Here we represent a formal statement and proof d **Weak Law of Large Numbers**.

Theorem 1 (Weak Law of Large Numbers)**.**

$$\lim_{n\to\infty} P\left[\left|\frac{\sum_{i=1}^{n} X_i}{n} - \mu\right| > \varepsilon\right] = 0$$

Proof will be based on Chebyshev's inequality which encode in Lean as *meas_ge_le_variance_div_sq*

Lemma 2 (Chebyshev's inequality)**.**

$$P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$$

The proof of Chebyshev's inequality we skip to be able to show the core of the Problem.

Proof. Apply Chebyshev's inequality to statement of a theorem and a random variable $Y_n = \frac{S_n}{n}$ with expected value μ_n and variance σ_n^2 we have

$$P\left[|Y_n - \mu_n| > \varepsilon\right] \leq \frac{\sigma_n^2}{\varepsilon^2} \tag{0.1}$$

Re-write μ_n and $_n$ in terms of μ and σ_n^2

$$\mu_n = E[Y_n] = E\left[\frac{S_n}{n}\right] = E\left[\frac{X_1 + \cdots + X_n}{n}\right] =$$
$$= E\left[\sum_{i=1}^{n} \frac{X_i}{n}\right] = \sum_{i=1}^{n} E\left[\frac{X_i}{n}\right] = \sum_{i=1}^{n} \frac{E[X_i]}{n} = \sum_{i=1}^{n} \frac{\mu}{n} = \mu$$

So $\mu_n = \mu$. Re-write σ_n^2 using the following fact about independent random variables which encode in Lean as *indep_fun.variance_sum*

Lemma 3 (Variance of independent Random Variables)**.** *For pairwise random variables* $X_1, \ldots, X_n$ *the following holds*

$$\mathrm{Var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \mathrm{Var}(X_i)$$

so using the above fact we have

$$\sigma_n^2 = \mathrm{Var}(Y_n) = \mathrm{Var}\left(\frac{S_n}{n}\right) = \mathrm{Var}\left(\frac{X_1 + \cdots + X_n}{n}\right) = \frac{\mathrm{Var}\left(\sum_{i=1}^{n} X_i\right)}{n^2}$$
$$= \sum_{i=1}^{n} \frac{\mathrm{Var}(X_i)}{n^2} = \sum_{i=1}^{n} \frac{\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

Re-write σ_n, μ_n into (0.1) we have

$$P\left[|Y_n - \mu| > \varepsilon\right] \leq \frac{\frac{\sigma^2}{n}}{\varepsilon^2}$$

Which can be presented as

$$P\left[|Y_n - \mu| > \varepsilon\right] \leq \frac{\sigma^2}{n\varepsilon^2}$$

so the theorem is proved. $\square$