Differentiation Formulas

Suppose that $\emptyset \neq D \subseteq \mathbb{R}$. If $f, g : D \to \mathbb{R}$ is differentiable at every point $x \in D$, therefore the following holds:

Basic Formulas

1. $\dfrac{\mathrm{d}}{\mathrm{d}x} c = 0$ where $c \in \mathbb{R}$.
2. $\dfrac{\mathrm{d}}{\mathrm{d}x} k f(x) = k \dfrac{\mathrm{d}}{\mathrm{d}x} f(x)$ where $k \in \mathbb{R}$.
3. $\dfrac{\mathrm{d}}{\mathrm{d}x} \left(f(x) \pm g(x)\right) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) \pm \dfrac{\mathrm{d}}{\mathrm{d}x} g(x)$.
4. $\dfrac{\mathrm{d}}{\mathrm{d}x} \left(f(x) \cdot g(x)\right) = f(x) \cdot \dfrac{\mathrm{d}}{\mathrm{d}x} g(x) + g(x) \cdot \dfrac{\mathrm{d}}{\mathrm{d}x} f(x)$.
5. $\dfrac{\mathrm{d}}{\mathrm{d}x} \left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x) \cdot \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) - f(x) \cdot \dfrac{\mathrm{d}}{\mathrm{d}x} g(x)}{(g(x))^2}$ where $g(x) \neq 0$.
6. $\dfrac{\mathrm{d}}{\mathrm{d}x} x^n = n \cdot x^{n-1}$ for any $n \in \mathbb{R}$.
7. $\dfrac{\mathrm{d}}{\mathrm{d}x} \exp(x) = \exp(x)$.
8. $\dfrac{\mathrm{d}}{\mathrm{d}x} \ln(x) = \dfrac{1}{x}$.
9. $\dfrac{\mathrm{d}}{\mathrm{d}x} \sin(x) = \cos(x)$.
10. $\dfrac{\mathrm{d}}{\mathrm{d}x} \cos(x) = \sin(x)$.

Additional Formulas

$$\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

Solution:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x} &= \frac{\mathrm{d}}{\mathrm{d}x}x^{\frac{1}{2}} \\ &= \frac{1}{2}\cdot x^{(\frac{1}{2}-1)} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}.\end{aligned}$$

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$$\frac{\mathrm{d}}{\mathrm{d}x}\tan(x) = \sec^2(x)$$

Solution:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\tan(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\frac{\sin(x)}{\cos(x)} \\ &= \frac{\cos(x)\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) - \sin(x)\frac{\mathrm{d}}{\mathrm{d}x}\cos(x)}{\cos^2(x)} \\ &= \frac{\cos(x)\cdot\cos(x) - \sin(x)\cdot(-\sin(x))}{\cos^2(x)} \\ &= \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x).\end{aligned}$$

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