

# Towards Formalising Trace Equivalence for Global and Local Types

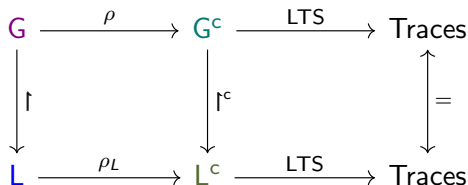
David Castro-Perez   Francisco Ferreira   Lorenzo Gheri  
Nobuko Yoshida

Imperial College London

Verification of Session Types Workshop - 4th June 2020

# Certifying the Semantics of Communication

This work is part of a bigger project for certifying, and reasoning about, programs in distributed systems.



- We formalize the meta-theory of multiparty session types<sup>1</sup>.
- We use the Coq<sup>2</sup> Proof Assistant.

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<sup>1</sup>Deniérou P.-M., Yoshida N. (2013) Multiparty Compatibility in Communicating Automata: Characterisation and Synthesis of Global Session Types. ICALP 2013.

<sup>2</sup><https://coq.inria.fr/>

# Global and Local Types

Inductively defined by the following syntaxes:

$G$	$::=$	$\text{end}$	end type
	$ $	$X$	variable
	$ $	$\mu X.G$	recursion
	$ $	$p \rightarrow q : \{\ell_i(S_i).G_i\}_{i \in I}$	message
$L$	$::=$	$\text{end}$	end type
	$ $	$X$	variable
	$ $	$\mu X.L$	recursion
	$ $	$![q]; \{\ell_i(S_i).L_i\}_{i \in I}$	send type
	$ $	$?[p]; \{\ell_i(S_i).L_i\}_{i \in I}$	receive type

Types are assumed *closed* (no free variables) and recursion variables are always assumed *guarded* in types (namely types like  $\mu X.X$  are not allowed).

# Projection

Projection Rules:

- $\text{end} \upharpoonright r = \text{end};$  [PROJ-END]
- $X \upharpoonright r = X;$  [PROJ-VAR]
- $(\mu X.G) \upharpoonright r = \mu X.(G \upharpoonright r)$  [PROJ-REC]
- $r = p$  implies  $p \rightarrow q : \{\ell_i(S_i).G_i\}_{i \in I} \upharpoonright r = ![q]; \{\ell_i(S_i).G_i \upharpoonright r\}_{i \in I};$  [PROJ-SEND]
- $r = q$  implies  $p \rightarrow q : \{\ell_i(S_i).G_i\}_{i \in I} \upharpoonright r = ?[p]; \{\ell_i(S_i).G_i \upharpoonright r\}_{i \in I};$  [PROJ-RECV]
- $r \neq p, r \neq q$  and, for all  $i, j \in I$ ,  $G_i \upharpoonright r = G_j \upharpoonright r$ ; implies  
 $p \rightarrow q : \{\ell_i(S_i).G_i\}_{i \in I} \upharpoonright r = G_1 \upharpoonright r;$  [PROJ-CONT]
- undefined otherwise.

# Warm-Up!

A global type for a simple protocol:

$$G = p \rightarrow q : \ell(S). q \rightarrow r : \ell'(S'). \text{end}$$

and its projection on participant q:

$$G \restriction q = ?[p]; \ell(S). ![r]; \ell'(S'). \text{end}$$

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Do these global types have well defined projections?

$$p \rightarrow q : \{ \begin{array}{l} \ell_1(\text{oranges}).\text{end}, \\ \ell_2(\text{bananas}).q \rightarrow p : \ell_3(\text{pears}).\text{end} \end{array}$$

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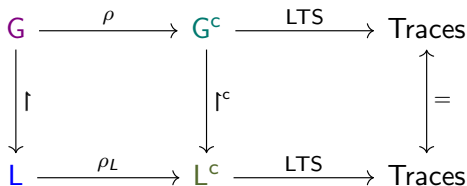
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# Certifying the Semantics of Communication



- Multipart Session Types ✓
- Coinductive Trees (equi-recursive point of view)
- Semantics by Traces



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...

with a coinductive unrolling process!

# Coinductive Asynchronous Trees

Global Trees:

$$\begin{array}{ll}
 G^c & ::= \text{end}^c & \text{end type} \\
 & | \quad p \rightarrow q : \{\ell_i(S_i).G^c_i\}_{i \in I} & \text{message send} \\
 & | \quad p \xrightarrow{\ell_i} q : \{\ell_i(S_i).G^c_i\}_{i \in I} & \text{message receive}
 \end{array}$$

Coinductive unrolling  $\neg\rho_-$ :

$$\frac{\frac{}{\text{end } \rho \text{ end}^c} \quad \frac{G\{\mu X.G/X\} \rho G^c}{\mu X.G \rho G^c}}{\frac{\forall i \in I. G_i \rho G^c_i}{p \rightarrow q : \{\ell_i(S_i).G_i\}_{i \in I} \rho p \rightarrow q : \{\ell_i(S_i).G^c_i\}_{i \in I}}}$$

# Coinductive Asynchronous Trees

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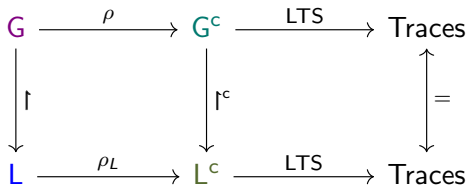
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Local Trees:

$$\begin{array}{lll} L^c & ::= & \text{end}^c \quad \text{end type} \\ & | & !^c[p]; \{\ell_i(S_i).L^c_i\}_{i \in I} \quad \text{send type} \\ & | & ?^c[q]; \{\ell_i(S_i).L^c_i\}_{i \in I} \quad \text{receive type} \end{array}$$



# Certifying the Semantics of Communication



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# Asynchronous Semantics

p sends a message to q with label  $\ell$ :

$$p \rightarrow q : \ell(S).G^c$$

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We keep track of this communication with a queue:

$$\epsilon \xrightarrow{\text{enqueue}} [(\ell, S)] \xrightarrow{\text{dequeue}} \epsilon$$

We have such a queue for each pair of participants.

# Queues for Asynchronous Local Coordination

p sends and q receive:

$$!^c[q];\ell(S).L^c \xrightarrow{\text{step}} L^c$$

$$Q(p, q) = \epsilon \xrightarrow{\text{enqueue}} Q(p, q) = [(\ell, S)] \xrightarrow{\text{dequeue}} Q(p, q) = \epsilon$$

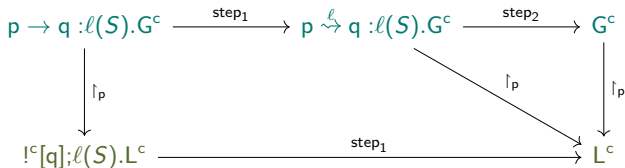
$$?^c[p];\ell(S).L^{c'} \xrightarrow{\text{step}} L^{c'}$$

## Queue Environments

A *queue environment* is a finitely supported function  $Q$  of type  $\text{role} \times \text{role} \rightarrow \mathcal{W}$ , where  $\mathcal{W}$  is the set of finite words  $w$  (*queues*) on the alphabet  $\text{labels} \times \text{sorts}$ .

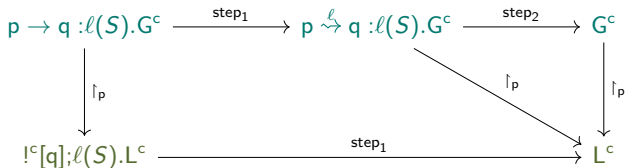
# Global and Local Steps

p sends:

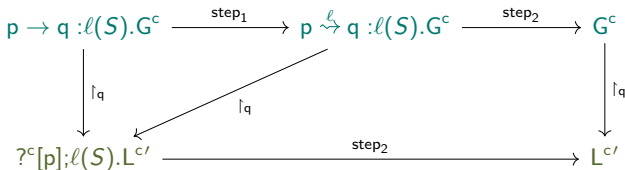


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p sends:



q receives:





# “Local” Environments

We want to consider altogether the different local types involved in the communication.

## Environments for Local Types

An *environment for local types* is a finitely supported function  $E$  of type  $\text{role} \rightarrow \text{l\_ty}^c$ .

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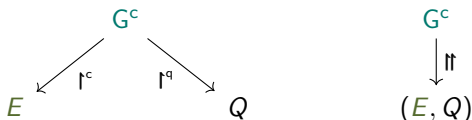
An *environment for local types* is a finitely supported function  $E$  of type  $\text{role} \rightarrow \text{lt}_p^c$ .

Why?!

It will be  $E(p) = L_p^c$ , where  $G^c \vdash_p^c L_p^c$ .

# One-Shot Projection

We get the one-shot projection of a global type both on environments of local types and on queue environments.



# Step Results

## Theorem (Step Soundness)

*If  $G^c \Vdash (E, Q)$  and  $G^c \xrightarrow{\text{step}} G^{c'}$ , then there exist  $E'$  and  $Q'$ , such that  $G^{c'} \Vdash (E', Q')$  and  $(E, Q) \xrightarrow{\text{step}} (E', Q')$ .*

## Theorem (Step Completeness)

*If  $G^c \Vdash (E, Q)$  and  $(E, Q) \xrightarrow{\text{step}} (E', Q')$ , then there exists  $G^{c'}$ , such that  $G^{c'} \Vdash (E', Q')$  and  $G^c \xrightarrow{\text{step}} G^{c'}$ .*

# Labelled Transition System

Keeping track of messages...

## Actions

An *action*  $a$  is an object of the shape:

- either  $pq!(\ell, S)$  (send action),
- or  $pq?(\ell, S)$  (receive action).

## Traces

A *trace*  $t$  is a coinductive, possibly infinite, stream of actions:

- $a_1 a_2 a_3 \dots$  is a trace.

# Traces for Trees

p sends and q receives:

$$p \rightarrow q : \ell(S).G^c \xrightarrow{\text{step}_1} p \xrightarrow{\ell} q : \ell(S).G^c \xrightarrow{\text{step}_2} G^c$$

Let  $t$  be a trace for  $G^c$ ,

$$pq!(\ell, S)pq?(\ell, S)t \xleftarrow{\text{step}_1} pq?(\ell, S)t \xleftarrow{\text{step}_2} t$$

# Non-Determinism

Let us consider two different executions:

- $p \rightarrow q : \{\ell_i(S_i).G^c_i\}_{i \in I} \xrightarrow{\text{step } \ell_1} p \xrightarrow{\ell_1} q : \{\ell_i(S_i).G^c_i\}_{i \in I} \xrightarrow{\text{step } \ell_1} G^c_1$
- $p \rightarrow q : \{\ell_i(S_i).G^c_i\}_{i \in I} \xrightarrow{\text{step } \ell_2} p \xrightarrow{\ell_2} q : \{\ell_i(S_i).G^c_i\}_{i \in I} \xrightarrow{\text{step } \ell_2} G^c_2$

If  $t_1$  is a trace admissible for  $G^c_1$  and  $t_2$  is admissible for  $G^c_2$ , both

- $pq!(\ell_1, S_1)pq?(\ell_1, S_1)t_1$  and
- $pq!(\ell_2, S_2)pq?(\ell_2, S_2)t_2$

are admissible for  $p \rightarrow q : \{\ell_i(S_i).G^c_i\}_{i \in I}$ .



# Trace Equivalence

And finally...

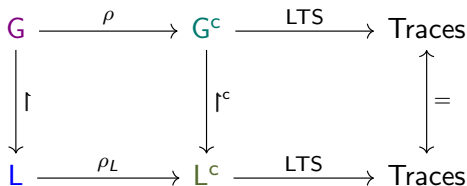
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## Theorem (Trace Equivalence)

*If  $G^c \vdash^c E$  then the set of traces admissible for  $G^c$  is equal to the set of traces admissible for  $E$ .*

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- Multipart Session Types ✓
- Coinductive Trees (equi-recursive point of view) ✓
- Semantics by Traces ✓ (almost)

# Things We Used

Formalisation in the Coq<sup>3</sup> Proof Assistant, in particular we have used:

- the SSReflect<sup>4</sup> proof language;
- the Mathematical Components<sup>5</sup> libraries;
- the PaCo library for parametrized coinduction<sup>6</sup>.

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<sup>3</sup><https://coq.inria.fr/>

<sup>4</sup><https://coq.inria.fr/refman/proof-engine/ssreflect-proof-language.html>

<sup>5</sup><https://math-comp.github.io/>

<sup>6</sup><https://github.com/snu-sf/paco>

# Conclusion

## Things we have got:

- a formalisation of the metatheory of mutliparty session types in Coq
- two birds with a (coinductive) stone: equi-recursion and no bindings
- (non-deterministic) semantics through labelled transition systems
- types that are ready for typing!

## From here what?

- a more comprehensive version of MPST (e.g., with merge operator)
- communicating finite state automata
- ... the future is unwritten!

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**Thank You!**