Determinant Static: Point Load - Free

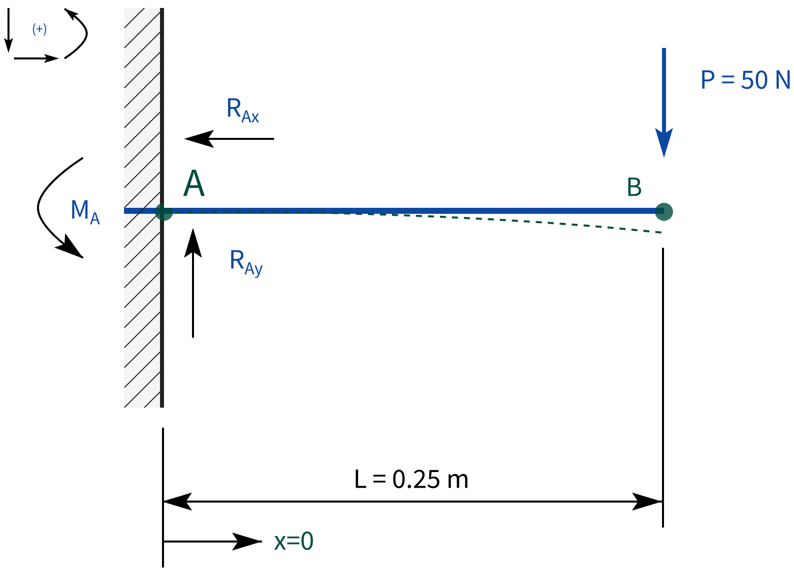
Table Lookup Method

Statically determinate

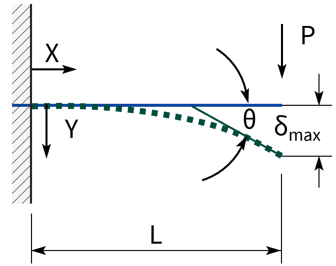
A: Fixed End

B: Point Load
B: Free End (No Support)

System Diagram



Sign/Convention



Lookup Table

$$\theta = \frac{PL^2}{2EI}$$

$$\delta_{max} = \frac{PL^3}{3EI}$$

$$y = \frac{Px^2}{6EI}(3L - x)$$

Equilibrium

$$\sum F_x = 0, \qquad R_{Ax} = 0$$

$$\sum F_y = 0, \qquad R_{Ay} - P = 0 \quad \Rightarrow \quad R_{Ay} = P = 50\,\text{N (Downward)}$$

$$\sum M_A = 0, \qquad M_A - (P \cdot L) = 0 \quad \Rightarrow \quad M_A = P \cdot L$$

$$F = 50 \text{ N}, \quad L_1 = 25 \text{ cm}, \quad L_2 = 5 \text{ cm}$$

$$R_{Ay} = 50\,\text{N}$$

$$M_A = (50\,\text{N}) \cdot (0.25\,\text{m}) = 12.5\,\text{N} \cdot \text{m} \quad \text{(Counter-Clockwise)}$$

Maximum Deflection (δ_{max})

$$\delta_{max} = \frac{(50\,\text{N})(0.25\,\text{m})^3}{3EI} = \frac{50 \cdot (0.015625)}{3EI} = \frac{0.78125}{3EI}\,\text{m}$$

Maximum Slope (θ_{max})

$$\theta_{max} = \frac{(50\,\text{N})(0.25\,\text{m})^2}{2EI} = \frac{50 \cdot (0.0625)}{2EI} = \frac{3.125}{2EI}\,\text{radian}$$

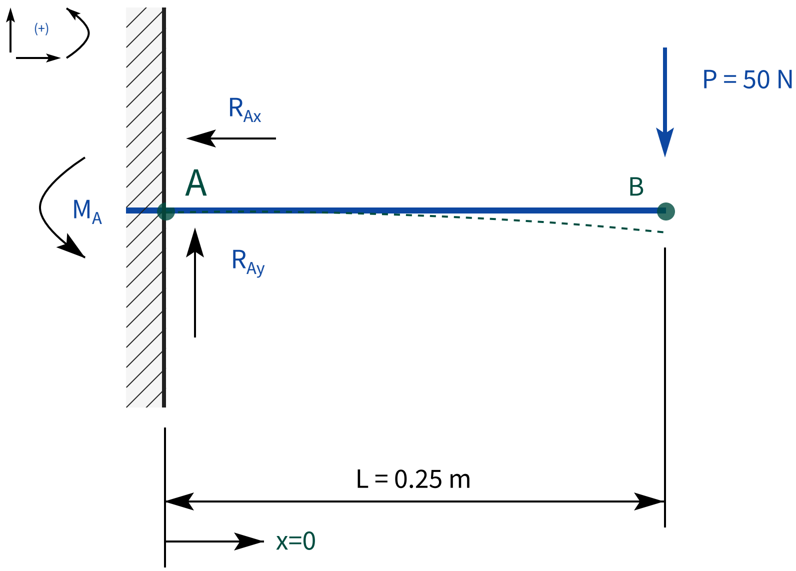
Determinant Static: Point Load - Free

Double Integration Method

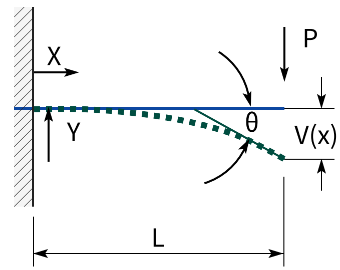
Statically determinate

A: Fixed End   B: Point Load
B: Free End (No Support)

System Diagram



Sign/Convention



Governing Differential Equation

$$EI\frac{d^2v}{dx^2} = M(x)$$

Boundary Conditions (BC)

Deflection is zero

$$v(0) = 0$$

Slope is zero

$$\frac{dv}{dx}(0) = 0$$

Moment Equation

$$M(x) = -P(L-x) = -PL + Px$$

First Integration (Slope Equation)

Substitute M(x)

$$EI\frac{d^2v}{dx^2} = -PL + Px$$

$$\int EI\frac{d^2v}{dx^2}dx = \int(-PL+Px)dx$$

$$EI\frac{dv}{dx} = -PLx + \frac{Px^2}{2} + C_1$$

Apply BC_A

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = 0$$

$$EI(0) = -PL(0) + \frac{P(0)^2}{2} + C_1$$

$$\implies \quad C_1 = 0$$

Slope Equation

$$EI\frac{dv}{dx} = -PLx + \frac{Px^2}{2}$$

Second Integration (Deflection Equation)

Integrate Slope Equation

$$\int EI\frac{dv}{dx}dx = \int\left(-PLx + \frac{Px^2}{2}\right)dx$$

$$EIv(x) = -\frac{PLx^2}{2} + \frac{Px^3}{6} + C_2$$

Apply BC_A

$$v = 0 \quad \text{at} \quad \text{x=0}$$

$$EI(0) = -\frac{PL(0)^2}{2} + \frac{P(0)^3}{6} + C_2$$

$$\implies \quad C_2 = 0$$

Deflection Curve Equation

$$v(x) = \frac{1}{EI}\left(-\frac{PLx^2}{2} + \frac{Px^3}{6}\right)$$

$$= \frac{PLx^2}{6EI}(x - 3L)$$

Find the Maximum Deflection

Free End (B), where x=L

$$v_{\max} = v(L) = \frac{1}{EI}\left(-\frac{PL(L)^2}{2} + \frac{P(L)^3}{6}\right)$$

$$v_{\max} = \frac{1}{EI}\left(-\frac{PL^3}{2} + \frac{PL^3}{6}\right)$$

$$= \frac{1}{EI}\left(-\frac{3PL^3}{6} + \frac{PL^3}{6}\right) = \frac{1}{EI} \cdot -\frac{2PL^3}{6}$$

$$v_{\max} = \frac{1}{EI}\left(-\frac{2PL^3}{6}\right) = -\frac{PL^3}{3EI}$$

Final Result

Maximum Slope at (x=L)

$$\boxed{\theta_{max} = \frac{PL^2}{2EI}}$$

$$\theta_{max} = \frac{(50\,\mathrm{N})(0.25\,\mathrm{m})^2}{2EI}$$

$$= \frac{50 \cdot (0.0625)}{2EI}$$

$$= \frac{3.125}{2EI}\,\mathrm{radian}$$

Maximum Deflection at (x=L)

$$\boxed{\Delta_{max} = \frac{PL^3}{3EI}}$$

$$\Delta_{max} = \frac{(50\,\mathrm{N})(0.25\,\mathrm{m})^3}{3EI}$$

$$= \frac{50 \cdot (0.015625)}{3EI}$$

$$= \frac{0.78125}{3EI}\,\mathrm{m}$$

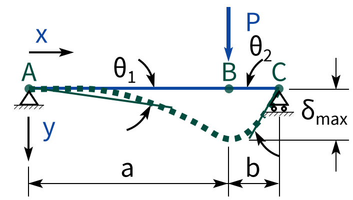
Point Load - Roller

Table Lookup Method

Determinant Static

A: Pinned   B: Point Load
C: Roller   B: No Support

Sign/Convention



Lookup Table

End Slopes (rotation at supports)

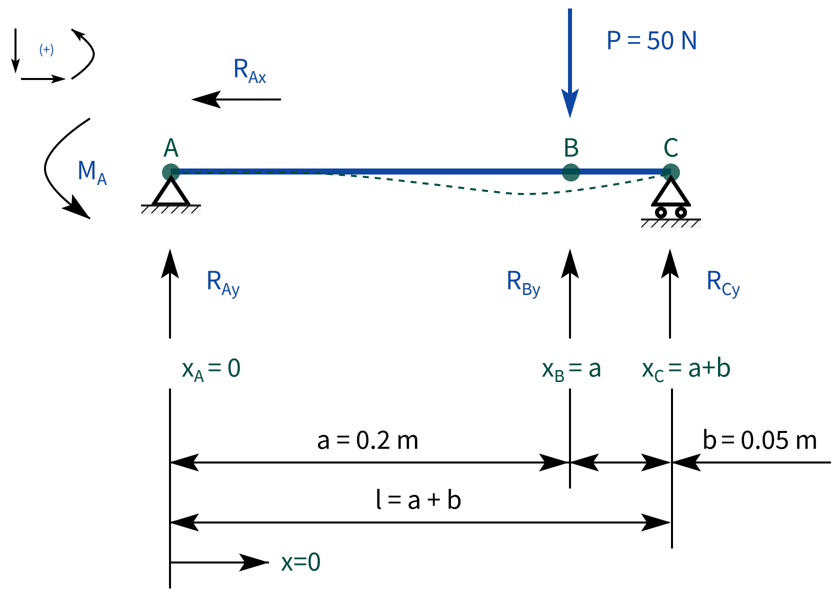
$$\theta_1 \quad \text{at} \quad x = 0,$$

$$\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI},$$

$$\theta_2 \quad \text{at} \quad x = l$$

$$\theta_2 = \frac{Pab(2l - b)}{6lEI}$$

System Diagram



Lookup Table

Deflection y(x) (piecewise)

For $\quad (0 < x < a):$

$$y(x) = \frac{Pbx}{6EIl}\left(l^2 - x^2 - b^2\right)$$

For $\quad (a < x < l):$

$$y(x) = \frac{Pb}{6EIl}\left[\frac{l}{b}(x-a)^3 + (l^2 - b^2), x - x^3\right]$$

Maximum Deflection

$$\delta_{\max} = \frac{Pb\left(l^2 - b^2\right)^{3/2}}{9\sqrt{3}\,l\,EI}$$

located at $\quad x = \sqrt{(l^2 - b^2)}/3$

Simplification at The Center

$$\delta_{\text{center}} = \frac{Pb\,(3l^2 - 4b^2)}{48\,EI}$$

(value at $x = l/2$)

Equilibrium

Sum of Forces

$$\sum F_y = 0 \implies R_A + R_C - P = 0$$

$$\Rightarrow R_A + R_C = P \qquad (1)$$

$$\sum M_A = 0 \implies R_C \cdot l - P \cdot a = 0$$

$$\Rightarrow R_C = \frac{Pa}{l} \qquad (2)$$

Substitutes

$$R_A + \frac{Pa}{l} = P$$

$$\Rightarrow R_A = P - \frac{Pa}{l}$$

$$\Rightarrow R_A = P\left(1 - \frac{a}{l}\right)$$

$$\Rightarrow R_A = \frac{P(l-a)}{l}$$

Considering (l−a=b)

$$\boxed{R_A = \frac{Pb}{l}},$$

$$\boxed{R_C = \frac{Pa}{l}}$$

Finding Result

Support Reactions

$$P = 50\,\text{N}, l = 0.25\,\text{m},$$
$$a = 0.20\,\text{m}, b = 0.05\,\text{m}$$

$$R_A = \frac{P\,b}{l} = \frac{50 \times 0.05}{0.25} = 10\,\text{N},$$

$$R_C = \frac{P\,a}{l} = \frac{50 \times 0.20}{0.25} = 40\,\text{N}.$$

End slopes (θ_{max})

$$\theta_1 = \frac{Pb\,(l^2 - b^2)}{6\,l\,EI} = \underbrace{0.002}_{\text{(rad) per } P/(EI)} \frac{P}{EI},$$

$$\theta_2 = \frac{Pab(2l - b)}{6\,l\,EI} = \underbrace{0.003}_{\text{(rad) per } P/(EI)} \frac{P}{EI}.$$

$$\boxed{\theta_{\max} = \theta_2 = 0.003;\ \frac{P}{EI}\ \text{(radians)}.}$$

Maximum Deflection (δ_{max})

$$\delta_{\max} = \frac{Pb\left(l^2 - b^2\right)^{3/2}}{9\sqrt{3}\,l\,EI}$$

$$= \underbrace{0.0001885618083}_{\text{m per } P/(EI)} \frac{P}{EI}$$

$$= 0.0001885618\,\frac{P}{EI}\ \text{(m)}.$$

$$x_{\delta_{\max}} = \frac{\sqrt{l^2 - b^2}}{3} \approx 0.08165\ \text{m}.$$

Deflection under The Load

$$y(a) = \frac{Pba}{6\,l\,EI}\left(l^2 - a^2 - b^2\right)$$

$$= \underbrace{0.0001333333333}_{\text{m per } P/(EI)} \frac{P}{EI}$$

$$= 0.0001333333\,\frac{P}{EI}\ \text{(m)}$$

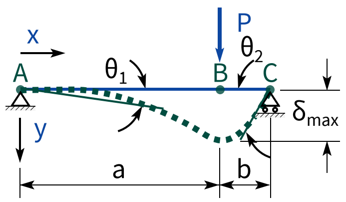
Point Load - Roller

Double Integration Method

Determinant Static

A: Pinned	B: Point Load
C: Roller	B: No Support

Sign/Convention



Governing Differential Equation

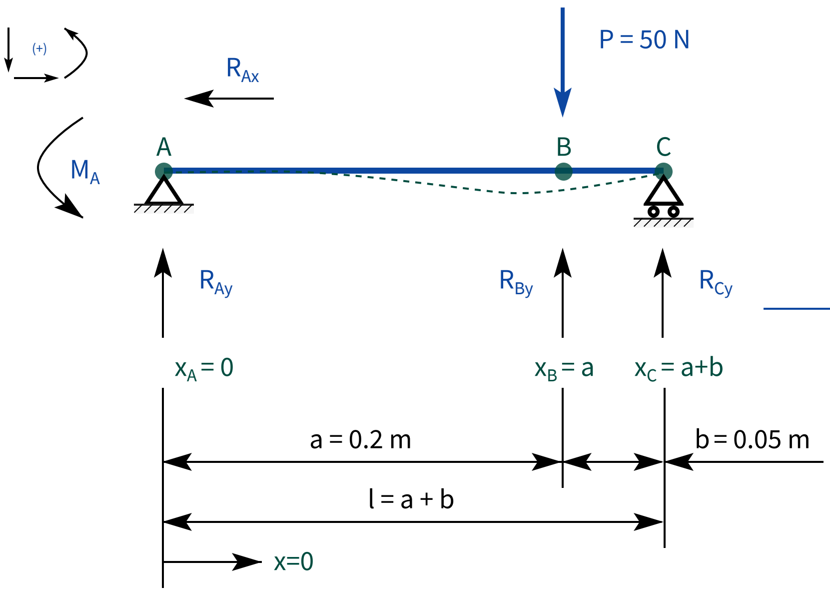
$$EI\,y''(x) = M(x)$$

Boundary Conditions (BC)

Deflection is zero

$$y(0) = 0 \quad y(l) = 0$$

System Diagram



Static Reactions

$$\sum F_y = 0 : R_A + R_C - P = 0,$$
$$\sum M_A = 0 : R_C\,l - Pa = 0.$$

where $\quad (b = l - a)$

Moment M(x) (piecewise)

$$R_A = \frac{Pb}{l}, \quad R_C = \frac{Pa}{l}$$

$$M(x) = \begin{cases} R_A\,x, & 0 \le x < a, \\ R_A\,x - P(x-a), & a \le x \le l. \end{cases}$$

Region	Range	Bending Moment M(x)
Left (1)	0 ≤ x ≤ a	$M_1 = R_A \cdot x$
Right (2)	a ≤ x ≤ l	$M_2 = R_A \cdot x - P \cdot (x-a)$

Double Integration (left segment 0<x<a)

$$y(0) = 0, \quad 0 < x < a \quad \Rightarrow C_2 = 0$$

$$EI\,y''(x) = R_A x.$$

$$EI\,y'(x) = \frac{R_A x^2}{2} + C_1$$
$$y_L'(x) = \frac{R_A x^2}{2EI} + \frac{C_1}{EI}$$

$$EI\,y(x) = \frac{R_A x^3}{6} + C_1 x + C_2$$
$$y_L(x) = \frac{R_A x^3}{6EI} + \frac{C_1}{EI}x$$

Boundary and Continuity Conditions

$$\begin{aligned} \text{At } A(x=0): &\quad y_1 = 0 \Rightarrow C_2 = 0, \\ \text{At } C(x=l): &\quad y_2 = 0, \\ \text{At } x=a: &\quad y_1(a) = y_2(a), \\ &\quad y_1'(a) = y_2'(a). \end{aligned}$$

Double Integration (right segment a<x<l)

For $(a < x < l)$

$$EI\,y''(x) = R_A x - P(x-a)$$

$$EI\,y'(x) = \frac{R_A x^2}{2} - \frac{Px^2}{2} + Pax + C_3$$
$$= \frac{(R_A - P)x^2}{2} + Pax + C_3$$
$$EI\,y(x) = \frac{(R_A - P)x^3}{6} + \frac{Pax^2}{2} + C_3 x + C_4$$
$$y_R(x) = \frac{(R_A - P)x^3}{6EI} + \frac{Pax^2}{2EI} + \frac{C_3}{EI}x + \frac{C_4}{EI}$$

Determine Integration Constants C_1, C_3, C_4

Deflection is zero

$$y_L(a) = y_R(a)$$

Slope is zero

$$y_L'(a) = y_R'(a)$$

Slope is zero

$$y_R(l) = 0$$

$$C_1 = -\frac{Pal}{3} - \frac{Pa^3}{6l} + \frac{Pa^2}{2},$$

$$C_3 = -\frac{Pal}{3} - \frac{Pa^3}{6l},$$

$$C_4 = C_2 + \frac{Pa^3}{6}.$$

Apply Continuity of Slope at (x=a)

$$EI\,y_1'(a) = \frac{R_A a^2}{2} + C_1,$$
$$EI\,y_2'(a) = \frac{(R_A - P)a^2}{2} + Pa^2 + C_3.$$

$$C_1 = C_3 + \tfrac{Pa^2}{2}$$

Apply Continuity of Deflection at (x=a)

$$EI\,y_1(x) = \frac{R_A x^3}{6} + C_1 x + C_2,$$
$$EI\,y_2(x) = \frac{(R_A - P)x^3}{6} + \frac{Pax^2}{2} + C_3 x + C_4.$$

Apply Continuity of Deflection at (x=a)

$$EI\,y_1(a) = EI\,y_2(a)$$
$$\frac{R_A a^3}{6} + C_1 a + C_2 = \frac{(R_A - P)a^3}{6} + \frac{Pa^3}{2} + C_3 a + C_4$$
$$\frac{R_A a^3}{6} + C_1 a + C_2 = \frac{R_A a^3}{6} - \frac{Pa^3}{6} + \frac{Pa^3}{2} + C_3 a + C_4$$
$$-\frac{Pa^3}{6} + \frac{Pa^3}{2} = +\frac{Pa^3}{3}$$
$$\frac{R_A a^3}{6} + C_1 a + C_2 = \frac{R_A a^3}{6} + \frac{Pa^3}{3} + C_3 a + C_4$$
$$C_1 a + C_2 = C_3 a + C_4 + \frac{Pa^3}{3}.$$

Substitute $C_1 = C_3 + \frac{Pa^2}{2}$

$$(C_3 + \frac{Pa^2}{2})a + C_2 = C_3 a + C_4 + \frac{Pa^3}{3}$$
$$C_3 a + \frac{Pa^3}{2} + C_2 = C_3 a + C_4 + \frac{Pa^3}{3}$$
$$\frac{Pa^3}{2} + C_2 = C_4 + \frac{Pa^3}{3}$$

$$C_4 = C_2 + \tfrac{Pa^3}{2} - \tfrac{Pa^3}{3} = C_2 + \tfrac{Pa^3}{6}$$

Deflection at Left Support (x=0)

$$EI\,y_1(x) = \frac{R_A x^3}{6} + C_1 x + C_2$$
$$EI\,y_1(0) = 0 = 0 + 0 + C_2 \implies C_2 = 0$$
$$C_4 = C_2 + \frac{Pa^3}{6} = \frac{Pa^3}{6}$$

Deflection at Right Support (x=l)

$$EI, y_2(x) = \frac{(R_A - P)x^3}{6} + \frac{Pax^2}{2} + C_3 x + C_4$$
$$0 = \frac{(R_A - P)l^3}{6} + \frac{Pal^2}{2} + C_3 l + C_4$$
$$0 = \frac{(\frac{Pb}{l} - P)l^3}{6} + \frac{Pal^2}{2} + C_3 l + \frac{Pa^3}{6}$$
$$(\tfrac{Pb}{l} - P)l^3 = P(b-l)l^2 = -Pal^2$$
$$0 = \frac{-Pal^2}{6} + \frac{Pal^2}{2} + C_3 l + \frac{Pa^3}{6}$$
$$\frac{-1}{6} + \frac{1}{2} = \frac{1}{3}$$
$$0 = \frac{Pal^2}{3} + C_3 l + \frac{Pa^3}{6}$$
$$C_3 = -\frac{Pal^2}{3l} - \frac{Pa^3}{6l} = -\frac{Pa}{l}\left(\frac{l^2}{3} + \frac{a^2}{6}\right)$$
$$C_3 = -\frac{Pa}{6l}(2l^2 + a^2)$$

Back-Substitute

$$C_1 = C_3 + \frac{Pa^2}{2}$$
$$= -\frac{Pa}{6l}(2l^2 + a^2) + \frac{Pa^2}{2}$$
$$= -\frac{Pa}{6l}(2l^2 + a^2 - 3al),$$
$$C_2 = 0,$$
$$C_3 = -\frac{Pa}{6l}(2l^2 + a^2),$$
$$C_4 = \frac{Pa^3}{6}.$$

Left Segment: Substitution

$$EI\, y_L(x) = \frac{R_A x^3}{6} + C_1 x + C_2$$

$$C_1 = -\frac{Pal}{3} - \frac{Pa^3}{6l} + \frac{Pa^2}{2}$$

$$EI\, y_L(x) = \frac{Pb}{l} \cdot \frac{x^3}{6} + \left(-\frac{Pal}{3} - \frac{Pa^3}{6l} + \frac{Pa^2}{2}\right) x$$

$$= \frac{P}{6l}\left(bx^3 + (-2al^2 - a^3 + 3a^2 l)\frac{x}{1}\right)$$

$$b = l - a$$

$$EI, y_L(x) = \frac{Pbx}{6l}\left(2al - a^2 - x^2\right)$$

$$b^2 = (l-a)^2 = l^2 - 2al + a^2$$

$$2al - a^2 - x^2 = l^2 - x^2 - b^2$$

$$\boxed{y_L(x) = -\frac{Pbx}{6EI\, l}\left(l^2 - x^2 - b^2\right)}$$

Deflection at Right Support (x=l)

$$y_L(x) = \frac{Pbx}{6EIl}\left(l^2 - x^2 - b^2\right)$$

$$K = \frac{Pb}{6EI\, l}$$

$$y_L(x) = K\, x\, (l^2 - x^2 - b^2)$$

$$y_L'(x) = K\left(l^2 - x^2 - b^2 + x(-2x)\right)$$

$$= K\left(l^2 - b^2 - 3x^2\right)$$

$$y_L'(x) = 0$$

$$l^2 - b^2 - 3x^2 = 0$$

$$\implies \quad x^2 = \frac{l^2 - b^2}{3}$$

$$\boxed{x_{\max} = \frac{\sqrt{l^2 - b^2,}}{\sqrt{3}}}$$

Maximum Deflection

$$\delta_{max} \equiv y_L(x_{max}) = K x_{max} \frac{2}{3}(l^2 - b^2)$$

$$= \frac{Pb}{6EI, l} \cdot \frac{\sqrt{l^2 - b^2}}{\sqrt{3}} \cdot \frac{2}{3}(l^2 - b^2)$$

$$= \frac{Pb\,(l^2 - b^2)^{3/2}}{6EI\, l} \cdot \frac{2}{3\sqrt{3}}$$

$$= \frac{Pb\,(l^2 - b^2)^{3/2}}{9\sqrt{3}\, EI, l}$$

$$\boxed{\delta_{\max} = \frac{Pb\,(l^2 - b^2)^{3/2}}{9\sqrt{3}\, EI\, l},}$$

Left end slope θ_1 (at x=0)

$$E\, y_L'(x) = \frac{R_A x^2}{2} + C_1$$

$$EI\, \theta_1 = EI, y_L'(0) = C_1$$

$$EI\, y_L'(x) = \frac{Pb}{6l}\left(l^2 - b^2 - 3x^2\right)$$

$$EI\, \theta_1 = \frac{Pb}{6l}\left(l^2 - b^2\right)$$

$$\boxed{\theta_1 = \frac{Pb\,(l^2 - b^2)}{6lEI}}$$

Right end slope θ2 (at x=l)

$$EI\, y_2'(x) = \frac{(R_A - P)x^2}{2} + Pax + C_3$$

$$\boxed{\theta_2 = \frac{Pab(2l - b)}{6lEI}}$$

Deflection (closed form)

For (0<x<a)

$$y(x) = \frac{Pb,x}{6EI\, l}\left(l^2 - x^2 - b^2\right)$$

For (a<x<l)

$$y(x) = \frac{Pb}{6EI\ l}\left[\frac{l}{b}(x-a)^3 + (l^2 - b^2)x - x^3\right]$$

Maximum Deflection (location and value)

$$y'(x) = 0 \quad \text{in the interval} \quad (0 < x < l)$$

$$x_{max} = \frac{\sqrt{l^2 - b^2}}{\sqrt{3}},$$

$$\delta_{max} = y(x_{max}) = \frac{Pb(l^2 - b^2)^{3/2}}{9\sqrt{3}\, l\, EI}$$

End slopes (closed form)

$$\theta_1 = y'(0) = \frac{Pb(l^2 - b^2)}{6\, l\, EI},$$

$$\theta_2 = y'(l) = \frac{Pab(2l - b)}{6\, l\, EI}$$

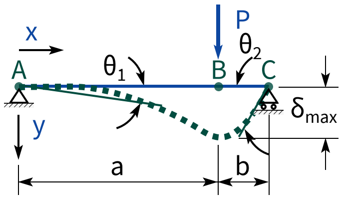
Point Load - Roller

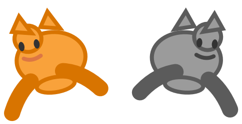
Segment by Segment Integration Method

Determinant Static

A: Pinned   B: Point Load
C: Roller   B: No Support

Sign/Convention





Setup / knowns

Reactions from statics

$$R_A = \frac{Pb}{l}, \qquad R_C = \frac{Pa}{l}$$

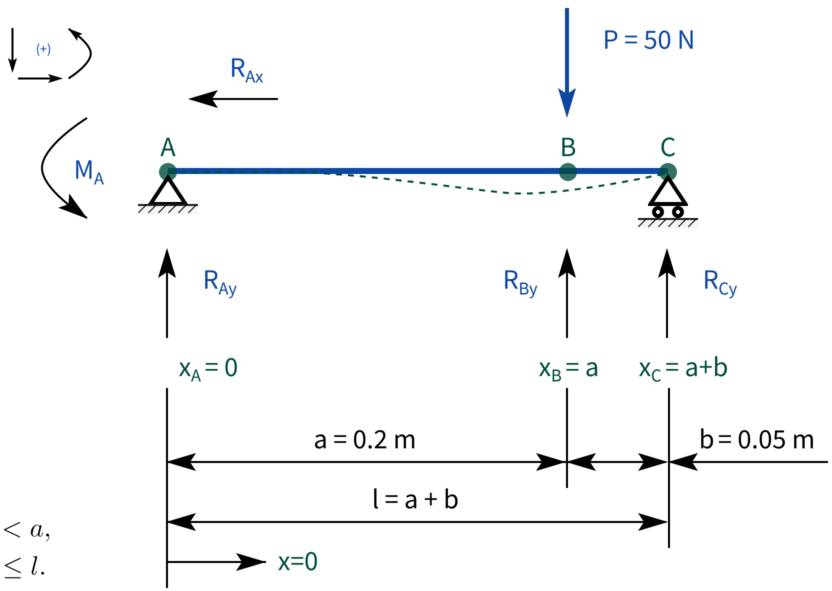
Governing Differential Equation

$$EI\,y''(x) = M(x)$$

Piecewise moment

$$M(x) = \begin{cases} R_A x, & 0 \le x < a, \\ R_A x - P(x-a), & a \le x \le l. \end{cases}$$

System Diagram



Left Segment (0 ≤ x ≤ a)

Integration

$$EI\,y_1''(x) = R_A x$$

$$EI\,y_1'(x) = \frac{R_A x^2}{2} + C_1$$

$$EI\,y_1(x) = \frac{R_A x^3}{6} + C_1 x + C_2$$

Boundary Conditions (BC)

$$x = 0 \Rightarrow y_1(0) = 0, \quad \text{gives} \quad C_2 = 0$$

$$\boxed{EI\,y_1(x) = \frac{R_A x^3}{6} + C_1 x \qquad 0 \le x \le a}$$

Evaluate at (x=a)

$$x = 0 \Rightarrow y_1(0) = 0, \quad \text{gives} \quad C_2 = 0$$

$$EI\,\theta_a^- = EI\,y_1'(a) = \frac{R_A a^2}{2} + C_1,$$

$$EI\,y_a^- = EI\,y_1(a) = \frac{R_A a^3}{6} + C_1 a.$$

Right Segment (a ≤ x ≤ l)

Integration

$$EI\,y_2''(x) = R_A x - P(x-a) = (R_A - P)x + Pa$$

$$EI\,y_2'(x) = \frac{(R_A - P)x^2}{2} + Pax + C_3$$

$$EI\,y_2(x) = \frac{(R_A - P)x^3}{6} + \frac{Pax^2}{2} + C_3 x + C_4$$

Boundary Conditions (BC)

$$x = a \Rightarrow y_2(a) = y_1(a), \quad y_2'(a) = y_1'(a)$$

$$EI\,y_2'(a) = \frac{(R_A - P)a^2}{2} + Pa^2 + C_3,$$

$$EI\,y_2(a) = \frac{(R_A - P)a^3}{6} + \frac{Pa^3}{2} + C_3 a + C_4.$$

Two Equations from Continuity

Equality of slopes at (x=a)

$$\frac{R_A a^2}{2} + C_1 = \frac{(R_A - P)a^2}{2} + Pa^2 + C_3$$

$$C_1 - C_3 = \frac{Pa^2}{2} \qquad \Longrightarrow \qquad \boxed{C_3 = C_1 - \frac{Pa^2}{2}}$$

Equality of deflections at (x=a)

$$\frac{R_A a^3}{6} + C_1 a = \frac{(R_A - P)a^3}{6} + \frac{Pa^3}{2} + C_3 a + C_4.$$

$$C_1 a - C_3 a - C_4 = \frac{Pa^3}{3}$$

$$C_1 a - (C_1 - \tfrac{Pa^2}{2})a - C_4 = \frac{Pa^3}{3}$$

$$\frac{Pa^3}{2} - C_4 = \frac{Pa^3}{3}$$

$$\Rightarrow \boxed{C_4 = \frac{Pa^3}{6}}$$

Apply Right-Support Boundary

$$y_2(l) = 0 \quad \text{to solve for} \quad C_1$$

$$0 = \frac{(R_A - P)l^3}{6} + \frac{Pal^2}{2} + C_3 l + C_4$$
$$0 = \frac{(R_A - P)l^3}{6} + \frac{Pal^2}{2} + \left(C_1 - \frac{Pa^2}{2}\right)l + \frac{Pa^3}{6}$$
$$C_1 l = -\frac{(R_A - P)l^3}{6} - \frac{Pal^2}{2} + \frac{Pa^2}{2} l - \frac{Pa^3}{6}$$

$$(R_A - P)l^3 = \left(\frac{Pb}{l} - P\right)l^3 = P(b-l)l^2 = -Pal^2$$

$$C_1 l = -\frac{(-Pal^2)}{6} - \frac{Pal^2}{2} + \frac{Pa^2 l}{2} - \frac{Pa^3}{6}$$
$$C_1 l = \frac{Pal^2}{6} - \frac{Pal^2}{2} + \frac{Pa^2 l}{2} - \frac{Pa^3}{6}$$
$$C_1 l = -\frac{Pal^2}{3} + \frac{Pa^2 l}{2} - \frac{Pa^3}{6}.$$

$$\boxed{C_1 = \frac{Pa}{6l}\left(-2l^2 + 3al - a^2\right)}$$

Back-Substitute to Find (C_3, C_4)

$$\boxed{C_3 = C_1 - \frac{Pa^2}{2} = -\frac{Pa}{6l}\left(2l^2 + a^2\right)}$$

$$\boxed{C_4 = \frac{Pa^3}{6}}$$

Final Piecewise Deflection Functions

Left Segment ($0 \le x \le a$)

$$EI\, y_1(x) = \frac{R_A x^3}{6} + C_1 x$$

$$\Longrightarrow \quad \boxed{y_1(x) = \frac{Pb\,x}{6EI\,l}\left(l^2 - x^2 - b^2\right)}$$

Right Segment ($a \le x \le l$)

$$EI\, y_2(x) = \frac{(R_A - P)x^3}{6} + \frac{Pax^2}{2} + C_3 x + C_4$$
$$\boxed{y_2(x) = \frac{Pb}{6EI\,l}\left[\frac{l}{b}(x-a)^3 + \left(l^2 - b^2\right)x - x^3\right]}$$

Stationary Point

$$y_1(x) = K\,x(l^2 - x^2 - b^2), \quad K = \tfrac{Pb}{6EI\,l}$$

$$\text{Set} \quad y_1'(x) = 0$$
$$l^2 - b^2 - 3x^2 = 0$$

$$\Rightarrow \quad x_{\max} = \frac{\sqrt{l^2 - b^2,}}{\sqrt{3}}$$

Maximum Deflection

$$\boxed{\delta_{\max} = \frac{Pb\,(l^2 - b^2)^{3/2}}{9\sqrt{3}\,EI\,l}}$$