



## Statistical Properties of Graphs

Network Science '21: Assignment 2

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## A02.1 Average degree of the nearest neighbours



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**Task:** For each dataset G, randomise it to obtain a network  $G_{rnd}$  and explore their assortativity properties

- 1. Plot the average degree of the nearest neighbours  $k_{nn}(k)$  as a function of the vertices degree k
- 2. Compute the assortativity coefficient of the real network
- Compute the assortativity coefficient of the randomised network





## A02.2 Clustering and randomisation



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For each dataset  $\mathcal{G}$ , randomise it to obtain a network  $\mathcal{G}_{rnd}$  and explore their clustering properties

- 1. Plot the degree distribution in double-logscale and mark with a vertical line the average degree  $\langle k \rangle$
- 2. For each node, compute the clustering coefficient in the graphs  $\mathcal{G}$  (denoted by C(i)) and  $\mathcal{G}_{rnd}$  (denoted as  $C_{rnd}(i)$ )
- 3. Do a scatter plot of C(i) vs.  $C_{rnd}(i)$



#### A02.2 Notes on randomisation

- The randomised networks are obtained via multiple edge swaps via the networkx function nx.algorithms.smallworld.random\_reference
- Make sure to set the parameter connectivity =
   False to have faster execution
- + If execution is still slow, we provide randomised datasets
- We will see next that this is a simple way to randomise a network by preserving its degree distribution (can you see why this is the case?)
- + This function does not ensure that the resulting graph is still connected (more on this later...)



#### Datasets provided:

- C. elegans interactomes: Nodes represent proteins and Edges represent protein-protein interactions in Caenorhabditis elegans (nematode) [1]
- AstroPhysics Arxiv collaborations: Nodes represent authors of papers submitted to arxiv.org in the AstroPh category and Edges represent co-authorship between two authors
   [2]
- + Condensed Matter Arxiv collaborations: Nodes represent authors of papers submitted to arxiv.org in the CondMat category and Edges represent co-authorship between two authors [3]



- Kaggle chess players: Nodes represent chess players and Edges represent chess match among the world's top chess players [4]
- Dolphin social network: Nodes represent dolphins and Edges represent frequent associations observed among a group of 62 individuals [5]
- + European airline network: Nodes represent airlines and Edges represent airline routes among European airports [6]



- Facebook friendships: Nodes represent Facebook users and Edges represent their friendship relations collected from survey participants [7]
- Florentine families: Nodes represent Florentine families during the Italian Renaissance and Edges represent marriage alliances and business relationships between two families [8]
- Game of Thrones coappearances: Nodes represent Game of Thrones characters and Edges represent coappearances of characters in the Game of Thrones series [9]



- Internet AS graph: Nodes represent autonomous systems and Edges represent connections between them [10]
- Jazz collaboration network: Nodes represent jazz musicians and Edges represent collaborations in bands that performed between 1912 and 1940 [11]
- + 9-11 terrorist network: Nodes represent individuals and Edges represent their known social associations, centered around the hijackers that carried out the September 11th, 2001 terrorist attacks [12]



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- [4] Kaggle, "Chess ratings Elo versus the Rest of the World." https://www.kaggle.com/c/chess/data (2010)
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