

In many applications of calculus, the quadratic formula is used to find the roots of a quadratic equation $ax^2 + bx + c = 0$. The solutions to this equation can be found using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This formula is derived by completing the square and provides a straightforward method to find the values of x that satisfy the equation. The discriminant, $\sqrt{b^2 - 4ac}$, determines the nature of the roots: if it is positive, the equation has two distinct real roots; if it is zero, the equation has exactly one real root; and if it is negative, the equation has two complex roots.

Understanding the behavior of functions often requires analyzing their derivatives. For a sequence $\{x_n\}$, the next term can be represented as x_{n+1} , indicating the dependence on the previous term. Similarly, for a function $f(x)$, the first derivative $f'(x)$ represents the rate of change of the function with respect to x . When dealing with polynomial functions, such as $f(x) = ax^n$, the power rule for differentiation states that $f'(x) = n \cdot ax^{n-1}$. This rule is fundamental in calculus and provides a straightforward method to find the slope of the tangent line at any point on the function. Additionally, higher-order derivatives, like $f''(x)$ and $f^{(3)}(x)$, can be found similarly, providing deeper insights into the function's curvature and concavity.

A well-known formula in calculus is the Taylor series expansion, which expresses a function as an infinite sum of terms calculated from the values of its derivatives at a single point. For a function $f(x)$ centered at $x = a$, the Taylor series is given by

$$f(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f^{(3)}(a)}{3!}(x - a)^3 + \dots \quad (1)$$

This formula allows us to approximate complex functions using polynomials, which are easier to work with analytically and computationally. The accuracy of the approximation increases with the number of terms used, making the Taylor series a powerful tool in both theoretical and applied mathematics.

For certain types of differential equations, solutions can be represented in terms of special functions. One such example is the Bessel function of the first kind, which satisfies Bessel's differential equation:

$$x^2 y'' + xy' + (x^2 - n^2)y = 0 \quad (2)$$

where $y = J_n(x)$ is the Bessel function and n is a constant. These functions frequently appear in problems with cylindrical or spherical symmetry in physics and engineering, demonstrating the broad applicability of advanced mathematical techniques.