Session 1

EpsilonDelta

August 2019

Problem 1: Zero Square (5)

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points P in the plane?

Problem 2: Stick 'em with the pointy end (10)

$2N + 1$ people are standing in an open field such that their pairwise distances are distinct. They decide to play a violent game. The rules of the game are as follows: Each person (simultaneously) throws a knife at the person standing closest to them. Anyone who has a knife thrown at them, dies. The winner(s) are those who survive. Notice that when you play the Game of Knives you win, or you die. Prove that there will always be at least one winner for all positive integer values of N.

Problem 3: Racial Segregation (20)

In the plane, 2019 red points and 2020 blue points are marked so that no three of the marked points are collinear. One needs to draw k lines not passing through the marked points and dividing the plane into several regions. The goal is to do it in such a way that no region contains points of both colors. Find the minimal value of k such that the goal is attainable for every possible configuration of 4039 points.

These two books contain the sum total of human knowledge.