

# Asynchronous Timed Session Types & Processes

**Laura Bocchi**

Maurizio Murgia  
Vasco Thudichum Vasconcelos  
Nobuko Yoshida

University of Kent

University of Kent  
University of Lisbon  
Imperial College London

# Agenda

- Session types ❤️ Time
  - Synchronous [Bartoletti,Cimoli&Murgia@FORTE'13]
  - Multiparty asynchronous [Bocchi,Yang,Yoshida@CONCUR'14]
    - restriction to types/protocols that could be used for type-checking
    - limitations to the expressiveness of the calculus
- Today
  - Designing timed protocols : asynchronous timed **duality**
  - Checking timed programs : a **time**-sensitive **calculus** & **typing** system

# Time & trouble

- A timed protocol is not correct by definition

$\mathbf{!Int.\mathbf{?String}}$

$\mathbf{!Int}(x \leq 3) . \mathbf{?String}(x \leq 2)$

- Usually this is handled by adding some conditions

*feasibility* [Bocchi, Yang, Yoshida@CONCUR'14],  
*interaction-enabledness (CTA)* [Bocchi, Lange, Yoshida@CONCUR'15],  
*compliance* [Bartoletti, Cimoli, Murgia@FORTE'16]  
*formation + duality* [Bocchi, Murgia, Yoshida, Vasconcelos'18]

$\mathbf{!Int}(x \leq 3) . \mathbf{?String}(x \leq 3)$

# Duality & progress

- Duality characterises well-behaved systems

$$S = !\text{Int}(x \leq 1, x) . ?\text{String}(x \leq 2) \quad \bar{S} = ?\text{Int}(y \leq 1, y) . !\text{String}(y \leq 2)$$

Synchronous

$$S \mid \bar{S} \xrightarrow{0.4} S \mid \bar{S} \xrightarrow{\text{Int}} ?\text{String}(x \leq 2) \mid !\text{String}(y \leq 2) \xrightarrow{2} \xrightarrow{\text{String}}$$

# Duality & progress

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$$S = !\text{Int}(x \leq 1, x) . ?\text{String}(x \leq 2) \quad \bar{S} = ?\text{Int}(y \leq 1, y) . !\text{String}(y \leq 2)$$

Synchronous

$$S \mid \bar{S} \xrightarrow{0.4} S \mid \bar{S} \xrightarrow{\text{Int}} ?\text{String}(x \leq 2) \mid !\text{String}(y \leq 2) \xrightarrow{2} \xrightarrow{\text{String}}$$

Asynchronous

$$\begin{aligned} S \mid \bar{S} &\xrightarrow{0.4} S \mid \bar{S} \xrightarrow{!\text{Int}} ?\text{String}(x \leq 2) \mid ?\text{Int}(y \leq 1, y) . !\text{String}(y \leq 2) \\ &\xrightarrow{0.6} \xrightarrow{?\text{Int}} ?\text{String}(x \leq 2) \mid !\text{String}(y \leq 2) \xrightarrow{2} \xrightarrow{!\text{String}} ?\text{String}(x \leq 2) \end{aligned}$$

$x$  2.6  $y$  2



# Receive & asynchrony (1/2)

$$S = \mu t. !\text{Int}(x \leq 1, x). ?\text{String}(x \leq 2). t_1$$

```
func s (a chan<- int, b <-chan string, start time.Time) {
    for {
        time.Sleep(400 * time.Millisecond)
        t := time.Now()
        a<-10
        fmt.Printf("sent int 10 at time %s\n", t.Sub(start))
        select{
            case c:=<-b :
                t := time.Now()
                fmt.Printf("received string %s at time %s\n", c, t.Sub(start))
            case <-time.After(2 * time.Second):
                fmt.Println("S Failed! String not received within deadline")
        }
    }
}
```

# Receive & asynchrony (2/2)

$$\overline{S} = \mu t.\ ?Int(y \leq 1, y).\ !String(y \leq 2).\ t$$

```
func Sd (a <-chan int, b chan<- string, start time.Time) {
    for{
        select{
            case c:=<-a :      t := time.Now()
                                fmt.Printf("received int %d at time %s\n", c, t.Sub(start))
            case <-time.After(1 * time.Second): fmt.Println("Sd Failed! ...")
        }
        time.Sleep(600 * time.Millisecond)
        t := time.Now()
        b<- "hello!"
        fmt.Printf("sent 'hello!' at time %s\n", t.Sub(start))
    }
}
```



# Urgent receive semantics

- Urgent receive semantics: *messages are **received** as soon as*
  - *they are in a channel, and*
  - *the time constraint of the receiver is satisfied*
- Urgent receive semantics yields executions that are
  - are a bit more synchronous ...
  - ... but as asynchronous as when using (common) receive primitives

$$S \mid \bar{S} \xrightarrow{0.4} S \mid \bar{S} \xrightarrow{!Int} ?String(x \leq 2) \mid ?Int(y \leq 1, y) . !String(y \leq 2)$$

$$\xrightarrow{0.6} \xrightarrow{?Int} ?String(x \leq 2) \mid !String(y \leq 2) \xrightarrow{2} \xrightarrow{!String} ?String(x \leq 2)$$

$x$  0       $y$  0

# Urgent receive semantics

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**Type Progress:**  $(\nu_1, S_1, M_1) \mid (\nu_2, S_2, M_2)$  satisfies progress if any reachable state is either **success** (end types and empty queues) or **allows an action**, possibly after some delay.

**Theorem (Duality Progress).**  $(\nu_0, S, \emptyset) \mid (\nu_0, \bar{S}, \emptyset)$  enjoys progress  
(when using urgent receive semantics).

# Subtyping

- Asymmetric as e.g., [Gay&Hole'05][Demangeon&Honda'11]  
[Chen,Dezani-Ciancaglini&Yoshida'14]

**Definition (Timed Simulation).** Fix  $\mathbf{s}_1 = (\nu_1, S_1)$  and  $\mathbf{s}_2 = (\nu_2, S_2)$ . A relation  $\mathcal{R} \in (\mathbb{V} \times \mathcal{S})^2$  is a timed simulation if  $(\mathbf{s}_1, \mathbf{s}_2) \in \mathcal{R}$  implies:

- $S_1 = \text{end}$  implies  $S_2 = \text{end}$
- $\mathbf{s}_1 \xrightarrow{t!m_1} \mathbf{s}'_1$  implies  $\exists \mathbf{s}'_2, m_2 : \mathbf{s}_2 \xrightarrow{t!m_2} \mathbf{s}'_2, (m_2, m_1) \in \mathcal{S}$ , and  $(\mathbf{s}'_1, \mathbf{s}'_2) \in \mathcal{R}$
- $\mathbf{s}_2 \xrightarrow{t?m_2} \mathbf{s}'_2$  implies  $\exists \mathbf{s}'_1, m_1 : \mathbf{s}_1 \xrightarrow{t?m_1} \mathbf{s}'_1, (m_1, m_2) \in \mathcal{S}$ , and  $(\mathbf{s}'_1, \mathbf{s}'_2) \in \mathcal{R}$
- $\mathbf{s}_1 \stackrel{?}{\Rightarrow}$  implies  $\mathbf{s}_2 \stackrel{?}{\Rightarrow}$  and  $\mathbf{s}_2 \stackrel{!}{\Rightarrow}$  implies  $\mathbf{s}_1 \stackrel{!}{\Rightarrow}$

$!\text{String}(x = 0)$	$<:$	$!\text{String}(x \leq 2)$
$? \text{String}(x \leq 2)$	$<:$	$? \text{String}(x = 0)$
$? \text{String}(x \leq 2) . ! \text{String}(x = 1)$	$\not<:$	$? \text{String}(x = 0) . ! \text{String}(x = 1)$
$! \text{String}(\text{true})$	$\not<:$	$? \text{String}(\text{true})$

# Subtyping

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[Chen,Dezani-Ciancaglini&Yoshida'14]

**Definition (Timed Simulation).** Fix  $\mathbf{s}_1 = (\nu_1, S_1)$  and  $\mathbf{s}_2 = (\nu_2, S_2)$ . A relation  $\mathcal{R} \in (\mathbb{V} \times \mathcal{S})^2$  is a timed simulation if  $(\mathbf{s}_1, \mathbf{s}_2) \in \mathcal{R}$  implies:

1.  $S_1 = \text{end}$  implies  $S_2 = \text{end}$
2.  $\mathbf{s}_1 \xrightarrow{t!m_1} \mathbf{s}'_1$  implies  $\exists \mathbf{s}'_2, m_2 : \mathbf{s}_2 \xrightarrow{t!m_2} \mathbf{s}'_2$ ,  $(m_2, m_1) \in \mathcal{S}$ , and  $(\mathbf{s}'_1, \mathbf{s}'_2) \in \mathcal{R}$
3.  $\mathbf{s}_2 \xrightarrow{t?m_2} \mathbf{s}'_2$  implies  $\exists \mathbf{s}'_1, m_1 : \mathbf{s}_1 \xrightarrow{t?m_1} \mathbf{s}'_1$ ,  $(m_1, m_2) \in \mathcal{S}$ , and  $(\mathbf{s}'_1, \mathbf{s}'_2) \in \mathcal{R}$
4.  $\mathbf{s}_1 \stackrel{?}{\Rightarrow}$  implies  $\mathbf{s}_2 \stackrel{?}{\Rightarrow}$  and  $\mathbf{s}_2 \stackrel{!}{\Rightarrow}$  implies  $\mathbf{s}_1 \stackrel{!}{\Rightarrow}$

**Theorem (Safe/Progressing Substitution).** Let  $S' <: \overline{S}$  then

- 1)  $(\nu_0, S, \emptyset) \mid (\nu_0, S', \emptyset) \lesssim (\nu_0, S, \emptyset) \mid (\nu_0, \overline{S}, \emptyset)$
- 2)  $(\nu_0, S, \emptyset) \mid (\nu_0, S', \emptyset)$  enjoys progress.

In [Bartoletti,Bocchi,Murgia@CONCUR'18] asymmetric refinement does not preserve behaviour/progress (it was “local” and did not assume duality)

# Implementing dual types

*“An SMTP server SHOULD have a timeout of at least 5 minutes while it is awaiting the next command from the sender” [RFC 5321]*

$$S = ?\text{Com}(x < 5, x).S' \quad C = !\text{Com}(y < 5, y).C'$$

- This protocol can be implemented e.g., in **Go**, **Erlang** (timeout pattern), **Real-Time Java**, ...

```
select{
    case <-b : \\ proceed as S'
    case <-time.After(5 * time.Second) : \\ explode
}
```

- This protocol cannot be correctly implemented with the calculus in [Bocchi,Yang&Yoshida'14]

# Implementing dual types

$$S = ?\text{Com}(x < 5, x).S'$$

$$C = !\text{Com}(y < 5, y).C'$$

i want to implement these types

$$\text{delay}(4.90).a(b).P'_s \mid \text{delay}(4.99).\bar{a}(\text{HELO}).P'_c$$

$$\longrightarrow a(b).P'_s \mid \text{delay}(0.09).\bar{a}(\text{HELO}).P'_c$$



**Wait-freedom** [Bocchi, Yang, Yoshida'14]: the solutions of the constraint of a receive action must be all after any solution of the corresponding send action

$$S = ?\text{Com}(x = 5, x).S'$$

$$C = !\text{Com}(y < 5, y).C'$$

# Programs

$P ::= \bar{a}v.P$

|  $a \triangleleft 1.P$

| if  $v$  then  $P$  else  $P$

|  $P | P$

| 0

| def  $D$  in  $P$

|  $X\langle \vec{a}; \vec{a} \rangle$

|  $(\nu ab) P$

|  $ab : h$

time-consuming

|  $\text{delay}(\delta).P$

|  $a^n(b).P$

|  $a^n \triangleright \{\mathbb{1}_i : P_i\}_{i \in I}$

$a^n(b).P$	$\dots\dots\dots$	$n = 0$	non-blocking
	$\dots\dots\dots$	$n = \infty$	blocking
	$\dots\dots\dots$	$n \in \mathbb{R}_{>0}$	blocking with timeout

$C = \text{!Com}(y < 5, y).C'$

$\text{delay}(x = 4.90).a^0(b).P'_s$

$a^5(b).P'_s$

# Programs

$P ::= \bar{a}v.P$

|  $a \triangleleft 1.P$

| if  $v$  then  $P$  else  $P$

|  $P | P$

| 0

| def  $D$  in  $P$

|  $X\langle \vec{a}; \vec{a} \rangle$

|  $(\nu ab) P$

|  $ab : h$

time-consuming

| delay( $\delta$ ). $P$

|  $a^n(b).P$

|  $a^n \triangleright \{\mathbb{1}_i : P_i\}_{i \in I}$

$S = ?\text{Com}(x < 5, x).S'$

delay( $x = 4.99$ ). $\bar{a}(\text{HELO}).P'_c$

delay( $4.8 \leq x < 5$ ). $\bar{a}(\text{HELO}).P'_c$

There are also typing rules...

$$\frac{\Gamma \vdash b : T \quad \nu \models \delta \quad \Gamma \vdash P \triangleright \Delta, a : (\nu[\lambda \mapsto 0], S)}{\Gamma \vdash \bar{a}b.P \triangleright \Delta, a : (\nu, !T(\delta, \lambda).S)} \quad [\text{send}]$$

$$\frac{\begin{array}{c} \forall t : \nu + t \models \delta \Leftrightarrow t \leq n \\ \forall t \leq n : \Gamma, b : T \vdash P \triangleright \Delta + t, a : (\nu + t[\lambda \mapsto 0], S) \quad \Delta \text{ not } t\text{-reading} \end{array}}{\Gamma \vdash a^n(b).P \triangleright \Delta, a : (\nu, ?T(\delta, \lambda).S)} \quad [\text{rcv}]$$

$$\frac{\forall n \in \delta : \Gamma \vdash \text{delay}(n).P \triangleright \Delta}{\Gamma \vdash \text{delay}(\delta).P \triangleright \Delta} \quad [\text{delay1}]$$

$$\frac{\Gamma \vdash P \triangleright \Delta + n \quad \Delta \text{ not } n\text{-reading}}{\Gamma \vdash \text{delay}(n).P \triangleright \Delta} \quad [\text{delay2}]$$

# What is a missed deadline?

$$S = ?\text{Com}(x < 5, x).S' \quad C = !\text{Com}(y < 5, y).C'$$

`delay(4.90).a(b).P'_s` | `delay(4.99).ā(HEL0).P'_c`

→ `a(b).P'_s` | `delay(0.09).ā(HEL0).P'_c`



is this really a violation of progress?

→ `failed` | `delay(0.09).ā(HEL0).P'_c`

- Failing semantics:
  - See system's behaviour beyond failure of some parts (-> error handling)
  - Reveals relationship between **untimed progress** and **time safety**

# Programs

$P ::= \bar{a}v.P$

|  $a \triangleleft 1.P$

| if  $v$  then  $P$  else  $P$

|  $P | P$

| 0

| def  $D$  in  $P$

|  $X\langle \vec{a}; \vec{a} \rangle$

|  $(\nu ab) P$

|  $ab : h$

time-consuming

| delay( $\delta$ ). $P$

|  $a^n(b).P$

|  $a^n \triangleright \{\mathbb{1}_i : P_i\}_{i \in I}$

| delay( $n$ ). $P$

| failed

run-time

# Subject reduction?

$$(\nu ab)(\nu cd) \text{ } \color{blue}{a^5(e). \bar{d}e.0 \mid c^5(e). \bar{b}e.0 \mid ab : \emptyset \mid ba : \emptyset \mid cd : \emptyset \mid cd : \emptyset}$$
$$\longrightarrow (\nu ab)(\nu cd)(\text{failed} \mid \text{failed} \mid ab : \emptyset \mid ba : \emptyset \mid cd : \emptyset \mid cd : \emptyset)$$

**Well typed**

$$\emptyset \vdash P \triangleright a : (\nu_0, S), b : (\nu_0, \bar{S}), c : (\nu_0, S), d : (\nu_0, \bar{S})$$
$$S = !\text{Int}(x \leq 5, \emptyset) . \text{end}$$

Subject reduction does not hold in general

# Subject reduction!

**Definition (Live process).**  $\hat{P}$  is live if, for each  $\hat{P}'$  such that  $\hat{P} \rightarrow^* \hat{P}'$  :

$$\hat{P}' \equiv (\nu ab)\hat{Q} \wedge a \in \text{Wait}(\hat{Q}) \implies \exists \hat{Q}' : \hat{Q} \longrightarrow^* \hat{Q}' \wedge a \in \text{NEQueue}(\hat{Q}')$$

**Theorem (Subject Reduction).** Let  $\text{erase}(P)$  be live. If  $\emptyset \vdash P \triangleright \emptyset$  and  $P \longrightarrow P'$  then  $\emptyset \vdash P \triangleright \emptyset$ .

**Theorem (Time Safety).** If  $\text{erase}(P)$  is live,  $\emptyset \vdash P \triangleright \emptyset$  and  $P \longrightarrow^* P'$  then  $P'$  is fail-free.

# In summary

- Duality, subtyping, & urgent receive.

In [Bartoletti,Bocchi,Murgia@CONCUR'18] asymmetric refinement does not preserve behaviour/progress (it was “local” and did not assume duality)

- Dual types cannot be (correctly) implemented with previous work on Multiparty Asynchronous Timed Session Types

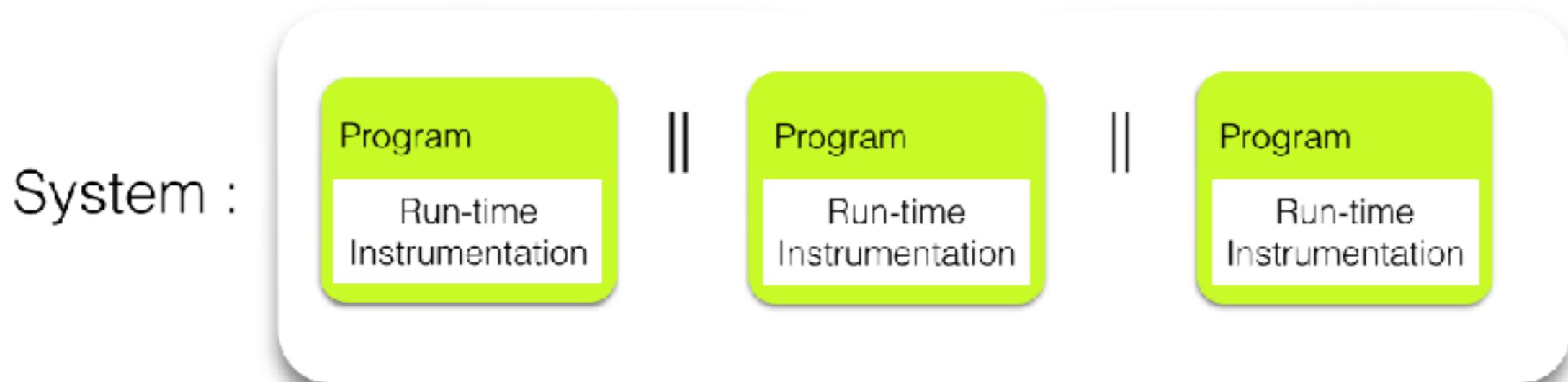
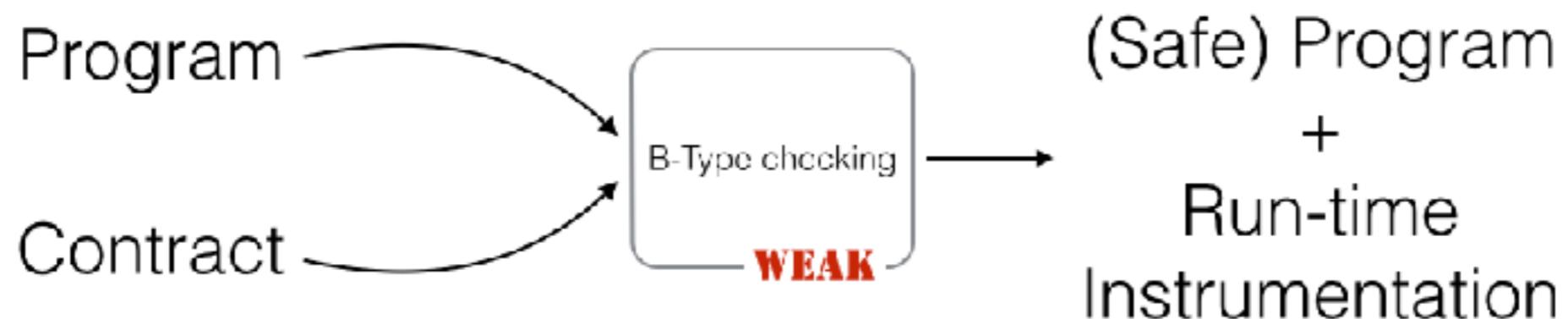
A **time-sensitive calculus** with: parametric receive, delays with arbitrary but constrained delays, and explicit failures upon timeout

A typing system for processes (with delegation) that satisfies subject reduction and time safety

Considerations on the meaning of progress and failure in a timed context

# Future work

- Time-sensitive protocol design and implementation EP/N035372/1
  - expressiveness (flexible timing schedules) + run-time adjustments



# Thank you!

