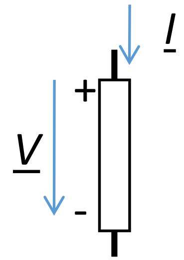
Impedance (Z) and admitance (Y)

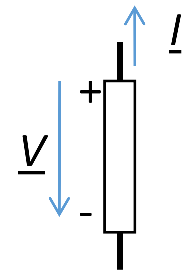
Association and sign criteria

- Sing criteria

$$\underline{V} = \underline{Z}\,\underline{I} \qquad \underline{Z} = \frac{1}{\underline{Y}}$$
$$\underline{I} = \underline{Y}\,\underline{V}$$

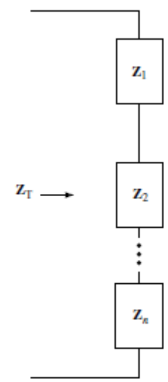


$$\underline{V} = -\underline{Z}\,\underline{I}$$



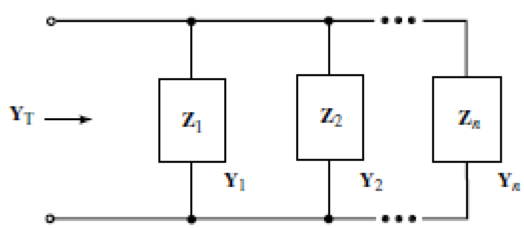
- Series association ($\underline{I}$ does not change)

$$\underline{Z}_{eq} = \underline{Z}_1 + \underline{Z}_2 + \ldots + \underline{Z}_N$$



- Parallel association ($\underline{V}$ does not change)

$$\underline{Y}_{eq} = \underline{Y}_1 + \underline{Y}_2 + \ldots + \underline{Y}_N \leftrightarrow \frac{1}{\underline{Z}_{eq}} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \ldots + \frac{1}{\underline{Z}_N}$$



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Activity III

Illustrative example in AC – Simulations to solve differential equations

- Circuit 1 – Resistive circuit (AC)
- Circuit 2 – Capacitive circuit (AC)
- Circuit 3 – Inductive circuit (AC)
- Simulations are useful, but analytically, **phasors** will simplify the analysis!
- See *.m file

AC power calculation

Derivation

$$v(t) = \sqrt{2}V\cos(\omega t + \theta_V) \rightarrow \underline{V} = V\angle\theta_V$$

$$\cos(\theta_1)\cos(\theta_2) = \frac{1}{2}(\cos(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2))$$

$$i(t) = \sqrt{2}I\cos(\omega t + \theta_I) \rightarrow \underline{I} = I\angle\theta_I$$

$$p(t) = v(t) \cdot i(t) = \sqrt{2}I\cos(\omega t + \theta_I) \cdot \sqrt{2}V\cos(\omega t + \theta_V) =$$

$$= 2VI\cos(\theta_V + \omega t)\cos(\theta_I + \omega t) =$$

$$= 2VI\frac{1}{2}\left(\cos(\theta_V + \omega t + \theta_I + \omega t) + \cos(\theta_V + \omega t - \theta_I - \omega t)\right)$$

$$= VI\left(\cos(\theta_V + \theta_I + 2\omega t) + \cos(\theta_V - \theta_I)\right) =$$

$$= VI\cos(\theta_V - \theta_I) + VI\cos(\theta_V + \theta_I + 2\omega t) = P + S\cos(\theta_V + \theta_I + 2\omega t)$$

- Average power

$$P = VI\left(\cancel{\cos(\theta_V + \theta_I + 2\omega t)} + \cos(\theta_V - \theta_I)\right) = VI\cos(\theta_V - \theta_I) = VI\cos(\varphi)$$

- Power factor

$$\cos(\theta_V - \theta_I) = \cos(\varphi)$$

- Apparent power

$$S = VI$$

Average: P

Max: $P + S$

Min: $P - S$

AC power calculation

Phasor-based calculation

- Power calculation (phasor domain)

$$\underline{V} = V\angle\theta_V$$

$$\underline{I} = I\angle\theta_I$$

$$p(t) = v(t)\cdot i(t) = \sqrt{2}I\cos(\omega t+\theta_I)\cdot\sqrt{2}V\cos(\omega t+\theta_V) =$$

$$= VI\left(\cos(\theta_V+\theta_I+2\omega t)+\cos(\theta_V-\theta_I)\right) =$$

$$= VI\cos(\theta_V-\theta_I) + \underbrace{VI}_{S}\left(\cos(\theta_V+\theta_I+2\omega t)\right) =$$

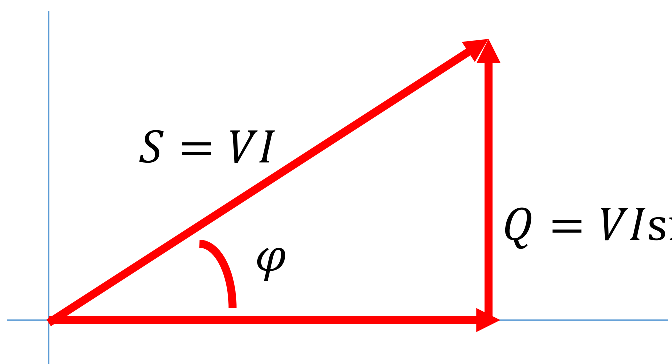
$$\underline{X}_1 \times \underline{X}_2 = X_1 X_{2\angle\alpha_1+\alpha_2}$$

$$\boxed{\begin{aligned}&\mathrm{Re}(X_1X_2\angle\alpha_1+\alpha_2) =\\ &= X_1X_2\cos(\alpha_1+\alpha_2)\end{aligned}}$$

cos(x)->Real part

$$VI\cos(\theta_V-\theta_I) = \mathrm{Re}(\underline{V}\,\underline{I}^*) = P$$

$$\boxed{\begin{aligned}&\mathrm{Imag}(X_1X_2\angle\alpha_1+\alpha_2) =\\ &= X_1X_2\sin(\alpha_1+\alpha_2)\end{aligned}}$$

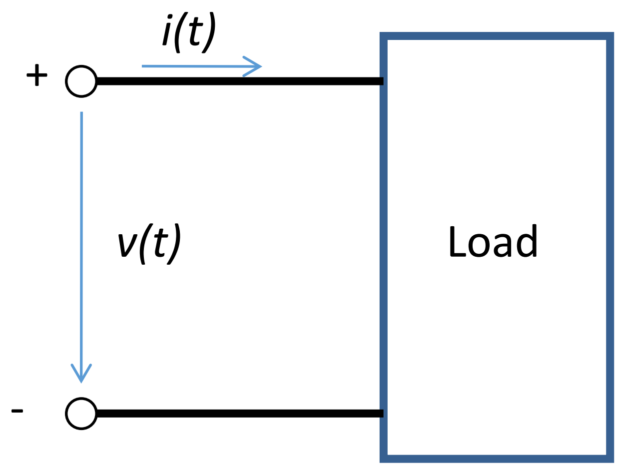


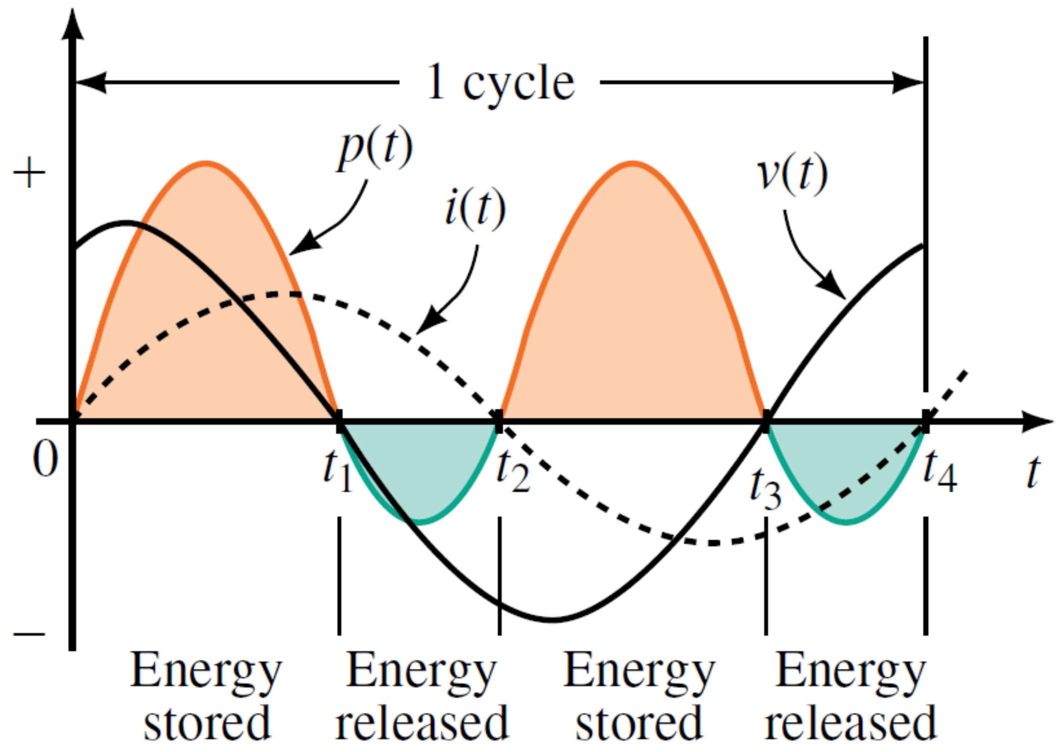
Power in AC circuits

AC power

Instantaneous power

- $p(t) = v(t)\, i(t)$





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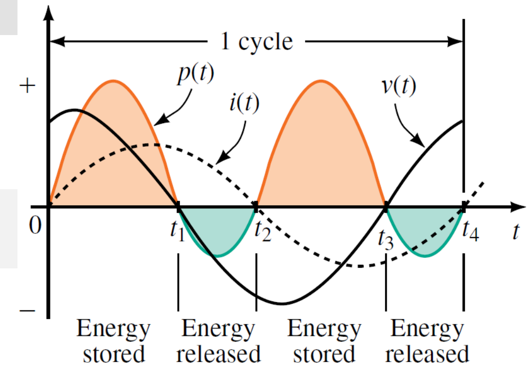
AC power



- Instantaneous power $p(t) = v(t)i(t)$
- Active power
  - $p(t)$ represents the power flowing to the load
  - P its the average power to the load.
  - More power flows to the load than is returned from it.
    - If P is zero, all power sent to the load is returned.
    - If P has a positive value, power is dissipated by the load.
  - For this reason, P is called real power. In modern terminology, real power is also called active power.
  - Thus, active power is the average value of the instantaneous power, and the terms real power, active power, and average power mean the same
- Reactive power
  - During the intervals that p is negative, power is being returned from the load. (This can only happen if the load contains reactive elements: L or C.)
  - The portion of power that flows into the load then back out is called reactive power.
  - Since it first flows one way then the other, its average value is zero; thus, reactive power contributes nothing to the average power to the load.
  - Although reactive power does no useful work, it cannot be ignored.
  - Extra current is required to create reactive power, and this current must be supplied by the source;

- It should be noted that real power and reactive power do not exist as separate entities.
- They are components of the power waveform. However, we are able to conceptually separate them for purposes of analysis.

AC power

Derivation

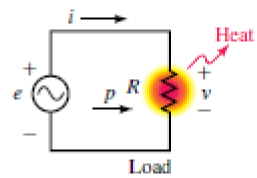
- Resistance

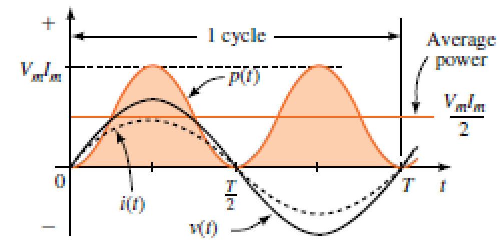
$z\,\underline{z} = |z|^2$

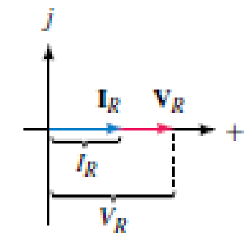
$$\underline{V}_R = R\underline{I}_R$$

$$\underline{S} = \underline{V}\,\underline{I}^* = R\underline{I}_R\underline{I}_R^* = RI_R^2$$

$$P = RI_R^2, Q = 0, \varphi = 0$$





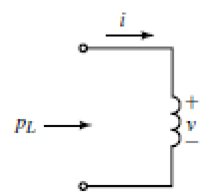


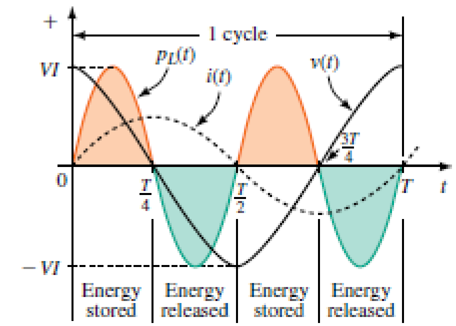
- Inductance

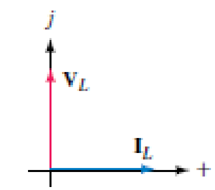
$$\underline{V}_L = jX_L\underline{I}_L$$

$$\underline{S} = \underline{V}\,\underline{I}^* = jX_L\underline{I}_L\underline{I}_L^* = jX_LI_L^2$$

$$P = 0, Q = X_LI_L^2, \varphi = \pi/2$$





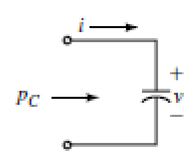


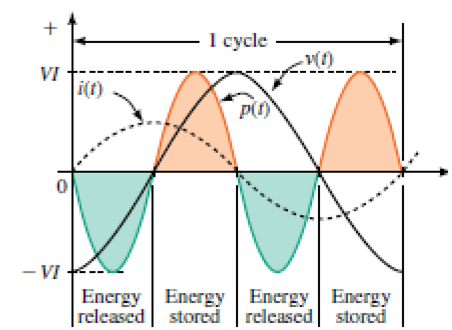
- Capacitor

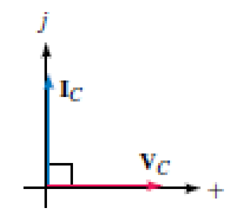
$$\underline{V}_C = -jX_C\underline{I}_C$$

$$\underline{S} = \underline{V}\,\underline{I}^* = -jX_C\underline{I}_C\underline{I}_C^* = -jX_CI_C^2$$

$$P = 0, Q = -X_CI_C^2, \varphi = -\pi/2$$







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Activity III – Part II

AC power analysis

- Circuit 1 – Resistive circuit (AC) – Active power
- Circuit 2 – Capacitive circuit (AC) – Capacitive reactive power
- Circuit 3 – Inductive circuit (AC) - Inductive reactive power
- Simulations are useful, but analytically, **phasors** will simplify the analysis!

Why three-phase AC?

Main characteristics and advantages

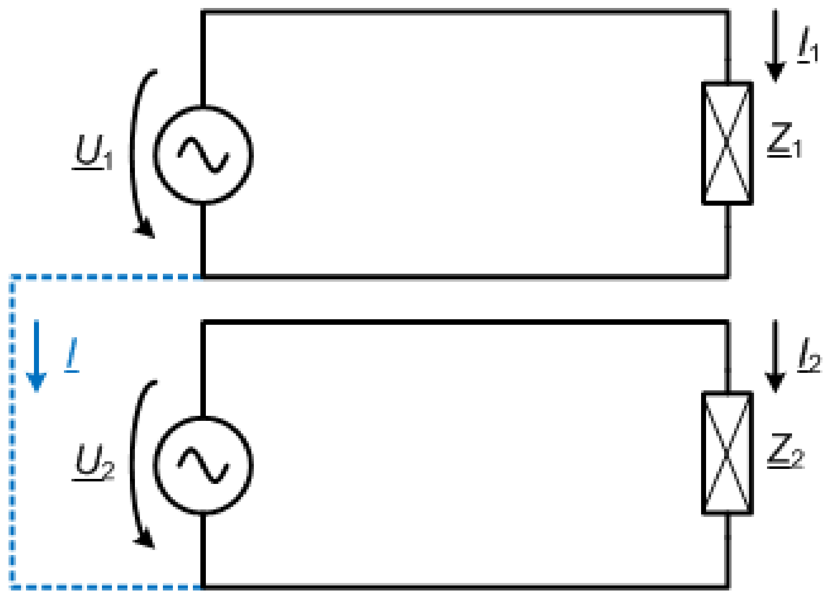
- Single-phase AC system – two conductors (phase + neutral)

- Three-phase AC power supply – three conductors (abc phases). Possible four wires if neutral is available.

- Main advantages of three-phase systems over single phase systems:
  - Increased power transfer capacity. With 1,5 more cables (from 2 to 3), three times more power can be transferred.
  - The power transferred is constant (for balanced systems and loads). Single phase systems show a pulsating power. This is specially relevant in mechanical devices as it can be transferred as torque oscillations.
- Three-phase systems can produce a rotating magnetic field with a specified direction and constant magnitude, which simplifies the design of electric motors, as no starting circuit is required.

Three-phase systems

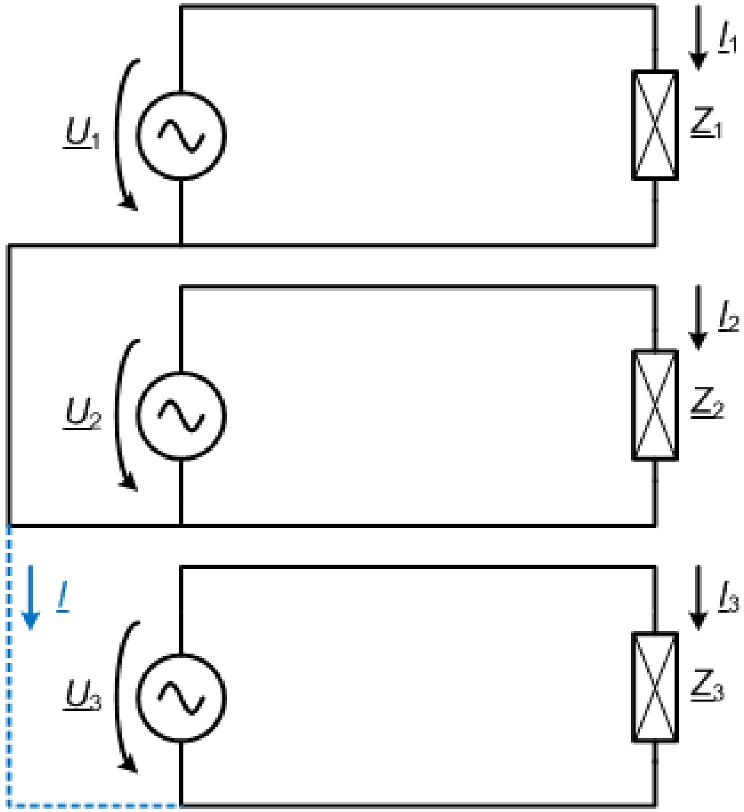
If we interconnect two single-phase circuits and interconnect them, which current *I* will be established?



Rull, Galceran, Gomis, “Electrical engineering”

Three-phase systems

What about a third circuit?



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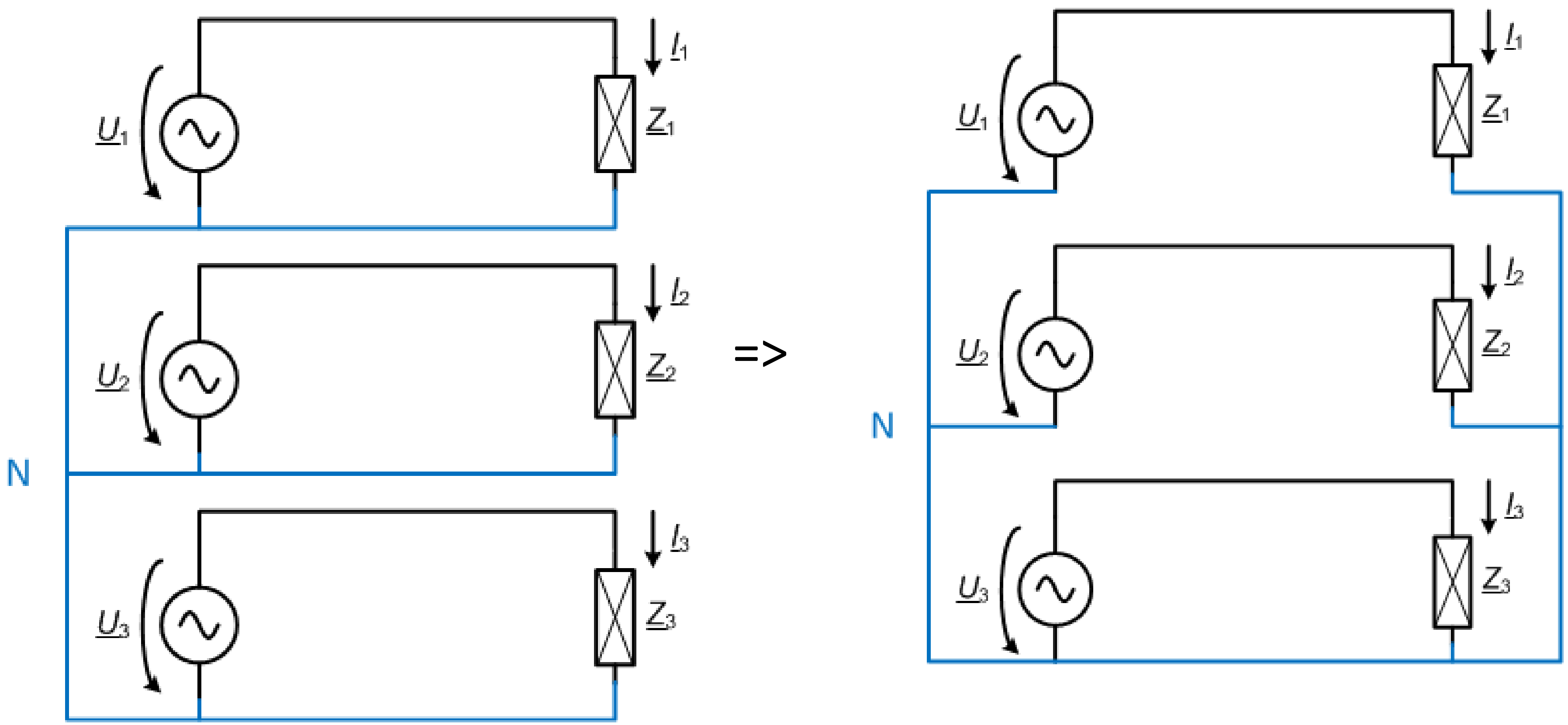
Three-phase systems

Three-phase system. The common point is called neutral.



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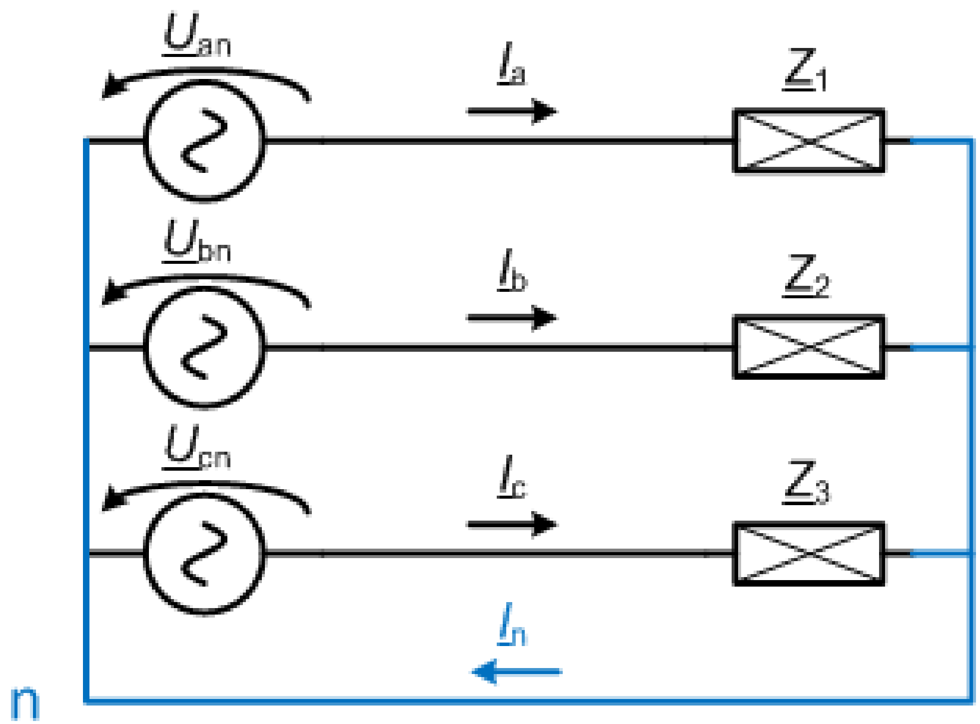
Three-phase systems

Typically, a three-phase system is created by a three-phase generator and not by three different generators.



$$\underline{I}_n = \underline{I}_a + \underline{I}_b + \underline{I}_c$$

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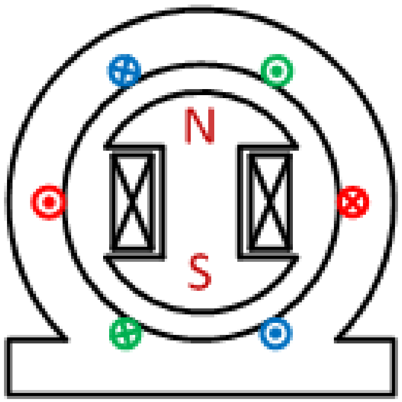
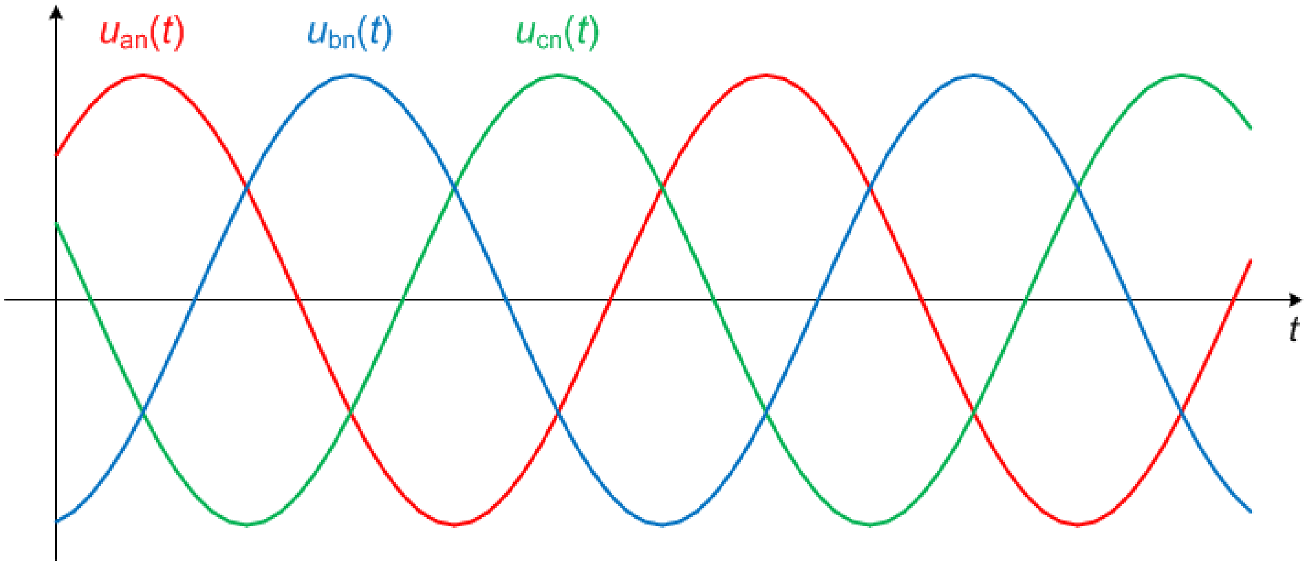
Three-phase systems

- Three-phase generator





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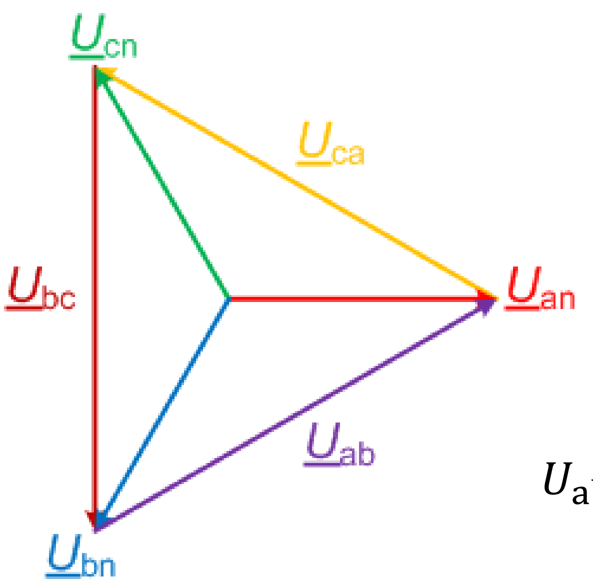
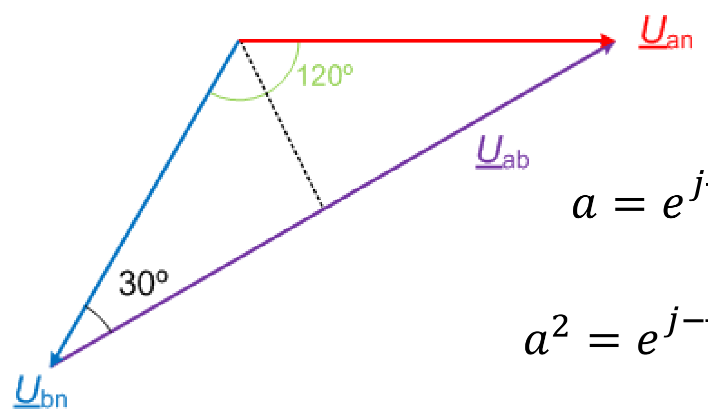
Three-phase systems

The generator provides a balanced three phase system (the phase-to-neutral voltages are equal).





$$a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$a^2 = e^{j-\frac{2\pi}{3}} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$U_{\mathrm{ab}} = \left|\underline{U}_{\mathrm{ab}}\right| = 2 \cdot \left|\underline{U}_{\mathrm{an}}\right| \cdot \cos 30º = 2 \cdot U_{\mathrm{an}} \cdot \frac{\sqrt{3}}{2} = \sqrt{3} \cdot U_{\mathrm{an}}$$

$$\text{Line voltage} = \sqrt{3}\ \text{Simple voltage}$$

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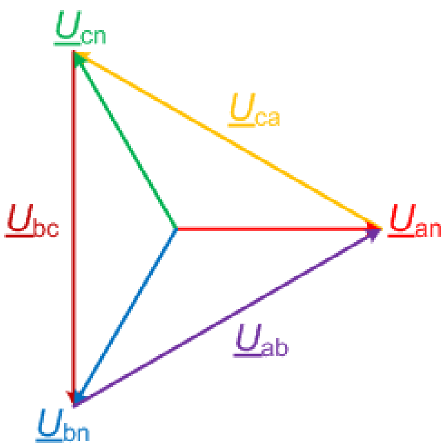
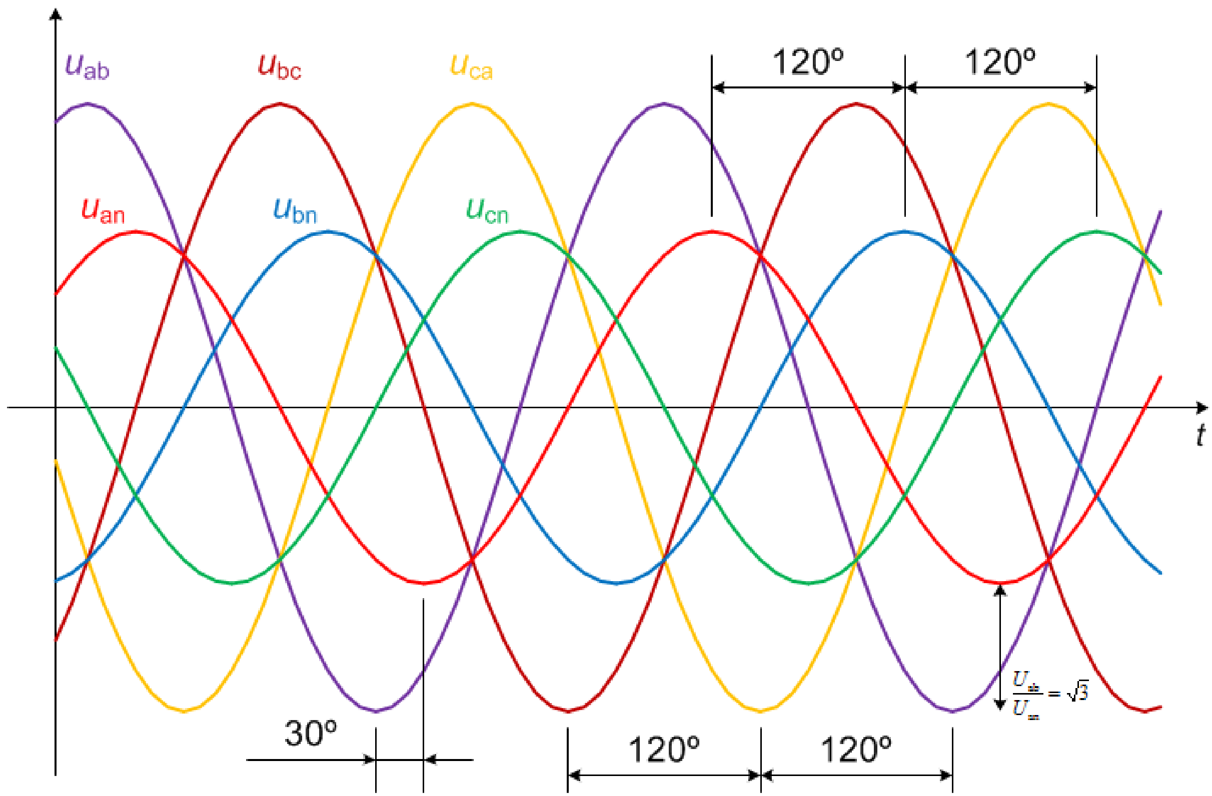
Three-phase systems



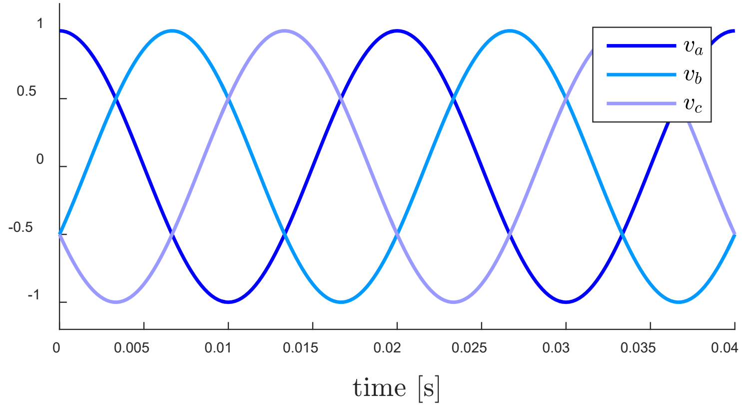


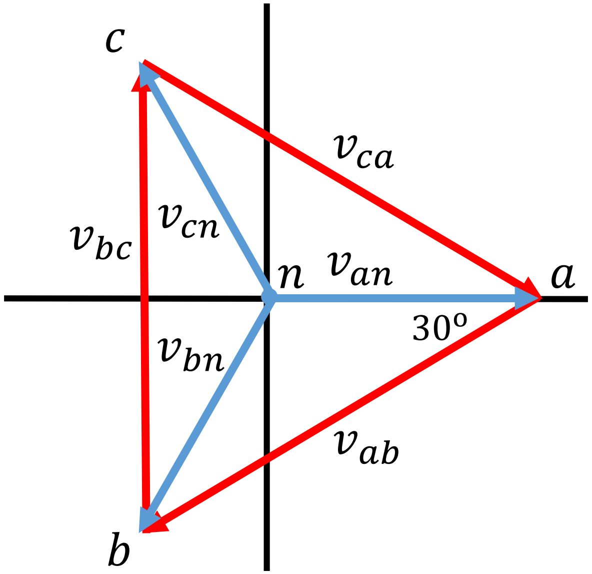
The nominal voltage of a three-phase system is the line voltage!

Three-phase systems

Voltages

- Then, we can write the following relations





$$v_{an} = \sqrt{2}\,V\cos(\omega t)$$

$$v_{bn} = \sqrt{2}\,V\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{cn} = \sqrt{2}\,V\cos(\omega t + \frac{2\pi}{3})$$

$\Leftrightarrow$

$$\underline{V}_{an} = V\angle 0^o$$

$$\underline{V}_{bn} = V\angle -120^o$$

$$\underline{V}_{cn} = V\angle +120^o$$

Activity IV

Time domain and space representation of three-phase systems

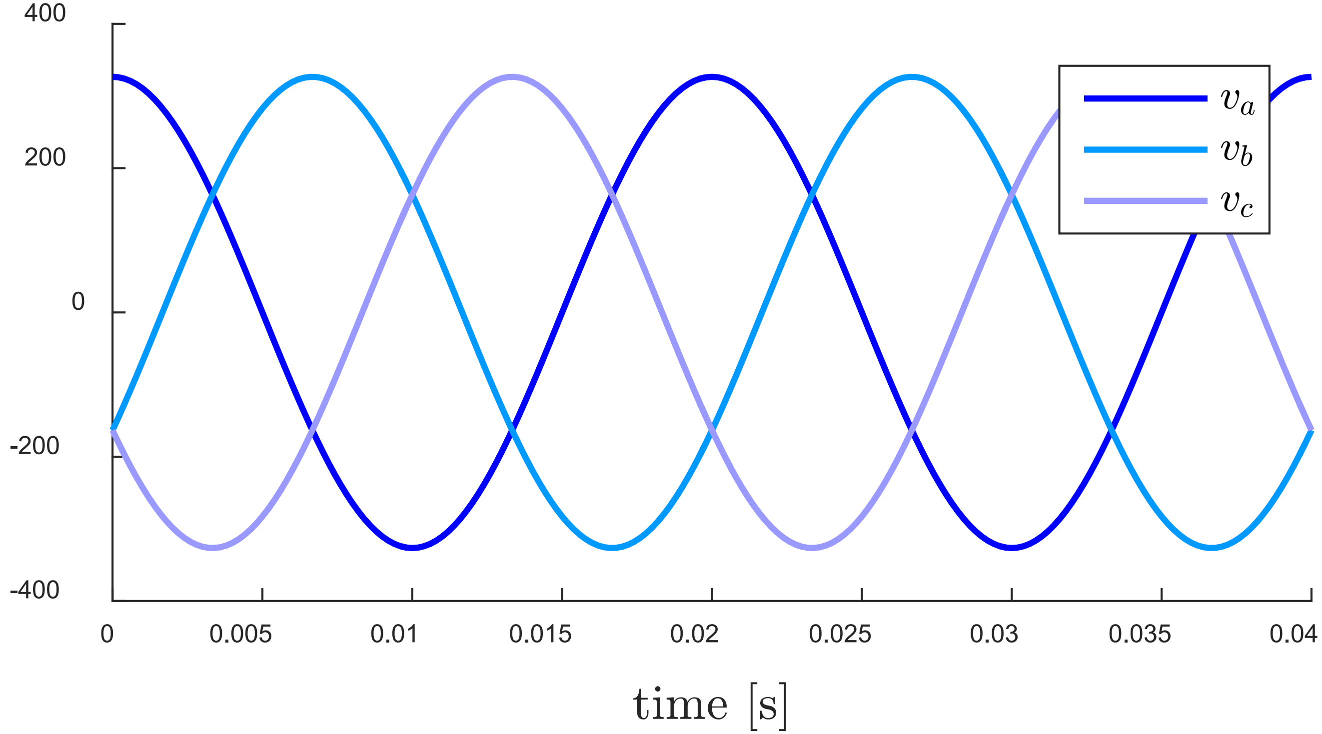
1. Create in a Matlab script a three-phase system with the following characteristics: nominal voltage: 400 V (phase-to-phase), nominal frequency: 50 Hz.
   - Draw it in the time domain (40 ms)
   - Change the following parameters and replot the drawings:
     - Frequency
     - Amplitude of the phases
     - Amplitude of one phase
     - Angle of one phase

2. Create a three-phase system in Simulink
   - Apply the same changes in the time domain.
   - Connect the voltage sources to a load and calculate the power for a balanced case.

Activity IV

Time domain and space representation of three-phase systems

Expected output



Activity IV

Time domain and space representation of three-phase systems

1. Useful tips for scripting
    1. Write the script in a *.m file
    2. Write close all, clear all, clc to:
        1. Close all figures
        2. Clear all variables
        3. Clear the command window
    3. Code selection + F9 – run selected code
    4. Use two %% to split the code by sections. Run each section with control+Enter

Activity IV

Backup code

```
%% Clear variables&workspace and close windows
clear all; clc; close all;

%% Voltages definition
w = 2*pi*50;
t = 0:0.04/200:0.04;

va = cos(w*t)%+0.1*cos(3*w*t)+0.2*cos(w*t);
vb = cos(w*t - 2*pi/3)%+0.1*cos(3*w*t)+0.2*cos(w*t + 2*pi/3);
vc = cos(w*t + 2*pi/3)%+0.1*cos(3*w*t)+0.2*cos(w*t - 2*pi/3);

%% Plot abc functions
f1 = figure(1);
set(f1, 'Position', [50 500 500 250]);
hold on;
plot(t,va,'LineWidth',1.5,'Color',[0 0 1]);
plot(t,vb,'LineWidth',1.5,'Color',[0 0.6 1]);
plot(t,vc,'LineWidth',1.5,'Color',[0.6 0.6 1]);
l1 = legend('$v_a$','$v_b$','$v_c$');
set(l1,'Interpreter','latex','FontSize',10,'Location','NorthEast');
xlabel('time [s]','Interpreter','latex','FontSize',12);
```

Interesting websites

Links

- http://stevenblair.github.io/pq/
- http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html