CHAPTER 4

Plotting 2

Before starting each exercise, you should restart Maple, either by entering the `restart;` command, or by clicking the button with the circulating arrow at the right hand end of the toolbar. After restarting Maple, enter `with(plots):` to (re)load the `plots` package. (It is safest to do this for all questions, although it is unnecessary for some. If the `display` command gives a big mess of coordinates instead of a picture, you probably forgot to enter `with(plots):`. Any warning messages produced by this command can safely be ignored.)

1. Parametric plotting

So far we have mostly drawn graphs where y is given as a function of x. We now instead draw some graphs where both x and y are given as functions of another variable, say t.

EXERCISE 4.1. (a) Plot the curve given by $x = \cos(10t)/(1+t^2)$ and $y = \sin(10t)/(1+t^2)$, as in **[10.8,10.9]**.

(b) Plot the curve $(x, y) = (\sin(3t), \sin(2t))$ for t from 0 to 2π (this is called a Lissajous figure). Replace the 2 and 3 by larger numbers, and investigate how the picture changes.

(c) Plot the curve $(x, y) = (t - \sin(t), 1 - \cos(t))$ for t from 0 to 8π. This is called a cycloid; it is the path traced out by a point on the edge of a wheel as the wheel rolls along the ground. It looks wrong because Maple draws it using different scales on the x and y axes. To fix this, click on the graph, and then click on the button marked “1:1” on the toolbar. You can click the button repeatedly to switch between the distorted and undistorted pictures. Alternatively, use the option `scaling=constrained` when you first draw the graph **[10.3]**, like this:

```
plot([t-sin(t),1-cos(t),t=0..8*Pi],scaling=constrained);
```

(d) Plot the curve $(x, y) = (2t/(1+t^2), (1-t^2)/(1+t^2))$, again using **[10.8]**. What do you see? Can you explain it?

EXERCISE 4.2. Consider the curve $(x, y) = (4t^2 - 1, 8t^3 - 8t)$, which is called a nodal cubic. In this exercise it will be convenient to enter these definitions separately from the `plot` command:

```
x := 4*t^2-1;
y := 8*t^3-8*t;
```

(You must remember to remove these definitions by restarting Maple **[1.9]** before you go on to the next exercise.)

(a) Plot the curve from $t = -1.5$ to $t = 1.5$: `plot([x,y,t=-1.5..1.5]);`

(b) Notice that the curve crosses over itself. What are the x and y coordinates of the point where this happens, and what are the corresponding values of t?

(c) The curve crosses the y-axis twice. What are the values of t and y at these two points?

(d) Now plot the function $(X - 3)\sqrt{X + 1}$ for X from -1 to 8. (Maple will not let us use x here because x has been defined in terms of t and so is no longer a free variable **[14.7]**. We could use almost any other letter, but X seems a natural choice.) To compare this precisely with (a), combine the two pictures like this **[10.14,10.10]**:

```
display(
  plot((X-3)*sqrt(X+1),X=-1..8,color=blue),
  plot([x,y,t=-2..2],color=red),
  view=[-2..8,-15..15]
);
```

(If this gives a big mess of coordinates instead of a picture, you probably forgot to enter `with(plots):`.) What do you observe? Can you test it algebraically?

EXERCISE 4.3. **Do not enter** the following commands. Instead, read them carefully, study the reference notes, and sketch what Maple would plot if you entered the commands.

(a) `plot([sin(x),cos(x),x=0..4*Pi]);`

(b) `plot([sin(x),cos(x)],x=0..4*Pi);`

(c) `plot([x,sin(x),x=0..4*Pi]);`
(d) `plot([y,sin(y),y=0..4*Pi]);`
(e) `plot([sin(y),y,y=0..4*Pi]);`
(e) `plot([sin(y),y],y=0..4*Pi);`

2. Implicit plotting

Often we want to plot a curve given by an equation like $x^2 + y^2 = 100$, where y is not explicitly a function of x. Here we might let x and y run from -11 to 11. Read **[10.11]** and ask Maple to plot the graph.

Exercise 4.1. Plot the curve $y^2 = x^3 - x$, for a range of values of x and y that shows the interesting features. The `implicitplot` command will give you a rather jagged picture; you can improve it as in **[10.12]**.

Now plot $y^2 = x^3 - x + a$ for various values of a between 0 and 1. You should see that at a certain value of a, the picture changes from being two separate curves to a single, connected curve. Find the relevant value of a approximately, by trial and error. Can you work out an exact formula?

Exercise 4.2. Plot the curve $x^2 + y^2 + a(\sin(2\pi x) + \cos(2\pi y)) = 100$ for various values of a, starting with $a = 1$, $a = 5$ and $a = 20$. You will need lots of points to get a good picture, so use the option `grid=[200,200]`. Describe the main features. (I have not yet worked out how to explain them; you can take that as a challenge!)

3. Plotting lists of points

Exercise 4.1. Read **[10.15...10.17]**. Ask Maple to draw the seven points at $x = 1, \ldots, 7$ with y-values 10, 40, 20, 30, 50, 10 and 20. Do this with a line joining the specified points **[10.15]** and then with just the points themselves **[10.16]**. Try changing the list of y-values in various ways and redrawing the graph.

Exercise 4.2. (a) Enter `ithprime(1);`, `ithprime(2);` and so on to list the first few prime numbers.
(b) Enter `seq(ithprime(i),i=1..100);` to list the first hundred primes in one go **[13.1]**.
(c) To plot this list, enter

```
listplot([seq(ithprime(i),i=1..100)],style=POINT);
```

Note that the `seq` command gives a list with no brackets, but the `listplot` command needs a list with square brackets, so we have put square brackets around the `seq` command.

To save this picture for future use, enter

```
pic1 := %:
```

Note that this ends with a colon, not a semicolon. If you use a semicolon, then Maple will print out all the coordinates of all the points in the picture, giving several pages of useless output.

(d) It is a very interesting and important fact that the n'th prime is approximately $n(\ln(\ln(n)) + \ln(n) - 1)$. To check this, plot the function $x(\ln(\ln(x)) + \ln(x) - 1)$ for x from 1 to 100, then enter `pic2:=%:` (with a colon, again) to save the picture. Then enter `display(pic1,pic2);` to compare the two plots.
(e) Repeat for the first 1000 primes. This can be done in one step, as follows:

```
display(
 listplot([seq(ithprime(i),i=1..1000)],style=POINT,symbol=POINT),
 plot(x*(ln(ln(x)) + ln(x) - 1),x=1..1000)
);
```

(To lay this out neatly, as above, hold the SHIFT key when you press RETURN at the end of each line. Maple will not attempt to carry out the command until you press RETURN without the SHIFT key.)

Exercise 4.3. (a) Enter `20!;` to calculate the number $20! = 1\times2\times3\times\cdots\times19\times20$. Then enter `evalf(20!);` to give the same answer in scientific notation, making it easier to see the approximate size: about 0.24×10^{19}.
(b) Now calculate $1!, 2!, \ldots, 20!$ using the `seq` command. Then plot these values, as in Exercise 4.2(c). The result is not very informative — why not?
(d) Instead, plot the values of $\ln(n!)$ for n from 1 to 20; this gives a more useful picture. Save the graph by entering `pic1:=%:`.
(e) Define $f(x) = \sqrt{2\pi}x^{x+1/2}e^{-x}$ (using **[9.1]**). It is an interesting and useful fact that $f(n)$ is a good approximation to $n!$, and so $\ln(f(n))$ is a good approximation to $\ln(n!)$. To check this, plot $\ln(f(x))$ for x from 1 to 20 (using the ordinary `plot()` command rather than `listplot()`). Save the result by entering `pic2:=%:`, then enter `display(pic1,pic2);` to combine the two graphs.

EXERCISE 4.4. Define a sequence of functions $f(n,c)$ by $f(0,c) = 0.5$ and $f(n+1,c) = c\,f(n,c)(1 - f(n,c))$, so for example

$$f(1,c) = c \times 0.5 \times (1 - 0.5) = 0.25c$$
$$f(2,c) = c \times (0.25c) * (1 - 0.25c) = 0.25c^2 - 0.0625c^3.$$

(This comes from a very simple model of population dynamics, where c is a parameter depending on the reproductive behaviour of a certain species, and $f_n(c)$ is the population density of that species in the n'th year.) It turns out that the long-term pattern of this sequence depends in an intricate and interesting way on c; this is the starting point of the theory of chaotic dynamics.

We can enter the definition in Maple as follows:

```
f := proc(n,c)
 option remember;
 if n = 0 then
  return 0.5;
 else
  return c * f(n-1,c) * (1 - f(n-1,c));
 end if;
end;
```

Now use the `listplot` command to plot the values $f(i, 2.9)$ for i from 1 to 500. Just plot the points, not the lines joining them [**10.16**]. You should see that the points bounce around a bit when i is small, but they settle down so that $f(i, 2.9)$ is close to 0.655 when i is large. Now change the parameter c from 2.9 to 3.1. How does the pattern change? Try various values between 2.9 and 3.1 to see in more detail what happens and when. Then look at the range $3.4 \leq c \leq 3.5$, then $3.52 \leq c \leq 3.56$, then $3.62 \leq c \leq 3.63$.