NATE STEMEN & KEVIN YEH

RUDIN: TRANSLATED

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Todo list

0 Introduction

This project is founded in the idea that mathematics, and resources for learning mathematics, should be available to all. While Walter Rudin' *Principles of Mathematical Analysis* is a beautiful book, there are many points in the book that may be unclear, especially to individuals first leanring analysis. It if of course, *extremely* important when learning mathematics, or any subject for that matter, to struggle through material. That said, the point at which struggle is helpful, is different for each individual. We find the level of struggle expected by Rudin, to be somewhat higher than is helpful for most new students to analysis, and also just for mathematics.

We hope that this book can serve as a great analysis book, as Rudin has written, but with a slighlty lower level of struggle expected. We hope we can provide more intuition and maybe some more figures than Rudin does.

talk more about who we expect to read this and what its main purpose is. a first book for analysis? a book to read WITH Rudin?

1

The Real and Complex Number System

Introduction

Before we can begin to talk about what analysis is about, we must have a clear notion of the objects that we want to study. Mathematics started with the study of numbers and that's what we shall start with here. Just like driving a car does not require knowledge of how an engine works, we will not enter into a discussion of all properties of numbers(an interested reader might look to for more information). We must take some things for granted if we wish to get off the runway. We will assume you to be familiar with rational, or decimal numbers such as

more rigorous definition of numbers reference

$$1, \qquad 2.56, \qquad \frac{1}{3}, \qquad 0.347 \tag{1.1}$$

This set of numbers is great for many purposes, including almost everything in our day to day life, but we can show it isn't quite satisfactory for all purposes. Take, for example, the following triangle.

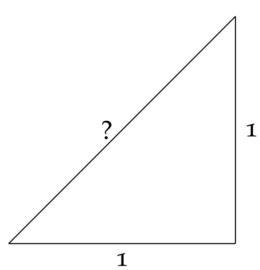

Figure 1.1: Simple Right Triangle with Side Length 1

Suppose we want to figure out how long the side is labelled by "?". Well, if we use the Pythagorean Theorem then we can write the following equation. We use the letter x to denote the length of the hypotenuse that we are interested in.

This simply says that if we have a right triangle with sides of length a and b, then the hypotenuse c is related to those numbers by $c^2 = a^2 + b^2$.

$$\begin{aligned} x^2 &= 1^2 + 1^2 \\ x^2 &= 2 \end{aligned} \tag{1.2}$$

If the rational numbers are "enough", then we should be able to write x as $\frac{n}{m}$ where n and m are integers $(\{\dots, -2, -1, 0, 1, 2, \dots\})$. However, as we will see in Example 1.1, this may not be the case.

meaning they represent everything we can construct geometrically

say rational numbers are fractions before this part

Example 1.1 (Existence of Gaps)**.** In this example we will set out to show that Equation 1.2 has no solutions that are rational. To show this we will employ a method known as proof by contradiction. We will assume that x can be written as a fraction.

$$x = \frac{n}{m} \tag{1.3}$$

We take the fraction to be as reduced as possible and hence n and m are *not both* even. We can plug this into Equation 1.2 to obtain

meaning we wouldn't have something like $\frac{5}{15}$ because that can be reduced further to $\frac{1}{3}$

$$x^2 = \frac{n^2}{m^2} = 2 \overset{\text{multiply by } m^2}{\Longrightarrow} n^2 = 2m^2$$

Ordered Sets

Fields

The Real Field

The Extended Real Number System

The Complex Field

Euclidean Spaces

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Exercises

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Basic Topology

Finite, Countable, and Uncountable Sets

Armed with a well-constructed real number system, we can begin to talk about some mathematical constructions. One of the simplest constructions (operations?) on the real numbers is the object called a function. A function is simply a gadget that takes an input number and outputs a number that is determined based on some rule associated to that specific function.

However, note that the notion of a function is not constrained to operating numbers. It can make as much sense to talk about a function on an arbitrary set, and the "outputs" need not even resemble the inputs in any shape or form. For example, one can define a function on the set of names of people in a class, which outputs the birthday of the person in binary.

It this case, we would call the set of names the domain of the function, and

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