Rudin: Translated

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Contents

Todo list

Preface

This project is founded on the glorious book by Walter Rudin; *Principle of Mathematical Analysis*[1].

This book is intended for...

who is this book intended for?

The prerequisites for this book are

what are the real prereqs? just some basic set theory?

Chapter 1

The Real and Complex Number System

1.1 Introduction

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1.2 Ordered Sets

Often when we talk about collections of things (people, cars, dogs, etc.) we talk about how they compare to each other (height, top speed, cuteness respectively). *But*, we can only do this because we have a way in which these objects relate to each other. My dog is *of course* cuter than yours, so I might say my dog is better than yours. Symbolically, I might write this as

$$\text{your dog} < \text{my dog}$$

where "$<$" can be read as "is less cute than" in this particular scenario, but in others, it might mean "has a lower top speed" or really anything else you can think of. We could even take all dogs and compare them in lots of different ways, such as by weight, or tail length, or number of hairs, or...

There are a lot of ways, but the idea is that with a collection of objects we often like to talk about how they relate to each other and how we can compare objects of this underlying collection or set. The following definition puts this in terms we will use through the rest of the book.

Definition 1.2.1. If we let S be a set, then an *order* on S is a relation, often denoted $<$, with two extra properties.

- If x and y are in S, then *only one* of the following is true.

$$x < y, \qquad x = y, \qquad y < x$$

- If x, y and z are in S and $x < y$ and $y < z$, then $x < z$.

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Note that we did not define what the symbol $>$ means, but as is often done we will use it because mathematics is nothing without some abuses of notation. If we write $x > y$, take that to mean $y < x$, but instead you may read it as x is "greater than" y or x is "larger than" y. Along with this notation, we will use $x \leq y$ to mean that x is either less than y or it is equal to y, but we don't know which. Similarly with $\geq$.

While in english (and many other languages) we rely on context to understand what set and order people are using when they talk, in mathematics we have to be very pedantic. Hence the following definition.

Definition 1.2.2. An *ordered set* is a set, together with an order defined on said set.

Going back to the dogs, people often say "Nate, you're dog is *the cutest*" which would imply, if taken literally, that there is no dog that is cuter than mine. People love these kind of extremes. We have a whole book dedicated to people who are the *most* at something (The Guiness World Records) and it comes out every year. We also have the olympics to find more of the *most* people. The fastest person on land, the fastest person in water, the fastest person on land/water/wheel (triathlon). We love this kind of thing, and of course some people are also intersested in the slowest.::

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1.6 The Complex Field

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2.3 Compact Sets

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Bibliography

[1] Walter Rudin. *Principles of Mathematical Analysis*. 3rd ed. McGraw Hill, 1976.