Solutions

1. Assume that the function $f(X)$ is non zero somewhere.
   With no loss in generality, we pick a point e such that $e \, \epsilon \, \mathbb{R}^2$ and $f(e) \neq 0$

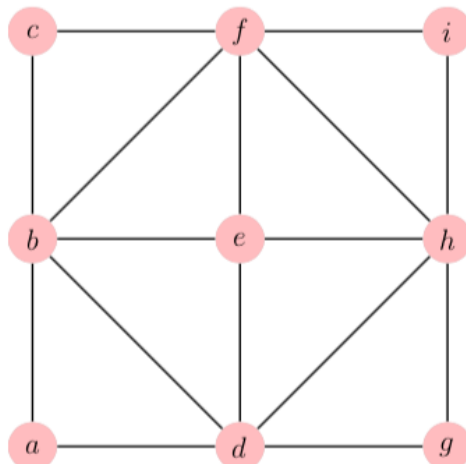

Figure 1: Points on $\mathbb{R}^2$

We construct the following figure in $\mathbb{R}^2$ with arbitrary measure.
Notice the fact that (c, f, e, b) (f, i, h, e) (e, h, g, d) (b, e, d, a) (c, i, g, a) (d, b, f, h) all form squares.
Therefore,
$f(c) + f(f) + f(e) + f(b) = 0$
$f(f) + f(i) + f(h) + f(e) = 0$
$f(e) + f(h) + f(g) + f(d) = 0$
$f(d) + f(b) + f(f) + f(h) = 0$
Adding all these equations we get
$(f(c) + f(i) + f(g) + f(a)) + 2(f(d) + f(b) + f(f) + f(h)) + 4f(e) = 0$
However $f(c) + f(i) + f(g) + f(a) = 0$ and $f(d) + f(b) + f(f) + f(h) = 0$ as the points constituting it form a square.
Hence we get $4f(e) = 0 \implies f(e) = 0$
Which is a contradiction to what follows from our initial assumption that $f(X)$ must be non zero somewhere.

$\implies \boxed{f(X) = 0 \; \forall X \, \epsilon \, \mathbb{R}^2}$

2. We present the proof via induction.
Base Case: $N = 3$ A scalene triangle is formed.
The two people who constitute the shortest edge of the triangle kill each other and one person survives.
Induction Step: Assume that for $2N + 1$ ($N \epsilon \mathbb{N}$) number of people, there will be at least one survivor after the game ends. Now, for $2N + 3$ Let us consider the two people whose pairwise distance between them is minimum compared to any other pairwise distance. Due to this fact it is guaranteed that they kill each other. Now, consider the remaining of the $2N + 1$ people.
Two cases emerges if we analyze the interaction.

    - Nobody from the group of $2N + 1$ throw a knife at the group of 2. In which case, the $2N + 1$ are essentially having an independent game. Thus, from induction somebody must survive in the end.
    - Knive(s) are thrown at the group of 2 in which case for the $2N + 1$ people
    Number of Knives < Number of People $= 2N + 1$.
    But as one knife can kill only one person there must be at least one survivor at the end among the $2N + 1$ people.(Pigeonhole Principle)

    Hence, for a game of odd players there will always exist at least one survivor at the end of the game.

3. k = 2019.

    Firstly, let us present an example showing that k $\geq$ 2019. Mark 2019 red and 2019 blue points on some circle alternately, and mark one more blue point somewhere in the plane. The circle is thus split into 4038 arcs, each arc having endpoints of different colors. Thus, if the goal is reached, then each arc should intersect some of the drawn lines. Since any line contains at most two points of the circle, one needs at least 4038 lines. It remains to prove that one can reach the goal using 2019 lines. First of all, let us mention that for every two points A and B having the same color, one can draw two lines separating these points from all other ones. Namely, it suffices to take two lines parallel to AB and lying on different sides of AB sufficiently close to it: the only two points between these lines will be A and B.Now, let P be the convex hull of all marked points. Two cases are possible.

    Case 1. Assume that P has a red vertex A. Then one may draw a line separating A from all the other points, pair up the other 2018 red points into 1009 pairs, and separate each pair from the other points by two lines. Thus, 2019 lines will be used.

    Case 2.Assume now that all the vertices of P are blue. Consider any two consecutive vertices of P, say A and B. One may separate these two points from the others by a line parallel to AB.Then, as in the previous case, one pairs up all the other 2018 blue points into 1009 pairs, and separates each pair from the other points by two lines. Again, 2019 lines will be used.