CHAPTER 3

Plotting 1

Before starting each exercise, you should restart Maple, either by entering the `restart;` command, or by clicking the button with the circulating arrow at the right hand end of the toolbar.

Numbers in square brackets refer to the "Maple reference" notes, which were distributed in the first lecture.

There are some hints at the end.

Exercise 3.1. Plot each of the following functions **[10.1]** for a suitable range of values of x. Experiment to find a range that displays the interesting features. Write two or three lines (for each function) describing those features.

(a) $2e^{-t}\sin(30t)$
(b) $2\sin(20t)+3\sin(21t)$
(c) $\sin(x)+\sin(3x)/3+\sin(5x)/5+\sin(7x)/7$
(d) $\cos(\pi x^2)$

Exercise 3.2. Consider the function

$$f(x)=(x^3-x)(x^2-4/9)(x^2-1/9).$$

Enter this definition, using the syntax explained in **[9.1]**. Plot the graph for various ranges of x, and describe the main features that you see. Compare $f(x)$ with the functions x, x^2, x^3 and so on, by plotting them together **[10.5]**. Which of these matches $f(x)$ most closely for large x? Can you explain why?

Exercise 3.3. Consider the function

$$g(x)=(2x+3)/(3x-4).$$

(a) Plot the graph for various ranges of x, and describe the main features that you see. You may find it helps to restrict the vertical range **[10.2]** as well as the horizontal one.
(b) What is the value of x where $g(x)$ is discontinuous? You can either work this out from the formula, or read this off from the graph, or use the Maple command `discont(g(x),x)`. You can also replot the graph, skipping over the discontinuity, as explained in **[10.6]**.
(c) Plot the line $y=5$ along with the function $g(x)$ [1]. You should see that the line crosses the graph. If you change the 5 to -1 and redo the plot, then again you see that the line $y=-1$ crosses the graph. However, there is one horizontal line (somewhere between $y=-1$ and $y=5$) that does not cross the graph. Can you find out which line it is? You can either use plotting and trial and error, or analyse the situation algebraically.
(d) Enter `limit(g(x),x=infinity);` and `limit(g(x),x=-infinity);`. How are these related to part (c)?

Exercise 3.4. Consider the function

$$f(x)=\frac{1-e^x+e^{2x}-e^{3x}}{\sin(x)}.$$

(Remember **[9.1]** when entering this in Maple.) What is $f(0)$? The formula above gives $f(0)=(1-1+1-1)/0=0/0$, which is meaningless. However, if we plot the graph of $f(x)$ (for x from -1 to 1, say) we see that there is a perfectly definite value for $f(0)$; what is it? (See the hints at the end for a more careful discussion of the logic.)

Exercise 3.5. We now use Maple to investigate the properties of a new function that you probably have not met before: the Bessel function $J_2(x)$. This is one of a whole family of Bessel functions, which have many applications, for example in studying the vibration of drums or the behaviour of fibre-optic cables. In Maple it is called `BesselJ(2,x)`.

(a) Plot $J_2(x)$ for $-50\le x\le 50$. Describe the main features.
(b) Plot $J_2(x)$ and $x^2/5$ in the same picture, from $x=-0.1$ to $x=+0.1$. Now change the 5 to something else, and repeat. Which value makes the two curves match up most closely?
(c) Plot $J_2(x)$ from $x=10$ to 300. You should see some strange wiggles in the graph, which are in fact not really there; they appear because Maple has not calculated enough points to draw an accurate picture. To fix this, we use the `numpoints` option (as in **[10.7]**):

[1] You should use the syntax in **[10.5]**. You should also remember that the command to plot $y=5$ is `plot(5,...)`, not `plot(y=5,...)`.

```
plot(BesselJ(2,x),x=10..300,numpoints=200);
pic := %:
```

(The second line here saves the picture, so we do not have to recalculate it later [**10.13**].)

You should see an oscillation dying slowly away. This leads us to ask how large the oscillations are, and what is their frequency.

(d) For the size of the oscillations, try plotting $0.15x^{-1}$ alongside $J_2(x)$, as in [**10.14**]:

```
plots[display](pic,plot(0.15*x^(-1),x=10..300,color=blue));
```

Do the same with $0.3x^{-1/3}$ and $0.5x^{-1/2}$ and some other similar functions. (Do not retype the whole line; just edit the relevant numbers, press ENTER, and Maple will redraw the graph.) If you get the numbers right, then the blue curve will just touch the tops of all the waves. (In fact it does not touch exactly, but you need to zoom in very close to see that.)

(e) For the frequency of the oscillations, try plotting $J_2(x)$ alongside $\sin(x - \pi/4)$, for various ranges of x. This should convince you that J_2 oscillates with approximately the same frequency as $\sin(x)$, at least when x is reasonably large.

(f) Can you combine (d) and (e) to find an easier function function $f(x)$ that is very close to $J_2(x)$ for large x? Plot f and J_2 together to check your answer.

Exercise 3.6. Put

$$g(a,x) = \frac{(x-1-\sin(a/4))(x-1-\cos(a/4))(x+1-\sin(a/4))(x+1-\cos(a/4))}{1+x^4}.$$

(Remember [**9.1,9.5**] when entering this in Maple.) The object of this exercise is to describe the properties of g.

(a) Plot $g(2,x)$ for x from -500 to 500. Write two or three lines describing the main features of the plot. Then plot $g(3,x)$ and $g(4,x)$; you should see that on this scale, they look almost exactly the same.

(b) Plot $g(2,x)$ and $g(4,x)$ for x from -2 to 2. In both cases, the curve dips below the x axis. Now plot $g(3,x)$ instead. It appears that the curve does not dip below the axis, but just touches it somewhere near $x = -0.3$, and again near $x = 1.7$. To investigate further, we zoom in. We plot only from $x = -0.4$ to $x = -0.2$, to focus on the region of interest. We also restrict the vertical range [**10.2**] to be from -0.01 to 0, so only the part of the curve below the axis will be shown. Finally, we specify `numpoints=1000` to make sure that Maple plots the graph very accurately [**10.7**].

We see that the graph still dips slightly below the axis. However, there is a certain number a close to 3 for which the curve does not dip below the axis at all — see if you can find it. Plot the graph for various different values of a, zooming in further if necessary, and also thinking about which values of a might make something special happen in the formula for $g(a,x)$.

(c) Plot $1 - 8g(a,0)$ for a reasonable range of values of a. You should recognize the resulting graph, and so should be able to give a simple formula for $1 - 8g(a,0)$. Can you derive this formula algebraically from the definition of $g(a,x)$?

(d) Plot $g(a,0.5)$ for a from -40 to 40. What are the main features? You should see three tall peaks and three lower peaks. Consider the tall peak closest to the x-axis. Click on it and copy down the coordinates that you see in the little box at the top left of the Maple window. They should be $(3,0.9)$ approximately. To get more accurate numbers, recall that the peak occurs where the derivative of the function is zero. Here we are thinking of g as a function of a, so the slope is dg/da, or in Maple terms:

```
slope := diff(g(a,0.5),a);
```

We need to solve numerically for the place where the slope is zero, close to $a = 3$:

```
fsolve(slope = 0,a=3);
```

You should recognize the answer. Now find the x-coordinates of the other two tall peaks in the same way. Can you give exact formulae for these as well? (Hint: divide them by the x-coordinate of the first peak.)

(e) Now try doing some 3-dimensional plots, for example

```
plot3d(g(a,x),a=-20..20,x=-15..15,axes=boxed,grid=[100,100]);
```

Hints

Exercise 3.4.

```
f := (x) -> (1 - exp(x) + exp(2*x) - exp(3*x))/sin(x);
plot(f(x),x=-1..1);
```

The graph crosses the y-axis at $y = -2$, suggesting that $f(0) = -2$.

If we want to be perfectly logical, we should say something a little bit different, however. This is our function f, so it is our right and privilege to define it however we want. The original formula does not make sense for $x = 0$, so

$f(0)$ is not yet defined. We can, if we choose, declare that $f(0)$ is simply undefined. Alternatively, we can, if we feel perverse, declare that $f(0) = 42$. Alternatively, we can declare that $f(0) = -2$. The graph shows only that the last option is the most reasonable one, not that we are forced to take it.

Exercise 3.5.

(b) `plot({BesselJ(2,x),x^2/5},x=-0.1..0.1);`
The best match occurs with $x^2/8$ instead of $x^2/5$.

(d) The graph of the function $y = 0.8x^{-1/2}$ touches the tops of the waves (not exactly, but to a very good approximation).

(f) The function $J_2(x)$ is very close to $-0.8\sin(x + \pi/4)/\sqrt{x}$ (for large x). Enter the following to plot them together:

```
plot({BesselJ(2,x),-0.8*sin(x+Pi/4)/sqrt(x)},x=1..40);
```

Exercise 3.6. The definition of g is

```
g := (a,x) -> (x-1-sin(a/4))*(x-1-cos(a/4))*
              (x+1-sin(a/4))*(x+1-cos(a/4))/(1+x^4);
```