



A05: Scale-Free Networks

Network Science '21: Assignment 5

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Objectives

1. Explore the scale-free property of real networks and the difference with random networks
2. Get familiar with the power law package



A05.1 Scale-Free Networks



A05.1 Everyday networks are not Poisson: parameter estimation and model selection

Task: For all given networks compute the degree distribution $P(k)$ and the average degree $\langle k \rangle$ and use the power law package to fit $P(k)$.

1. Compute the power-law exponent γ of the degree distribution $P(k) \sim k^{-\gamma}$ and the corresponding error
2. Superimpose in the same plot of the power-law fit (a) the Poisson and (b) the exponential distribution with mean value equal to $\langle k \rangle$. Which distribution is more likely to describe the data?
3. What does it mean in practice for the Internet?



A05.1 Hints

- + The Poisson distribution with mean value $\langle k \rangle = \lambda$ can be computed as:

$$P(k, \lambda) = \frac{\lambda^k}{k!} \exp(-\lambda)$$

- + The exponential distribution with mean value $\langle k \rangle = \lambda$ can be computed as:

$$P(k, \lambda) = \lambda^{-1} \exp(-k/\lambda)$$

- + For the factorial $k!$ you can use the `factorial` function that can be imported from the module `scipy.special`



A06.1 Datasets provided

Datasets provided:

- + Sex Escorts: Nodes represent female (sex-sellers) and male (sex-buyers) and Edges represent a sexual encounter between a male and a female [1]
- + Internet: Nodes represent autonomous systems (AS) , i.e. collections of connected IP routing prefixes controlled by independent network operators, and Edges represent physical connections between them [2]
- + Amazon: Nodes represent Amazon products and Edges represent frequently co-purchased products [3]
- + Actor-actor collaborations: Nodes represent actors and Edges represent appearances in the same movie [4]



A06.1 Datasets provided

- [1] L E C Rocha, F Liljeros, and P Holme, Information dynamics shape the sexual networks of Internet-mediated prostitution, Proceedings of the National Academy of Sciences of the USA 107 (13), 5706-5711 (2010)
- [2] B. Zhang, R. Liu, D. Massey, and L. Zhang, Collecting the Internet AS-level topology, SIGCOMM Computer Communication Review, 35 (2005), pp. 53–61.
- [3] J. Yang and J. Leskovec. Defining and Evaluating Network Communities based on Ground-truth. ICDM, 2012.
- [4] A-L. Barabasi and R. Albert , Emergence of Scaling in Random Networks, Science (286), 509–512 (1999)



A05.2 Preferential Attachment model



A05.2 Barabási-Albert model

Task: Build networks with the Barabási-Albert model. Connect $m = 3$ for each new node and grow until $N = 50, 100, 500, 1000, 5000$, then compute some network properties. Compare them with randomised versions of the networks.

1. Compute the average clustering coefficient, assortativity, average shortest path length and diameter
2. Compare them by scatterplots with the same measures on randomised versions of the networks (use log x axis)
3. Which of these measures is relatively unchanged by randomisation? Why?



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