LaTeX Typesetting of Theorems and Proofs

Professional Document Preparation System

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Introduction

Mathematical documents include elements that require special formatting and numbering such as theorems, definitions, propositions, remarks, corollaries, lemmas and so on.

Definition 1 (Tautology). A *tautology* is a proposition that is always true for any of its variables.

Definition 2 (Contradiction). A *contradiction* is a proposition that is always false for any value of its variables.

Theorem 5.3. *If proposition P is a tautology then $\sim P$ is a contradiction, and conversely.*

Proof. If P is a tautology, then all elements of its true table are true (by Definition 1), so all elements of the truth table for $\sim P$ are false, therefore $\sim P$ is a contradiction (by Definition 2). □

Example 1. "It is raining or it is not raining" is a tautology, but "it is not raining and it is raining" is a contradiction.

Remark 1. Example 1 used De Morgan's Law $\sim (p \vee q) \equiv \sim p \wedge \sim q$.

The `amsthm` Package

Theorem environments in LaTeX can be defined by means of using the package:

```
\usepackage{amsthm}
```

and the command `\newtheorem` which takes two arguments:

```
\newtheorem{theorem}{Theorem}
```

- the first one is the name of the environment that is defined;
- the second one is the word that will be printed, in boldface font, at the beginning of the environment.

Once this new environment is defined it can be used normally within the document, delimited by `\begin{theorem}` and `\end{theorem}`.

Example

```latex
\documentclass{article}
\usepackage{amsthm}
\newtheorem{theorem}{Theorem}
\begin{document}

\section{Introduction}
Theorems can easily be defined:

\begin{theorem}
Let \(f\) be a function whose derivative exists in every point, then \(f\) is a continuous function.
\end{theorem}
\end{document}
```

Example

1 Introduction

Theorems can easily be defined:

Theorem 1 *Let f be a function whose derivative exists in every point, then f is a continuous function.*

Numbered theorems, definitions, corollaries and lemmas

The numbering of the environments can be controlled by means of two additional parameters in the `\newtheorem` command.

```
\newtheorem{theorem}{Theorem}[section]
\newtheorem{corollary}{Corollary}[theorem]
\newtheorem{lemma}[theorem]{Lemma}
```

```
\begin{theorem}
Let \(f\) be a function whose derivative exists in every point, then
↪ \(f\) is a continuous function.
\end{theorem}

\begin{theorem}[Pythagorean theorem]
\label{pythagorean}
This is a theorem about right triangles and can be summarised in the
↪ next equation \[ x^2 + y^2 = z^2 \]
\end{theorem}
```

Numbered theorems, definitions, corollaries and lemmas

The numbering of the environments can be controlled by means of two additional parameters in the `\newtheorem` command.

```
\newtheorem{theorem}{Theorem}[section]
\newtheorem{corollary}{Corollary}[theorem]
\newtheorem{lemma}[theorem]{Lemma}
```

```
\begin{corollary}
There's no right rectangle whose sides measure 3cm, 4cm, and 6cm.
\end{corollary}
```

```
\begin{lemma}
Given two line segments whose lengths are \(a\) and \(b\) respectively
↪  there is a real number \(r\) such that \(b=ra\).
\end{lemma}
```

Numbered theorems, definitions, corollaries and lemmas

1 Introduction

Theorems can easily be defined:

Theorem 1.1 *Let f be a function whose derivative exists in every point, then f is a continuous function.*

Theorem 1.2 (Pythagorean theorem) *This is a theorem about right triangles and can be summarised in the next equation*

$$x^2 + y^2 = z^2$$

And a consequence of theorem 1.2 is the statement in the next corollary.

Corollary 1.2.1 *There's no right rectangle whose sides measure 3cm, 4cm, and 6cm.*

You can reference theorems such as 1.2 when a label is assigned.

Lemma 1.3 *Given two line segments whose lengths are a and b respectively there is a real number r such that $b = ra$.*

Unnumbered theorem-like environments

It can be useful to have an unnumbered theorem-like environment to add remarks, comments or examples to a mathematical document. The `amsthm` package provides this functionality.

```
\newtheorem*{remark}{Remark}
```

```
\begin{remark}
This statement is true, I guess.
\end{remark}
```

The syntax of the command `\newtheorem*` is the same as the non-starred version, except for the counter parameters. In this example a new unnumbered environment called remark is created.

Unnumbered theorem-like environments

Unnumbered theorem-like environments are also possible.

Remark. *This statement is true, I guess.*

Theorem styles

The package amsthm provide special commands to accomplish a feature that is telling apart definitions from theorems by its formatting.

```
\theoremstyle{definition}
\newtheorem{definition}{Definition}[section]

\theoremstyle{remark}
\newtheorem*{remark}{Remark}
```

```
\begin{remark}
This statement is true, I guess.
\end{remark}
```

```
\begin{definition}[Fibration]
A fibration is a mapping between two topological spaces that has the
↪  homotopy lifting property for every space \(X\).
\end{definition}
```

1 Introduction

Unnumbered theorem-like environments are also possible.

Remark. This statement is true, I guess.

And the next is a somewhat informal definition

Definition 1.1 (Fibration)**.** A fibration is a mapping between two topological spaces that has the homotopy lifting property for every space X.

Proofs

Proofs are the core of mathematical papers and books and it is customary to keep them visually apart from the normal text in the document. The `amsthm` package provides the environment proof for this.

```
\begin{lemma}
Given two line segments whose lengths are \(a\) and \(b\) respectively
↪  there is a real number \(r\) such that \(b=ra\).
\end{lemma}

\begin{proof}
To prove it by contradiction try and assume that the statement is
↪  false, proceed from there and at some point you will arrive to a
↪  contradiction.
\end{proof}
```

Proofs

1 Introduction

Lemma 1.1. *Given two line segments whose lengths are a and b respectively there is a real number r such that $b = ra$.*

Proof. To prove it by contradiction try and assume that the statement is false, proceed from there and at some point you will arrive to a contradiction. $\square$