502 ASSIGNMENTS
SPRING 2017

Assignment I

1. Let $\mathcal{V}$ be a vector space over the field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$. A *seminorm* on $\mathcal{V}$ is a map $\rho : \mathcal{V} \to \mathbb{R}$ satisfying

(i) $\rho(v) \geq 0$ for all $v \in \mathcal{V}$
(ii) $\rho(\alpha v) = |\alpha|\rho(v)$ for all $\alpha \in \mathbb{F}$ and $v \in \mathcal{V}$.
(iii) $\rho(v + w) \leq \rho(v) + \rho(w)$ for all $v, w \in \mathcal{V}$.

Let φ be a continuous function $[0, 1] \to \mathbb{R}$.

For $f \in C[0, 1]$ define $\rho(f) = \int_0^1 |f(x)|\varphi(x)\, dx$. What conditions must φ satisfy so that ρ is a seminorm? What conditions must φ satisfy so that ρ is a norm?

2. Let $C^1[0, 1]$ denote the vector space of functions defined on the interval $[0, 1]$ which have continuous derivatives. (The derivative at an endpoint is the one-sided derivative.) For $f \in C^1[0, 1]$, let $\rho(f) = \max_{0 \leq x \leq 1} |f'(x)|$. Is ρ a norm? Is it a seminorm?

3, Determine which of the following formulas define a metric.

(i) On $\mathbb{R}$, $d(x, y) = \sqrt{|x - y|}$
(ii) On $\mathbb{R}$, $d(x, y) = (x - y)^2$
(iii) On $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$, $d(x, y) = |\log(y/x)|$

4. Do exercises 1, 2, and 4 in section 2.1, p. 27

due 1/18

Assignment II

1. In $(\mathbb{Z}, |\cdot|)$, let $a \in \mathbb{Q}$, describe

(i) $\partial B_r(a)$ if $r \in \mathbb{Q}$, $r > 0$.
(ii) $\partial B_r(a)$ if $r \in \mathbb{R} \setminus \mathbb{Q}$, $r > 0$.

2. Do problems 1, 2, 4, and 5 in Sec. 2.2, p. 34.

3. Do problems 1 and 3 in Sec. 2.3, p. 40.

due 1/25