NATE STEMEN & KEVIN YEH

RUDIN: TRANSLATED

Contents

Todo list

0

Introduction

This book is for ...

who is this book actually for?

The expected prerequisites are ...

what do we actually expect the reader to know?

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The Real and Complex Number System

Introduction

Before we can begin to talk about what analysis is about, we must have a clear notion of the objects that we want to study. Mathematics started with the study of numbers and that's what we shall start with here. Just like driving a car does not require knowledge of how an engine works, we will not enter into a discussion of all properties of numbers (an interested reader might look to for more information). We must take some things for granted if we wish to get off the runway. We will assume you to be familiar with rational, or decimal numbers such as

more rigorous definition of numbers reference

$$1, \qquad 2.56, \qquad \frac{1}{3}, \qquad 0.347 \tag{1.1}$$

This set of numbers is great for many purposes, including almost everything in our day to day life, but we can show it isn't quite satisfactory for all purposes. Take, for example, the following triangle.

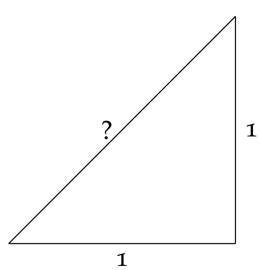

Figure 1.1: Simple Right Triangle with Side Length 1

Suppose we want to figure out how long the side is labelled by "?". Well, if we use the Pythagorean Theorem[1] then we can write the following equation. We use the letter x to denote the length of the hypotenuse that we are interested in.

[1] This simply says that if we have a right triangle with sides of length a and b, then the hypotenuse c is related to those numbers by $c^2 = a^2 + b^2$.

$$x^2 = 1^2 + 1^2$$
$$x^2 = 2 \tag{1.2}$$

If the rational numbers are "enough"[2], then we should be able to write x as $\frac{n}{m}$ where n and m are integers ($\{\ldots, -2, -1, 0, 1, 2, \ldots\}$). However, as we will see in Example **??**, this may not be the case.

[2] meaning they represent everything we can construct geometrically

say rational numbers are fractions before this part

Example 1.1 (Existence of Gaps)**.** In this example we will set out to show that Equation **??** has no solutions that are rational. To show this we will employ a method known as proof by contradiction. We will assume that x can be written as a fraction.

$$x = \frac{n}{m} \tag{1.3}$$

We take the fraction to be as reduced as possible[3] and hence n and m are *not both* even. We can plug this into Equation **??** to obtain

[3] meaning we wouldn't have something like $\frac{5}{15}$ because that can be reduced further to $\frac{1}{3}$

$$x^2 = \frac{n^2}{m^2} = 2 \overset{\text{multiply by } m^2}{\Longrightarrow} n^2 = 2m^2$$

Ordered Sets

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