

TYPESETTING THEOREMS AND PROOFS

L^AT_EX: A Professional Document Preparation System

Ratthaprom PROMKAM
ratthaprom_p@rmutt.ac.th

Department of Mathematics and Computer Science
Rajamangala University of Technology Thanyaburi

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AGENDA

1. The `amsthm` Package

2. Typesetting Theorems

3. Theorem Styles

4. Typesetting Proofs

INTRODUCTION

Mathematical documents include elements required special formatting and numbering, e.g., theorems, definitions, propositions, remarks, corollaries, lemmas and so on.

Definition 1 (Tautology). A *tautology* is a proposition that is always true for any of its variables.

Definition 2 (Contradiction). A *contradiction* is a proposition that is always false for any value of its variables.

Theorem 5.3. *If proposition P is a tautology then $\sim P$ is a contradiction, and conversely.*

Proof. If P is a tautology, then all elements of its true table are true (by Definition 1), so all elements of the truth table for $\sim P$ are false, therefore $\sim P$ is a contradiction (by Definition 2). \square

Example 1. “It is raining or it is not raining” is a tautology, but “it is not raining and it is raining” is a contradiction.

Remark 1. Example 1 used De Morgan’s Law $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

THE `amsthm` PACKAGE

Theorem environments in L^AT_EX can be defined by means of using the package:

```
\usepackage{amsthm}
```

and the command `\newtheorem` which takes two arguments:

```
\newtheorem{theorem}{Theorem}
```

- The first one is the name of the environment;
- The second one is the word that will be printed, in boldface font, at the beginning of the environment.

It can be used normally within the document, delimited by `\begin{theorem}` and `\end{theorem}`.

EXAMPLE

```
1 \documentclass{article}
2 \usepackage{amsthm}
3 \newtheorem{theorem}{Theorem}
4 \begin{document}

5
6 \section{Introduction}
7 Theorems can easily be defined:

8
9 \begin{theorem}
10 Let  $\backslash(f\backslash)$  be a function whose derivative
11   ↳ exists in every point, then  $\backslash(f\backslash)$  is a
12   ↳ continuous function.
13 \end{theorem}
14 \end{document}
```

EXAMPLE

1 Introduction

Theorems can easily be defined:

Theorem 1 *Let f be a function whose derivative exists in every point, then f is a continuous function.*

NUMBERED THEOREMS

The numbering of the environments is controlled by means of two additional parameters in the `\newtheorem`.

```
\newtheorem{theorem}{Theorem}[section]
\newtheorem{corollary}{Corollary}[theorem]
\newtheorem{lemma}[theorem]{Lemma}
```

```
\begin{theorem}
```

Let f be a function whose derivative exists in every point, then f is a continuous function.

```
\end{theorem}
```

```
\begin{theorem}[Pythagorean theorem]
```

```
\label{pythagorean}
```

This is a theorem about right triangles and can be summarized in the next equation $x^2 + y^2 = z^2$

```
\end{theorem}
```

NUMBERED THEOREMS

The numbering of the environments is controlled by means of two additional parameters in the `\newtheorem`.

```
\newtheorem{theorem}{Theorem}[section]
\newtheorem{corollary}{Corollary}[theorem]
\newtheorem{lemma}[theorem]{Lemma}
```

```
\begin{corollary}
```

There's no right rectangle whose sides measure 3cm, 4cm, and
→ 6cm.

```
\end{corollary}
```

```
\begin{lemma}
```

Given two line segments whose lengths are $\text{\textbackslash}(a\text{\textbackslash})$ and $\text{\textbackslash}(b\text{\textbackslash})$
→ respectively there is a real number $\text{\textbackslash}(r\text{\textbackslash})$ such that
→ $\text{\textbackslash}(b=ra\text{\textbackslash}).$

```
\end{lemma}
```

EXAMPLES

1 Introduction

Theorems can easily be defined:

Theorem 1.1 *Let f be a function whose derivative exists in every point, then f is a continuous function.*

Theorem 1.2 (Pythagorean theorem) *This is a theorem about right triangles and can be summarised in the next equation*

$$x^2 + y^2 = z^2$$

And a consequence of theorem 1.2 is the statement in the next corollary.

Corollary 1.2.1 *There's no right rectangle whose sides measure 3cm, 4cm, and 6cm.*

You can reference theorems such as 1.2 when a label is assigned.

Lemma 1.3 *Given two line segments whose lengths are a and b respectively there is a real number r such that $b = ra$.*

UNNUMBERED THEOREMS

It can be useful to have an unnumbered theorem-like environment to add remarks, comments or examples to a mathematical document. The `amsthm` package provides this functionality.

```
\newtheorem*{remark}{Remark}
```

```
\begin{remark}  
This statement is true, I guess.  
\end{remark}
```

The syntax of the command `\newtheorem*` is the same as the non-starred version, except for the counter parameters. In this example a new unnumbered environment called `remark` is created.

EXAMPLE

Unnumbered theorem-like environments are also possible.

Remark. *This statement is true, I guess.*

THEOREM STYLES

Here are commands to accomplish a feature telling apart definitions from theorems by its formatting.

```
\theoremstyle{definition}
\newtheorem{definition}{Definition}[section]

\theoremstyle{remark}
\newtheorem*{remark}{Remark}
```

```
\begin{remark}
This statement is true, I guess.
\end{remark}

\begin{definition}[Fibration]
A fibration is a mapping between two topological spaces that
→ has the homotopy lifting property for every space \(\mathbf{x}\).
\end{definition}
```

THEOREM STYLES

1 Introduction

Unnumbered theorem-like environments are also possible.

Remark. This statement is true, I guess.

And the next is a somewhat informal definition

Definition 1.1 (Fibration). A fibration is a mapping between two topological spaces that has the homotopy lifting property for every space X .

PROOFS

Proofs are the core of mathematical papers and books and it is customary to keep them visually apart from the normal text in the document. The `amsthm` package provides the environment `proof` for this.

```
\begin{lemma}
Given two line segments whose lengths are \(\mathbf{a}\) and \(\mathbf{b}\)
→ respectively there is a real number \(\mathbf{r}\) such that
→ \(\mathbf{b=ra}\).
\end{lemma}
```

```
\begin{proof}
To prove it by contradiction try and assume that the
→ statement is false, proceed from there and at some point
→ you will arrive to a contradiction.
\end{proof}
```

EXAMPLE

1 Introduction

Lemma 1.1. *Given two line segments whose lengths are a and b respectively there is a real number r such that $b = ra$.*

Proof. To prove it by contradiction try and assume that the statement is false, proceed from there and at some point you will arrive to a contradiction. \square

Ratthaprom PROMKAM

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ratthaprom_p@rmutt.ac.th