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On the Diophantine equation $a^x + b^y = z^2$ where $a \equiv 1 \pmod 3$ and $b \equiv 1 \pmod 3$

Somnuk Srisawat, Wanna Sripad

Department of Mathematics and Computer Science
Faculty of Science and Technology
Rajamangala University of Technology Thanyaburi
Pathum Thani 12110, Thailand

email: somnuk_s@rmutt.ac.th, wanna_sriprad@rmutt.ac.th

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Abstract

In this article, we show that the Diophantine equation $a^x + b^y = z^2$ has no non-negative integer solution, where a, b are positive odd integers with $a \equiv 1 \pmod 3$, $b \equiv 1 \pmod 3$, and x, y, z are non-negative integers.

1 Introduction

Diophantine equations are equations in which the solutions, are required to be integers. It is a popular topic in Number theory. The Diophantine equations of the form $a^x + b^y = z^2$, where a and b are fixed integers have been studied by many mathematicians (see for instance [1, 2, 3, 4]) and the references therein. In 2020, Dokchann and Pakapongpun [2] showed that the Diophantine equation $a^x + (a+2)^y = z^2$, where a is a positive integer with $a \equiv 5 \pmod{42}$, has no non-negative integer solution. In 2022, N. Viriyapong and C. Viriyapong [4] showed that the Diophantine equation $n^x + 19^y = z^2$ has only one solution $(n, x, y, z) = (2, 3, 0, 3)$, where n is a positive integer with $n \equiv 2 \pmod{57}$ and x, y, z are non-negative integers.

Key words and phrases: Diophantine equation, congruence.
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The corresponding author is Wanna Sriprad.

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Some identities of (s,t)-Pell and (s,t)-Pell-Lucas polynomials by matrix methods

Somnuk Srisawat, Wanna Sripad

Department of Mathematics and Computer Science
Faculty of Science and Technology
Rajamangala University of Technology Thanyaburi
Pathum Thani 12110, Thailand

email: somnuk_s@rmutt.ac.th
wanna_sriprad@rmutt.ac.th

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Abstract

In this paper, we explore the extensions of Pell and Pell-Lucas polynomials known as (s,t)-Pell and (s,t)-Pell-Lucas polynomials. We introduce the 2×2 matrices denoted as A and B to facilitate our investigation. By using these matrices, we derive various identities and summation formulas for (s,t)-Pell and (s,t)-Pell-Lucas polynomials.

1 Introduction

For several years, many researchers have extensively investigated numerous polynomial sequences. The most famous polynomials are Fibonacci, Lucas, Pell, and Pell-Lucas polynomials which are also renowned for their diverse range of remarkable properties and wide-ranging applications in mathematics, physics, and computer science [1, 2]. These polynomials have garnered

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Novel inertial methods for fixed point problems in reflexive Banach spaces with applications

Pongsakorn Sunthrayuth[1] · Kunrada Kankam[2] · Ratthaprom Promkam[1] · Somnuk Srisawat[1]

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Abstract

In this paper, we suggest and analyze four inertial algorithms for solving fixed point problems of Bregman quasi-nonexpansive mappings in the framework of reflexive Banach spaces. Our first two algorithms, we propose inertial-like methods based on Mann-type and Halpern-type iterations, and in the others, we propose relaxed inertial-like methods based on Mann-type and Halpern-type iterations. The weak and strong convergence of the algorithms are established under some appropriate conditions on the parameters. As an application, we utilize our main results to find a zero of the sum of Bregman inverse strongly monotone mappings and maximal monotone operators in real reflexive Banach spaces. Also, we provide several numerical experiments to show the convergence behaviour of our algorithms in both finite-dimensional and infinite-dimensional spaces. Finally, we further utilize our algorithms to numerically solve the data classification problems of lung cancer.

Keywords Reflexive Banach space · Weak convergence · Strong convergence · Bregman quasi-nonexpansive mapping · Fixed point problem

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✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

Kunrada Kankam
kunradazzz@gmail.com

Ratthaprom Promkam
ratthaprom_p@rmutt.ac.th

Somnuk Srisawat
somnuk_s@rmutt.ac.th

1 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

2 School of Science, University of Phayao, Phayao 56000, Thailand

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On the Vieta–Jacobsthal-like polynomials

Wanna Sriprad[1], Somnuk Srisawat[2] and Kitsana Charoenchaianan[3]

[1] Department of Mathematics and Computer Science, Faculty of Science and Technology,
Rajamangala University of Technology Thanyaburi, Pathum Thani 12110 Thailand
e-mail: `wanna_sriprad@rmutt.ac.th`

[2] Department of Mathematics and Computer Science, Faculty of Science and Technology,
Rajamangala University of Technology Thanyaburi, Pathum Thani 12110 Thailand
e-mail: `somnuk_s@rmutt.ac.th`

[3] Department of Mathematics and Computer Science, Faculty of Science and Technology,
Rajamangala University of Technology Thanyaburi, Pathum Thani 12110 Thailand
e-mail: `1160109010224@rmutt.ac.th`

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Abstract: In this paper, we first introduce the generalization of the Vieta–Jacobsthal polynomial, which is called the Vieta–Jacobsthal-like polynomial. After that, we give the generating function, the Binet formula, and some well-known identities for this polynomial. Finally, we also present the relation between this polynomial and the previously famous Vieta-polynomials.

Keywords: Vieta–Jacobsthal polynomial, Vieta–Jacobsthal–Lucas polynomial, Generalized Vieta–Jacobsthal polynomial.
2010 Mathematics Subject Classification: 11C08, 11B39, 33C45

1 Introduction

The theory of the Vieta polynomials was first introduced in 1991 by Robbins [6]. The recursive sequence of the Vieta-Fibonacci polynomial $V_n(x)$ and Vieta-Lucas polynomials $v_n(x)$ were introduced by Horadam [2]. These polynomials are defined by $V_n(x) = xV_{n-1}(x) - V_{n-2}(x)$, and $v_n(x) = xv_{n-1}(x) - v_{n-2}(x)$ for $n \geq 2$, with the initial conditions $V_0(x) = 0, V_1(x) = 1$, and

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Research Article

An Iterative Method for Solving Split Monotone Variational Inclusion Problems and Finite Family of Variational Inequality Problems in Hilbert Spaces

Wanna Sriprad and Somnuk Srisawat

Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology, Thanyaburi, Pathum Thani 12110, Thailand

Correspondence should be addressed to Wanna Sriprad; wanna_sriprad@rmutt.ac.th

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The purpose of this paper is to study the convergence analysis of an intermixed algorithm for finding the common element of the set of solutions of split monotone variational inclusion problem (SMIV) and the set of a finite family of variational inequality problems. Under the suitable assumption, a strong convergence theorem has been proved in the framework of a real Hilbert space. In addition, by using our result, we obtain some additional results involving split convex minimization problems (SCMPs) and split feasibility problems (SFPs). Also, we give some numerical examples for supporting our main theorem.

1. Introduction

Let H_1 and H_2 be real Hilbert spaces whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$, respectively, and let C, Q be nonempty closed convex subsets of H_1 and H_2, respectively. For a mapping $S\colon C \longrightarrow C$, we denoted by $F(S)$ the set of fixed points of S (i.e., $F(S) = \{x \in C\colon Sx = x\}$). Let $A\colon C \longrightarrow H$ be a nonlinear mapping. The variational inequality problem (VIP) is to find $x^* \in C$ such that

$$\langle Ax^*, y - x^*\rangle \geq 0, \quad \forall y \in C, \tag{1}$$

and the solution set of problem (1) is denoted by $VI(C, A)$. It is known that the variational inequality, as a strong and great tool, has already been investigated for an extensive class of optimization problems in economics and equilibrium problems arising in physics and many other branches of pure and applied sciences. Recall that a mapping $A\colon C \longrightarrow C$ is said to be α-inverse strongly monotone if there exists $\alpha > 0$ such that

$$\langle Ax - Ay, x - y\rangle \geq \alpha\|Ax - Ay\|^2, \quad \forall x, y \in C. \tag{2}$$

A multivalued mapping $M\colon H_1 \longrightarrow 2^{H_1}$ is called monotone if for all $x, y \in H_1$, $\langle x - y, u - v\rangle \geq 0$, for any $u \in Mx$ and $v \in My$. A monotone mapping $M\colon H_1 \longrightarrow 2^{H_1}$ is maximal if the graph $G(M)$ for M is not properly contained in the graph of any other monotone mapping. It is generally known that M is maximal if and only if for $(x, u) \in H_1 \times H_1, \langle x - y, u - v\rangle \geq 0$ for all $(y, v) \in G(M)$ implies $u \in Mx$. Let $M\colon H_1 \longrightarrow 2^{H_1}$ be a multivalued maximal monotone mapping. The resolvent mapping $J_\lambda^M\colon H_1 \longrightarrow H_1$ associated with M is defined by

$$J_\lambda^M(x) \coloneqq (I + \lambda M)^{-1}(x), \quad \forall x \in H_1, \lambda > 0, \tag{3}$$

where I stands for the identity operator on H_1. We note that for all $\lambda > 0$, the resolvent J_λ^M is single-valued, nonexpansive, and firmly nonexpansive.

ORIGINAL PAPER

VIETA-PELL-LIKE POLYNOMIALS AND SOME IDENTITIES

WANNA SRIPRAD[1], SOMNUK SRISAWAT[1], KANTIDA TUITAKU[1]

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Abstract. *In this paper, we introduce some new generalizations of the Vieta-Pell polynomial, which is called the Vieta-Pell-Like polynomial. We also give the generating function, the Binet's formula, the sum formula, and some well-known identities for this Vieta polynomial. Furthermore, the relations between the Vieta-Pell-Like polynomial and the previously well-known identities are presented.*

Keywords: *Vieta-Pell polynomials; Vieta-Pell-Lucas polynomials; Vieta-Pell-Like polynomial.*

1. INTRODUCTION

The Vieta polynomials were first introduced in 1991 by Robbins [1]. After that, in 2002, Horadam [2] introduced and studied the Vieta-Fibonacci polynomial $V_n(x)$ and Vieta-Lucas polynomials $v_n(x)$. These polynomials are defined respectively by

$$V_0(x) = 0, V_1(x) = 1, V_n(x) = xV_{n-1}(x) - V_{n-2}(x), \text{ for } n \geq 2$$

and

$$v_0(x) = 2, v_1(x) = x, V_n(x) = xv_{n-1}(x) - v_{n-2}(x), for n \geq 2.$$

The Vieta-Pell polynomials $t_n(x)$ and Vieta-Pell-Lucas polynomials $s_n(x)$ were studied in 2013 by Tasci and Yalcin [3]. They defined these polynomials for $|x| > 1$ by

$$t_0(x) = 0, t_1(x) = 1, t_n(x) = 2xt_{n-1}(x) - t_{n-2}(x), \text{ for } n \geq 2$$

and

$$s_0(x) = 2, s_1(x) = 2x, s_n(x) = 2xs_{n-1}(x) - s_{n-2}(x), \text{ for } n \geq 2.$$

They obtained the Binet form and generating functions of Vieta-Pell and Vieta-Pell-Lucas polynomials. Also, they received some differentiation rules and the finite summation formulas. Moreover, they show that Vieta-Pell and Vieta-Pell-Lucas polynomials are closely related to the well-known Chebyshev polynomials of the first kinds $T_n(x)$ and the second kinds $U_n(x)$. The related features of Vieta-Pell, Vieta-Pell-Lucas polynomials, and Chebyshev polynomials are given as

$$s_n(x) = 2T_n(x),$$

and

$$t_{n+1}(x) = U_n(x).$$

[1] Rajamangala University of Technology Thanyaburi, Faculty of Science and Technology, Department of Mathematics, 12110 Pathum Thani, Thailand. E-mail: wanna_sriprad@rmutt.ac.th; somnuk_s@rmutt.ac.th; 1160109010166@mail.rmutt.ac.th.

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Vieta–Fibonacci-like polynomials and some identities*

Wanna Sriprad, Somnuk Srisawat, Peesiri Naklor

Department of Mathematics and computer science,
Faculty of Science and Technology,
Rajamangala University of Technology Thanyaburi,
Pathum Thani 12110, Thailand
wanna_sriprad@rmutt.ac.th
somnuk_s@rmutt.ac.th
1160109010257@mail.rmutt.ac.th

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Abstract

In this paper, we introduce a new type of the Vieta polynomial, which is Vieta–Fibonacci-like polynomial. After that, we establish the Binet formula, the generating function, the well-known identities, and the sum formula of this polynomial. Finally, we present the relationship between this polynomial and the previous well-known Vieta polynomials.

Keywords: Vieta–Fibonacci polynomial, Vieta–Lucas polynomial, Vieta–Fibonacci-like polynomial

AMS Subject Classification: 11C08, 11B39, 33C45

1. Introduction

In 2002, Horadam [1] introduced the new types of second order recursive sequences of polynomials which are called Vieta–Fibonacci and Vieta–Lucas polynomials respectively. The definition of Vieta–Fibonacci and Vieta–Lucas polynomials are defined as follows:

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On the (s,t)-Pell and (s,t)-Pell-Lucas Polynomials

Somnuk Srisawat[1*] and Wanna Sriprad[1]
[1] *Department of Mathematics, Faculty of Science and Technology,*
Rajamangala University of Technology Thanyaburi, Pathum Thani 12110 Thailand
*E-mail: somnuk_s@rmutt.ac.th

Abstract

In this paper, we introduced the generalizations of Pell and Pell-Lucas polynomials, which are called (s,t)-Pell and (s,t)- Pell-Lucas polynomials. We also give the Binet formula and the generating function for these polynomials. Finally, we obtain some identities by using the Binet formulas.

Keywords: (s,t)-Pell Number, (s,t)-Pell-Lucas Number, (s,t)-Pell Polynomial, (s,t)-Pell-Lucas Polynomial.

1. Introduction

For over years, many recursive sequences have been studied in the literature. The famous examples of these sequences are Fibonacci, Lucas, Pell, and Pell-Lucas. They are used in many research areas such as Engineering, Architecture, Nature, and Art (for examples, see: (1-6)). The classical Fibonacci $\{F_n\}_{n=0}^{+\infty}$, Lucas $\{L_n\}_{n=0}^{+\infty}$, Pell $\{P_n\}_{n=0}^{+\infty}$, and Pell-Lucas $\{Q_n\}_{n=0}^{+\infty}$ sequences are defined by

$$F_0 = 0, F_1 = 1 \text{ and } F_n = F_{n-1} + F_{n-2},$$
$$L_0 = 2, L_1 = 1 \text{ and } L_n = L_{n-1} + L_{n-2},$$
$$P_0 = 0, P_1 = 1 \text{ and } P_n = 2P_{n-1} + P_{n-2},$$
$$Q_0 = 2, Q_1 = 2 \text{ and } Q_n = 2Q_{n-1} + Q_{n-2},$$

for $n \geq 2$, respectively. For more detailed information on the Fibonacci, Lucas, Pell, and Pell-Lucas sequences can be found in (1-2).

Recently, Fibonacci, Lucas, Pell, and Pell-Lucas sequences were generalized and studied by many authors in different ways to derive many identities. In 2012, Gulec and Taskara (7) introduced new generalizations of Pell and Pell-Lucas sequences, which are called (s,t)-Pell and (s,t)-Pell-Lucas sequences as in the following definition:

Definition 1.1 Let s,t be any real numbers with $s^2 + t > 0, s > 0$, and $t \neq 0$. Then the (s,t)-Pell sequence $\{P_n(s,t)\}_{n=0}^{+\infty}$ and the (s,t)-Pell-Lucas sequence $\{Q_n(s,t)\}_{n=0}^{+\infty}$ are defined respectively by

$$P_n(s,t) = 2sP_{n-1}(s,t) + tP_{n-2}(s,t),$$
$$Q_n(s,t) = 2sQ_{n-1}(s,t) + tQ_{n-2}(s,t)$$

for $n \geq 2$, with initial conditions $P_0(s,t) = 0$, $P_1(s,t) = 1$ and $Q_0(s,t) = 2$, $Q_1(s,t) = 2s$.

The terms of these sequences are called (s,t)-Pell and (s,t)-Pell-Lucas numbers, respectively. Also, they introduced the matrices sequences, which have elements of (s,t)-Pell and (s,t)-Pell-Lucas sequences. Further, they obtained some identities of (s,t)-Pell and (s,t)-Pell-Lucas matrices sequences by using elementary matrix algebra. After that, the (s,t)-Pell and (s,t)-Pell-Lucas numbers were studied in different ways to obtain many identities of these numbers. (See: (8-10))

On the other hand, the theory of the second-order recursive sequence of the polynomials has been studied in the literature. In 1883 E.C. Catalan and E. Jacobsthal introduced and studied the polynomials, which are defined by Fibonacci-like recurrence relations. Such polynomials, called the Fibonacci polynomials.

The Fibonacci polynomials studied by Catalan are defined by the recurrence relation.

$$F_n(x) = xF_{n-1}(x) + F_{n-2}(x), n \geq 3$$

with initial conditions $F_1(x) = 1, F_2(x) = x$.

The Fibonacci polynomials studied by Jacobsthal are defined by the recurrence relation.

$$J_n(x) = J_{n-1}(x) + xJ_{n-2}(x), n \geq 3$$

with initial conditions $J_1(x) = J_2(x) = 1$.

In 1965, V.E. Hoggatt (11) introduced Lucas polynomials defined by the recurrence relation.

$$L_n(x) = xL_{n-1}(x) + L_{n-2}(x), n \geq 2$$

with initial conditions $L_0(x) = 2$, $L_1(x) = x$.

Pell polynomials sequence $\{P_n(x)\}_{n=0}^{+\infty}$ and Pell-Lucas polynomials sequence $\{P_n(x)\}_{n=0}^{+\infty}$ were studied in 1985 by A.F. Horadam and J.M. Mahon (12), and these polynomials are defined by

$$P_n(x) = 2xP_{n-1}(x) + P_{n-2}(x), n \geq 2$$
$$Q_n(x) = 2xQ_{n-1}(x) + Q_{n-2}(x)\ , n \geq 2$$

with initial conditions $P_0(x) = 0$, $P_1(x) = 1$ and $Q_0(x) = 2$, $Q_1(x) = 2x$.

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Three novel inertial subgradient extragradient methods for quasi-monotone variational inequalities in Banach spaces

Zhong-bao Wang[1,2,3] · Pongsakorn Sunthrayuth[4] · Ratthaprom Promkam[4] · Abubakar Adamu[5,6]

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Abstract

In this paper, we introduce three new inertial subgradient extragradient methods for solving variational inequalities involving quasi-monotone operators in the setting of 2-uniformly convex and uniformly smooth Banach spaces. We dispense with the well-known requirement of the stepsizes of the subgradient extragradient method on the prior knowledge of the Lipschitz constant of the cost function in our proposed algorithms. Furthermore, we give many numerical examples to test the robustness of our proposed algorithms and compare their performance with several algorithms in the literature. In addition, we use our proposed algorithms in the restoration process of some degraded images and compare the quality of the restored images using our proposed algorithms and some recent algorithms in the literature. Finally, from the results of the numerical simulations, our proposed algorithms are competitive and promising.

Keywords Banach space · Weak convergence · Variational inequalities · Quasi-monotone mapping

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✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

1 Department of Mathematics, Southwest Jiaotong University, Chengdu 611756, Sichuan, China

2 National Engineering Laboratory of Integrated Transportation Big Data Application Technology, Chengdu 611756, Sichuan, China

3 School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu 611731, Sichuan, China

4 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi 12110, Pathumthani, Thailand

5 Operational Research Center in Healthcare, Near East University, TRNC Mersin 10, Nicosia 99138, Turkey

6 Charles Chidume Mathematics Institute, African University of Science and Technology, Abuja 900107, Nigeria

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Novel inertial methods for fixed point problems in reflexive Banach spaces with applications

Pongsakorn Sunthrayuth[1] · Kunrada Kankam[2] · Ratthaprom Promkam[1] · Somnuk Srisawat[1]

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Abstract

In this paper, we suggest and analyze four inertial algorithms for solving fixed point problems of Bregman quasi-nonexpansive mappings in the framework of reflexive Banach spaces. Our first two algorithms, we propose inertial-like methods based on Mann-type and Halpern-type iterations, and in the others, we propose relaxed inertial-like methods based on Mann-type and Halpern-type iterations. The weak and strong convergence of the algorithms are established under some appropriate conditions on the parameters. As an application, we utilize our main results to find a zero of the sum of Bregman inverse strongly monotone mappings and maximal monotone operators in real reflexive Banach spaces. Also, we provide several numerical experiments to show the convergence behaviour of our algorithms in both finite-dimensional and infinite-dimensional spaces. Finally, we further utilize our algorithms to numerically solve the data classification problems of lung cancer.

Keywords Reflexive Banach space · Weak convergence · Strong convergence · Bregman quasi-nonexpansive mapping · Fixed point problem

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✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

Kunrada Kankam
kunradazzz@gmail.com

Ratthaprom Promkam
ratthaprom_p@rmutt.ac.th

Somnuk Srisawat
somnuk_s@rmutt.ac.th

1 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

2 School of Science, University of Phayao, Phayao 56000, Thailand

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RESEARCH ARTICLE

WILEY

Inertial-like Bregman projection method for solving systems of variational inequalities

Lateef Olakunle Jolaoso[1,2] | **Nattawut Pholasa**[3] | **Pongsakorn Sunthrayuth**[4] | **Prasit Cholamjiak**[3]

[1]School of Mathematical Sciences, University of Southampton, Southampton, UK

[2]Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Pretoria, South Africa

[3]School of Science, University of Phayao, Phayao, Thailand

[4]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, Thailand

Correspondence
Pongsakorn Sunthrayuth, Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, 12110, Thailand.
Email: pongsakorn_su@rmutt.ac.th

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In this paper, we propose a *self-adaptive inertial-like algorithm* with Bregman distance for approximating a common solution of systems of variational inequalities for a class of monotone and Lipschitz continuous mappings in real reflexive Banach spaces. Our algorithm is constructed without using hybrid projection method and shrinking projection method, and its strong convergence is proved without the prior information of the Lipschitz constant of the mapping. Finally, we provide some numerical experiments to illustrate the performance of the newly proposed method including a comparison with related works in solving signal restoration problems.

KEYWORDS

Bregman distance, monotone mapping, reflexive Banach space, strong convergence, variational inequality problem

MSC CLASSIFICATION

47H09, 47H10, 47J25, 47J05

1 | INTRODUCTION

Let $\mathcal{K}$ be a nonempty, convex, and closed set in a real Banach space; $\mathcal{X}$ and $\mathcal{X}^*$ be the dual of $\mathcal{X}$. We denote the duality pairing between $x \in \mathcal{X}$ and $y^* \in \mathcal{X}^*$ by $\langle x, y^* \rangle$. Let $\mathcal{A} : \mathcal{K} \to \mathcal{X}^*$ be a mapping, the *variational inequality problem* (shortly, VIP) is to find a point $v \in \mathcal{K}$ such that

$$\langle x - v, \mathcal{A}v \rangle \geq 0 \ \forall x \in \mathcal{K}. \tag{1.1}$$

The solution set of VIP is denoted by $VI(\mathcal{K}, \mathcal{A})$. The variational inequality theory is well-known for its crucial roles in many fields of applied science, which include economics, physics, engineering, and other related fields. In particular, it can be applied to mathematical problems, such as optimization problems, game theory, complementarity problems, systems of nonlinear equations, including fixed point problems (see, e.g., earlier studies [1–5]). This explains why a considerable research effort has been wildly devoted in both theory and applications for solving the VIP. Consequently, many

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Inertial projection and contraction methods for solving variational inequalities with applications to image restoration problems

LATEEF OLAKUNLE JOLAOSO[1], PONGSAKORN SUNTHRAYUTH[2], PRASIT CHOLAMJIAK[3] and YEOL JE CHO[4,5]

ABSTRACT. In this paper, we introduce two inertial self-adaptive projection and contraction methods for solving the pseudomonotone variational inequality problem with a Lipschitz-continuous mapping in real Hilbert spaces. The adaptive stepsizes provided by the algorithms are simple to update and their computations are more efficient and flexible. Also we prove some weak and strong convergence theorems without prior knowledge of the Lipschitz constant of the mapping. Finally, we present some numerical experiments to demonstrate the effectiveness of the proposed algorithms by comparisons with related methods and some applications of the proposed algorithms to the image deblurring problem.

1. INTRODUCTION

Let H be a real Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and the norm $\| \cdot \|$. Let C be a nonempty closed and convex subset of H and $A : H \to H$ be a continuous mapping.

The *variational inequality problem* (shortly, VIP) is defined as follows:

$$\text{Find} \quad z \in C \quad \text{such that} \quad \langle Az, y - x \rangle \geq 0, \quad \forall y \in C. \tag{1.1}$$

We denote the solution set of the VIP (1.1) by $VI(C, A)$. Several important applications of the VIP (1.1) have been discussed in, for instance, [2, 4, 12, 22, 23, 24, 28]. It is well known that a point z is a solution of the VIP (1.1) if and only if z solves the fixed point equation:

$$z = P_C(z - \lambda Az), \quad \forall \lambda > 0,$$

where P_C is the projection operator from H onto C. One of the earliest projection methods for solving VIP is the *extragradient method* (EGM) introduced independently by Antipin [3] and Korpelevich [25] as follows:

$$\begin{cases} y_n = P_C(x_n - \lambda Ax_n), \\ x_{n+1} = P_C(x_n - \lambda Ay_n), \end{cases}$$

where $A : H \to H$ is monotone and L-Lipschitz continuous and suitable stepsize $\lambda \in \left(0, \frac{1}{L}\right)$. It was proved that the EGM converges weakly to a solution of VIP in finite dimensional spaces. However, the EGM requires two projections onto the feasible set C which can be computationally costly if A is not simple.

A question of interest in projection-type algorithms is how to reduce the number of projections in the algorithm. This has led to many modifications and improvements of the EGM by many authors.

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Corresponding author: Pongsakorn Sunthrayuth; pongsakorn_su@rmutt.ac.th

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Research article

Two-step inertial method for solving split common null point problem with multiple output sets in Hilbert spaces

Chibueze C. Okeke[1], **Abubakar Adamu**[2,3], **Ratthaprom Promkam**[4] **and Pongsakorn Sunthrayuth**[4,*]

[1] School of Mathematics, University of the Witwatersrand, Johannesburg, South Africa
[2] Operational Research Center in Healthcare, Near East University, Nicosia, TRNC
[3] Mathematics Institute, African University of Science and Technology, Abuja, Nigeria
[4] Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani, Thailand

* **Correspondence:** Email: pongsakorn_su@rmutt.ac.th.

Abstract: In this paper, an algorithm with two-step inertial extrapolation and self-adaptive step sizes is proposed to solve the split common null point problem with multiple output sets in Hilbert spaces. Weak convergence analysis are obtained under some easy to verify conditions on the iterative parameters in Hilbert spaces. Preliminary numerical tests are performed to support the theoretical analysis of our proposed algorithm.

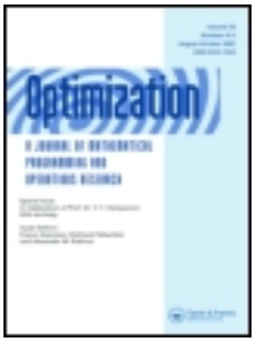
Keywords: Hilbert space; metric projection; self-adaptive step size; two-step inertial; split common null point problem
Mathematics Subject Classification: 47H09, 47H10, 49J53, 90C25

1. Introduction

Throughout this paper, $\mathcal{H}$ denotes a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and the induced $\| \cdot \|$, I the identity operator on $\mathcal{H}$, $\mathbb{N}$ the set of all natural numbers and $\mathbb{R}$ the set of all real numbers. For a self-operator T on $\mathcal{H}$, $F(T)$ denotes the set of all fixed points of T.

Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be real Hilbert spaces and let $\mathcal{T} : \mathcal{H}_1 \to \mathcal{H}_2$ be bounded linear operator. Let $\{U_j\}_{j=1}^t : \mathcal{H}_1 \to \mathcal{H}_1$ and $\{T_i\}_{i=1}^r : \mathcal{H}_2 \to \mathcal{H}_2$ be two finite families of operators, where $t, r \in \mathbb{N}$. The split common fixed point problem (SCFPP) is formulated as finding a point $x^* \in \mathcal{H}_1$ such that

$$x^* \in \cap_{j=1}^t F(U_j) \quad \text{such that} \quad \mathcal{T} x^* \in \cap_{i=1}^r F(T_i). \tag{1.1}$$



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Modified accelerated Bregman projection methods for solving quasi-monotone variational inequalities

Zhong-bao Wang, Pongsakorn Sunthrayuth, Abubakar Adamu & Prasit Cholamjiak

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Modified accelerated Bregman projection methods for solving quasi-monotone variational inequalities

Zhong-bao Wang[a,b,c], Pongsakorn Sunthrayuth [d], Abubakar Adamu [e,f] and Prasit Cholamjiak[g]

[a]Department of Mathematics, Southwest Jiaotong University, Chengdu, People's Republic of China; [b]National Engineering Laboratory of Integrated Transportation Big Data Application Technology, Chengdu, People's Republic of China; [c]School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, People's Republic of China; [d]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani, Thailand; [e]Mathematics Institute, African University of Science and Technology, Abuja, Nigeria; [f]Operational Research Center in Healthcare, Near East University, Nicosia, Northern Cyprus; [g]School of Science, University of Phayao, Phayao, Thailand

ABSTRACT
In this paper, we introduce three new inertial-like Bregman projection methods with a nonmonotone adaptive step-size for solving quasi-monotone variational inequalities in real Hilbert spaces. Under some suitable conditions, the weak convergence of these methods is proved without the prior knowledge of the Lipschitz constant of the operator and the strong convergence of some proposed methods under a strong quasi-monotonicity assumption of the mapping is also provided. Finally, several numerical experiments and applications in image restoration problems are provided to illustrate the performance of the proposed methods.

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KEYWORDS
Bregman projection; Hilbert space; weak convergence; variational inequality problem; quasi-monotone mapping

1. Introduction

Throughout this paper, let H be a real Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and the norm $\| \cdot \|$, and let C be a nonempty, closed and convex subset of H. Recall that a mapping $F : C \to H$ is said to be *monotone* if

$$\langle Fx - Fy, x - y \rangle \geq 0, \quad \forall\, x, y \in C.$$

A mapping $F : C \to H$ is said to be *pseudo-monotone* if

$$\langle Fx, y - x \rangle \geq 0 \Rightarrow \langle Fy, y - x \rangle \geq 0, \quad \forall\, x, y \in C.$$

A mapping $F : C \to H$ is said to be *quasi-monotone* if

$$\langle Fx, y - x \rangle > 0 \Rightarrow \langle Fy, y - x \rangle \geq 0, \quad \forall\, x, y \in C$$

CONTACT Pongsakorn Sunthrayuth pongsakorn_su@rmutt.ac.th

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Modified inertial extragradient methods for finding minimum-norm solution of the variational inequality problem with applications to optimal control problem

Bing Tan, Pongsakorn Sunthrayuth, Prasit Cholamjiak & Yeol Je Cho

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RESEARCH ARTICLE

Modified inertial extragradient methods for finding minimum-norm solution of the variational inequality problem with applications to optimal control problem

Bing Tan [a,b], Pongsakorn Sunthrayuth [c], Prasit Cholamjiak [d] and Yeol Je Cho [e,f]

[a]Institute of Fundamental and Frontier Sciences, University of Electronic Science and Technology of China, Chengdu, China; [b]Department of Mathematics, University of British Columbia, Kelowna, B.C., Canada; [c]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani, Thailand; [d]School of Science, University of Phayao, Phayao, Thailand; [e]Department of Mathematics Education and the RINS, Gyeongsang National University, Jinju, Korea; [f]Center for General Education, China Medical University, Taichung, Taiwan

ABSTRACT

In order to discover the minimum-norm solution of the pseudomonotone variational inequality problem in a real Hilbert space, we provide two variants of the inertial extragradient approach with a novel generalized adaptive step size. Two of the suggested algorithms make use of the projection and contraction methods. We demonstrate several strong convergence findings without requiring the prior knowledge of the Lipschitz constant of the mapping. Finally, we give a number of numerical examples that highlight the benefits and effectiveness of the suggested algorithms and how they may be used to solve the optimal control problem.

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Strong convergence; variational inequality problem; pseudomonotone mapping; minimum-norm solution; optimal control problem

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1. Introduction

The primary goal of this study is to construct several accelerated iterative methods with adaptive step sizes for finding the solutions of variational inequality problems in infinite-dimensional Hilbert spaces. Let $A : \mathcal{H} \to \mathcal{H}$ be an operator and let $\mathcal{H}$ be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. Take $C \subset \mathcal{H}$ is a nonempty, closed, and convex subset of $\mathcal{H}$. The *variational inequality problem* (shortly, VIP) is find $x^* \in C$ such that

$$\langle Ax^*, x - x^* \rangle \geq 0, \quad \forall\, x \in C. \tag{VIP}$$

Variational inequality theory provides a fundamental model for many areas; for example engineering, economics, traffic management, operations optimization, and mathematical programming, and it constructs a unified framework for many optimization problems (see, e.g. [1,6,22,28,42]). Therefore, the theory and solution methods of variational inequalities have received more and more attention from scholars.

CONTACT Pongsakorn Sunthrayuth pongsakorn_su@rmutt.ac.th Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

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Analysis of two versions of relaxed inertial algorithms with Bregman divergences for solving variational inequalities

Lateef Olakunle Jolaoso[1] · Pongsakorn Sunthrayuth[2] · Prasit Cholamjiak[3] · Yeol Je Cho[4,5]

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Abstract

In this paper, we introduce and analyze two new inertial-like algorithms with the Bregman divergences for solving the pseudomonotone variational inequality problem in a real Hilbert space. The first algorithm is inspired by the Halpern-type iteration and the subgradient extragradient method and the second algorithm is inspired by the Halpern-type iteration and Tseng's extragradient method. Under suitable conditions, we prove some strong convergence theorems of the proposed algorithms without assuming the Lipschitz continuity and the sequential weak continuity of the given mapping. Finally, we give some numerical experiments with various types of Bregman divergence to illustrate the main results. In fact, the results presented in this paper improve and generalize the related works in the literature.

Keywords Bregman divergence · Hilbert space · Strong convergence · Variational inequality problem · Pseudomonotone mapping

Mathematics Subject Classification 47H09 · 47H10 · 47J25 · 47J05

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✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

1 School of Mathematical Sciences, University of Southampton, Southampton SO17 1BJ, UK

2 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

3 School of Science, University of Phayao, Phayao 56000, Thailand

4 Department of Mathematics Education, Gyeongsang National University, Jinju, Korea

5 Center for General Education, China Medical University, Taichung 40402, Taiwan

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Research Article

The Analysis of Fractional-Order System Delay Differential Equations Using a Numerical Method

Pongsakorn Sunthrayuth,[1] **Hina M. Dutt**,[2] **Fazal Ghani**,[3] **and Mohammad Asif Arefin**[4]

[1]*Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, Thailand*
[2]*Department of Humanities and Sciences, School of Electrical Engineering and Computer Science (SEECS), National University of Sciences and Technology (NUST), Islamabad, Pakistan*
[3]*Department of Mathematics, Abdul Wali Khan University, Mardan 23200, Pakistan*
[4]*Department of Mathematics, Jashore University of Science and Technology, Jashore-7408, Bangladesh*

Correspondence should be addressed to Mohammad Asif Arefin; asif.math@just.edu.bd

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To solve fractional delay differential equation systems, the Laguerre Wavelets Method (LWM) is presented and coupled with the steps method in this article. Caputo fractional derivative is used in the proposed technique. The results show that the current procedure is accurate and reliable. Different nonlinear systems have been solved, and the results have been compared to the exact solution and different methods. Furthermore, it is clear from the figures that the LWM error converges quickly when compared to other approaches. When compared with the exact solution to other approaches, it is clear that LWM is more accurate and gets closer to the exact solution faster. Moreover, on the basis of the novelty and scientific importance, the present method can be extended to solve other nonlinear fractional-order delay differential equations.

1. Introduction

In 1965, a mathematician named L'Hopital asked Leibniz what would be the solution to the problem if the derivatives and integrals were fractional order. This L'Hopital question has resulted in the creation of new mathematical knowledge, but no one has been able to deal with it for a long time [1]. Mathematicians began to conduct study in the field of fractional derivatives, integration, and the development of a new field of fractional calculus after a period of time. In mathematics, this domain is known as fractional calculus, and it is a significant branch of mathematics that deals with the study of fractional derivatives and integration. Mathematicians have recently started working on fractional calculus because of its wide applications in all fields of research such as economics [2], viscoelastic materials [3], dynamics of interfaces between soft nanoparticles and rough substrates [4], continuum and statistical mechanics [5], solid mechanics [6], and many other topics.

Many natural problems can be solved using mathematical formulations by transforming physical facts into equation form. Differential equations (DEs) are a type of equation that is used to model a variety of phenomena. However, certain cases are too complicated to be solved using a differential equation. In this case, the researchers used fractional differential equations (FDEs), which are more accurate than differential equations with order integers in modelling the phenomenon. FDEs have realised the importance of real-world modelling challenges in recent years. Such as electrochemistry of corrosion [7], electrode-electrolyte polarization [8], heat conduction [9], optics and signal processing [10], diffusion wave [11], circuit systems [12], control theory of dynamical systems [6], probability and statistics [14, 15], fluid flow [16], and so on.

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Research Article

Solving Fractional-Order Diffusion Equations in a Plasma and Fluids via a Novel Transform

Pongsakorn Sunthrayuth[1] **Haifa A. Alyousef,**[2] **S. A. El-Tantawy**[3,4] **Adnan Khan,**[5] **and Noorolhuda Wyal**[6]

[1]*Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, Thailand*
[2]*Department of Physics, College of Science, Princess Nourah Bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia*
[3]*Department of Physics, Faculty of Science, Port Said University, Port Said 42521, Egypt*
[4]*Research Center for Physics (RCP), Department of Physics, Faculty of Science and Arts, Al-Mikhwah, Al-Baha University, Al Bahah 1988, Saudi Arabia*
[5]*Department of Mathematics, Abdul Wali Khan University Mardan, Mardan 23200, Pakistan*
[6]*Department of Mathematics, Polytechnical University of Kabul, Kabul, Afghanistan*

Correspondence should be addressed to Noorolhuda Wyal; noorolhuda.wyal@kpu.edu.af

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Motivated by the importance of diffusion equations in many physical situations in general and in plasma physics in particular, therefore, in this study, we try to find some novel solutions to fractional-order diffusion equations to explain many of the ambiguities about the phenomena in plasma physics and many other fields. In this article, we implement two well-known analytical methods for the solution of diffusion equations. We suggest the modified form of homotopy perturbation method and Adomian decomposition methods using Jafari-Yang transform. Furthermore, illustrative examples are introduced to show the accuracy of the proposed methods. It is observed that the proposed method solution has the desire rate of convergence toward the exact solution. The suggested method's main advantage is less number of calculations. The proposed methods give series form solution which converges quickly towards the exact solution. To show the reliability of the proposed method, we present some graphical representations of the exact and analytical results, which are in strong agreement with each other. The results we showed through graphs and tables for different fractional-order confirm that the results converge towards exact solution as the fractional-order tends towards integer-order. Moreover, it can solve physical problems having fractional order in different areas of applied sciences. Also, the proposed method helps many plasma physicists in modeling several nonlinear structures such as solitons, shocks, and rogue waves in different plasma systems.

1. Introduction

The integer-order differentiation operators are used to study local phenomena, whereas fractional-order operators are used to studying nonlocal phenomena [1]. The mathematical groundwork for fractional-order derivatives was laid by the collective struggles of various mathematicians, such as Riemann, Liouville, Caputo, Podlubny, Miller, and Ross. Afterward, numerous mathematicians dedicated their efforts to this area. Fractional calculus (FC) can be described as very successful in many phenomena in applied sciences, fluid mechanics, physics of plasmas [2, 3], and other biology utilising mathematical tools of FC. [4, 5]. Other numerous applications of FC in the field of science and technology are related to solid mechanics [6, 7], anomalous transport [8], continuum and statistical mechanics [9], economics

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Research Article

A Novel Multicriteria Decision-Making Approach for Einstein Weighted Average Operator under Pythagorean Fuzzy Hypersoft Environment

Pongsakorn Sunthrayuth,[1] Fahd Jarad,[2,3,4] Jihen Majdoubi,[5] Rana Muhammad Zulqarnain,[6] Aiyared Iampan,[7] and Imran Siddique[8]

[1]*Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand*
[2]*Department of Mathematics, Cankaya University, Etimesgut, Ankara, Turkey*
[3]*Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia*
[4]*Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan*
[5]*Department of Computer Science, College of Science and Humanities at Alghat Majmaah University, Al-Majmaah 11952, Saudi Arabia*
[6]*Department of Mathematics, University of Management and Technology, Lahore, Sialkot Campus, Pakistan*
[7]*Department of Mathematics, School of Science, University of Phayao, Mae Ka, Mueang, Phayao 56000, Thailand*
[8]*Department of Mathematics, University of Management and Technology, Lahore, Pakistan*

Correspondence should be addressed to Fahd Jarad; fahd@cankaya.edu.tr

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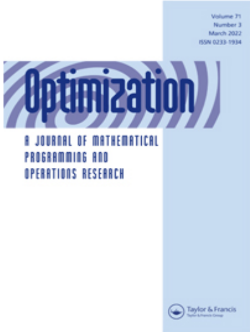
The experts used the Pythagorean fuzzy hypersoft set (PFHSS) in their research to discourse ambiguous and vague information in decision-making processes. The aggregation operator (AO) plays a prominent part in the sensitivity of the two forefront loops and eliminates anxiety from that perception. The PFHSS is the most influential and operative extension of the Pythagorean fuzzy soft set (PFSS), which handles the subparameterized values of alternatives. It is also a generalized form of Intuitionistic fuzzy hypersoft set (IFHSS) that provides better and more accurate assessments in the decision-making (DM) process. In this work, we present some operational laws for Pythagorean fuzzy hypersoft numbers (PFHSNs) and then formulate Pythagorean fuzzy hypersoft Einstein weighted average (PFHSEWA) operator based on developed operational laws. We discuss essential features such as idempotency, boundedness, and homogeneity for the proposed PFHSEWA operator. Furthermore, a DM approach has been developed based on the built-in operator to address multicriteria decision-making (MCDM) issues. A numerical case study of decision-making problems in real-life agricultural farming is considered to validate the settled technique's dominance and applicability. The consequences display that the planned model is more operative and consistent to handle inexact data based on PFHSS.

1. Introduction

In farming history, the subjugation of vegetations, wildlife, and the manufacturing and propagation methods used for high-yielding cultivation have been recorded. Farming started independently in numerous parts of the world, including a wide range of taxa. By 8000 BC, farming along the Nile was widely known. Around this time, farming developed autonomously in the Far East, most likely in China, and the main crop was rice instead of wheat. Modern agricultural practices result from an excessive water supply, extensive deforestation, and reduced soil fertility. Since there is lacking water to endure farming, it is compulsory to reexamine how to use essential water, land, and environmental resources to



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Weak and strong convergence results for solving monotone variational inequalities in reflexive Banach spaces

Jun Yang, Prasit Cholamjiak & Pongsakorn Sunthrayuth

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Weak and strong convergence results for solving monotone variational inequalities in reflexive Banach spaces

Jun Yang[a], Prasit Cholamjiak[b] and Pongsakorn Sunthrayuth [c]

[a]School of Mathematics and Information Science, Xianyang Normal University, Xianyang, People's Republic of China; [b]School of Science, University of Phayao, Phayao, Thailand; [c]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani, Thailand

ABSTRACT
In this paper, we introduce two modified Tseng's extragradient algorithms with a new generalized adaptive stepsize for solving monotone variational inequalities (VI) in reflexive Banach spaces. The advantage of our methods is that stepsizes do not require prior knowledge of the Lipschitz constant of the cost mapping. Based on Bregman projection-type methods, we prove weak and strong convergence of the proposed algorithms to a solution of VI. Some numerical experiments to show the efficiency of our methods including a comparison with related methods are provided.

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1. Introduction

Let E be a real Banach space with a norm $\|\cdot\|$ and E^* be a dual of E. We denote by $\langle x, f\rangle$ the value of $f \in E^*$ at $x \in E$, that is, $\langle x, f\rangle = f(x)$. Let C be a nonempty, closed and convex subset of E and $A : C \to E^*$ be a continuous mapping. The *variational inequalities* (VI) is to find a point $z \in C$ such that

$$\langle x - z, Az\rangle \geq 0 \quad \forall\, x \in C. \tag{1}$$

The solution set of VI (1) is denoted by $VI(C, A)$. Variational inequality theory is an important tool in physics, control theory, engineering, economics, management science, mathematical programming, and so on. Several iterative methods have been proposed for solving the variational inequalities. A classical method for solving the VI in a Hilbert space H is the *gradient projection method* which is given by

$$x_{n+1} = P_C(x_n - \lambda A x_n), \tag{2}$$

CONTACT Pongsakorn Sunthrayuth pongsakorn_su@rmutt.ac.th

Waves in Random and Complex Media

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Phenomena of thermo-sloutal time's relaxation in mixed convection Carreau fluid with heat sink/source

Muhammad Irfan, Pongsakorn Sunthrayuth, Amjad Ali Pasha, Muhammad Shoaib Anwar & Waqar Azeem Khan

WAVES IN RANDOM AND COMPLEX MEDIA
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Phenomena of thermo-sloutal time's relaxation in mixed convection Carreau fluid with heat sink/source

Muhammad Irfan[a], Pongsakorn Sunthrayuth[b], Amjad Ali Pasha[c], Muhammad Shoaib Anwar[d] and Waqar Azeem Khan[e,f]

[a]Department of Mathematical Sciences, Federal Urdu University of Arts, Sciences & Technology, Islamabad, Pakistan; [b]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathum Thani, Thailand; [c]Aerospace Engineering Department, King Abdulaziz University, Jeddah, Saudi Arabia; [d]Department of Mathematics, University of Jhang, Jhang, Pakistan; [e]Nonlinear Analysis and Applied Mathematics (NAAM) Research Group Department of Mathematics, Faculty of Science King AbdulAziz University, Jeddah, Saudi Arabia; [f]Department of Mathematics, Mohi-ud-Din Islamic University, Islamabad, Pakistan

ABSTRACT

Recently, with the fast growth of new engineering technologies various researchers have presented fantastic dynamism in sightseeing the heat propagation through a wave mechanism instead of essentially by dispersion. Further studies insist that this is not a little temperature phenomenon; however, relatively one which has probable vital physical uses. This topic caught exceptional thought because of its generous uses in industrial and current built-up processes. Thus, here advanced Cattaneo–Christov heat and mass fluxes have been reported for the study of thermo-sloutal time relations. Aspects of MHD, heat sink/soure, variable properties of mass diffusivity and thermal conductivity in mixed convection Carreau fluid have been elaborated. The bvp4c numerical process has been exploited for the solutions of influential factors graphically. The mixed convection factor improves the fluid velocity of Carreau fluid. Our outcomes report the diminishing performance for thermo-relaxation factor on temperature field and sloutal-relaxation factor in concentration field. The escalating performance of mass diffusivity factor has been detected on concentration field. Tables for comparison with former studies with good agreement have been also disclosed.

Abbreviations: α: ratio of stretching rates parameter ε_1: thermal conductivity factor ε_2: mass diffusivity factor HAM: homotopy analysis method M: magnetic factor ODEs: ordinary differential equations PDEs: partial differential equations Sc: Schmidt number

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Cattaneo–Christov double diffusions; Carreau fluid; mixed convection; heat sink/source; variable mass diffusivity

Nomenclature

$(u,\ v,\ w)$	velocity components
Γ	material constant
$(\rho c)_f$	heat capacity of fluid

CONTACT Muhammad Irfan mirfan@math.qau.edu.pk; Pongsakorn Sunthrayuth pongsakorn_su@rmutt.ac.th

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ORIGINAL ARTICLE

A New Self-Adaptive Method for the Multiple-Sets Split Common Null Point Problem in Banach Spaces

Truong Minh Tuyen[1] · **Prasit Cholamjiak**[2] · **Pongsakorn Sunthrayuth**[3]

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Abstract

In this paper, we study the multiple-sets split common null point problem (MSCNPP) in Banach spaces. We introduce a new self-adaptive algorithm for solving this problem. Under suitable conditions, we prove a strong convergence theorem of the proposed algorithm. An application of the main theorem to the multiple-sets split feasibility problem in Banach spaces is also presented. Finally, we provide the numerical experiments which show the efficiency and implementation of the proposed method.

Keywords Banach space · Strong convergence · Maximal monotone · Split common null point problem

Mathematics Subject Classification (2010) 47H09 · 47H10 · 47J25 · 47J05

1 Introduction

Throughout this paper, we denote by $\mathbb{N}$ the set of positive integers and by $\mathbb{R}$ the set of real numbers. Let E and F are p-uniformly convex Banach spaces which are also uniformly smooth. Let $A_i : E \multimap E^*$, $i = 1, 2, \ldots, M$ and $B_j : F \multimap F^*$, $j = 1, 2, \ldots, N$ be set-valued mappings and $T : E \to F$ be a bounded linear operator. Consider the following so-called *multiple-sets split common null point problem* (MSCNPP): Find $z \in E$ such that

$$z \in \left(\bigcap_{i=1}^{M} A_i^{-1}0\right) \cap T^{-1}\left(\bigcap_{j=1}^{N} B_j^{-1}0\right). \tag{1.1}$$

✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

[1] Department of Mathematics and Informatics, Thai Nguyen University of Sciences, Thai Nguyen, Viet Nam

[2] School of Science, University of Phayao, Phayao, 56000, Thailand

[3] Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, 12110, Thailand

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Analysis of non-singular fractional bioconvection and thermal memory with generalized Mittag-Leffler kernel

Muhammad Imran Asjad [a], Pongsakorn Sunthrayuth [b,*], Muhammad Danish Ikram [a], Taseer Muhammad [c], Ali Saleh Alshomrani [d]

[a] Department of Mathematics, University of Management and Technology Lahore, 54770, Pakistan
[b] Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand
[c] Department of Mathematics, College of Sciences, King Khalid University, Abha 61413, Saudi Arabia
[d] Mathematical Modelling and Applied Computation (MMAC) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

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ABSTRACT

This paper deals with the application of non-singular fractional operator in the bioconvection flow of a MHD viscous fluid for vertical surface. The Laplace transform method is used for dimensionless governing equations of momentum, energy and diffusion respectively. Classical governing model is extended to non-integer order approach with non-singular kernel which can be used to describe the memory for natural phenomena. The main advantage is to use this fractional operator can it measure the rate of change at all points of the considered interval, therefore, the present fractional operator incorporate the previous history/memory effects of any system. For the prediction of physical behavior of embedded parameters, some graphs are presented in the graphical section. At the end some remarkable results are found. It is found that non-singular fractional operator measures the memory better in comparison with singular fractional operator. Further, on comparison between different kinds of viscous fluid (Water, Air, Kerosene), it is found that temperature and velocity of air is higher than water and kerosene respectively. The results are validated with the recent published work.

1. Introduction

Bioconvection is a charming fact of fluid mechanics which is motivated by the spinning motion of microorganisms. The term bioconvection is characterized by hydrodynamics instability and patterns in suspensions of biased swimming microorganisms. The velocity and spatial range of fluid motions are typically considerable higher than those associated with the velocity and size of individual cells, resulting in rapid cell transfer and the formation of specialized cell concentration visualization patterns [1,2]. Biotechnology has evolved to integrate new and diverse sciences in the early 21st century introduced by Platt [3]. Bioconvection can be defined as the phenomenon of macroscopic convection motion of the fluid generated by the density gradient which was developed by directional collective swimming of microorganisms. Bioconvection can be found in wide range of applications such as biological applications and biomicrosystems, the pharmaceutical industry, biological polymer synthesis, environmentally-friendly applications, sustainable fuel cell technologies, microbial enhanced oil recovery, bio sensors, biotechnology and continuous refinements in mathematical modeling.

Recently, many researchers have presented precious works on bioconvection. Rao et al. [4] analyzed Darcy free convection with an isothermal upright cone with static apex half angle, indicating downward in a nanofluid-soaked porous medium. Khan et al. [5] examined couple stress nanofluid flow with magnetic effect and flow properties like radiation, activation energy and chemical reaction. Abdelmalek et al. [6] investigated the bioconvection movement of cross nanofluid under the magnetic dipole exposed to a cylinder numerically. Alshomrani et al. [7] studied the movement of non-Newtonian nanofluid with heat and mass transfer rates with bioconvection, activation energy and motile microorganisms numerically. Kuznetsov et al. [8] inspected bioconvection of gyrotactic motile microorganisms in a fluid saturated porous medium. Muhammad et al. [9] calculated a time-dependent movement of magnetized rheological Carreau nanofluid carrying microorganisms on a moving wedge. Mondal et al. [10] studied bioconvection movement of nanofluid holding gyrotactic microorganisms over an expanding wedge fixed in a porous medium with binary chemical reaction numerically. Furthermore readings on bioconvection can be seen in the references [11–20].

* Corresponding author.
E-mail address: pongsakorn_su@rmutt.ac.th (P. Sunthrayuth).

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Research article

Numerical solution of stochastic and fractional competition model in Caputo derivative using Newton method

Huang Meihua[1], **Pongsakorn Sunthrayuth**[2,*], **Amjad Ali Pasha**[3] **and Muhammad Altaf Khan**[4]

[1] School of Marxism Studies (Department of Public Education), Nantong Open University, Nantong, Jiangsu, China

[2] Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, 12110, Thailand

[3] Aerospace Engineering Department, King Abdulaziz University, Jeddah 21589, Saudi Arabia

[4] Institute for Groundwater Studies, Faculty of Natural and Agricultural Sciences, University of the Free State, South Africa

* **Correspondence:** Email: pongsakorn_su@rmutt.ac.th.

Abstract: Many useful numerical algorithms of the numerical solution are proposed due to the increasing interest of the researchers in fractional calculus. A new discretization of the competition model for the real statistical data of banking finance for the years 2004–2014 is presented. We use a novel numerical method that is more reliable and accurate which is introduced recently for the solution of ordinary differential equations numerically. We apply this approach to solve our model for the case of Caputo derivative. We apply the Caputo derivative on the competition system and obtain its numerical results. For the numerical solution of the competition model, we use the Newton polynomial approach and present in detail a novel numerical procedure. We utilize the numerical procedure and present various numerical results in the form of graphics. A comparison of the present method versus the predictor corrector method is presented, which shows the same solution behavior to the Newton Polynomial approach. We also suggest that the real data versus model provide good fitting for both the data for the fractional-order parameter value $\rho = 0.7$. Some more values of ρ are used to obtain graphical results. We also check the model in the stochastic version and show the model behaves well when fitting to the data.

Keywords: Caputo derivative; Newton polynomial approach; real data 2004–2014; numerical algorithm; stochastic version
Mathematics Subject Classification: 34A08, 37N30

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Research Article

Unsteady MHD Flow for Fractional Casson Channel Fluid in a Porous Medium: An Application of the Caputo-Fabrizio Time-Fractional Derivative

Pongsakorn Sunthrayuth[1] **A. A. Alderremy,**[2] **Fazal Ghani,**[3] **Ayékotan M. J. Tchalla**[4] **Shaban Aly,**[5] **and Yasser Elmasry**[6]

[1]*Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathum Thani, Thailand*
[2]*Department of Mathematics, Faculty of Science, King Khalid University, Abha 61413, Saudi Arabia*
[3]*Department of Mathematics, Abdul Wali Khan University Mardan, Pakistan*
[4]*Département de Mathématiques, Faculté des Sciences, Université de Lomé, 01 BP 1515, Lomé, Togo*
[5]*Department of Mathematics, Faculty of Science, Al-Azhar University, Assiut, Egypt*
[6]*Department of Mathematics, Faculty of Science, Mansoura University, Egypt*

Correspondence should be addressed to Ayékotan M. J. Tchalla; atchalla@univ-lome.tg

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Theoretically, this work describes the exact solutions of fractional Casson fluid through a channel under the effect of MHD and porous medium. The unsteady fluid motion of the bottom plate, which is confined by parallel but perpendicular sidewalls, supports the flow. By introducing the dimensionless parameters and variables, the momentum equation, as well as the initial and boundary conditions, has been transformed to a dimensionless form. A mix of Laplace and Fourier transformations is used to get the exact solution for the momentum equation. The constitutive equations for Caputo-Fabrizio's time-fractional derivative are also incorporated for recovering the exact solutions of the flow problem under consideration. After recovering the exact solutions for flow characteristics, three different cases at the surface of the bottom plate are discussed, by addressing the limiting cases under the influence of the side walls. Moreover, these solutions are captured graphically, and the effects of the Reynolds number Re, fractional parameter α, effective permeability K_{eff}, and dimensionless parameter for Casson fluid β on the fluid's motion are observed.

1. Introduction

Fractional calculus plays a critical part in the solving of complicated engineering issues. Because of its significance, a fractional model solution for flow issues is preferred by many scientists and researchers. A fraction model correctly depicts the motion of a flow issue when compared to ordinary differential equations (ODEs). It recovers an ODE's solution, which explains minute flow system fluctuations. Even for Newtonian fluids, the older scientists' operators including the Caputo operator employed a solitary kernel that resulted in complicated series solutions. In 2015, Caputo and Fabrizio [1] suggested a new fractional operator that may be employed in simple ways to solve this problem. Following that, a lot of scholars have applied this notion to numerous sorts of flows using varied geometries. Alshabanat et al. [2] proposed a new fractional derivative utilizing a nonsingular form kernel of exponential and trigonometric functions. Singh et al. [3] investigated a fractional epidemiological model for viral determination in the computer utilizing fractional derivatives and numerically solved the modeled issue using the iterative technique. Shah and Khan [4] have discussed heat transmission for an oscillating second-grade fluid upon a vertical surface by employing

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Impact of nanoparticle aggregation on heat transfer phenomena of second grade nanofluid flow over melting surface subject to homogeneous-heterogeneous reactions

Pongsakorn Sunthrayuth [a], Shaimaa A.M. Abdelmohsen [b], M.B. Rekha [c], K. R. Raghunatha [c], Ashraf M.M. Abdelbacki [d], M.R. Gorji [e], B.C. Prasannakumara [c,*]

[a] *Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, 12110, Thailand*
[b] *Department of Physics, College of Science, Princess Nourah Bint Abdulrahman University, P.O. Box 84428, Riyadh, 11671, Saudi Arabia*
[c] *Department of Studies and Research in Mathematics, Davanagere University, Davanagere, Karnataka, India*
[d] *Deanship of Skill Development, King Saud University, Riyadh, 11451, Saudi Arabia*
[e] *Faculty of Medicine and Health Sciences, Ghent University, Ghent, 9000, Belgium*

A R T I C L E I N F O

Keywords:
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Melting heat transfer
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A B S T R A C T

It is well known that the inclusion of a certain quantity of nanoparticles boosts the thermal conductivity of the nanofluid. The reason for this tremendous improvement is yet unknown. Consequently, finding the proper thermal effect of nanoscale particles requires an understanding of nanoparticle aggregation kinematics. The utilization of nanomaterials may be seen in a variety of technological and industrial applications. The influence of homogeneous and heterogeneous chemical reactions on an incompressible flow of second-grade nanofluid through a stretched cylinder with NP aggregation is investigated in this work. Similarity transformations are used to change partial differential equations (PDEs) into a system of ordinary differential equations (ODEs). The Runge Kutta Fehlberg fourth fifth-order (RKF 45) technique and shooting approach are used to numerically solve these ODEs. The influence of major elements on flow fields and heat transfer rates is investigated and addressed using graphical representations. The results suggest that the fluid flow without NPs aggregation has better heat transmission than when the melting parameter increases. Furthermore, the higher mass transfer for fluid flow with aggregation condition is detected for increased values of strength of heterogeneous and homogeneous reaction parameters.

Nomenclature:

α_1	Normal stress moduli
α	Thermal diffusivity
T_∞	Ambient temperature
k	Thermal conductivity

* Corresponding author.
E-mail addresses: pongsakorn_su@rmutt.ac.th (P. Sunthrayuth), shamohamed@pnu.edu.sa (S.A.M. Abdelmohsen), aabdelbacki@ksu.edu.sa (A.M.M. Abdelbacki), mohammad.rahimigorji@ugent.be, m69.rahimi@yahoo.com (M.R. Gorji), prasannakumarabc@davangereuniversity.ac.in (B.C. Prasannakumara).

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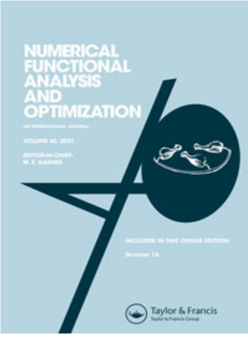

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Two New Inertial Algorithms for Solving Variational Inequalities in Reflexive Banach Spaces

Simeon Reich, Truong Minh Tuyen, Pongsakorn Sunthrayuth & Prasit Cholamjiak

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Two New Inertial Algorithms for Solving Variational Inequalities in Reflexive Banach Spaces

Simeon Reich[a], Truong Minh Tuyen[b], Pongsakorn Sunthrayuth[c], and Prasit Cholamjiak[d]

[a]Department of Mathematics, The Technion — Israel Institute of Technology, Haifa, Israel; [b]Department of Mathematics and Informatics, Thai Nguyen University of Sciences, Thai Nguyen, Vietnam; [c]Faculty of Science and Technology, Department of Mathematics and Computer Science, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani, Thailand; [d]School of Science, University of Phayao, Phayao, Thailand

ABSTRACT

The purpose of this paper is to introduce and analyze two inertial algorithms with self-adaptive stepsizes for solving variational inequalities in reflexive Banach spaces. Our algorithms are based on inertial hybrid and shrinking projection methods. Knowledge of the Lipschitz constant of the cost operator is not required. Under appropriate conditions, the strong convergence of the algorithms is established. We also present several numerical experiments which bring out the efficiency and the advantages of the proposed algorithms. Our work provides extensions of many known results from Hilbert spaces to reflexive Banach spaces.

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Bregman projection; extragradient method; Legendre function; reflexive banach space; strong convergence

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1. Introduction

Let E be a real Banach space with norm $||\cdot||$ and let E^* be the dual of E. We denote by $\langle x, f\rangle$ the value of $f \in E^*$ at $x \in E$, that is, $\langle x, f\rangle = f(x)$. Let C be a nonempty, closed and convex subset of E and $A : C \to E^*$ be a continuous mapping. The *variational inequality problem* (VIP) is to find a point $z \in C$ such that

$$\langle x - z, Az\rangle \geq 0 \quad \forall x \in C. \tag{1.1}$$

The solution set of VIP is denoted by $VI(C, A)$. It is well known that the VIP is a general problem formulation that encompasses a plethora of mathematical problems, including systems of nonlinear equations, optimization

CONTACT Truong Minh Tuyen tuyentm@tnus.edu.vn Department of Mathematics and Informatics, Thai Nguyen University of Sciences, Thai Nguyen, Vietnam; Pongsakorn Sunthrayuth pongsakorn_su@rmutt.ac.th Faculty of Science and Technology, Department of Mathematics and Computer Science, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani, Thailand.

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An iterative algorithm with inertial technique for solving the split common null point problem in Banach spaces

Yan Tang* and Pongsakorn Sunthrayuth†,‡

*College of Mathematics and Statistics
Chongqing Technology and Business University
Chongqing, P. R. China

†Department of Mathematics and Computer Science
Faculty of Science and Technology
Rajamangala University of Technology Thanyaburi (RMUTT)
Thanyaburi, Pathumthani 12110, Thailand
‡pongsakorn_su@rmutt.ac.th

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In this work, we introduce a modified inertial algorithm for solving the split common null point problem without the prior knowledge of the operator norms in Banach spaces. The strong convergence theorem of our method is proved under suitable assumptions. We apply our result to the split feasibility problem, split equilibrium problem and split minimization problem. Finally, we provide some numerical experiments including compressed sensing to illustrate the performances of the proposed method. The result presented in this paper improves and generalizes many recent important results in the literature.

Keywords: Inertial method; Banach space; strong convergence; resolvent operator.

AMS Subject Classification: 47H09, 47H10, 47J25, 47J05

1. Introduction

Let H_1 and H_2 be two real Hilbert spaces. Let $A : H_1 \multimap H_1$ and $B : H_2 \multimap H_2$ be two set-valued operators and let $T : H_1 \to H_2$ be a bounded linear operator with its adjoint operator T^* of T. Byrne *et al.* [8] considered the following so-called *split common null point problem* (SCNPP): find $x^* \in H_1$ such that

$$x^* \in A^{-1}0 \cap T^{-1}(B^{-1}0). \tag{1.1}$$

‡Corresponding author.

CONVERGENCE RESULTS OF ITERATIVE ALGORITHMS FOR THE SUM OF TWO MONOTONE OPERATORS IN REFLEXIVE BANACH SPACES

Yan Tang, Chongqing, Ratthaprom Promkam, Pathumthani,
Prasit Cholamjiak, Phayao, Pongsakorn Sunthrayuth, Pathumthani

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Abstract. The aim of this paper is to propose two modified forward-backward splitting algorithms for zeros of the sum of a maximal monotone operator and a Bregman inverse strongly monotone operator in reflexive Banach spaces. We prove weak and strong convergence theorems of the generated sequences by the proposed methods under some suitable conditions. We apply our results to study the variational inequality problem and the equilibrium problem. Finally, a numerical example is given to illustrate the proposed methods. The results presented in this paper improve and generalize many known results in recent literature.

Keywords: maximal operator; Bregman distance; reflexive Banach space; weak convergence; strong convergence

MSC 2020: 47H09, 47H10, 47J25, 47J05

1. Introduction

Let E be a real Banach space with its dual space E^*. We study the so-called *quasi-inclusion problem*: find $z \in E$ such that

$$0 \in (A + B)z, \tag{1.1}$$

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ORIGINAL PAPER

A Generalized Self-Adaptive Algorithm for the Split Feasibility Problem in Banach Spaces

Pongsakorn Sunthrayuth[1] · Truong Minh Tuyen[2]

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Abstract
In this paper, we propose a generalized self-adaptive method for solving the multiple-set split feasibility problem in the framework of certain Banach spaces. Under some suitable conditions, we prove the strong convergence of the sequence generated by our method with a new way to select the step-sizes without prior knowledge of the operator norm. Several numerical experiments to illustrate the convergence behavior are presented. The results presented in this paper improve and extend the corresponding results in the literature.

Keywords Metric projection · Banach space · Strong convergence · Self-adaptive method · Multiple-set split feasibility problem

Mathematics Subject Classification 47H09 · 47H10 · 47J25 · 47J05

1 Introduction

Let E and F be two real p-uniformly convex Banach spaces which are also uniformly smooth. Let C_i, $i = 1, 2, \ldots, M$ and Q_j, $j = 1, 2, \ldots, N$ be nonempty, closed and convex subsets of E and F, respectively. Let $A : E \to F$ be a bounded linear operator with its adjoint $A^* : F^* \to E^*$. We consider the following so-called *multiple-set split*

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✉ Truong Minh Tuyen
tuyentm@tnus.edu.vn

Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

1 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

2 Department of Mathematics and Informatics, Thai Nguyen University of Sciences, Thai Nguyen, Vietnam

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ORIGINAL RESEARCH

An inertial self-adaptive algorithm for the generalized split common null point problem in Hilbert spaces

Truong Minh Tuyen[1] · **Pongsakorn Sunthrayuth**[2] · **Nguyen Minh Trang**[3]

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Abstract

In this paper, we propose an inertial self-adaptive algorithm for solving the generalized split common null point problem introduced by Reich and Tuyen (Optimization 69(5): 1013–1038, 2020). The strong convergence theorem of our proposed method is established in real Hilbert spaces. As applications, we apply our main result to the generalized split feasibility problem, the generalized split equilibrium problem and the generalized split minimization problem. Finally, we provide numerical experiments to show the efficiency and advantage of the proposed method.

Keywords Maximal monotone operator · Hilbert space · Strong convergence · Self adaptive method

Mathematics Subject Classification 47H09 · 47H10 · 47J25 · 47J05

1 Introduction

Let C and Q be nonempty, closed and convex subsets of real Hilbert spaces H_1 and H_2, respectively. Let $T : H_1 \to H_2$ be a bounded linear operator with its adjoint T^* and T. Recall the following *split feasibility problem* (SFP): find an element

$$z \in C \text{ such that } Tz \in Q. \tag{1.1}$$

The SFP was first introduced by Censor and Elfving [9] in 1994 for modeling inverse problems which arise from phase retrievals, medical image reconstruction and recently in modeling of intensity modulated radiation therapy. Moreover, it plays an important role in medical image reconstruction and signal processing (see [6, 7]).

✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

1 Department of Mathematics and Informatics, Thai Nguyen University of Sciences, Thai Nguyen, Viet Nam

2 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

3 Faculty of International training, Thai Nguyen University of Technology, Thai Nguyen, Vietnam

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ORIGINAL RESEARCH

New Bregman projection methods for solving pseudo-monotone variational inequality problem

Pongsakorn Sunthrayuth[1] · Lateef Olakunle Jolaoso[2] · Prasit Cholamjiak[3]

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Abstract
In this work, we introduce two Bregman projection algorithms with self-adaptive stepsize for solving pseudo-monotone variational inequality problem in a Hilbert space. The weak and strong convergence theorems are established without the prior knowledge of Lipschitz constant of the cost operator. The convergence behavior of the proposed algorithms with various functions of the Bregman distance are presented. More so, the performance and efficiency of our methods are compared to other related methods in the literature.

Keywords Bregman projection · Hilbert space · Strong convergence · Variational inequality · Pseudo-monotone mapping

Mathematics Subject Classification 47H09 · 47H10 · 47J25 · 47J05

1 Introduction

Let H be a real Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and the norm $\| \cdot \|$. Let C be a nonempty, closed and convex subset of H and $A : C \to H$ be a given mapping. The *variational inequality problem* (VIP) is to find a point $z \in C$ such that

$$\langle Az, x - z \rangle \geq 0, \ \forall x \in C. \tag{1.1}$$

✉ Prasit Cholamjiak
prasitch2008@yahoo.com

1 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

2 Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Pretoria, South Africa

3 School of Science, University of Phayao, Mae Ka, Phayao 56000, Thailand

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Mann-type algorithms for solving the monotone inclusion problem and the fixed point problem in reflexive Banach spaces

Pongsakorn Sunthrayuth[1] · Nattawut Pholasa[2] · Prasit Cholamjiak[2]

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Abstract
In this paper, we introduce two algorithms for finding a common solution of the monotone inclusion problem and the fixed point problem for a relatively nonexpansive mapping in reflexive Banach spaces. The weak convergence results for both algorithms are established without the prior knowledge of the Lipschitz constant of the mapping. An application to the variational inequality problem is considered. Finally, some numerical experiments of the proposed algorithms including comparisons with other algorithms are provided.

Keywords Maximal monotone operator · Banach space · Weak convergence · Fixed point problem

Mathematics Subject Classification 47H09 · 47H10 · 47J25 · 47J05

1 Introduction

Let E be a real Banach space with its dual space E^*. Let $A : E \to E^*$ be a monotone operator and $B : E \to 2^{E^*}$ be a maximal monotone operator. The *monotone inclusion problem* is to find an element $x^* \in E$ such that

$$0 \in (A + B)x^*. \tag{1.1}$$

✉ Prasit Cholamjiak
prasitch2008@yahoo.com

[1] Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

[2] School of Science, University of Phayao, Phayao 56000, Thailand

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symmetry

Article

The Comparative Study for Solving Fractional-Order Fornberg–Whitham Equation via ρ-Laplace Transform

Pongsakorn Sunthrayuth [1], Ahmed M. Zidan [2,3], Shao-Wen Yao [4,*], Rasool Shah [5] and Mustafa Inc [6,7,8,*]

[1] Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani 12110, Thailand; pongsakorn_su@rmutt.ac.th
[2] Department of Mathematics, College of Science, King Khalid University, P.O. Box 9004, Abha 61413, Saudi Arabia; ahmoahmed@kku.edu.sa
[3] Department of Mathematics, Faculty of Science, Al-Azhar University, Assuit 71511, Egypt
[4] School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo 454000, China
[5] Department of Mathematics, Abdul Wali khan University, Mardan 23200, Pakistan; rasoolshahawkum@gmail.com
[6] Department of Computer Engineering, Biruni University, Istanbul 34096, Turkey
[7] Department of Mathematics, Science Faculty, Firat University, Elazig 23119, Turkey
[8] Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan
* Correspondence: yaoshaowen@hpu.edu.cn (S.-W.Y.); minc@firat.edu.tr (M.I.)

Abstract: In this article, we also introduced two well-known computational techniques for solving the time-fractional Fornberg–Whitham equations. The methods suggested are the modified form of the variational iteration and Adomian decomposition techniques by ρ-Laplace. Furthermore, an illustrative scheme is introduced to verify the accuracy of the available methods. The graphical representation of the exact and derived results is presented to show the suggested approaches reliability. The comparative solution analysis via graphs also represented the higher reliability and accuracy of the current techniques.

Keywords: ρ-Laplace variational iteration method; ρ-Laplace decomposition method; partial differential equation; caputo operator; fractional Fornberg–Whitham equation (FWE)

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1. Introduction

With engineering and science development, non-linear evolution models have been analyzed as the problems to define physical phenomena in plasma waves, fluid mechanics, chemical physics, solid-state physics, etc. For the last few years, therefore, a lot of interest has been paid to the result (both numerical and analytical) of these significant models [1–4]. Different methods are available in the literature for the approximate and exact results of these models. In current years, fractional calculus (FC) applied in many phenomena in applied sciences, fluid mechanics, physics and other biology can be described as very effective using mathematical tools of FC. The fractional derivatives have occurred in many applied sciences equations such as reaction and diffusion processes, system identification, velocity signal analysis, relaxation of damping behaviour fabrics and creeping of polymer composites [5–8].

The investigation of non-linear wave models and their application is significant in different areas of engineering. Travelling wave notions are between the most attractive results for non-linear fractional-order partial differential equations (NLFPDEs). NLFPDEs are usually identified as mechanical processes and complex physical. Therefore, it is important to get exact results for non-linear time-fractional partial differential equations [9–12]. Overall, travelling wave results are between the exciting forms of products for NFPDEs. On the other hand, other NLFPDEs, such as the Camassa–Holm or the Kortewegde–Vries equa-

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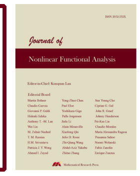

A MODIFIED POPOV'S SUBGRADIENT EXTRAGRADIENT METHOD FOR VARIATIONAL INEQUALITIES IN BANACH SPACES

PONGSAKORN SUNTHRAYUTH[1,*], HABIB UR REHMAN[2], POOM KUMAM[2]

[1]Department of Mathematics and Computer Science, Faculty of Science and Technology,
Rajamangala University of Technology Thanyaburi (RMUTT),
39 Rangsit-Nakhonnayok Rd., Klong 6, Thanyaburi, Pathumthani, 12110, Thailand
[2]KMUTT Fixed Point Research Laboratory, KMUTT-Fixed Point Theory and Applications Research Group,
SCL 802 Fixed Point Laboratory, Department of Mathematics, Faculty of Science,
King Mongkut's University of Technology Thonburi (KMUTT), Thrung Khru, Bangkok 10140, Thailand

Abstract. In this paper, we propose a new modification of Popov's subgradient extragradient method for solving the variational inequality problem involving pseudo-monotone and Lipschitz-continuous mappings in the framework of Banach spaces. The weak convergence theorem of the proposed method is established without the knowledge of the Lipschitz constant of the Lipschitz continuous mapping. Finally, we provide several numerical experiments of the proposed method including comparisons with other related methods. Our result generalizes and extends many related results in the literature from Hilbert spaces to Banach spaces.

Keywords. Popov's method; Variational inequality problem; Pseudo-monotone mapping; Banach space.

1. Introduction

Let E be a real Banach space with norm $\|\cdot\|$ and let E^* be the dual space of E. We denote by $\langle x, f\rangle$ the value of $f \in E^*$ at $x \in E$, that is, $\langle x, f\rangle = f(x)$. Let C be a nonempty, closed and convex subset of E and let $A : C \to E^*$ be a mapping. The *variational inequality problem* (VIP) is to find an element $z \in C$ such that

$$\langle x - z, Az\rangle \geq 0, \quad \forall x \in C. \tag{1.1}$$

The solution set of the VIP is denoted by $VI(C,A)$. The VIP has been studied widely in many real-world problems, such as, artificial intelligence, computer science, control engineering, management science and operations research, and differential equations, fluid flow through

*Corresponding author.

E-mail addresses: pongsakorn_su@rmutt.ac.th (P. Sunthrayuth), hrehman.hed@gmail.com (H.U. Rehman), poom.kum@kmutt.ac.th (P. Kumam)

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Research article

Modified Tseng's splitting algorithms for the sum of two monotone operators in Banach spaces

Jun Yang[1], **Prasit Cholamjiak**[2] **and Pongsakorn Sunthrayuth**[3,*]

[1] School of Mathematics and Information Science, Xianyang Normal University, Xianyang 712000, Shaanxi, China

[2] School of Science, University of Phayao, Phayao 56000, Thailand

[3] Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

* **Correspondence:** Email: pongsakorn_su@rmutt.ac.th; Tel: +6625494139; Fax: +6625494119.

Abstract: In this work, we introduce two modified Tseng's splitting algorithms with a new non-monotone adaptive step size for solving monotone inclusion problem in the framework of Banach spaces. Under some mild assumptions, we establish the weak and strong convergence results of the proposed algorithms. Moreover, we also apply our results to variational inequality problem, convex minimization problem and signal recovery, and provide several numerical experiments including comparisons with other related algorithms.

Keywords: maximal monotone operator; Banach space; strong convergence; self adaptive method
Mathematics Subject Classification: 47H09, 47H10, 47J25

1. Introduction

Let E be a real Banach space with its dual space E^*. In this paper, we study the so-called *monotone inclusion problem*:

$$\text{find } z \in E \ \text{ such that } \ 0 \in (A + B)z, \tag{1.1}$$

where $A : E \to E^*$ is a single mapping and $B : E \to 2^{E^*}$ is a multi-valued mapping. The set of solutions of the problem (1.1) is denoted by $(A + B)^{-1}0 := \{x \in E : 0 \in (A + B)x\}$. This problem draws much attention since it stands at the core of many mathematical problems, such as: variational inequalities, split feasibility problem and minimization problem with applications in machine learning, statistical regression, image processing and signal recovery (see [17, 33, 44]).

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Dedicated to Prof. Suthep Suantai on the occasion of his 60^{th} anniversary

Iterative Methods for Solving the Monotone Inclusion Problem and the Fixed Point Problem in Banach Spaces

Prasit Cholamjiak[1], Pongsakorn Sunthrayuth[2], Akarate Singta[2] and Kanikar Muangchoo[3,*]

[1] *School of Science, University of Phayao, Phayao 56000, Thailand*
e-mail : prasitch2008@yahoo.com (P. Cholamjiak)

[2] *Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT) 39 Rangsit-Nakhonnayok Rd., Klong 6, Thanyaburi, Pathumthani, 12110, Thailand*
e-mail : pongsakorn_su@rmutt.ac.th (P. Sunthrayuth)

[3] *Faculty of Science and Technology, Rajamangala University of Technology Phra Nakhon (RMUTP) Pracharat 1 Road, Wongsawang, Bang Sue, Bangkok 10800, Thailand*
e-mail : kanikar.m@rmutp.ac.th (K. Muangchoo)

Abstract In this work, we propose two iterative algorithms for solving the monotone inclusion problem and the fixed point problem of a relatively nonexpansive mapping in the framework of Banach spaces. We prove the strong convergence theorems of the proposed algorithms under some suitable assumptions. Furthermore, some numerical experiments of proposed algorithms to compressed sensing in signal recovery are presented. Our results improve and generalize many recent and important results in the literature.

MSC: 47H09; 47H10; 47J25; 47J05

Keywords: maximal monotone operator; Banach space; strong convergence; extragradient algorithm

Submission date: 28.05.2020 / Acceptance date: 11.09.2020

1. Introduction

Let E be a real Banach space. Consider the following so-called *monotone inclusion problem*: find $x^* \in E$ such that

$$0 \in (A + B)x^*, \tag{1.1}$$

where $A : E \to E$ and $B : E \to 2^E$ are single and set-valued mappings, respectively and 0 is a zero vector in E. In particular case, when $A = 0$, then the problem (1.1) becomes the inclusion problem introduced by Rockafellar [1] and when $E = \mathbb{R}^n$, then the problem

*Corresponding author.

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Strong convergence of a generalized forward–backward splitting method in reflexive Banach spaces

Truong Minh Tuyen , Ratthaprom Promkam & Pongsakorn Sunthrayuth

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Strong convergence of a generalized forward–backward splitting method in reflexive Banach spaces

Truong Minh Tuyen[a], Ratthaprom Promkam [b] and Pongsakorn Sunthrayuth [b]

[a]Department of Mathematics and Informatics, Thai Nguyen University of Sciences, Thai Nguyen, Viet Nam; [b]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT) Thanyaburi, Pathumthani, Thailand

ABSTRACT

In this paper, we study the so-called *generalized monotone quasi-inclusion problem* which is a generalization and extension of well-known monotone quasi-inclusion problem. We propose a forward–backward splitting method for solving this problem in the framework of reflexive Banach spaces. Based on Bregman distance function, we prove a strong convergence result of the proposed algorithm to a common zero of the problem. As an application, we apply the main result to the variational inequality problem. Finally, we provide some numerical examples to demonstrate our algorithm performance. The results presented in this paper improve and extend many known results in the literature.

ARTICLE HISTORY

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KEYWORDS

Maximal monotone operator; Legendre function; reflexive banach space; strong convergence; Bregman inverse strongly monotone

2010 MATHEMATICS SUBJECT CLASSIFICATIONS

47H09; 47H10; 47J25; 47J05

1. Introduction

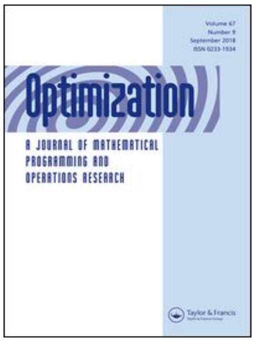
Let E be a real Banach space. Let $A : E \to E$ and $B : E \multimap E$ be single and set-valued operators, respectively. Consider the following so-called *monotone quasi-inclusion problem*:

$$\text{find } z \in E \text{ such that } 0 \in (A + B)z, \tag{1}$$

where 0 is the zero vector in E. The solutions set of the problem (1) is denoted by $(A + B)^{-1}0 = \{x \in E : 0 \in (A + B)x\}$. Many practical nonlinear problems arising in applied sciences such as in image recovery, signal processing and machine learning can be formulated as this problem (see [1–3]). Moreover, this problem includes the core of many mathematical problems, as special cases, such as: variational inequalities, split feasibility problem, minimization problem, Nash equilibrium problem in noncooperative games and so on (see [4–6]).

A well-known method for approximating a solution of the problem (1) is the *forward–backward splitting algorithm* which was introduced in [7,8]. This

CONTACT Pongsakorn Sunthrayuth pongsakorn_su@rmutt.ac.th



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The generalized viscosity explicit rules for solving variational inclusion problems in Banach spaces

Prasit Cholamjiak , Nattawut Pholasa , Suthep Suantai & Pongsakorn Sunthrayuth

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The generalized viscosity explicit rules for solving variational inclusion problems in Banach spaces

Prasit Cholamjiak[a], Nattawut Pholasa[a], Suthep Suantai[b] and Pongsakorn Sunthrayuth[c]

[a]School of Science, University of Phayao, Phayao, Thailand; [b]Research Center in Mathematics and Applied Mathematics, Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai, Thailand; [c]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, Thailand

ABSTRACT

In this paper, we propose a generalized viscosity explicit method for finding zeros of the sum of two accretive operators in the framework of Banach spaces. The strong convergence theorem of such method is proved under some suitable assumption on the parameters. As applications, we apply our main result to the variational inequality problem, the convex minimization problem and the split feasibility problem. The numerical experiments to illustrate the behaviour of the proposed method including compare it with other methods are also presented.

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Received 15 April 2019
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KEYWORDS
m-accretive operator; forward–backward algorithm; Banach space; strong convergence

2010 MATHEMATICS SUBJECT CLASSIFICATIONS
47H09; 47H10; 47J25; 47J05

1. Introduction

The starting point of this paper, we consider initial value problem (IVP) in the following form:

$$x'(t) = f\big(x(t)\big), \quad x(t_0) = x_0. \tag{1}$$

In real life, many mathematical model have been formulated as this problem. It is well known that most of ordinary differential equations (ODEs) are not analytically solvable. Numerical methods have become a powerful method for numerically solving time-dependent ordinary and partial differential equations, as is required in computer simulations of physical processes such as groundwater flow and the wave equation. One of famous method is known as *implicit midpoint method* (or modified Euler's method) (see [1–3] for more detail). Given a time interval $[t_0, T]$, the method firstly computes the step size $h = (T - t_0)/N$, where N is the number of steps of h and select the mesh $\{t_n\}_{n=0}^N$ of time steps $t_n \in [t_0, T]$, through the formula $t_n = t_0 + nh$ for $n = 0, 1, \ldots, N - 1$. It provides to generate a sequence $\{y_n\}_{n=0}^N$ of approximation of solution at each time

CONTACT Pongsakorn Sunthrayuth pongsakorn_su@rmutt.ac.th

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STRONG CONVERGENCE OF A GENERAL VISCOSITY EXPLICIT RULE FOR THE SUM OF TWO MONOTONE OPERATORS IN HILBERT SPACES

Prasit Cholamjiak[1], Suthep Suantai[2]
and Pongsakorn Sunthrayuth[3,†]

Abstract In this paper, we study a general viscosity explicit rule for approximating the solutions of the variational inclusion problem for the sum of two monotone operators. We then prove its strong convergence under some new conditions on the parameters in the framework of Hilbert spaces. As applications, we apply our main result to the split feasibility problem and the LASSO problem. We also give some numerical examples to support our main result. The results presented in this paper extend and improve the corresponding results in the literature.

Keywords Monotone operator, Hilbert space, strong convergence, iterative method.

MSC(2010) 47H09, 47H10, 47J25, 47J05.

1. Introduction

Let H be a real Hilbert space. In this paper, we study the *variational inclusion problem* (VIP) which is the problem of finding $z \in H$ such that

$$0_H \in (A+B)z \tag{1.1}$$

where $A : H \to H$ is an operator, $B : H \multimap H$ is a set-valued operator and 0_H is a zero vector in H. The set of solutions of VIP is denoted by $(A+B)^{-1}0_H$.

It is known that the variational inclusion problem is a generalization of variational inequalities, equilibrium problem, split feasibility problem, convex minimization problem and linear inverse problem (see [23, 33, 37]). Moreover, the variational inclusion problem has many applications in applied sciences, engineering, economics and medical sciences especially image and signal processing, statistical regression and machine learning (see, *e.g.* [6, 34, 39]).

†the corresponding author. Email address: pongsakorn_su@rmutt.ac.th (P. Sunthrayuth)
[1]School of Science, University of Phayao, Phayao 56000, Thailand
[2]Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand
[3]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani, Thailand

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ORIGINAL PAPER

An explicit parallel algorithm for solving variational inclusion problem and fixed point problem in Banach spaces

Prasit Cholamjiak[1] · **Suthep Suantai**[2] · **Pongsakorn Sunthrayuth**[3]

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Abstract

In this article, we introduce an explicit parallel algorithm for finding a common element of zeros of the sum of two accretive operators and the set of fixed point of a nonexpansive mapping in the framework of Banach spaces. We prove its strong convergence under some mild conditions. Finally, we provide some applications to the main result. The results presented in this paper extend and improve the corresponding results in the literature.

Keywords Variational inclusion · Banach space · Strong convergence · Accretive operator

Mathematics Subject Classification 47H09; 47H10 · 47H17

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✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

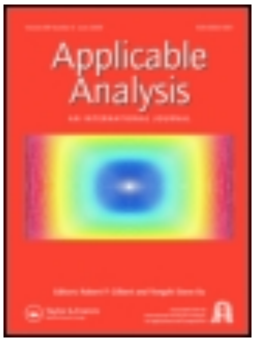
Prasit Cholamjiak
prasitch2008@yahoo.com

Suthep Suantai
suthep.s@cmu.ac.th

1 School of Science, University of Phayao, Phayao 56000, Thailand

2 Research Center in Mathematics and Applied Mathematics, Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand

3 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani 12110, Thailand



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A modified extragradient method for variational inclusion and fixed point problems in Banach spaces

Pongsakorn Sunthrayuth & Prasit Cholamjiak

APPLICABLE ANALYSIS
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A modified extragradient method for variational inclusion and fixed point problems in Banach spaces

Pongsakorn Sunthrayuth[a] and Prasit Cholamjiak[b]

[a]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Thailand; [b]School of Science, University of Phayao, Phayao, Thailand

ABSTRACT
In this work, we introduce a modified extragradient method for solving the fixed point problem of a nonexpansive mapping and the variational inclusion problem for two accretive operators in the framework of Banach spaces. We then prove its strong convergence under certain assumptions imposed on the parameters. As applications, we apply our main result to the variational inequality problem, split feasibility problem and the LASSO problem.

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KEYWORDS
m-accretive operator; Banach space; strong convergence; fixed point problem; variational inclusion

2010 MATHEMATICS SUBJECT CLASSIFICATIONS
47H09; 47H10; 47J25; 47J05

1. Introduction

Variational inequality theory has been studied widely in several branches of pure and applied sciences. In particular, applications of variational inequalities span as diverse disciplines as differential equations, time-optimal control, optimization, mathematical programming, mechanics, economic and other applied science problems. Let C be a nonempty, closed and convex subset of a real Hilbert space H. The *variational inequality problem* is to find a point $x^* \in C$ such that

$$\langle Ax^*, y - x^*\rangle \geq 0, \quad \forall y \in C, \tag{1}$$

where $A : C \to H$ is a mapping. The set of solutions of problem (1) is denoted by $VI(C, A)$. In recent years, several methods have been invented and modified for solving the variational inequality problem.

A simple method for solving problem (1) is *projection method* which is defined by the following manner: For a given $x_0 \in C$ and

$$x_{n+1} = P_C(x_n - \lambda Ax_n),\ \forall n \geq 0, \tag{2}$$

where P_C is the metric projection of H into C, λ is a positive real number. In fact, this method requires a slightly strong assumption that operators are strongly monotone or inverse strongly monotone [1].

To avoid this assumption, Korpelevich [2] (see also [3]) introduced the *extragradient method* for solving saddle point problems, and later, this method was successfully studied and extended for

CONTACT Prasit Cholamjiak prasitch2008@yahoo.com

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Article

Convergence Theorems for Generalized Viscosity Explicit Methods for Nonexpansive Mappings in Banach Spaces and Some Applications

Pongsakorn Sunthrayuth [1], Nuttapol Pakkaranang [2,3], Poom Kumam [2,3], Phatiphat Thounthong [4,*] and Prasit Cholamjiak [5,*]

1 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand; pongsakorn_su@rmutt.ac.th

2 KMUTTFixed Point Research Laboratory, Room SCL 802 Fixed Point Laboratory, Science Laboratory Building, Department of Mathematics, Faculty of Science, King Mongkut's University of Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thung Khru, Bangkok 10140, Thailand; nuttapol.pakka@gmail.com (N.P.); poom.kum@kmutt.ac.th (P.K.)

3 KMUTT-Fixed Point Theory and Applications Research Group (KMUTT-FPTA), Theoretical and Computational Science Center (TaCS), Science Laboratory Building, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thung Khru, Bangkok 10140, Thailand

4 Renewable Energy Research Centre & Department of Teacher Training in Electrical Engineering, Faculty of Technical Education, King Mongkut's University of Technology North Bangkok (KMUTNB), Wongsawang, Bangsue, Bangkok 10800, Thailand

5 School of Science, University of Phayao, Phayao 56000, Thailand

* Correspondence: phatiphat.t@fte.kmutnb.ac.th (P.T.); prasitch2008@yahoo.com (P.C.)

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Abstract: In this paper, we introduce a generalized viscosity explicit method (GVEM) for nonexpansive mappings in the setting of Banach spaces and, under some new techniques and mild assumptions on the control conditions, prove some strong convergence theorems for the proposed method, which converge to a fixed point of the given mapping and a solution of the variational inequality. As applications, we apply our main results to show the existence of fixed points of strict pseudo-contractions and periodic solutions of nonlinear evolution equations and Fredholm integral equations. Finally, we give some numerical examples to illustrate the efficiency and implementation of our method.

Keywords: nonexpansive mapping; Banach space; strong convergence; viscosity iterative method; nonlinear evolution equation; Fredholm integral equation

MSC: 47H09; 47H10; 47J25; 47J05

1. Introduction

In the real world, many engineering and science problems can be reformulated as ordinary differential equations. Several numerical methods have been developed for solving ordinary differential equations (ODEs) by numerous authors. The major method in order to solve ODEs is the implicit midpoint rule, also well known as the second-order Runge–Kutta method or improved the Euler method. It is a forceful numerical method for numerically solving ODEs (in particular, stiff equations) (see [1–6]) and differential algebraic equations (see [4]). Consider the following initial value problem for the following time-dependent ordinary differential equation:

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An iterative method with residual vectors for solving the fixed point and the split inclusion problems in Banach spaces

Prasit Cholamjiak[1] · Suthep Suantai[2] · Pongsakorn Sunthrayuth[3]

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Abstract

In this paper, we propose an iterative technique with residual vectors for finding a common element of the set of fixed points of a relatively nonexpansive mapping and the set of solutions of a split inclusion problem (SIP) with a way of selecting the stepsizes without prior knowledge of the operator norm in the framework of p-uniformly convex and uniformly smooth Banach spaces. Then strong convergence of the proposed algorithm to a common element of the above two sets is proved. As applications, we apply our result to find the set of common fixed points of a family of mappings which is also a solution of the SIP. We also give a numerical example and demonstrate the efficiency of the proposed algorithm. The results presented in this paper improve and generalize many recent important results in the literature.

Keywords Resolvent operator · Relatively nonexpansive mapping · Strong convergence · Iterative methods · Banach spaces

Mathematics Subject Classification 47H09 · 47H10 · 47J25 · 47J05

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✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

Prasit Cholamjiak
prasitch2008@yahoo.com

Suthep Suantai
suthep.s@cmu.ac.th

1 School of Science, University of Phayao, Phayao 56000, Thailand

2 Centre of Excellence in Mathematics and Applied Mathematics, Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand

3 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), 39 Rangsit-Nakhonnayok Rd., Klong 6, Thanyaburi, Pathumthani 12110, Thailand

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A Halpern-Type Iteration for Solving the Split Feasibility Problem and the Fixed Point Problem of Bregman Relatively Nonexpansive Semigroup in Banach Spaces

Prasit Cholamjiak[a], Pongsakorn Sunthrayuth[b]

[a]*School of Science, University of Phayao, Phayao 56000, Thailand*
[b]*Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT) Thanyaburi, Pathumthani, 12110, Thailand*

Abstract. We study the split feasibility problem (SFP) involving the fixed point problems (FPP) in the framework of p-uniformly convex and uniformly smooth Banach spaces. We propose a Halpern-type iterative scheme for solving the solution of SFP and FPP of Bregman relatively nonexpansive semigroup. Then we prove its strong convergence theorem of the sequences generated by our iterative scheme under implemented conditions. We finally provide some numerical examples and demonstrate the efficiency of the proposed algorithm. The obtained result of this paper complements many recent results in this direction.

1. Introduction

Throughout in this paper, we let E_1 and E_2 be two p-uniformly convex real Banach spaces which are also uniformly smooth. Let C and Q be nonempty, closed and convex subsets of E_1 and E_2, respectively. Let $A : E_1 \to E_2$ be a bounded linear operator and $A^* : E_2^* \to E_1^*$ be the adjoint of A which is defined by

$$\langle A^* y, x\rangle := \langle y, Ax\rangle, \tag{1}$$

for all $x \in C$ and $y \in E_2^*$. We consider the following *split feasibility problem* (SFP): find an element

$$\hat{x} \in C \text{ such that } A\hat{x} \in Q. \tag{2}$$

The set of solutions of problem (2) is denoted by $\Gamma := C \cap A^{-1}(Q) = \{x \in C : Ax \in Q\}$. We assume that Γ is nonempty. Then, we have Γ is a closed and convex subset of E_1. It is clear that $\hat{x}$ is a solution to the split feasibility problem (2) if and only if $\hat{x} \in C$ and $A\hat{x} - P_Q A\hat{x} = 0$. The split feasibility problem originally introduced in Censor and Elfving [11] in finite-dimensional Hilbert spaces for modeling inverse problems which arise from phase retrievals and in medical image reconstruction. Recently, SFP can also be used to model the intensity-modulated radiation therapy [10, 12–14].

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Email addresses: prasitch2008@yahoo.com (Prasit Cholamjiak), pongsakorn_su@rmutt.ac.th (Pongsakorn Sunthrayuth)

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On Solving the Split Feasibility Problem and the Fixed Point Problem in Banach Spaces

Prasit Cholamjiak† and Pongsakorn Sunthrayuth‡[1]

‡ School of Science, University of Phayao, Phayao, Thailand
e-mail : prasitch2008@yahoo.com
‡Department of Mathematics and Computer Science,
Faculty of Science and Technology,
Rajamangala University of Technology Thanyaburi, Pathumthani, Thailand
e-mail : pongsakorn_su@rmutt.ac.th

Abstract : In this paper, we introduce an iterative method for solving the split feasibility problem and the fixed point problem of countable family of Bregman relatively nonexpansive mappings in the framework of p-uniformly convex and uniformly smooth Banach spaces. Then, we prove strong convergence theorem of the sequence generated by our iterative scheme with a new way of selecting the step-size which does not require the computation on the norm of the bounded linear operator. The obtained result of this paper complements many recent results in this direction.

Keywords : Split feasibility problem; Banach space; Strong convergence; Iterative method; Fixed point
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1 Introduction

Let E_1 and E_2 be two p-uniformly convex real Banach spaces which are also

[1]Corresponding author email: pongsakorn_su@rmutt.ac.th (P. Sunthrayuth)

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Convergence analysis of generalized viscosity implicit rules for a nonexpansive semigroup with gauge functions

Check for updates

Pongsakorn Sunthrayuth[a], Nuttapol Pakkaranang[b], Poom Kumam[b,c,d,*]

[a]*Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, 12110, Thailand.*

[b]*KMUTTFixed Point Research Laboratory, Department of Mathematics, Room SCL 802 Fixed Point Laboratory, Science Laboratory Building, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thung Khru, Bangkok 10140, Thailand.*

[c]*KMUTT-Fixed Point Theory and Applications Research Group, Theoretical and Computational Science Center (TaCS), Science Laboratory Building, Facuty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thung Khru, Bangkok 10140, Thailand.*

[d]*Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan.*

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Abstract

In this paper, we introduce an iterative algorithm for finding the set of common fixed points of nonexpansive semigroups by the generalized viscosity implicit rule in certain Banach spaces which has a uniformly Gâteaux differentiable norm and admits the duality mapping j_φ, where φ is a gauge function. We prove strong convergence theorems of proposed algorithm under appropriate conditions. As applications, we apply main result to solving the fixed point problems of countable family of nonexpansive mappings and the problems of zeros of accretive operators. Furthermore, we give some numerical examples for supporting our main results.

Keywords: Nonexpansive semigroup, Banach spaces, strong convergence, fixed point problem, iterative method.

2010 MSC: 47H09, 47H10, 47J05, 47J25.

1. Introduction

In this paper, we assume that E is a real Banach space with dual space E^* and C is a nonempty subset of E. Let $T : C \to C$ be a mapping. We denote the set of all fixed points of T by $F(T) = \{x \in C : x = Tx\}$. A mapping $T : C \to C$ is called *nonexpansive* if for each $x, y \in C$ such that

$$\|Tx - Ty\| \leqslant \|x - y\|.$$

*Corresponding author
Email addresses: pongsakorn_su@rmutt.ac.th (Pongsakorn Sunthrayuth), nuttapol.pak@mail.kmutt.ac.th (Nuttapol Pakkaranang), poom.kum@kmutt.ac.th (Poom Kumam)

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Some New Results on Fixed Points for ϖ-Distances in Complex-Valued Metric Spaces

Wongvisarut Khuangsatung[1], Anteneh Getachew Gebrie[2], Cholatis Suanoom[3,*]

[1] *Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani 12110, Thailand*
[2] *Department of Mathematics, College of Computational and Natural Science, Debre Berhan University, Debre Berhan 445, Ethiopia*
[3] *Program of Mathematics, Faculty of Science and Technology, Kamphaengphet Rajabhat University, Kamphaengphet 62000, Thailand*

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ABSTRACT

In this paper, we introduce the notion of a ϖ-distance in complete complex-valued metric spaces and prove some fixed point theorems for mappings satisfying some appropriate inequalities in complete complex-valued metric spaces. Moreover, we deduce new fixed point results in complete complex-valued metric spaces and provide some examples to illustrate the usability of the obtained results.

Keywords: ϖ-distance; Ω-distance; c-distance; Complex-valued metric spaces; Fixed point; Generalized c-distance; wt-distance; w-distance

1. Introduction

The Banach's contraction mapping principle is widely recognized as the source of a metric fixed point theory. The existence and uniqueness of a fixed point of operators or mappings has been a subject of a great interest since the work of a Banach in 1922 [1]. The concept of nonexpansive mappings has also been widely studied in the following works [2, 3]. This principle has been applied in different spaces by mathematicians, for example D-metric spaces, quasimetric spaces, quasi b-metric spaces, b-metric-like spaces, Dislocated quasi-b-metric spaces, and G-metric spaces (see [4-8]) have already been obtained. A new space called the complex valued metric space which is more general than well-know metric spaces has been introduced by Azam et al. [9]. Naturally, this new idea

*Corresponding author: cholatis.suanoom@gmail.com doi: 10.14456/scitechasia.2024.35

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A MODIFIED KRASNOSELSKII-TYPE SUBGRADIENT EXTRAGRADIENT ALGORITHM WITH INERTIAL EFFECTS FOR SOLVING VARIATIONAL INEQUALITY PROBLEMS AND FIXED POINT PROBLEM

Araya Kheawborisut[1] and Wongvisarut Khuangsatung[2]

[1]Department of Mathematics, School of Science,
King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand
e-mail: `araya.kheaw@gmail.com`

[2]Department of Mathematics and Computer Science, Faculty of Science and Technology,
Rajamangala University of Technology Thanyaburi,
Pathumthani, 12110, Thailand
e-mail: `wongvisarut_k@rmutt.ac.th`

Abstract. In this paper, we propose a new inertial subgradient extragradient algorithm with a new linesearch technique that combines the inertial subgradient extragradient algorithm and the KrasnoselskiiMann algorithm. Under some suitable conditions, we prove a weak convergence theorem of the proposed algorithm for finding a common element of the common solution set of a finitely many variational inequality problem and the fixed point set of a nonexpansive mapping in real Hilbert spaces. Moreover, using our main result, we derive some others involving systems of variational inequalities. Finally, we give some numerical examples to support our main result.

1. Introduction

Throughout this paper, let H be a real Hilbert space and C be a nonempty closed convex subset of H with the inner product $\langle \cdot, \cdot \rangle$ and norm $\|\cdot\|$. Let

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[0]Corresponding author: W. Khuangsatung(`wongvisarut_k@rmutt.ac.th`).

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An intermixed algorithm for solving fixed point problems of proximal operators in Hilbert Spaces

WONGVISARUT KHUANGSATUNG[1] and ATID KANGTUNYAKARN[2]

ABSTRACT. The aim of this paper is to modify proximal operators in Hilbert spaces. We introduce an intermixed algorithm with viscosity technique to find the solution of fixed point problem of two proximal operators in a real Hilbert space, utilizing the modified proximal operators. Under some mild conditions, a strong convergence theorem is established for the proposed algorithm. We also apply our main result to the split feasibility problem. Finally we provide numerical examples for supporting the main result.

1. INTRODUCTION

Let H be a real Hilbert space with an inner product $\langle \cdot, \cdot \rangle$ and an induced norm $\| \cdot \|$ and let $\Gamma_0(H)$ be a class of convex, lower semicontinuous, and proper functions from a Hilbert space H to $(-\infty, +\infty]$. Let C be closed convex subset of H. Let $S : C \to C$ be a nonlinear mapping, a point $x \in C$ is called a fixed point of S if $Sx = x$. We denote by $Fix(S)$, the set of all fixed points of S, i.e. $Fix(S) = \{x \in C : S(x) = x\}$. Consider the following convex minimization problem

$$\min_{x \in H}(f(x) + g(x)), \tag{1.1}$$

where $f \in \Gamma_0(H)$, $g : H \to \mathbb{R}$ is convex and differentiable with the Lipschitz continuous gradient denoted by ∇g. The solution set of (1.1) will be denoted by $\mathrm{Argmin}(f + g)$. In 2014, Xu [29] presented an important mathematical tool to demonstrate that the solution set of (1.1) is equivalent to the fixed point equation as follows:

$$\tilde{x} = \mathrm{Prox}_{\gamma f}(\tilde{x} - \gamma \nabla g(\tilde{x})), \tag{1.2}$$

where $\gamma > 0$ and $\mathrm{Prox}_{\gamma f}\, x := \mathrm{argmin}_{u \in H}\{f(u) + \frac{1}{2\gamma}\|u - x\|^2\}$ is the proximal mapping of f (see [2] for more informations on the proximal mapping). The most widely used algorithm for solving the convex minimization problem (1.1) is the so-called proximal-gradient algorithm. This proximal-gradient algorithm is given by: $x_1 \in H$ and

$$x_{n+1} = \mathrm{Prox}_{\gamma f}(I - \gamma \nabla g)(x_n), \quad \forall n \geq 1. \tag{1.3}$$

where Prox_f is the proximal operator of f, $\gamma \in (0, 2/L)$ and L is the Lipschitz constant of ∇g, then the sequence $\{x_n\}$ generated by algorithm (1.3) converges weakly to an element of $\mathrm{Argmin}(f + g)$ [see [2], Corollary 28.9]. This method is sometimes called the forward-backward algorithm. The proximal-gradient algorithm can be used in real-world applications, for example, in signal recovery, in image deblurring, and in machine learning (regression on highdimensional datasets) (see, [4], [20], [14], [22]). Recently there are extensive works in studying proximal gradient algorithm, see [19], [13], [28], [27], [10], [1], [25] and the references therein. For a set C, we denote by δ_C the indicator function of the set, that is, $\delta_C(x) = 0$ if $x \in C$ and ∞ otherwise. We denote the metric projection

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RESEARCH **Open Access**

A regularization method for solving the G-variational inequality problem and fixed-point problems in Hilbert spaces endowed with graphs

Wongvisarut Khuangsatung[1], Akarate Singta[1] and Atid Kangtunyakarn[2*]

*Correspondence:
beawrock@hotmail.com
[2]Department of Mathematics, School of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok, 10520, Thailand
Full list of author information is available at the end of the article

Abstract

This article considers and investigates a variational inequality problem and fixed-point problems in real Hilbert spaces endowed with graphs. A regularization method is proposed for solving a G-variational inequality problem and a common fixed-point problem of a finite family of G-nonexpansive mappings in the framework of Hilbert spaces endowed with graphs, which extends the work of Tiammee et al. (Fixed Point Theory Appl. 187, 2015) and Kangtunyakarn, A. (Rev. R. Acad. Cienc. Exactas Fís. Nat., Ser. A Mat. 112:437–448, 2018). Under certain conditions, a strong convergence theorem of the proposed method is proved. Finally, we provide numerical examples to support our main theorem. The numerical examples show that the speed of the proposed method is better than some recent existing methods in the literature.

Mathematics Subject Classification: Primary 47H09; 47H10; secondary 90C33

Keywords: G-variational inequality problem; G-inverse strongly monotone mapping; G-nonexpansive mapping; Regularization method; Directed graph

1 Introduction

Assume that H is a real Hilbert space with an inner product $\langle\cdot,\cdot\rangle$ and its induced norm $\|\cdot\|$. Let C be a nonempty, closed, and convex subset of H and $\mathcal{T}: C \to C$ be a nonlinear mapping. A point $x \in C$ is called a *fixed point* of $\mathcal{T}$ if $\mathcal{T}x = x$. Let $F(\mathcal{T}) := \{x \in C : \mathcal{T}x = x\}$ be the set of fixed points of $\mathcal{T}$. The mapping $\mathcal{T}$ is *nonexpansive* if $\|\mathcal{T}x - \mathcal{T}y\| \le \|x - y\|$ for all $x, y \in C$.

Denote by $G = (V(G), E(G))$ a directed graph, where $V(G)$ and $E(G)$ are the set of its vertices and edges, respectively. Assuming that G has no parallel edges, we denote G^{-1} as the directed graph derived from G by reversing the direction of its edges, i.e.,

$$E(G^{-1}) = \{(x,y) : (y,x) \in E(G)\}.$$

In 2008, Jachymski [1] studied fixed-point theory in a metric space endowed with a directed graph by combining the concepts of fixed-point theory and graph theory. The

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Self-adaptive CQ-type algorithms for the split feasibility problem involving two bounded linear operators in Hilbert spaces

PACHARA JAILOKA[1], CHOLATIS SUANOOM[2], WONGVISARUT KHUANGSATUNG[3] and SUTHEP SUANTAI[4]

ABSTRACT. In this article, we consider and investigate a split convex feasibility problem involving two bounded linear operators in Hilbert spaces. We introduce a self-adaptive CQ-type algorithm by selecting the stepsize which is independent of the operator norms and establish a strong convergence result of the proposed algorithm under some mild control conditions. Moreover, we propose a self-adaptive relaxed CQ-type algorithm for solving the problem constrained by sub-level sets of convex functions. A numerical example and an application in compressed sensing are also given to illustrate the convergence behaviour of our proposed algorithms. Our results in this paper improve and generalize some existing results in the literature.

1. INTRODUCTION

Let C and Q be two nonempty closed convex subsets of Hilbert spaces H_1 and H_2, respectively. The *split feasibility problem* (shortly, SFP) is to find a point

$$x \in C \quad \text{such that} \quad Ax \in Q, \tag{1.1}$$

where $A : H_1 \to H_2$ is a bounded linear operator. The SFP is the first instance of the split inverse problem (referred to [13, Sect. 2]), which was first introduced by Censor and Elfving [11] in Euclidean spaces. The SFP model can be applied to solving many mathematical problems such as the constrained least-squares problem, the linear split feasibility problem, and the linear programming problem and it can be used in real-world applications, for example, in signal processing, in image recovery, in intensity-modulated therapy, in pattern recognition and in data prediction (see [3, 5, 10, 12, 20, 22]). Consequently, the SFP has been widely studied and various methods for solving such a problem have been invented and developed by many authors, see [2, 9, 17, 24, 25, 35, 36, 37, 38, 41, 43, 44] and the references therein. One of the powerful methods for approximating solutions of (1.1) is known as the *CQ algorithm* introduced by Byrne [2] as follows:

$$\begin{cases} x_1 \in H_1, \\ x_{k+1} = P_C\left(x_k - \lambda A^*(I - P_Q)Ax_k\right), \quad k \geq 1, \end{cases} \tag{1.2}$$

where $\lambda \in \left(0, 2/\|A\|^2\right)$, P_C and P_Q are the metric projections onto C and Q, respectively, and A^* stands for the adjoint operator of A. After that, various kinds of the split inverse problem, which are generalizations of the SFP were introduced and studied, see [4, 12, 13, 14, 28, 32] for instance.

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Corresponding author: Wongvisarut Khuangsatung; wongvisarut_k@rmutt.ac.th

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Impact of pretreatment with dielectric barrier discharge plasma on the drying characteristics and bioactive compounds of jackfruit slices

Weerasak Seelarat,[a] Sujarinee Sangwanna,[b] Tawan Chaiwon,[c,d] Teerapap Panklai,[e] Natthaphon Chaosuan,[f] Atipong Bootchanont,[g,h] Chakkaphan Wattanawikkam,[g,h] Porramain Porjai,[g,h*] Wongvisarut Khuangsatung[i] and Dheerawan Boonyawan[j]

Abstract

BACKGROUND: Hot-air drying is a popular method for preserving the production of jackfruit, but heat treatment damages its nutritional qualities. Cold plasma is one of the pretreatment methods used to preserve quality attributes of fruits before drying. In the present work, we studied the effect of dielectric barrier discharge (DBD) plasma on the drying characteristics, microstructure, and bioactive compounds of jackfruit slices with different pretreatment times (15, 30, 45, and 60 s), followed by hot-air drying at 50, 60, and 70 °C. A homemade DBD device was operated via three neon transformers.

RESULTS: Optical emission spectrophotometry revealed the emitted spectra of the reactive species in DBD plasma, including the N_2 second positive system, N_2 first negative system, nitrogen ion, and hydroxyl radical. The results showed that the DBD plasma promoted moisture transfer and enhanced the drying rate, related to the changes in the surface microstructure of samples damaged by DBD plasma. The modified Overhults model was recommended for describing the drying characteristics of jackfruit slices. The contents of ascorbic acid, total phenolics, total flavonoids, total polysaccharides, and antioxidant activity in pretreated jackfruit slices were improved by 9.64%, 42.59%, 25.77%, 27.00%, and 23.13%, respectively. However, the levels of color and carotenoids were reduced.

CONCLUSION: Thus, the bioactive compounds in dried jackfruit slices can be improved using the DBD plasma technique as a potential pretreatment method for the drying process.

Supporting information may be found in the online version of this article.

Keywords: dielectric barrier discharge plasma; dried jackfruit slices; Total phenolic contents; ascorbic acid; antioxidant activity

INTRODUCTION

Jackfruit (*Artocarpus heterophyllus*) is a tropical composite fruit and is widely grown in Thailand, Bangladesh, Indonesia, and Malaysia,[1] where the total production of jackfruit is almost 3.11 million tons per year.[2] Jackfruit is rich in sugar, vitamin C, protein, carotenoid, total phenolics, and antioxidants.[3] However, jackfruit is difficult to preserve and store because it is easily perishable.

Drying is a popular preservation method in fruits and vegetables for preventing microorganism growth, reducing the bulk and extending the shelf life of foods, and decreasing transportation costs.[4,5] Some methods have been used for drying jackfruits, such as osmo-convective dehydration,[6] hot-air drying,[7] and infrared drying.[8] Hot-air drying is the most economical technique for jackfruit slices because of its inexpensive and uncomplicated operation. However, higher temperature and long interval time for drying have contrary effects on nutritional qualities.[9] Therefore, research focused on preventing the degradation of food quality has become attractive.

* Correspondence to: P Porjai, Division of Physics, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani, 12110, Thailand. E-mail: porramain_p@rmutt.ac.th

a Food and Beverage Innovation for Health, Faculty of Science and Technology, Valaya Alongkorn Rajabhat University under the Royal Patronage Pathum Thani Province, Pathum Thani, Thailand

b Nutrition and Dietetics, Faculty of Science and Technology, Valaya Alongkorn Rajabhat University under the Royal Patronage Pathum Thani Province, Pathum Thani, Thailand

c Office of Research Administration, Chiang Mai University, Chiang Mai, Thailand

d Bioplastics Production Laboratory for Medical Applications, Faculty of Science, Chiang Mai University, Chiang Mai, Thailand

e Food and nutrition, Faculty of Home Economics Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani, Thailand

f Division of Biology, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani, Thailand

g Division of Physics, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani, Thailand

h Smart Materials Research Unit, Rajamagala University of Technology Thanyaburi, Pathum Thani, Thailand

i Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani, Thailand

j Plasma and Beam Physics Research Facility, Chiang Mai University, Chiang Mai, Thailand

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An intermixed method for solving the combination of mixed variational inequality problems and fixed-point problems

Wongvisarut Khuangsatung[1] and Atid Kangtunyakarn[2*]

*Correspondence:
beawrock@hotmail.com
[2]King Mongkut's Institute of Technology Ladkrabang, Bangkok, 10520, Thailand
Full list of author information is available at the end of the article

Abstract

In this paper, we introduce an intermixed algorithm with viscosity technique for finding a common solution of the combination of mixed variational inequality problems and the fixed-point problem of a nonexpansive mapping in a real Hilbert space. Moreover, we propose the mathematical tools related to the combination of mixed variational inequality problems in the second section of this paper. Utilizing our mathematical tools, a strong convergence theorem is established for the proposed algorithm. Furthermore, we establish additional conclusions concerning the split-feasibility problem and the constrained convex-minimization problem utilizing our main result. Finally, we provide numerical experiments to illustrate the convergence behavior of our proposed algorithm.

Keywords: Mixed variational inequality problems; Intermixed algorithm; Strong convergence

1 Introduction

Let C be a nonempty, closed, and convex subset of a real Hilbert space H. Let $T : C \to C$ be a nonlinear mapping. A point $x \in C$ is called a *fixed point* of T if $Tx = x$. The set of fixed points of T is the set $\mathrm{Fix}(T) := \{x \in C : Tx = x\}$. A mapping T of C into itself is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in C.$$

Note that the mapping $I - T$ is demiclosed at zero iff $x \in \mathrm{Fix}(T)$ whenever $x_n \rightharpoonup x$ and $x_n - Tx_n \to 0$ (see, [1]). It is widely known that if $T : H \to H$ is nonexpansive, then $I - T$ is demiclosed at zero. A mapping $g : C \to C$ is said to be a *contraction* if there exists a constant $\alpha \in (0, 1)$ such that

$$\left\|g(x) - g(y)\right\| \leq \alpha \|x - y\|, \quad \forall x, y \in C.$$

Let $A : C \to H$ be a mapping and $f : H \to \mathbb{R} \cup \{+\infty\}$ be a proper, convex, and lower semicontinuous function on H. Now, we consider the mixed variational inequality prob-

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Strong Convergence for the Modified Split Monotone Variational Inclusion and Fixed Point Problem

Wongvisarut Khuangsatung[1] **and Atid Kangtunyakarn**[2,*]

[1] *Department of Mathematics and Computer Science, Faculty of Science and Technology Rajamangala University of Technology Thanyaburi, Pathumthani 12110, Thailand*
e-mail : wongvisarut_k@rmutt.ac.th

[2] *Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand*
e-mail : beawrock@hotmail.com

Abstract The purpose of this research is to modify the split monotone variational inclusion problem and prove a strong convergence theorem for finding a common element of the set of solutions of this problem and the set of fixed points of a nonexpansive mapping in Hilbert space. We also apply our main result involving a κ-strictly pseudo-contractive mapping. Moreover, we give the numerical example to support some of our results.

MSC: 47H09; 47H10
Keywords: split monotone variational inclusion; fixed point problem; nonexpansive mapping

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1. Introduction

Throughout this article, let H be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. Let C be a nonempty closed convex subset of H. Let $T : C \to C$ be a nonlinear mapping. A point $x \in C$ is called a *fixed point* of T if $Tx = x$. The set of fixed points of T is the set $F(T) := \{x \in C : Tx = x\}$.
A mapping T of C into itself is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \forall x, y \in C.$$

It is well known that if $T : H \to H$ is a nonexpansive mapping, we have

$$\langle Ty - Tx, (I - T)x - (I - T)y \rangle \leq \frac{1}{2}\|(I - T)x - (I - T)y\|^2, \forall x, y \in H.$$

Moreover, we also know that

$$\langle y - Tx, (I - T)x \rangle \leq \frac{1}{2}\|(I - T)x\|^2,$$

*Corresponding author.

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Original research article

On an Open Problem in Complex Valued Rectangular b-Metric Spaces with an Application

Cholatis Suanoom[1], Wongvisarut Khuangsatung[2], Thanatporn Bantaojai[3,*]

[1] *Program of Mathematics, Faculty of Science and Technology, Kamphaengphet Rajabhat University, Kamphaengphet 62000, Thailand*
[2] *Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani 12110, Thailand*
[3] *Mathematics English Program, Faculty of Education, Valaya Alongkorn Rajabhat University under the Royal Patronage, Pathum Thani 13180, Thailand*

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ABSTRACT

The purpose of the paper is to solve problem 1. Moreover, we prove fixed point theorems for contraction mappings in complete rectangular b-metrics and give examples as a satisfying the theorems in such spaces and give examples as a satisfying the theorems in rectangular b-metric spaces. Finally, we apply our result to examine the existence and uniqueness of solution for a system of Fredholm integral equation.

Keywords: Fixed point; Contraction mapping; Rectangular b-metric spaces; Integral equation; Fredholm type

1. Introduction

In 2015, George et al. [1] established the concept of rectangular b-metric space as a generalization of metric space (MS) [2], rectangular metric space (RMS) [3] and b-metric space (bMS) [4].

In the same year, Ege [6] established the complex valued rectangular b-metric space (CRbMS) as a generalization of a complex valued metric space (CMS) [5] and rectangular b-metric space (RbMS) [1] and proved an analogue of Banach contraction principle. Author also proved a different contraction principle with a new condition and a fixed point theorem in this space. Finally, author gave an application of Banach contraction principle to linear equations.

The complex metric space was initiated by Azam et al. [5]. Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$.

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Convergence results for modified SP-iteration in uniformly convex metric spaces

Check for updates

Pakeeta Sukprasert, Vitou Yang, Rattana Khunprasert, Wongvisarut Khuangsatung*

Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathumthani, 12110, Thailand.

Abstract

In this paper, we prove a strong convergence theorem of a Modified SP-iteration for finding a common fixed point of the combination of a finite family of nonexpansive mappings in a convex metric space. Moreover, we give some numerical example for supporting our main theorem and compare convergence rate between the modified SP-iteration and the Ishikawa iteration.

Keywords: Convergence theorem, SP-iteration, convex metric space.

2020 MSC: 47H09, 47H10.

1. Introduction

Let C be a nonempty closed convex subset of a metric space (X, d). Let $T : C \to C$ be a mapping. The set of all fixed points of a mapping T is denoted by $\mathrm{Fix}(T)$, that is, $\mathrm{Fix}(T) = \{x \in C : Tx = x\}$. A mapping $T : C \to C$ is called *nonexpansive* if

$$d(Tx, Ty) \leqslant d(x, y)$$

for all $x, y \in C$.

Let (X, d) be a metric space. A mapping $W : X \times X \times [0, 1] \to X$ is said to be a *convex structure* on X if for each $x, y \in X$ and $\lambda \in [0, 1]$,

$$d(z, W(x, y, \lambda)) \leqslant \lambda d(z, x) + (1 - \lambda)d(z, y)$$

for all $z \in X$. A metric space (X, d) together with a convex structure W is called a *convex metric space* which is denoted by (X, d, W). A nonempty subset C of X is said to be *convex* if $W(x, y, \lambda) \in C$ for all $x, y \in C$ and $\lambda \in [0, 1]$. The concept of convex metric spaces was introduced by Takahashi [18]. Takahashi [18] also studied some fixed point theorems for nonexpansive mappings in convex metric spaces. Note that a normed space and each of its convex subsets are convex metric spaces, but the converse does not hold. Many authors have increasingly investigated a convex metric space; see for instance [3, 4, 11, 15] and references therein.

*Corresponding author

Email addresses: pakeeta s@rmutt.ac.th (Pakeeta Sukprasert), yangvitou999@gmail.com (Vitou Yang), Rattanabow1998@gmail.com (Rattana Khunprasert), wongvisarut_k@rmutt.ac.th (Wongvisarut Khuangsatung)

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The Convergence Results for an AK-Generalized Nonexpansive Mapping in Hilbert Spaces

Cholatis Suanoom[1,2] and Wongvisarut Khuangsatung[3,*]

[1] *Program of Mathematics, Faculty of Science and Technology, Kamphaeng Phet Rajabhat University, Kamphaeng Phet 62000, Thailand*
e-mail : cholatis.suanoom@gmail.com

[2] *Science and Applied Science Center, Kamphaeng Phet Rajabhat University, Kamphaeng Phet 62000, Thailand*

[3] *Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathumthani 12110, Thailand*
e-mail : wongvisarut_k@rmutt.ac.th

Abstract In this paper, we introduce a new class of nonexpansive type of mapping namely, AK-generalized nonexpansive mapping, which is more general than an α-nonexpansive mapping. Moreover, we obtain convergence results of the viscosity approximation method for an AK-generalized nonexpansive semigroups under some assumptions in Hilbert spaces. Furthermore, we prove a strong convergence theorem for a family of AK-generalized nonexpansive mapping in Hilbert spaces.

MSC: 47H05; 47H10; 47J25

Keywords: fixed point; AK-generalized nonexpansive mapping; convergence theorem; Hilbert space

Submission date: 01.02.2019 / Acceptance date: 10.02.2021

1. Introduction

Throughout this article, let H be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. Let C be a nonempty closed convex subset of H. Let $T : C \to C$ be a nonlinear mapping. A point $x \in C$ is called a *fixed point* of T if $Tx = x$. The set of fixed points of T is the set $F(T) := \{x \in C : Tx = x\}$. The mapping $T : C \to C$ is said to be nonexpansive if $||Tx - Ty|| \leq ||x - y||$ for any $x, y \in C$. In 1965, Browder [1] shown that if a nonexpansive mapping $T : H \to H$ of a Hilbert space H into itself is asymptotically regular and has at least one fixed point then, for any $x \in H$, a weak limit of a weakly convergent subsequence of the sequence of successive approximations $T^n x$ is a fixed point of T.

In 2011, Aoyama and Kohsaka [2] introduced the class of α-nonexpansive mappings in Banach spaces as follows: Let E be a Banach space and let C be a nonempty subset of

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A Method for Solving the Variational Inequality Problem and Fixed Point Problems in Banach Spaces

Wongvisarut Khuangsatung and Atid Kangtunyakarn

Abstract. The purpose of this research is to modify Halpern iteration's process for finding a common element of the set of solutions of a variational inequality problem and the set of fixed points of a strictly pseudo contractive mapping in q-uniformly smooth Banach space. We also introduce a new technique to prove a strong convergence theorem for a finite family of strictly pseudo contractive mappings in q-uniformly smooth Banach space. Moreover, we give a numerical result to illustrate the main theorem.

1 Introduction

For the last decades, fixed point theory is a very importance tool for solving the problems in economic, computer science, physics, etc. Throughout this paper, let E be a Banach space with dual space of E^* and let C be a nonempty closed convex subset of E. We use the norm of E and E^* by the same symbol $\|\cdot\|$. We denote weak and strong convergence by notations " $\rightharpoonup$ " and " $\rightarrow$ ", respectively. Let q be a given real number with $q > 1$. The *generalized duality mapping* $J_q : E \to 2^{E^*}$ is defined by

$$J_q(x) = \{x^* \in E^* : \langle x, x^* \rangle = \|x\|^q, \|x^*\| = \|x\|^{q-1}\},$$

for all $x \in E$. If $q = 2$, then $J_2 = J$ is called *normalized duality mapping.*

Remark 1. If J_q is generalized duality mapping of E into 2^{E^*}. Then the following properties are holds:

1. $J_q(tx) = t^{q-1}J_q(x)$, for all $x \in E$ and $t \in [0, \infty)$;

2. $J_q(-x) = -J_q(x)$, for all $x \in E$.

2010 *Mathematics Subject Classification*. 46B25, 47H05, 47H06, 47H10.
Key words and phrases. Strictly pseudo contractive mapping, Inverse strongly monotone accretive operator, Variational inequality problem, q-uniformly smooth Banach space.
Corresponding author: Atid Kangtunyakarn.

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The Modification of Generalized Mixed Equilibrium Problems for Convergence Theorem of Variational Inequality Problems and Fixed Point Problems

Wongvisarut Khuangsatung[1], **Suthep Suantai**[2] **and Atid Kangtunyakarn**[3,*]

[1] *Department of Mathematics and Computer Science, Faculty of Science and Technology Rajamangala University of Technology Thanyaburi, Pathumthani, 12110, Thailand*
e-mail : wongvisarut_k@rmutt.ac.th

[2] *Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand*
e-mail : suthep.s@cmu.ac.th

[3] *Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand*
e-mail : beawrock@hotmail.com

Abstract The purpose of this research, we modify generalized mixed equilibrium problems and prove a strong convergence theorem for approximating a common element of the set of such a problem and variational inequality problems and the set of fixed points of infinite family of strictly pseudo contractive mappings. Utilizing our main result, we also prove a strong convergence theorem involving generalized equilibrium problems and variational inequality problems.

MSC: 47H09; 47H10; 90C33
Keywords: strictly pseudo contractive mapping; generalized mixed equilibrium problem; inverse-strongly monotone

Submission date: 31.03.2016 / Acceptance date: 27.12.2020

1. Introduction

Throughout this article, we assume that H is a real Hilbert space and let C be a nonempty closed convex subset of H. Let $T : C \to C$ be a nonlinear mapping. A point $x \in C$ is called a *fixed point* of T if $Tx = x$. The set of fixed points of T is the set $Fix(T) := \{x \in C : Tx = x\}$.

Definition 1.1. Let $T : C \to C$ be a nonlinear mapping, then

(1) T is said to be *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \forall x, y \in C,$$

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Research Article

Fixed Point Theorems for a Demicontractive Mapping and Equilibrium Problems in Hilbert Spaces

Wongvisarut Khuangsatung[1] and Sarawut Suwannaut[2*]

[1] *Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathumtani 12110, Thailand*
[2] *Department of Mathematics, Faculty of Science, Lampang Rajabhat University, Lampang 50100, Thailand*
[*] **Corresponding author:** sarawut-suwan@hotmail.co.th

Abstract. In this research, we introduce some properties of demicontractive mapping and the combination of equilibrium problem. Then, we prove a strong convergence for the iterative sequence converging to a common element of fixed point set of demicontractive mapping and a common solution of equilibrium problems. Finally, we give a numerical example for the main theorem to support our results.

Keywords. The combination of equilibrium problem; Fixed point; Demicontractive mapping

MSC. 47H09; 47H10; 90C33

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1. Introduction

Let C be a nonempty closed convex subset of a real Hilbert space H. Let $F : C \times C \to \mathbb{R}$ be bifunction. *The equilibrium problem* for F is to determine its equilibrium point, i.e., the set

$$EP(F) = \{x \in C : F(x, y) \geq 0, \forall\, y \in C\}. \tag{1.1}$$

Equilibrium problems were introduced by [1] in 1994 where such problems have had a significant impact and influence in the development of several branches of pure and applied sciences. Various problems in physics, optimization, and economics are related to seeking some

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Dedicated to Prof. Suthep Suantai on the occasion of his 60^{th} anniversary

The Convergence Theorem for a Square α-Nonexpansive Mapping in a Hyperbolic Space

Thanatporn Bantaojai[1], **Cholatis Suanoom**[2] **and Wongvisarut Khuangsatung**[3,*]

[1] *Mathematics English Program, Faculty of Education, Valaya Alongkorn Rajabhat University under the Royal Patronage, Pathumtani 13180, Thailand*
e-mail : thanatporn.ban@vru.ac.th (T. Bantaojai)

[2] *Program of Mathematics, Faculty of Science and Technology, Kamphaengphet Rajabhat University, Kamphaengphet 62000,Thailand*
e-mail : cholatis.suanoom@gmail.com (C. Suanoom)

[3] *Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathumthani, 12110, Thailand*
e-mail : wongvisarut_k@rmutt.ac.th (W. Khuangsatung)

Abstract In this paper, we prove Δ-convergence theorems of the generalized Picard normal S_5-iterative process to approximate a fixed point for square α-nonexpansive mappings. Moreover, we obtain some properties of such mappings on a nonempty subset of a hyperbolic space.

MSC: 47H09; 47H10

Keywords: fixed point set; square α-nonexpansive mapping; generalized Picard normal S_5-iterative; hyperbolic spaces

Submission date: 26.05.2020 / Acceptance date: 01.09.2020

1. Introduction

Let X be a metric space and let M be a nonempty closed convex subset of X. A mapping $T : M \to M$ is said to be nonexpansive, if $d(Tx, Ty) \leq d(x, y)$, for each $x, y \in M$. In 2011, Aoyama and Kohsaka [1] introduced the class of α-nonexpansive mappings in Banach spaces as follow: Let X be a Banach space and M be a nonempty closed and convex subset of X. A mapping $T : M \to M$ is said to be α-nonexpansive if for all $x, y \in M$ and $\alpha < 1$, $\|Tx - Ty\|^2 \leq \alpha \|Tx - y\|^2 + \alpha \|x - Ty\|^2 + (1 - 2\alpha) \|x - y\|^2$. This class contains the class of nonexpansive mappings and is related to the class of firmly nonexpansive mappings in Banach spaces. Then $F(T)$ is nonempty if and only if there exists $x \in M$ such that $\{T^n x\}$ is bounded, where X is a uniformly convex Banach

*Corresponding author.

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The Rectangular Quasi-Metric Space and Common Fixed Point Theorem for ψ-Contraction and ψ-Kannan Mappings

Wongvisarut Khuangsatung[†], Saowapak Chan-iam[‡], Patchara Muangkarn[‡] and Cholatis Suanoom[‡,§,1]

[†]Department of Mathematics and Computer Science,
Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathumthani, 12110, Thailand,
e-mail : `wongvisarutk@9rmutt.ac.th (W. Khuangsatung)`
[‡]Program of Mathematics, Faculty of Science and Technology,
Kamphaengphet Rajabhat University, Kamphaengphet 62000, Thailand,
e-mail : `booklnw23@hotmail.com (S. Chan-iam)`
`putchara_31@windowslive.com (P. Muangkarn)`
[§]Science and Applied Science center, Faculty of Science and Technology,
Kamphaengphet Rajabhat University, Kamphaengphet 62000, Thailand,
e-mail : `cholatis.suanoom@gmail.com (C. Suanoom)`

Abstract : In this work, we extend and improve rectangular metric spaces to rectangular quasi-metric spaces by using the concept of quasi-metric spaces. Next, we obtain fixed point theorems in rectangular quasi-metric spaces. Moreover, we present some examples to illustrate and support our results.

Keywords : fixed point; quasi-metric space; rectangular metric space; rectangular quasi-metric space.

2010 Mathematics Subject Classification : 47H09; 47H10.

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[1] Corresponding author.

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THE METHOD FOR SOLVING FIXED POINT PROBLEM OF G-NONEXPANSIVE MAPPING IN HILBERT SPACES ENDOWED WITH GRAPHS AND NUMERICAL EXAMPLE

Wongvisarut Khuangsatung* and Atid Kangtunyakarn**

**Department of Mathematics and Computer Science,Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathumthani*, 12110, *Thailand*

***Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok* 10520, *Thailand*

e-mails: wongvisarut_k@rmatt.ac.th; k@rmutt.ac.th; beawrock@hotmail.com

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The main aim of this paper is to study a strong convergence theorem of viscosity approximation method for G-nonexpansive mapping defined on a Hilbert space endowed with a directed graph. By using our main result, we give a numerical expample to approximate the value of π.

Key words : G-nonexpansive mappings; viscosity approximation; edge-preserving.

2010 Mathematics Subject Classification: 47H09, 47H10, 05C69

1. Introduction

The fixed point theory plays an important role in nonlinear functional analysis and is a very useful tool in various fields. In particular, fixed point theorem has been applied in many branches of sciences. For a recent trend of fixed point problem, one of the most interesting problems is the combination of fixed point theory and graph theory. In the past few years, many researchers have studied fixed point theorems in a metric space endowed with a graphs; see [1-4] and references cited therein.

Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be contraction if there is $0 < k < 1$ such that

$$d(Tx, Ty) \leq kd(x, y) \text{ for all } x, y \in X.$$

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Three novel inertial subgradient extragradient methods for quasi-monotone variational inequalities in Banach spaces

Zhong-bao Wang[1,2,3] · Pongsakorn Sunthrayuth[4] · Ratthaprom Promkam[4] · Abubakar Adamu[5,6]

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Abstract

In this paper, we introduce three new inertial subgradient extragradient methods for solving variational inequalities involving quasi-monotone operators in the setting of 2-uniformly convex and uniformly smooth Banach spaces. We dispense with the well-known requirement of the stepsizes of the subgradient extragradient method on the prior knowledge of the Lipschitz constant of the cost function in our proposed algorithms. Furthermore, we give many numerical examples to test the robustness of our proposed algorithms and compare their performance with several algorithms in the literature. In addition, we use our proposed algorithms in the restoration process of some degraded images and compare the quality of the restored images using our proposed algorithms and some recent algorithms in the literature. Finally, from the results of the numerical simulations, our proposed algorithms are competitive and promising.

Keywords Banach space · Weak convergence · Variational inequalities · Quasi-monotone mapping

Mathematics Subject Classification 47H09 · 47H10 · 47J25 · 47J05

✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

1 Department of Mathematics, Southwest Jiaotong University, Chengdu 611756, Sichuan, China

2 National Engineering Laboratory of Integrated Transportation Big Data Application Technology, Chengdu 611756, Sichuan, China

3 School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu 611731, Sichuan, China

4 Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi 12110, Pathumthani, Thailand

5 Operational Research Center in Healthcare, Near East University, TRNC Mersin 10, Nicosia 99138, Turkey

6 Charles Chidume Mathematics Institute, African University of Science and Technology, Abuja 900107, Nigeria

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Novel inertial methods for fixed point problems in reflexive Banach spaces with applications

Pongsakorn Sunthrayuth[1] · Kunrada Kankam[2] · Ratthaprom Promkam[1] · Somnuk Srisawat[1]

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Abstract

In this paper, we suggest and analyze four inertial algorithms for solving fixed point problems of Bregman quasi-nonexpansive mappings in the framework of reflexive Banach spaces. Our first two algorithms, we propose inertial-like methods based on Mann-type and Halpern-type iterations, and in the others, we propose relaxed inertial-like methods based on Mann-type and Halpern-type iterations. The weak and strong convergence of the algorithms are established under some appropriate conditions on the parameters. As an application, we utilize our main results to find a zero of the sum of Bregman inverse strongly monotone mappings and maximal monotone operators in real reflexive Banach spaces. Also, we provide several numerical experiments to show the convergence behaviour of our algorithms in both finite-dimensional and infinite-dimensional spaces. Finally, we further utilize our algorithms to numerically solve the data classification problems of lung cancer.

Keywords Reflexive Banach space · Weak convergence · Strong convergence · Bregman quasi-nonexpansive mapping · Fixed point problem

Mathematics Subject Classification 47H09, 47H10, 47J25, 47J05

✉ Pongsakorn Sunthrayuth
pongsakorn_su@rmutt.ac.th

Kunrada Kankam
kunradazzz@gmail.com

Ratthaprom Promkam
ratthaprom_p@rmutt.ac.th

Somnuk Srisawat
somnuk_s@rmutt.ac.th

[1] Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani 12110, Thailand

[2] School of Science, University of Phayao, Phayao 56000, Thailand

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RESEARCH Open Access

New inertial self-adaptive algorithms for the split common null-point problem: application to data classifications

Ratthaprom Promkam[1], Pongsakorn Sunthrayuth[1*], Suparat Kesornprom[2] and Ekapak Tanprayoon[1]

*Correspondence:
pongsakorn_su@rmutt.ac.th
[1]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathumthani, 12110, Thailand
Full list of author information is available at the end of the article

Abstract

In this paper, we propose two inertial algorithms with a new self-adaptive step size for approximating a solution of the split common null-point problem in the framework of Banach spaces. The step sizes are adaptively updated over each iteration by a simple process without the prior knowledge of the operator norm of the bounded linear operator. Under suitable conditions, we prove the weak-convergence results for the proposed algorithms in p-uniformly convex and uniformly smooth Banach spaces. Finally, we give several numerical results in both finite- and infinite-dimensional spaces to illustrate the efficiency and advantage of the proposed methods over some existing methods. Also, data classifications of heart diseases and diabetes mellitus are presented as the applications of our methods.

Mathematics Subject Classification: 47H09; 47H10; 47J25; 47J05

Keywords: p-uniformly convex Banach spaces; Weak convergence; Split common null-point problem; Maximal monotone operator; Self-adaptive algorithm

1 Introduction

In this paper, we consider the following *split common null-point problem* [13] (see also [29]): find $z \in H_1$ such that

$$z \in A^{-1}0 \cap T^{-1}(B^{-1}0), \tag{1.1}$$

where $A : H_1 \to 2^{H_1}$ and $B : H_2 \to 2^{H_2}$ are set-valued maximal monotone operators, $T : H_1 \to H_2$ is a bounded linear operator, and H_1 and H_2 are real Hilbert spaces. We denote the solution set of the split common null-point problem (1.1) by Ω. The split common null-point problem can be applied to solving many real-life problems, for instance, in practices as a model in intensity-modulated radiation-therapy treatment planning [14, 15] and in sensor networks in computerized tomography and data compression [19]. In addition, the split common null-point problem also generalizes several split-type problems that is the core the modeling of many inverse problems such as the split feasibility problem, the split equilibrium problem, and the split minimization problem as special cases.

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Research article

Two-step inertial method for solving split common null point problem with multiple output sets in Hilbert spaces

Chibueze C. Okeke[1], **Abubakar Adamu**[2,3], **Ratthaprom Promkam**[4] **and Pongsakorn Sunthrayuth**[4,*]

[1] School of Mathematics, University of the Witwatersrand, Johannesburg, South Africa
[2] Operational Research Center in Healthcare, Near East University, Nicosia, TRNC
[3] Mathematics Institute, African University of Science and Technology, Abuja, Nigeria
[4] Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani, Thailand

* **Correspondence:** Email: pongsakorn_su@rmutt.ac.th.

Abstract: In this paper, an algorithm with two-step inertial extrapolation and self-adaptive step sizes is proposed to solve the split common null point problem with multiple output sets in Hilbert spaces. Weak convergence analysis are obtained under some easy to verify conditions on the iterative parameters in Hilbert spaces. Preliminary numerical tests are performed to support the theoretical analysis of our proposed algorithm.

Keywords: Hilbert space; metric projection; self-adaptive step size; two-step inertial; split common null point problem
Mathematics Subject Classification: 47H09, 47H10, 49J53, 90C25

1. Introduction

Throughout this paper, $\mathcal{H}$ denotes a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and the induced $\|\cdot\|$, I the identity operator on $\mathcal{H}$, $\mathbb{N}$ the set of all natural numbers and $\mathbb{R}$ the set of all real numbers. For a self-operator T on $\mathcal{H}$, $F(T)$ denotes the set of all fixed points of T.

Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be real Hilbert spaces and let $\mathcal{T}:\mathcal{H}_1\to\mathcal{H}_2$ be bounded linear operator. Let $\{U_j\}_{j=1}^t:\mathcal{H}_1\to\mathcal{H}_1$ and $\{T_i\}_{i=1}^r:\mathcal{H}_2\to\mathcal{H}_2$ be two finite families of operators, where $t,r\in\mathbb{N}$. The split common fixed point problem (SCFPP) is formulated as finding a point $x^*\in\mathcal{H}_1$ such that

$$x^*\in\cap_{j=1}^t F(U_j)\quad\text{such that}\quad\mathcal{T}x^*\in\cap_{i=1}^r F(T_i).\tag{1.1}$$

CONVERGENCE RESULTS OF ITERATIVE ALGORITHMS FOR THE SUM OF TWO MONOTONE OPERATORS IN REFLEXIVE BANACH SPACES

Yan Tang, Chongqing, Ratthaprom Promkam, Pathumthani,
Prasit Cholamjiak, Phayao, Pongsakorn Sunthrayuth, Pathumthani

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Abstract. The aim of this paper is to propose two modified forward-backward splitting algorithms for zeros of the sum of a maximal monotone operator and a Bregman inverse strongly monotone operator in reflexive Banach spaces. We prove weak and strong convergence theorems of the generated sequences by the proposed methods under some suitable conditions. We apply our results to study the variational inequality problem and the equilibrium problem. Finally, a numerical example is given to illustrate the proposed methods. The results presented in this paper improve and generalize many known results in recent literature.

Keywords: maximal operator; Bregman distance; reflexive Banach space; weak convergence; strong convergence

MSC 2020: 47H09, 47H10, 47J25, 47J05

1. Introduction

Let E be a real Banach space with its dual space E^*. We study the so-called *quasi-inclusion problem*: find $z \in E$ such that

$$0 \in (A+B)z, \tag{1.1}$$

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Strong convergence of a generalized forward–backward splitting method in reflexive Banach spaces

Truong Minh Tuyen[a], Ratthaprom Promkam [b] and Pongsakorn Sunthrayuth [b]

[a]Department of Mathematics and Informatics, Thai Nguyen University of Sciences, Thai Nguyen, Viet Nam; [b]Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT) Thanyaburi, Pathumthani, Thailand

ABSTRACT

In this paper, we study the so-called *generalized monotone quasi-inclusion problem* which is a generalization and extension of well-known monotone quasi-inclusion problem. We propose a forward–backward splitting method for solving this problem in the framework of reflexive Banach spaces. Based on Bregman distance function, we prove a strong convergence result of the proposed algorithm to a common zero of the problem. As an application, we apply the main result to the variational inequality problem. Finally, we provide some numerical examples to demonstrate our algorithm performance. The results presented in this paper improve and extend many known results in the literature.

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Maximal monotone operator; Legendre function; reflexive banach space; strong convergence; Bregman inverse strongly monotone

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1. Introduction

Let E be a real Banach space. Let $A : E \to E$ and $B : E \multimap E$ be single and set-valued operators, respectively. Consider the following so-called *monotone quasi-inclusion problem*:

$$\text{find } z \in E \text{ such that } 0 \in (A + B)z, \tag{1}$$

where 0 is the zero vector in E. The solutions set of the problem (1) is denoted by $(A + B)^{-1}0 = \{x \in E : 0 \in (A + B)x\}$. Many practical nonlinear problems arising in applied sciences such as in image recovery, signal processing and machine learning can be formulated as this problem (see [1–3]). Moreover, this problem includes the core of many mathematical problems, as special cases, such as: variational inequalities, split feasibility problem, minimization problem, Nash equilibrium problem in noncooperative games and so on (see [4–6]).

A well-known method for approximating a solution of the problem (1) is the *forward–backward splitting algorithm* which was introduced in [7,8]. This

CONTACT Pongsakorn Sunthrayuth pongsakorn_su@rmutt.ac.th

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Some Matrices with Padovan Q-matrix and the Generalized Relations

Mongkol Tatong
Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathumthani 12110, Thailand
E-mail: mongkol_t@rmutt.ac.th

Abstract

In this paper, we establish a new Q-matrix for Padovan numbers and the multiplies between the Q-matrix and the A-matrix. Moreover, we investigate the n^{th} of Q_2, the n^{th} of Q_1 multiply the A-matrix, and the n^{th} of Q_2 multiply the A-matrix. Finally, we use these matrices to obtain elementary identities for Padovan, Perrin, and relations between numbers.

Keywords: Q-matrix, Padovan Number, Perrin Number

1. Introduction

The Fibonacci $\{F_n\}$ and Lucas sequences $\{L_n\}$ are well–known sequences. For $n \geq 2$, the Fibonacci and Lucas sequences are defined respectively by $F_n = F_{n-1} + F_{n-2}$ and $L_n = L_{n-1} + L_{n-2}$, with an initial value $F_0 = 0$, $F_1 = 1$, $L_0 = 2$, and $L_1 = 1$. The Padovan $\{P_n\}$ and Perrin sequences $\{R_n\}$ are the favorable third-order sequences. For $n \geq 3$, the Padovan and Perrin sequences are defined respectively by $P_n = P_{n-2} + P_{n-3}$ and $R_n = R_{n-2} + R_{n-3}$, with an initial value $P_0 = P_1 = P_2 = 1$, $R_0 = 3$, $R_1 = 0$, and $R_2 = 2$. The first few values of P_n and R_n are 1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16,… and 3, 0, 2, 3, 2, 5, 5, 7, 10, 12, 17, 22, …., respectively (see (6)).

In 1963, S. L. Basin and Verner E. Hoggatt, Jr. (1) studied the Fibonacci Q-matrix Q_F, which is defined as $Q_F = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. After that, they showed that $Q_F^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$, for all $n \geq 1$. Moreover, they obtained some identities of Fibonacci numbers.

In 2013, Kritsana Sokhuma (3) studied the Padovan Q-matrix Q_1, defined as

$$Q_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \tag{1.1}$$

such that the n^{th} of Q-matrix is defined by

$$Q_1^n = \begin{pmatrix} P_{n-5} & P_{n-3} & P_{n-4} \\ P_{n-4} & P_{n-2} & P_{n-3} \\ P_{n-3} & P_{n-1} & P_{n-2} \end{pmatrix}, \text{ for all } n \geq 1. \tag{1.2}$$

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The Differential Equation in Terms of Jacobsthal and Jacobsthal-Lucas Numbers

Mongkol Tatong
Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani 12110, Thailand
E-mail: mongkol_t@rmutt.ac.th

Abstract

In this paper, we study Jacobsthal sine, Jacobsthal-Lucas sine, Jacobsthal cosine, Jacobsthal-Lucas cosine, Jacobsthal tangent, Jacobsthal-Lucas tangent, Jacobsthal cotangent, Jacobsthal-Lucas cotangent, Jacobsthal secant, Jacobsthal-Lucas secant, Jacobsthal cosecant, and Jacobsthal-Lucas cosecant. Furthermore, we establish some identities of Jacobsthal sine, Jacobsthal-Lucas sine, Jacobsthal cosine, Jacobsthal-Lucas cosine, Jacobsthal tangent, Jacobsthal-Lucas tangent, Jacobsthal cotangent, Jacobsthal-Lucas cotangent, Jacobsthal secant, Jacobsthal-Lucas secant, Jacobsthal cosecant, and Jacobsthal-Lucas cosecant.

Keywords: differential equations, Jacobsthal number, Jacobsthal-Lucas number

1. Introduction

The well-known Fibonacci $\{F_n\}$, Lucas $\{L_n\}$, Pell $\{P_n\}$, and Pell-Lucas $\{Q_n\}$ sequences have been found for several years. Their Binet's formulas are $F_n = \dfrac{a^n - b^n}{a-b}$, $L_n = a^n + b^n$, $J_n = \dfrac{\alpha^n - \beta^n}{\alpha - \beta}$, and $j_n = \alpha^n + \beta^n$, where n is an integer, $a = \dfrac{1+\sqrt{5}}{2}$, $b = \dfrac{1-\sqrt{5}}{2}$ and $\alpha = 2$, $\beta = -1$ are the root of the characteristic equation $r^2 - r - 1 = 0$ and $r^2 - r - 2 = 0$, respectively [1,2,4]. So $a > b$, $a + b = 1$ $a - b = \sqrt{5}$, $ab = -2$ and $\alpha > \beta$, $\alpha + \beta = 1$, $\alpha - \beta = 3$, $\alpha\beta = -2$.

Recently, the general solution of a second-order homogeneous linear differential equation in terms of numbers was studied by many authors in different ways to derive many identities. In 1964, Verner E. Hoggatt, Jr. [3] studied a general solution of a second-order homogeneous linear differential equation $y'' - y' - y = 0$ with an initial value $y(0) = 0$ $y'(0) = 1$, which is defined by

$$y = \frac{e^{ax} - e^{bx}}{a-b} = \sum_{n=0}^{\infty} \frac{a^n - b^n}{a-b} \frac{x^n}{n!}, \tag{1.1}$$

where $a = \dfrac{1+\sqrt{5}}{2}$ and $b = \dfrac{1-\sqrt{5}}{2}$ are the roots of the characteristic equation $r^2 - r - 1 = 0$. They obtained some identities of these (5, 7).

เอกลักษณ์เบื้องต้นของจำนวนโมดิฟายด์ (s,t) จาคอปทอล และ จำนวนโมดิฟายด์ (s,t) จาคอปทอล-ลูคัสโดยเมทริกซ์

Some Identities of the Modified (s,t) Jacobsthal and Modified (s,t) Jacobsthal – Lucas Numbers by the Matrix Method

มงคล ทาทอง*

Mongkol Tatong*

ภาควิชาคณิตศาสตร์และวิทยาการคอมพิวเตอร์ คณะวิทยาศาสตร์และเทคโนโลยี มหาวิทยาลัยเทคโนโลยีราชมงคลธัญบุรี

Department of Mathematics and Computer Science, Faculty of Science and Technology,

Rajamangala University of Technology Thanyaburi

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บทคัดย่อ

ในงานวิจัยนี้เราได้ศึกษาจำนวนโมดิฟายด์ (s,t) จาคอปทอล และ จำนวนโมดิฟายด์ (s,t) จาคอปทอล-ลูคัส และนิยามเมทริกซ์มิติ 2 x 2 A B และ W ซึ่งสอดคล้องกับความสัมพันธ์ $A^2=(s-t)A+stI$ $B^2=(s-t)B+stI$ และ $W^2=(s+t)^2 I$ พร้อมทั้งพิสูจน์เอกลักษณ์เบื้องต้นของจำนวนโมดิฟายด์ (s,t) จาคอปทอล และ จำนวนโมดิฟายด์ (s,t) จาคอปทอล-ลูคัส เอกลักษณ์เบื้องต้นของความสัมพันธ์ระหว่างจำนวนโมดิฟายด์ (s,t) จาคอปทอล และ จำนวนโมดิฟายด์ (s,t) จาคอปทอล-ลูคัส และสูตรผลรวมเบื้องต้นสำหรับจำนวนโมดิฟายด์ (s,t) จาคอปทอล และ จำนวนโมดิฟายด์ (s,t) จาคอปทอล-ลูคัสโดยใช้เมทริกซ์

คำสำคัญ: จำนวนโมดิฟายด์ (s,t) จาคอปทอล; จำนวนโมดิฟายด์ (s,t) จาคอปทอล-ลูคัส; วิธีเมทริกซ์; สูตรไบเนต

Abstract

In this paper, we study the modified (s,t) Jacobsthal and modified (s,t) Jacobsthal – Lucas numbers, and we define the 2 x 2 matrices A, B, W, which satisfy the relation $A^2=(s-t)A+stI$, $B^2=(s-t)B+stI$, and $W^2=(s+t)^2 I$. Moreover, we prove some identities of modified (s,t) Jacobsthal and modified (s,t) Jacobsthal – Lucas numbers, some of the relation between modified (s,t) Jacobsthal and modified (s,t) Jacobsthal – Lucas numbers, and some sum formulas for modified (s,t) Jacobsthal and modified (s,t) Jacobsthal – Lucas numbers by using these matrices.

Keywords: modified (s,t) Jacobsthal number; modified (s,t) Jacobsthal – Lucas number; matrix method; Binet's formulas

*Corresponding author. E-mail: mongkol_t@rmutt.ac.th

วารสารวิทยาศาสตร์บูรพา ปีที่ 26 (ฉบับที่ 1) มกราคม – เมษายน พ.ศ. 2564
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ลำดับเมทริกซ์ในพจน์ของพหุนามเกาส์เซียนเพลล์ พหุนามเกาส์เซียนโมดิฟายด์เพลล์ จำนวนเกาส์เซียนเพลล์ จำนวนเกาส์เซียนเพลล์-ลูคัส จำนวนเกาส์เซียนโมดิฟายด์เพลล์ พหุนามเพลล์ พหุนามเพลล์-ลูคัส และพหุนามโมดิฟายด์เพลล์

Matrix Sequences in Terms of Gaussian Pell Polynomial, Gaussian Modified Pell Polynomial, Gaussian Pell Number, Gaussian Pell-Lucas Number, Gaussian Modified Pell Number, Pell Polynomial, Pell-Lucas Polynomial and Modified Pell Polynomial

มงคล ทาทอง*

Mongkol Tatong*

ภาควิชาคณิตศาสตร์และวิทยาการคอมพิวเตอร์ คณะวิทยาศาสตร์และเทคโนโลยี มหาวิทยาลัยเทคโนโลยีราชมงคลธัญบุรี

Department of Mathematics and Computer Science, Faculty of Science and Technology,

Rajamangala University of Technology Thanyaburi

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บทคัดย่อ

ในบทความนี้เราได้ศึกษาลำดับเมทริกซ์พหุนามเกาส์เซียนเพลล์ ลำดับเมทริกซ์พหุนามเกาส์เซียนโมดิฟายด์เพลล์ ลำดับเมทริกซ์เกาส์เซียนเพลล์ ลำดับเมทริกซ์เกาส์เซียนเพลล์-ลูคัส ลำดับเมทริกซ์เกาส์เซียนโมดิฟายด์เพลล์ ลำดับเมทริกซ์พหุนามเพลล์ ลำดับเมทริกซ์พหุนามเพลล์-ลูคัส และลำดับเมทริกซ์พหุนามโมดิฟายด์เพลล์ พร้อมทั้งพิสูจน์เอกลักษณ์บางอย่างของความสัมพันธ์ระหว่างลำดับเมทริกซ์และเอกลักษณ์บางอย่างของผลบวก

คำสำคัญ : ความสัมพันธ์เวียนเกิด ; ลำดับเมทริกซ์ ; สูตรไบเนต ; พจน์ที่ n

Abstract

In this paper, we study Gaussian Pell polynomial, Gaussian modified Pell polynomial, Gaussian Pell, Gaussian Pell-Lucas, Gaussian modified Pell, Pell polynomial, Pell-Lucas polynomial, and modified Pell polynomial matrix sequences. Furthermore, we prove some identities of the relation between matrix sequences and summations.

Keywords : recurrence relations ; matrix sequences ; Binet's formulas ; n^{th} terms

*Corresponding author. E-mail : mongkol_t@rmutt.ac.th

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Generalized Identities for third order Pell Number, Pell-Lucas Number and Modified Pell Number

Mongkol Tatong[1*] and Tawan Ampawa[1]

[1]Department of Mathematics, Faculty of Science and Technology,

Rajamangala University of Technology Thanyaburi, Khlong Luang, Pathum Thani 12110, Thailand

*E-mail: mongkol_t@rmutt.ac.th

Abstract

In this paper, we first presented the generalized Pell Number, Pell-Lucas Number and modified Pell Number, which are the recurrence relation by from the previous three terms. We have the Binet's formula generating functions and generating functions of all three sequences. We establish some of the interesting properties involving of sequences those sequences.

Keywords: Pell sequence Pell-Lucas sequence, Modified Pell sequence, Binet's formula

1. Introduction

We will refer to the sequence of occurrences starting in the recurring relationship from the previous second terms: Fibonacci and Lucas number. Because of their general characteristics, there are many interesting properties and application to almost every fields of science and art.

Previously, the sequence mentioned above is a sequence of positive integers that have been studied for many years. Many researchers have therefore examined about these sequences and also some properties that are excellent research topics. These sequences are examples of a sequences defined by a recurrence relation of second terms. It is well known that the Fibonacci sequence $\{F_n\}$, Lucas sequence $\{L_n\}$, Fibonacci-

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