Statistics 520, Fall 2016

Assignment 0 Solution

<u>The Answer</u>

The major difficulty with the study design was a mis-identification of experimental units. The experimental units in this study are the classes (or sections) and there was only one experimental unit per treatment (course delivery method) making this an unreplicated study.

<u>Additional Comments</u>

Many individuals identify the lack of randomized assignment of students to treatments (modes of instruction) as the primary problem with the study. While this is not quite correct, the lack of randomized treatment assignment is, in fact, a major problem in addition to improper identification of experimental units. Given that experimental units are classes, the lack of randomization here damages primarily the uniformity that we would like to assume exists among experimental units, aside from the treatment. An experimental approach to statistical analysis would require random assignment of treatments to experimental units. While random assignment of students to classes would automatically ensure that, random assignment of treatments to classes could in principle be accomplished without random assignment of students to treatments. Of course, even that could not be accomplished by allowing students to select the identity of the treatment they will receive.

What if we abandon a pure experimental approach to analysis, and fit a model to student-level responses, similar to what was actually done? Could that be justified in some way? Perhaps. Disregarding difficulties caused by lack of randomization of students to treatments, which would now become a major concern, we could fit a model that assumes independent responses of students within classes. What would be the result of doing so in the analysis of this study? Primarily to render the scope of inference to be exceedingly small. Conclusions would apply to these three classes which, given that these three classes constitute an unreplicated experiment, amounts to an anecdote – a single occurence of something.

Now, suppose the study had been designed such that there was replication (more than one class per treatment) and randomization of students to classes (which ensures random assignment of treatments to experimental units, which are still classes). We could analyze the resultant data according to either a pure experimental approach, or based on a model. A pure experimental approach would aggregate student responses to the level of experimental units and then we might conduct either a permutation test or fit a one-way ANOVA (note that this would amount to using a model within an experimental approach to analysis). An alternative more strictly model-based approach could take student-level responses to be nested within classes. If there were a small number of classes we might model the effects of those classes as fixed parameter values and all student responses as independent. If there were a larger number of classes we might model the class effects as random, which would then result in non-zero covariances among students within the same class.

A Closer Look at Two Models

Since you have already seen the two models mentioned in the previous section, let's go ahead and look at how these models are representing the situation that produces observed data. Start to think of models as "data generating mechanisms" that are conceptualizations of the underlying substantive mechanisms that did actually produce the data.

Let $Y_{i,j,k}$ denote random variables associated with the response of student k ($k = 1, \ldots, n_{i,j}$) in class j ($j = 1, \ldots, m_i$) that receives treatment i ($i = 1, \ldots, K$).

1. Model 1.

   A model that takes class effects to be fixed parameters could be formulated as

   $$Y_{i,j,k} = \mu + \alpha_i + \gamma_j + \sigma\epsilon_{i,j,k}, \tag{1}$$

   where μ, σ, $\alpha_i; i = 1, \ldots, K$ and $\gamma_j; \jmath = 1, \ldots, m_i$ are fixed parameters to be estimated, and $\epsilon_{i,j,k} \sim iidN(0, 1)$. Basic moments for this model are:

   $$\begin{aligned} E(Y_{i,j,k}) &= \mu + \alpha_i + \gamma_j \\ var(Y_{i,j,k}) &= \sigma^2 \\ cov(Y_{i,j,k}, Y_{i,j,l}) &= 0 \end{aligned}$$

   and all other covariances are 0.

2. Model 2.

   $$Y_{i,j,k} = \mu + \alpha_i + \tau\gamma_j + \sigma\epsilon_{i,j,k}, \tag{2}$$

   where μ, σ, and $\alpha_i; i = 1, \ldots, K$ are fixed parameters to be estimated, $\gamma_j \sim iidN(0, 1)$, and $\epsilon_{i,j,k} \sim iidN(0, 1)$.

   Conditional moments for this model are:

   $$\begin{aligned} E(Y_{i,j,k}|\gamma_j) &= \mu + \alpha_i + \tau\gamma_j \\ var(Y_{i,j,k}|\gamma_j) &= \sigma^2 \\ cov(Y_{i,j,k}, Y_{i,j,l}|\gamma_j) &= 0 \end{aligned}$$

   and all other covariances are 0. Marginal moments for this model are:

   $$\begin{aligned} E(Y_{i,j,k}) &= \mu + \alpha_i \\ var(Y_{i,j,k}) &= \tau^2 + \sigma^2 \\ cov(Y_{i,j,k}, Y_{i,j,l}) &= \tau^2 \end{aligned}$$

   and all other covariances are 0.

Notice that the conditional moments of Model 2 are essentially the same as the basic moments of Model 1. Also note that Model 1 and Model 2 partition what I call the structure of the data in different probabilistic ways. In particular, both models take the responses of students within a class to be probabilistically more similar than responses of students from different classes. Model 1 attributes this to a fixed difference in expectations, while Model 2 attributes this to covariance within classes.