

# Approaches to Mechanising Behavioural Types

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with the contributions in the spreadsheet

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# Representations of Binders

## Five approaches

- ▶ de Bruijn indices
- ▶ polymorphic HOAS
- ▶ proof-assistant binders / separation logic
- ▶ nominal sets
- ▶ syntactic

# de Bruijn indices

Common cons: not necessarily “human-readable”  
(should we care? it’s to be checked by machines...)

- ▶ type level (Uma & Ornela)  
well-scoped by construction, easily mechanised
- ▶ tree-shaped (James & Robert)  
metatheory just like  $ST\lambda C$  (well-known)
- ▶ co-indices (Robert)  
simple in the linear case, readily translated to BCI combinators
- ▶ Agda-binders in types (Luca<sup>2</sup>)  
+ just Agda functions      - heavy notation, limited inference
- ▶ proof of context membership in types (Edwin)  
+ well-scoped by construction, type-driven implementation of context manipulation      - sometimes costly at compile-time

## Other Representations of Binders

- ▶ polymorphic HOAS (Uma & Ornela – why another approach?)
  - + transparent to the user
  - cannot reason about in host language
- ▶ proof-assistant binders (Jonas et al.)
  - + Binders like HOAS, no meta theory needed (“apart” from SL)
  - semantic approach
- ▶ nominal sets (Kirstin)
  - + binders treated just like in paper proofs
  - requires equivariance proofs, not “easily” portable
- ▶ syntactic (Petros / Antonio et al.)
  - + quick & easy, no libraries required
  - nightmare to reason about

# Dealing with linearity - I

(almost) as many approaches as entries in the sheet...

- ▶ leftover typing on partial commutative monoids (Uma & Ornela)
  - + no extrinsic context splits
  - needs to be cancelative
- ▶ predicate on processes with polymorphic channels (Uma & Ornela)
  - + trivial to define
  - user loses access to channel type info
- ▶ semiring usage annotations (James & Robert / Lucas)
  - + standard linear algebra
  - no clear general approach, users have to supply proofs
- ▶ linear co-de Bruijn indices (Robert)
  - + no algebra
  - (no algebra) only syntactic linearity, user supplies permutations

# Dealing with linearity - II

(almost) as many approaches as entries in the sheet...

- ▶ separation logic (Iris) (Jonas et al.)
  - + resource reasoning encapsulated
  - unclear how to do other properties than safety
- ▶ quantitative type theory (Edwin)
  - + session types definable directly
  - "read only" operations are syntactically noisy
- ▶ linear logic (Petros)
  - + correct-by-construction
  - allowed patterns severely limited
- ▶ addition relation + proof of well-formedness (Antonio et al.)
  - + operations on types are partial functions, natural to implement
  - operations on type environments require proofs; non-determinism in the type system complicates soundness proofs

# Questions / Challenges

## Are each of the approaches scalable?

- ▶ from linear types to binary session types
- ▶ from binary to multiparty session types
- ▶ from protocol to functional correctness
- ▶ from safety to liveness properties
- ▶ ...

## How to relate these approaches?

- ▶ Encodings?
- ▶ Sharing definitions and results
- ▶ Characterisations to find the right setting to the problem of interest
- ▶ ...