

Modular Minimalist Grammar

Edward Stabler UCLA

MG+2 2024

- Modular minimalist grammar composed over workspaces
- Interfaces
- Language model composed from grammar and interfaces
- Language model as 1 transduction (modularity breached!)

These slides, code (Haskell, Python), and (soon) draft paper:

<https://github.com/epstabler/mgt>

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© Grammar
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These slides, code (Haskell, Python), and (soon) draft paper:
<https://github.com/epstabler/mgt>

MG derivational steps are broken up then reassembled,

This approach makes MGs *easier to change*!

For many changes you might want to make, only one or two components will need to be modified, leaving the rest intact.

Last step similar Collins&Stabler 2015 . . .

raising similar issues! The issues are clearer here.

(De)composing each derivational step

5 grammatical functions: mrg, ck, t, smc, match
+ 5 bureaucratic functions:

$1 : [2,3] = [1,2,3]$ aka 'cons'

$\text{tail } [1,2,3] = [2,3]$

$\text{zip } [(1,2), (3,4), (5,6)] = [(1,3,5), [2,4,6]]$

$\text{unzip } [(1,3,5), [2,4,6]] = [(1,2), (3,4), (5,6)]$

$[1,2] ++ [3,4] = [1,2,3,4]$ aka 'concatenate'

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(De)composing each derivational step
5 grammatical functions: mrg, ck, t, smc, match
+ 5 bureaucratic functions

└ (De)composing each derivational step

1 : [2,3] = [1,2,3] aka 'cons'
tail [1,2,3] = [2,3]
zip [(1,2), (3,4), (5,6)] = [(1,3,5), [2,4,6]]
unzip [(1,3,5), [2,4,6]] = [(1,2), (3,4), (5,6)]
[1,2] ++ [3,4] = [1,2,3,4] aka 'concatenate'

One MG derivational step will be composed from these 10 simple functions. We cover all the subcases of earlier MG, but without breaking the rule up that way.

The 5 bureaucratic functions are familiar to programmers.

Setup

Grammar: a finite set of lexical items, (ph form, label) pairs:
 $g = \{ (jo, D), (which, N \multimap D.Wh), (likes, D.D \multimap V) \}$

Syntactic object: a lexical item or set of syntactic objects:
 $SO = g \mid \{SO\} \mid [SO]$

Labeled syntactic object: a pair
 $LSO = (SO, label)$

Workspace: a set of LSOs:
 $WS = \{LSO\}$

Our first reformulation of MG will derive workspaces.

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Our first reformulation of MG will derive workspaces

We will define 5 linguistic functions over these types of things.

5 linguistic functions

- (match) $\frac{WS_1, \dots, WS_i \text{ for } i \in \{1, 2\}}{\text{head LSO: complement LSOs, movers}}$
- (mrg) $\frac{SO_1, \dots, SO_n}{\{SO_1, \dots, SO_n\}}$
- (ck) $\frac{f.\alpha \multimap \beta_1, \quad f.\beta_2}{\alpha \multimap \beta_1, \quad \beta_2}$
- (t) $\frac{LSOs}{\text{LSOs - 'inert' LSOs, those with no features left}}$
- (smc) $\frac{LSO_0 \quad \dots \quad LSO_i}{\{LSO_0, \dots, LSO_i\} \text{ if no 2 have same 1st feature}}$

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5 linguistic functions

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Our first reformulation of MG will derive workspaces

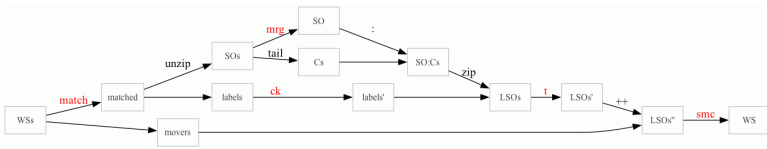
Linearization is left to PF interface...
collapsing 'merge1' and 'merge2'

match pulls out LSOs with matching features, returning 2 sequences:
matching elements with head first; remainder, aka 'movers'.
It also enforces move-over-merge: $\exists!1$ WS if 'move' is possible.

Function ck is *modus ponens*, the *law of detachment*

Function t removes 'trivial' LSOs, those with no features...
This collapses 'merge1/2' and 'merge3', 'move1' and 'move 2'.
An example will make this clear, below

MG composed: The derivational step



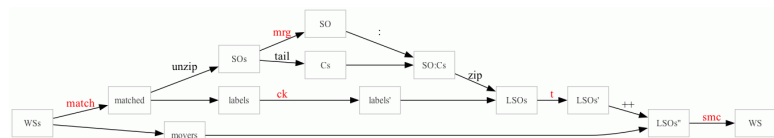
10 steps, but each simple and fast.

After match selects out LSOs with matching first features, 'bureaucratic' functions separate SOs from labels for mrg, ck, t and smc, then reassociate the derived SO with a label for later steps.

```
d WSs = let (matched,movers) = match WSs in
  let (SOs,labels) = unzip matched in
  smc ((t (zip (mrg SOs:tail SOs) (ck labels)))) ++ movers)
```

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MG composed: The derivational step



```
d WSs = let (matched,movers) = match WSs in
  let (SOs,labels) = unzip matched in
  smc ((t (zip (mrg SOs:tail SOs) (ck labels)))) ++ movers)
```

$$\text{derived WSs} = \text{closure of } \{\{i\} | i \in g\} \text{ wrt } d$$

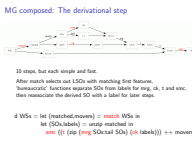
complete WS, SO = derived $\{(\text{SO}, \text{label})\}$ where label has 1 feature

(0) Derived workspaces are connected – a head and ‘active’ substructures

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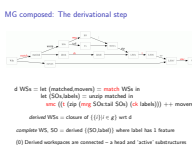
- └ MG composed: The derivational step



This is the actual Haskell definition in my [github](#) implementation.

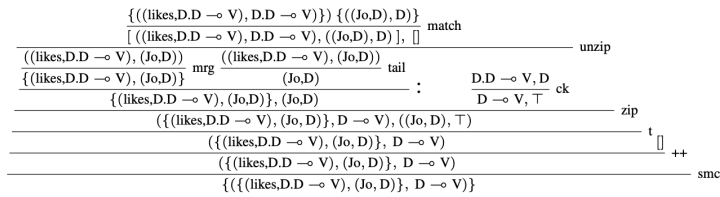
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└MG composed: The derivational step



(0) means that we can view the workspace as a moving, labeled 'window' on the head SO , a kind of 'locus of attention'

Derivational step: example

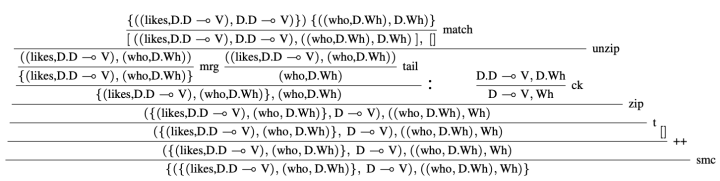


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Derivational step: example

Instead of merging a verb with a direct object in 1 specialized step, we do it in 10 simpler steps that cover all the cases.

Derivational step: example



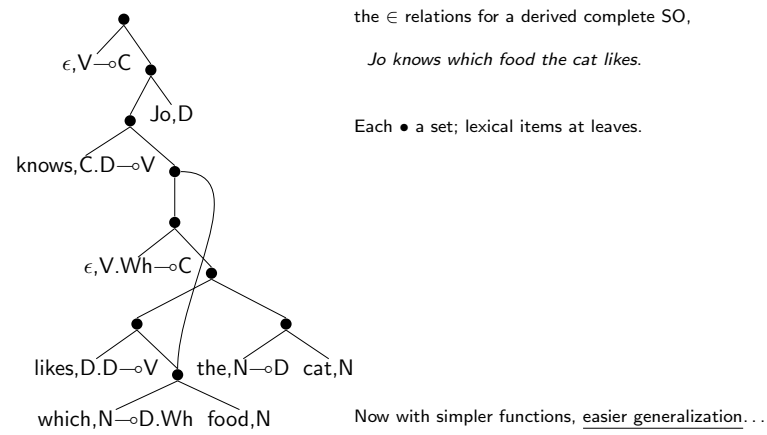
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Derivational step: example

The same pattern of rules can combine *likes* and *who*, but because *who* has two positive features, the effect of step t is different, and as a result, the resulting workspace has 2 LSOs instead of just 1. This would be accomplished with a different rule in many early MGs, but here it is exactly the same pattern of 10 steps

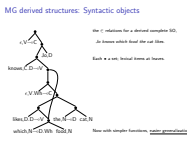
Since these 10 steps are exactly d, and d is the only rule, we just show them as one step, and we do not need to label it!

MG derived structures: Syntactic objects



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MG derived structures: Syntactic objects

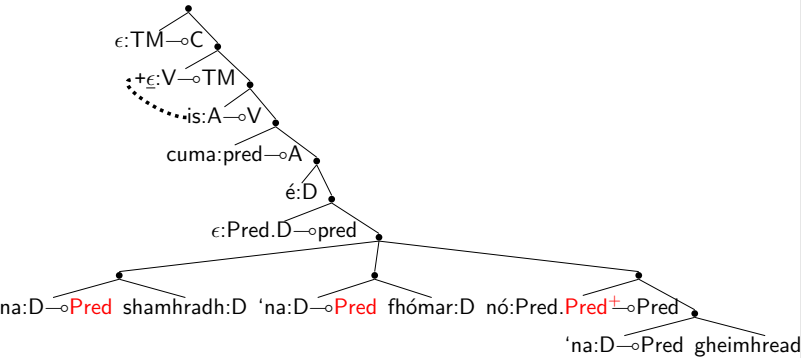


Since the feature checking done by d is so simple, after some practice, I find the SO itself to be the most readable notation for the derivation, though I can understand why linguists prefer highly redundant X-bar-like notations in the literature.

Each set represents 1 application of d to the workspaces corresponding to its children. (This is formalized as transduction ℓ below.)

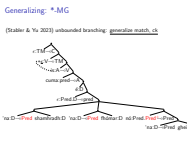
Generalizing: *-MG

(Stabler & Yu 2023) unbounded branching: generalize match, ck



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Generalizing: *-MG



One other detail is discussed in the written version of this talk: To allow multiple identical elements sisters of head, instead of the usual (global) indexing, merge is also adjusted: it builds *multisets*. Using multisets is essentially equivalent to indexing, but local.

One other detail: What's that dotted line? Head movement! A transduction

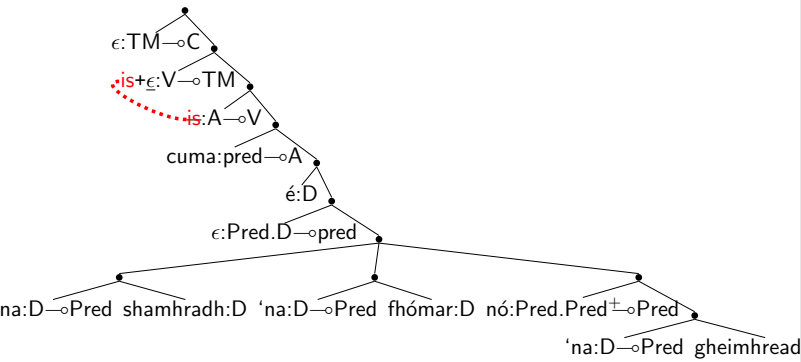
Interface as transduction: ℓ from SO to WS

```
 $\ell$  lex = { (lex,label) }  
 $\ell$  {SO1,...,SOi} =  
  find unique negWS among  $\ell$  SO1,..., $\ell$  SOi;  
  if negWS has matching pos SO=SO2 and i=2,  
  then: d negWS  
  else: d negWS posWS1... posWSi-1
```

- (1) ℓ finds leaves by descent from root,
 then applies d recursively, bottom-up
- (2) deterministic, linear
 if WS regarded as 'window' on head

Interfaces: ℓ in Irish

(McCloskey 2022) V + Asp + TM complexes



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Interface as transduction: ℓ from SO to WS

Before defining a transduction in any new notation, it is good practice to define a very simple one, like id.

A transduction computes id by traversing the whole structure, at each point outputting the very symbol it is looking at.

Here, instead of id, we define the mapping from SO to its workspace (if it has one). At each point, if IM, we check that the attached sister is in fact the element moving from inside the head. Else if EM, d gives us as output the symbol we are looking at, plus copy of (or really, a pointer to) any newly incorporated non-inert complements, with their labels. (Other moving elements are carried along untouched).

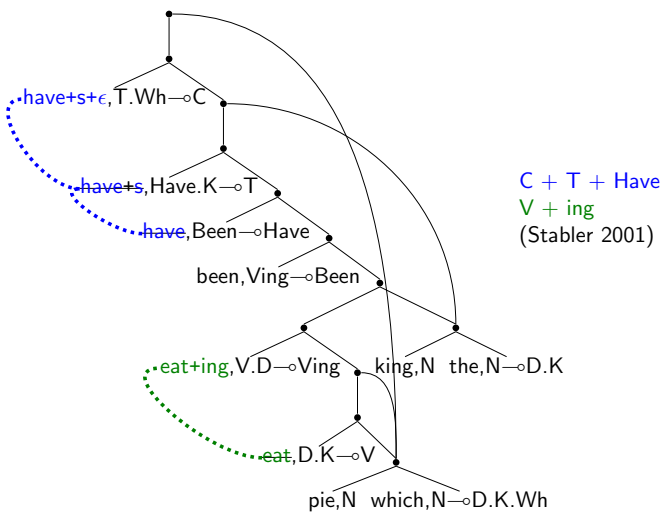
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Interfaces: ℓ in Irish

V raises to TM

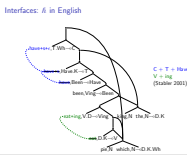
McCloskey 2022 discusses some more complex examples, left for future work

Interfaces: *h* in English



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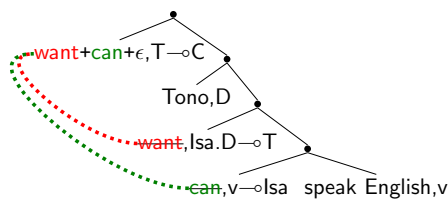
Interfaces: *h* in English



The simplified treatment of Eng aux from Stabler 2001.
which pie have +s +ε the king been eat +ing

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Interfaces: *h* in Javanese



Dheen gelem isa ngomong Inggris
he want can speak English
Gelem isa dheen ngomong Inggris?
want can he speak English?
(Cole&al 2008, Branigan 2023) 'tucking in'

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Interfaces: *h* in Javanese



More than 1 verb can move up to C, 'tucking in' so that linear order
corresponds to linear order of the linearized sources

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Interface: *h* as transduction

Basic idea: Lexical items marked - must associate with X0 stem.
If selector is marked -, move there.

In BU transduction: 'Look ahead' to selector!

Strategy of precise definition:

- a. On path from root to leaf, at an - marked node, pass - down
- b. On way up, at each node:
 - if -: then delete head and pass it (and upcoming) up
 - else: combine head with upcoming (if any)

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Interface: Linear ordering *o*

A linearized MG = (MG, *o*).

*o*_{SVO}: first merge <; nonfirst >

*o*_{SOV}: all >

Both: Very minor variations on *ℓ*.
Like first/nonfirst in SVO, easy to add category-sensitivity.Z

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Interface: Linear ordering *o*

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Chomsky (1995:340) "we take the LCA to be a principle of the phonological component"

Chomsky&al (2019:4) "a matter of externalization of internally generated expressions"

Still no consensus on constituent order:
cf. LCA vs. Abels&Neeleman 2012 and many others

Interface: ϕ_{SVO}

```
 $\phi_{SVO} \text{ lex} = \{ (\text{lex}, \text{label}) \}$   
 $\phi_{SVO} \{SO_1, \dots, SO_i\} =$   
  find unique negWS among  $\ell SO_1, \dots, \ell SO_i$ ; #ineff!  
  if negWS lexical,  
  then: if  $> 1$  pos f:  
    then head(negWS)  
    else head(negWs):heads(posWSs)  
  else: if  $> 1$  pos f:  
    then head(negWS)  
    else heads(posWSs) ++ [head(negWS)]
```

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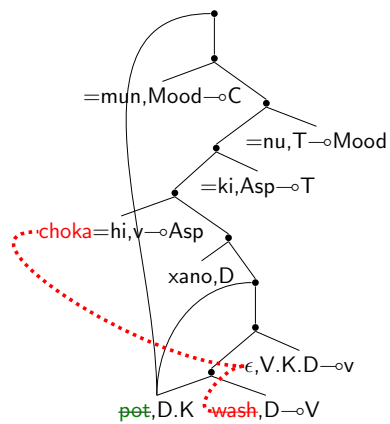
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```
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```

Inefficient: While ℓ goes to leaved and then applies d on the way up, this one goes to leaves, and then on its way up, repeatedly calls ℓ which goes to the leaves again to compute the label and workspace.

This inefficiency can be repaired when language model is reformulated as a single transduction

Interfaces: m in Amahuaca



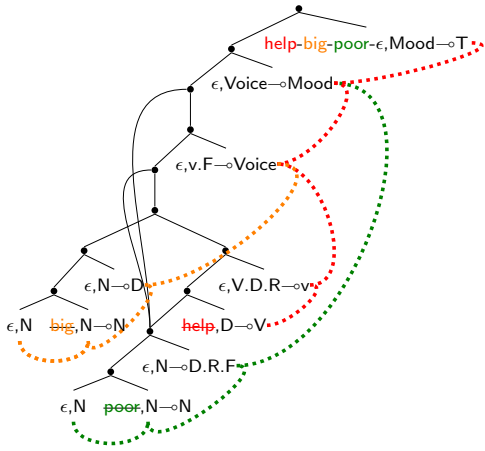
Kuntii=mun choka=hi xano =ki =nu
pot wash woman =3.PRES =DECL
(Clem 2022) – An apparent FOFC violation

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Interfaces: m in Amahuaca

```
Interfaces:  $m$  in Amahuaca  
  
We distinguish head movement from 'm-merger' or 'amalgamation', which can form words from elements that are not heads selected along a single extended projection (Harizanov&Gribanova 2019, Branigan 2023, inter alia)  
  
Here a head movement is shown in red, but notice e.g. that pot/kuntii, linearized as the left child of the root, forms a word with =mun.  
  
Should  $m$  be formalized as a map from the linearized tree to a word sequence? ... Working on this now
```

Interfaces: *h* and *m* in Aleut



kidu -l̥gu -ḡsiida -ku -u
help big poor pres A[sg]:3
'The big one is helping the poor one'
(Snigaroff 2024 (41-3), Merchant 2011, Yuan 2018,2022)

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Interfaces: *h* and *m* in Aleut

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More complicated cases like this one are appearing in the literature.

Cf., e.g., Branigan 2023, Oxford 2014 on the Algonquian language Innu-aimûn.

The language model

Given any set (of sets. . .) of lexical items:

- first** $\ell: SO \rightarrow WS$
- then** to head of output WS, $h: SO \rightarrow SO$
- then** $o: SO \rightarrow (\text{ordered}) \text{ tree}$
- then** $m: \text{tree} \rightarrow \text{word sequence}$

That is, compute $(m \circ o \circ h \circ \text{head} \circ \ell)$.
Full definitions in paper, and in code. Simpler than any previous!

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Full definitions in paper, and in code. Simpler than any previous!

The language model as a single transduction

Better idea?: interleave steps and traverse SO once:

When ℓ applies d, apply h, o, m immediately

Breaches modularity: h, o, m become pre- and post-syntactic because they affect SO, which d tests

How to get the required lookahead on h ?
delay (to max, or phase?)

Both h, o break (0), connectedness

**Idea – apply not to results of d, but at t
't' for 'transfer'**

Still breaks connectedness.

Not an issue if labels completely determine d

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The language model as a single transduction

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Not an issue if labels completely determine d

This comes close to Collins&Stabler 2015. And I think a number of Chomskian, minimalist proposals are trying to do closely related things informally.

Collins&Stabler (2015, §11) note problems that the breach causes for remnant movement, since that is a case where an element in the workspace can be complete, dropped from the workspace by t, even when it contains another element that is not inert.

But it is not obvious that the problem is restricted to that case. We need to watch for anything d tests that is possibly affected by interfaces. E.g. if heads are found by size, then deletion of structure by linearization can be problematic; etc. If we want incremental structure building, then we want even interface transductions that are not counted as part of 'transfer' to be applying at every step.

For me, I like to have a crisp formulation and a running implementation to avoid having to do so many tedious checks by hand.

A Implementations: Haskell

One goal of the present project is to simplify the definition of minimalist grammars, and that goal should really be assessed by looking at a complete, formal statement. Haskell stays close to higher order logic and provides static typing which helps to keep things simple. Each function definition is preceded by a declaration of its type.

See <https://github.com/epstabler/mgt> for this code and examples that use these functions.

A.1. Haskell: Derivations with binary merge

```
module MgBin where
import Data.Set (Set)
import qualified Data.Set as Set
import Data.List
import qualified Data.List as List

data Ft = C | D | N | V | A | P | Wh deriving (Show, Eq, Ord)
type Label = ([Ft], [Ft])
type Lex = ([String], Label)
data PhTree = Pl Lex | Ps [PhTree] deriving (Show, Eq, Ord)
data SO = L Lex | S (Set (SO)) | O PhTree deriving (Show, Eq, Ord)
type LSO = (SO, Label)
type WS = Set (LSO)

-- merge
mrg :: [SO] -> SO
mrg sos = S (Set.fromList sos)

-- already matched features can now be 'forgotten'
ck :: [Label] -> [Label]
ck [(_:nns,nps), ([],_:pps)] = [(nns,nps), ([],pps)]

-- constituents with no remembered features can be 'forgotten'
t :: [LSO] -> [LSO]
t = filter (\lso -> snd lso /= ([],[]))

-- return number of nodes in SO (here we can disregard multidominance)
soSize :: SO -> Int
soSize (S so) = foldr (\x y -> (soSize x) + y) 1 (Set.toList so)
soSize (O (Ps ts)) = foldr (\x y -> (soSize (O x)) + y) 1 ts
soSize _ = 1

-- given LSOs from a positive WS, return largest, i.e. the head
maxx :: [LSO] -> LSO
maxx lsos = foldr1 (\x y -> if ((soSize.fst) x) >= ((soSize.fst) y) then x else y) lsos

-- given WS, return matching ([head,comp], [other LSOs])
match :: [WS] -> ([LSO],[LSO])
match (ws0:wss) = case List.partition ((/= []).fst.snd) (Set.toList ws0) of
  ([h], others') -> let f = ((head.fst.snd) h) in case (List.partition ((== f).head.snd.snd) others', wss) of
    ([c], others'') -> ([h,c], others'')
    ([], wss1) -> case List.partition ((== f).head.snd.snd) (Set.toList wss1) of
      ([c], others'') -> ([h,c], others' ++ others'')

-- if LSOs satisfy shortest move constraint, return WS
smc :: [LSO] -> WS
smc lsos = if smc' [] lsos then (Set.fromList lsos) else (error "smc violation")
  where
    smc' _ [] = True
    smc' sofar ((s,([],p:ps)): lsos) = if elem p sofar then False else smc' (p:sofar) lsos
    smc' sofar (_: lsos) = smc' sofar lsos

-- the derivational step: binary merge and label
d :: [WS] -> WS
d wss = let (matched,movers) = match wss in
  let (sos, labels) = unzip matched in
  smc (t (zip (mrg sos:tail sos) (ck labels) ++ movers))
```

A.2. Haskell: Derivations with unbounded merge

Replacing Data.Set with Data.MultiSet, and extending match and ck for Kleene-plus features.

```

module Mg where    -- Multiset needed. E.g., start ghci with: stack ghci --package multiset
import Data.MultiSet (MultiSet)
import qualified Data.MultiSet as MultiSet
import Data.List
import qualified Data.List as List

data F = C | D | N | V | A | P | Wh | Pred | Predx | T | K | Vx | Scr |
        Modal | Have | Be | Been | Ving | Ven deriving (Show, Eq, Ord)
data Ft = One F | Plus F deriving (Show, Eq, Ord) -- build features from base F
type Label = ([Ft], [Ft])
type Lex = ([String], Label)
data PhTree = Pl Lex | Ps [PhTree] deriving (Show, Eq, Ord)
data SO = L Lex | S (MultiSet (SO)) | O PhTree deriving (Show, Eq, Ord)
type LSO = (SO, Label)
type WS = MultiSet (LSO)

mrg :: [SO] -> SO
mrg sos = S (MultiSet.fromList sos)

ck :: [Label] -> [Label] -- already matched features can be 'forgotten'
ck ((_:nns,nps):(_:_:pps):more) = [(nns,nps), ([],pps)] ++ (map (\label -> ([],[])) more)

t :: [LSO] -> [LSO] -- constituents with no remembered features can be 'forgotten'
t = filter (\lso -> snd lso /= ([],[]))

soSize :: SO -> Int
soSize (S so) = foldr (\x y -> (soSize x) + y) 1 (MultiSet.toList so)
soSize (O (Ps ts)) = foldr (\x y -> (soSize (O x)) + y) 1 ts
soSize _ = 1

maxx :: [LSO] -> LSO
maxx lsos = foldr1 (\x y -> if ((soSize.fst) x) >= ((soSize.fst) y) then x else y) lsos

match :: [WS] -> ([LSO],[LSO])
match (ws0:wss) = case List.partition ((/= []).fst.snd) (MultiSet.toList ws0) of
  ([h], others) -> let (f, plus) = ((fplus.head.fst.snd) h) in
    case (List.partition ((== (One f)).head.snd.snd) others, wss) of
      ([c], others') -> ([h,c], others') -- IM for One feature; otherwise EM:
      ([], _), ws1:wss1 -> let lsos = (MultiSet.toList ws1) in let c = maxx lsos in
        if (((/= One f).head.snd.snd) c)
        then (error "match complement feature clash")
        else let others' = List.filter (/= c) lsos in
          if plus && others' == others
          then (h:c:(atb (One f) others wss), others)
          else if wss == [] then ([h,c], others ++ others') else (error ("match: too many wss"))
  where
    fplus :: Ft -> (F, Bool) -- parse the feature
    fplus ft = case ft of (One f) -> (f, False); (Plus f) -> (f, True)

    atb :: Ft -> [LSO] -> [WS] -> [LSO] -- collect comps with first feature f and others=movers
    atb _ _ [] = []
    atb ft movers (ws:wss) = case List.partition ((== ft).head.snd.snd) (MultiSet.toList ws) of
      ([c'], others) -> if others == movers then c':(atb ft movers wss) else (error "match: ATB error")

smc :: [LSO] -> WS
smc lsos = if smc' [] lsos then (MultiSet.fromList lsos) else (error "smc violation")
  where
    smc' _ [] = True
    smc' sofar ((s,(_:p:ps)):lsos) = if elem p sofar then False else smc' (p:sofar) lsos
    smc' sofar (_:lsos) = smc' sofar lsos

-- the derivational step: unbounded merge and label
d :: [WS] -> WS
d wss = let (matched,movers) = match wss in
  let (sos, labels) = unzip matched in
  smc (t (zip (mrg sos:tail sos) (ck labels) ++ movers))

```

A.3. Haskell: Derivation as transduction

```

module MgTransduction where -- Multiset needed. E.g., start ghci with: stack ghci --package multiset
import Data.MultiSet (MultiSet)
import qualified Data.MultiSet as MultiSet
import Data.List
import qualified Data.List as List

import Mg

negWS :: WS -> Bool
negWS ws = fst (MultiSet.partition ((/= []).fst.snd) ws) /= (MultiSet.empty)

ell :: SO -> WS
ell (L (w,l)) = (MultiSet.singleton (L (w,l), l))
ell (S s) = case List.partition negWS (map ell (MultiSet.toList s)) of
  ([nws],pws:pwss) -> case List.partition ((/= []).fst.snd) (MultiSet.toList nws) of
    ([nso,(f:ns,ps)], others) -> case List.filter ((== f).head.snd.snd) others of
      [(pso,plabel)] -> if (fst.maxx) (MultiSet.toList pws) /= pso -- IM
        then (error "ell : move-over-merge violation")
        else d [nws]
    [] -> d (nws:pws:pwss) -- EM

```

A.5. Haskell: Linearization

```

module MgLinearization where -- Multiset needed. E.g., start ghci with: stack ghci --package multiset
import Data.MultiSet (MultiSet)
import qualified Data.MultiSet as MultiSet
import Data.List
import qualified Data.List as List

import Mg
import MgTransduction

ord_svo :: SO -> SO
ord_svo (O t) = (O t) -- NB! recurse only as deeply as necessary
ord_svo (L (w,l)) = (O (Pl (w,l)))
ord_svo so = let (nso, pso, _, posfs, pwss) = ord(so) in
  let plsos = (map (maxx.(MultiSet.toList)) pwss) in
  let ts = map (\x -> case (ord_svo.fst) x of (O t) -> t) plsos in
  case (ord_svo nso, ord_svo pso, posfs) of
    -- (O (Pl i), O t, _:_:_:_ -> (O (Ps ((Pl i):(map silent (t:ts)))))
    (O (Pl i), O t, _:_:_:_ -> (O ((Pl i))))
    (O (Pl i), O t, _ -> (O (Ps ((Pl i):t:ts))))
    -- (O t, O t', _:_:_:_ -> (O (Ps (((silent t'):(map silent ts))++[t])))
    (O t, O t', _:_:_:_ -> (O t))
    (O t, O t', _ -> (O (Ps (t':ts++[t])))

ord_sov :: SO -> SO
ord_sov (O t) = (O t) -- NB! recurse only as deeply as necessary
ord_sov (L (w,l)) = (O (Pl (w,l)))
ord_sov so = let (nso, pso, _, posfs, pwss) = ord(so) in
  let plsos = (map (maxx.(MultiSet.toList)) pwss) in
  let ts = map (\x -> case (ord_sov.fst) x of (O t) -> t) plsos in
  case (ord_sov nso, ord_sov pso, posfs) of
    -- (O t, O t', _:_:_:_ -> (O (Ps ((silent t'):(map silent ts))++[t])))
    (O t, O t', _:_:_:_ -> (O t))
    (O t, O t', _ -> (O (Ps ((t':ts)++[t])))

-- map SO to what ordering usually depends on: (head, comp, head_features, comp_features, otherCompWSs)
ord :: SO -> (SO,SO,[Ft],[Ft],[WS])
ord (S s) = case List.partition negWS (map ell (MultiSet.toList s)) of -- NB! inefficient
  ([nws],pws:pwss) -> case List.partition ((/= []).fst.snd) (MultiSet.toList nws) of
    ([nso,(f:ns,ps)], others) -> case ((List.filter ((== f).head.snd.snd) others),pwss) of
      ([pso,([],posfs)], []) -> (nso, pso, ps, posfs, pwss) -- for IM, else EM:
      ([],_) -> let (pso,([],posfs)) = (maxx.(MultiSet.toList)) pws in (nso, pso, ps, posfs, pwss)

-- silent :: PhTree -> PhTree
-- silent (Pl (ws,label)) = (Pl ((map (\w -> "(" ++ w ++ ")") ws),label))
-- silent (Ps phs) = Ps (map silent phs)

```

B Implementations: Python

Implementing minimalist grammars in Python is slightly harder than in Haskell. Keeping the two implementations similar, this Python code is written in a functional style, and so ‘type hints’ are added to allow each file to be checked with `mypy` after any changes.

See <https://github.com/epstabler/mgt> for this code and examples that use these functions, with command-line and nltk-based graphical display.

B.0. Python setup

Unlike Haskell, Python cannot have lexical items in a set if those items are built with lists of features. (Python lists are not hashable.) So lexical items are (SO, label) pairs, where label is also a pair (posFeatures, negFeatures), where both positive and negative features are given in tuples.

Python tuples are written with parentheses. Since parentheses are also used for grouping, an extra comma must be added to length 1 tuples for disambiguation. The empty sequence is ().

The files `mgBinTests.py` and `mgTests.py` have functions to print our data structures in more readable form. So, for example, here is the tuple-based representation of the grammar from §1.1.2 of the paper, and the ‘pretty-printed’ version of that grammar:

```
> python
>>> from mgBinTests import *

>>> for i in g112: print(i)

((), (('V',), ('C',)))
((), (('V', 'Wh'), ('C',)))
(('Jo',), ((), ('D',)))
(('the',), (('N',), ('D',)))
(('which',), (('N',), ('D', 'Wh')))
(('who',), ((), ('D', 'Wh')))
(('cat',), ((), ('N',)))
(('dog',), ((), ('N',)))
(('food',), ((), ('N',)))
(('likes',), (('D', 'D'), ('V',)))
(('knows',), (('C', 'D'), ('V',)))

>>> ppMg(g112)

(, V → C)
(, V.Wh → C)
(Jo, D)
(the, N → D)
(which, N → D.Wh)
(who, D.Wh)
(cat, N)
(dog, N)
(food, N)
(likes, D.D → V)
(knows, C.D → V)

>>>
```

We will use these common list functions

```
""" listfs .py
    It is convenient to have these two list functions
    Since written in a functional style, annotated for mypy.
"""

def partition(f, lst: list) -> tuple:
    """ given boolean function f, partition list into two lists : (f-elements, non-f-elements) """
    yes, no = [], []
    for d in lst:
        if f(d): yes.append(d)
        else: no.append(d)
    return (yes, no)

def unzip(pairs: list) -> tuple:
    """ we could use zip(*pairs), but mypy typing does not understand """
    firsts, seconds = [], []
    for (x, y) in pairs:
        firsts.append(x)
        seconds.append(y)
    return (firsts, seconds)
```

Python also does not allow sets of sets. So we use frozensets as our sets, and define these functions:

```

""" fset.py (f is for "frozen", i.e. immutable)
Since python sets cannot contain sets, we use frozensets .
>>> x = set([1,2])
>>> y = set([3,4])
>>> z = set([x,y])
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: unhashable type: 'set'

>>> import fset
>>> x = fset.fromList ([1,2])
>>> y = fset.fromList ([3,4])
>>> z = fset.fromList ([x,y])

since written in a functional style, annotated for mypy.
"""

def fromList( lst:list) -> frozenset:
    """ given list, return fset (i.e. frozenset) of its elements """
    return frozenset(lst)

def toList( fs:frozenset) -> list:
    """ given fset (i.e. frozenset), return its elements in a list """
    return list(fs)

def partition( f,fs:frozenset) -> tuple:
    """ given boolean function f and fset (i.e. frozenset), return (f-elements,non-f-elements) """
    yes, no = [], []
    for d in fs:
        if f(d): yes.append(d)
        else: no.append(d)
    return (frozenset(yes), frozenset(no))

```

Python also does not allow multisets of multisets, so an alternative implementation of multisets is provided here, based on frozendicts. The frozendict module can be installed with ‘pip install frozendict’. Then, the needed multiset functions can be defined like this:

```

""" fmultiset.py (f is for "frozen", i.e. immutable)
Since python multisets cannot contain multisets, we use frozendicts .

NB: mypy does not know the datatype frozendict, so no type hints here.
"""
import frozendict

def fromList( lst ):
    """ given list, return multiset (i.e. frozendict) of those elements """
    counts = {}
    for e in lst:
        if not (e in counts.keys()): counts[e] = 1
        else: counts[e] += 1
    return frozendict.frozendict (counts)

def toList(fms):
    """ given fmultiset (i.e. frozendict), return its elements in a list """
    lst = []
    for y in fms.keys():
        lst.extend(fms[y] * [y])
    return lst

def partition( f,fms):
    """ given boolean function f and fmultiset (i.e. frozendict), return (f-elements,non-f-elements) """
    yes, no = {}, {}
    for d in fms.keys():
        if f(d): yes[d] = fms[d]
        else: no[d] = fms[d]
    return (frozendict.frozendict (yes), fd.frozendict (no))

```


B.1. Python: Derivations with binary merge

```

""" mgBin.py (since written in a functional style, annotated for mypy) """
import fset
from listfs import * # for partition, unzip

def mrg(lst: list) -> frozenset:
    """ merge """
    return fset.fromList( lst )

def ck(labels: list) -> list:
    """ remove first features from already matched labels [0] and labels [1] """
    [(nns,nps), ((),.pps)] = labels
    return [(nns[1:],nps), ((),.pps[1:])]

def t(lsos: list) -> list:
    """ remove LSOs with no features in their label """
    return [x for x in lsos if x[1][1]]

def soSize(so) -> int:
    """ calculate the size of a syntactic object """
    if isinstance(so,tuple) and len(so) == 2 and \
        isinstance(so[0],tuple) and ( len(so[0]) == 0 or isinstance(so[0][0],str) ): # lex
        return 1
    elif isinstance(so,frozenset): # fset
        return 1 + sum(map(soSize, fset.toList( so )))
    else: # phtree
        return 1 + sum(map(soSize, so))

def maxx(lsos: list) -> tuple:
    """ given LSOs (of a WS), return LSO with largest SO (the head) """
   sofar = 0
    maxLSO = (((),), ((),.))
    for lso in lsos:
        lsoSize = soSize( lso [0])
        if lsoSize > sofar :
            sofar = lsoSize
            maxLSO = lso
    return maxLSO

def smc(lsos: list) -> frozenset:
    """ if LSOs have no pos feature in common, return them """
    firstPosFeatures = [ lso [1][1][0] for lso in lsos if lso [1][0] == () ]
    if len( firstPosFeatures ) < len(set( firstPosFeatures )):
        return fset.fromList( [] )
    else:
        return fset.fromList( lsos )

def match(wss: list) -> tuple:
    """ given list of workspaces, return ([head, complement], other LSOs) """
    (ws0, wss0) = (wss[0], wss[1:])
    (heads, others) = partition( lambda lso: lso [1][0] != (), fset.toList( ws0 ))
    if not( len(heads) == 1 ):
        raise RuntimeError('match: 0 or > 1 neg lsos in ws0')
    h = heads[0]
    f = h [1][0][0]
    (ics, iothers) = partition( lambda lso: lso [1][1][0] == f, others )
    if ics and wss0 == []: # im
        return ([h, ics [0]], iothers )
    elif ics == [] and len(wss0) == 1: # em
        ws1 = wss0[0]
        lsos = fset.toList( ws1 )
        c = maxx( lsos )
        if c [1][0] == () and c [1][1][0] == f:
            others1 = [x for x in lsos if x != c]
            return ([h, c], others + others1 )
        else: RuntimeError('match: complement')
    else:
        raise RuntimeError('match')

def d(wss: list) -> frozenset:
    """ the derivational step: given list of workspaces, return derived workspace """
    (matched, movers) = match(wss)
    (sos, labels) = unzip(matched)
    return smc( t( list( zip( [mrg(sos)] + sos[1:], ck( labels ) ) ) + movers ) )

```

B.2. Python: Derivations with unbounded merge

Replacing fset.py (i.e. frozensets) with fmultiset.py (i.e. frozendicts), and extending match, ck.

```
""" mg.py (not annotated for mypy, since mypy does not know frozendict) """
import frozendict, fmultiset
from listfs import * # for partition, unzip

def mrg(lst: list):
    """ merge """
    return fmultiset.fromList( lst )

def ck(labels: list) -> list:
    """ remove first features from already matched labels [0], labels [1] and T for rest """
    (nns,nps),((),pps)=labels[0], labels[1]
    return ((nns[1:],nps), ((),pps[1:])) + tuple((((),()) for label in labels[2:]))

def t(lsos:list) -> list:
    """ remove LSOs with no features in their label """
    return [x for x in lsos if x[1][1]]

def soSize(so) -> int:
    """ calculate the size of a syntactic object """
    if isinstance(so,tuple) and len(so)==2 and \
        isinstance(so[0],tuple) and ( len(so[0]) == 0 or isinstance(so[0][0],str) ): # lex
        return 1
    elif isinstance(so,frozendict.frozendict): # fmultiset
        return 1 + sum(map(soSize, fmultiset.toList( so )))
    else: # phtree
        return 1 + sum(map(soSize, so))

def maxx(lsos: list) -> tuple:
    """ given LSOs (of a WS), return LSO with largest SO (the head) """
   sofar = 0
    maxLSO = (((),()),((),()))
    for lso in lsos:
        lsoSize = soSize(lso[0])
        if lsoSize > sofar:
            sofar = lsoSize
            maxLSO = lso
    return maxLSO

def smc(lsos: list):
    """ if LSOs have no pos feature in common, return them """
    firstPosFeatures = [lso[1][1][0] for lso in lsos if lso[1][1][0] == () ]
    if len( firstPosFeatures ) < len(set( firstPosFeatures )):
        return fmultiset.fromList( [] )
    else:
        return fmultiset.fromList( lsos )

def match(wss:list) -> tuple:
    """ given list of workspaces, return ([head,complement],otherLSOs) """
    (ws0,wss0) = (wss[0],wss[1:])
    (heads, others) = partition( lambda lso: lso[1][1][0] != (), fmultiset.toList( ws0 ))
    if not(len(heads)==1): raise RuntimeError("match: 0 or >1 neg lsos in ws0")
    h = heads[0]
    f = h[1][0][0]
    if f[-1] == '+':
        f, plus = f[:-1], True
    else:
        plus = False
    (ics, iothers) = partition( lambda lso: lso[1][1][0] == f, others )
    if ics and wss0 == []: # im
        return ([h,ics[0]], iothers )
    elif ics==[] and len(wss0)>0: # em
        ws1 = wss0[0]
        lsos = fmultiset.toList( ws1 )
        c = maxx(lsos)
        if c[1][1][0] != f:
            raise RuntimeError("match: %r != %r" % (f,c[1][1][0]))
        else:
            others1 = [x for x in lsos if x != c]
            if plus and others == others1:
                return ([h,c]+atb(f, others, wss0[1:]), others )
            elif wss0[1:] == []:
                return ([h,c], others + others1 )
            else:
                raise RuntimeError("match: too many wss")

def atb(f, movers, wss):
    """ collect comps with first feature f and others==movers """
    if wss == []:
        return []
    else:
        ws1 = wss[0]
        lsos = fmultiset.toList( ws1 )
        c = maxx(lsos)
```

```

    if c[1][1][0] != f:
        raise RuntimeError("match: complement clash")
    else:
        others = [x for x in lsos if x != c]
        if others != movers:
            raise RuntimeError("match: mover clash")
        else:
            return [c] + atb(f, movers, wss[1:])

def d(wss:list):
    """ the derivational step: given list of workspaces, return derived workspace """
    (matched, movers) = match(wss)
    (sos, labels) = unzip(matched)
    return smc( t( list(zip( [mrg(sos)]+sos[1:], ck(labels ))) + movers))

```

B.3. Python: Derivation as transduction

```

""" mgTransduction.py (not annotated for mypy, since mypy does not know frozendict) """
import frozendict
from mg import *

def negWS(ws:list) -> bool:
    return [lso for lso in fmultiset.toList(ws) if lso[1][0] != () ] != []

def ell(so):
    """ map so to its derived workspace """
    if isinstance(so,tuple) and len(so) == 2 and \
        isinstance(so[0],tuple) and ( len(so[0]) == 0 or isinstance(so[0][0],str) ):
        return fmultiset.fromList([(so,so[1])])
    elif isinstance(so,frozendict.frozendict):
        ([nws],pwss) = partition (negWS, [ell(s) for s in fmultiset.toList(so)])
        pws = pwss[0]
        (nlsos, others) = partition ( lambda lso: lso[1][0] != (), fmultiset.toList(nws) )
        if len(nlsos) != 1:
            raise RuntimeError("ell: zero or >1 neg LSOs in workspaces")
        (nso,(nfs,pfs)) = nlsos[0]
        imatches = [lso for lso in others if lso[1][1][0] == nfs[0]]
        if len(imatches) == 1:
            (pso,plabel) = imatches[0]
            if maxx(fmultiset.toList(pws))[0] != pso or pwss[1:] != []:
                raise RuntimeError("move-over-merge error")
            else: # im
                return d([nws])
        else: # em
            return d([nws] + pwss)
    else:
        raise TypeError("ell type error")

```

B.5. Interfaces in Python: Linearization

```

""" mgLinearization.py (not annotated for mypy, since mypy does not know frozendict) """
import frozendict
from mgTransduction import *

def ord_svo(so) -> tuple:
    """ maps so to ordered phTree """
    if isinstance(so,tuple) and len(so) == 2 and \
        isinstance(so[0],tuple) and ( len(so[0]) == 0 or isinstance(so[0][0],str) ): # so is lex
        return so
    else: # so is mset
        (nso, pso, _, posfs, pwss) = ord(so)
        nt, pt = ord_svo(nso), ord_svo(pso)
        plsos = list(map(maxx, pwss))
        psos = [x[0] for x in plsos]
        pts = map(ord_svo, psos)
        if isinstance(nt,tuple) and len(nt) == 2 and \
            isinstance(nt[0],tuple) and ( len(nt[0]) == 0 or isinstance(nt[0][0],str) ): # nt is lex
            if len(posfs) > 1: return (nt,) + silent((pt,) + tuple(pts))
            else: return (nt, pt) + tuple(pts)
        else: # nt is mset
            if len(posfs) > 1: return silent((pt,) + tuple(pts)) + (nt,)
            else: return (pt,) + tuple(pts) + (nt,)

def ord_sov(so) -> tuple:
    """ maps so to ordered phTree """
    if isinstance(so,tuple) and len(so) == 2 and \
        isinstance(so[0],tuple) and ( len(so[0]) == 0 or isinstance(so[0][0],str) ): # so is lex
        return so
    else: # so is mset
        (nso, pso, _, posfs, pwss) = ord(so)
        nt, pt = ord_sov(nso), ord_sov(pso)
        plsos = list(map(maxx, pwss))
        psos = [x[0] for x in plsos]
        pts = map(ord_sov, psos)
        if len(posfs) > 1: return silent((pt,) + tuple(pts)) + (nt,)
        else: return (pt,) + tuple(pts) + (nt,)

def ord(so) -> tuple:
    """ maps so to (head, comp, head_pos, features, comp_positive-features, otherPosWSs) """
    if not(isinstance(so,frozendict.frozendict)): raise TypeError("ord type error")
    ([nws],pwss) = partition (negWS, [ell(s) for s in fmultiset.toList(so)]) # NB! inefficient
    (pws,pwss1) = (pwss[0],pwss[1:])
    (nlsos, others) = partition ( lambda lso: lso[1][0] != (), fmultiset.toList(nws) )
    (nso,(fns,ps)) = nlsos[0]
    f = fns[0]
    imatches = [lso for lso in others if lso[1][1][0] == f]
    if len(imatches) == 1: # im
        (pso,([] ,posfs)) = imatches[0]
        return (nso, pso, ps, posfs, pwss1)
    else: # em
        (pso,([] ,posfs)) = maxx(fmultiset.toList(pws))
        return (nso, pso, ps, posfs, pwss1)

def silent(ph: tuple) -> tuple:
    """ maps phTree (nested tuple of lexical items)
        to isomorphic phTree in which lexical ph content is marked with parens """
    if isinstance(ph,tuple) and len(ph) == 2 and \
        isinstance(ph[0],tuple) and ( len(ph[0]) == 0 or isinstance(ph[0][0],str) ): # lex
        return ( tuple([w if (w[0]=='(' and w[-1]==')') else '('+w+')' for w in ph[0]]), ph[1] )
    else:
        return tuple([silent(x) for x in ph])

```