Wealth, Uninsurable Idiosyncratic Risk and International Risk Sharing*

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Abstract

This paper evaluates the role of uninsurable idiosyncratic risk and wealth distribution in international risk sharing. The dynamics of net international capital flows in a HANK model featuring idiosyncratic income risk, distribution of wealth, and two types of assets (liquid and illiquid) traded with the rest of the world are determined by the relative strength of three effects: First, the low consumption-smoothing ability of borrowing-constrained households dampens aggregate savings responses to business cycles. Second, idiosyncratic risk increases the savings of unconstrained households. Finally, endogenous changes in the share of constrained households amplify savings responses when idiosyncratic risk is high; in a low-risk economy responses are dampened. As a result of these three opposing effects, aggregate consumption volatility, in the presence of borrowing constraints, is a U-shaped function of the level of idiosyncratic risk. Two household-level surveys were used to calibrate the model devised in this paper to wealth-rich low idiosyncratic risk and wealth-poor high idiosyncratic risk economies. Results show that aggregate consumption volatility would decrease with the increase in idiosyncratic risk in a wealthy country due to a decrease in the share of borrowing-constrained households. In contrast, in a wealth-poor country, consumption volatility would be lower if the risk was lower. Differential responses of the share of borrowing-constrained households in respective model economies match empirical responses obtained using local projections.

JEL: F32, F41, E21, E25, E32, D31, D52

Keywords: intertemporal approach, current account, HANK, precautionary savings, distributional effects, RLMS, PSID

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1. Introduction

What drives international capital flows is a central question in international macroeconomics. Traditionally, intertemporal consumption smoothing of a representative household is viewed as the key driver of the current account (Obstfeld and Rogoff ,1996). However, a growing literature[1] underscores the importance of wealth and uninsurable idiosyncratic income risk within a country for understanding aggregate savings dynamics and their impact on the macroeconomy. This paper studies the business cycle fluctuations of net and gross capital flows between countries driven by a distribution of households that face aggregate and idiosyncratic risks and differ in their levels of wealth.

Employing a Heterogeneous Agent New Keynesian (HANK) model, three novel effects are uncovered when a distribution of consumption-smoothing households saves intertemporally. First, borrowing-constrained households' low marginal propensities to save lead to a decrease in aggregate savings responses to total factor productivity (TFP) shocks. Second, optimizing households save more when accounting for idiosyncratic risk, due to precautionary motives, and save more the wealthier they are. Third, endogenous changes in the distribution of households in response to shocks can amplify or dampen the effects of TFP shocks on aggregate savings.

In an economy where idiosyncratic risk is high, changes in the share of borrowing-constrained households magnify business cycle volatility of aggregate savings. In response to a positive TFP shock households accumulate liquid buffers, to insure against idiosyncratic and aggregate risks, and the share of consumption-smoothing households increases. In contrast, during an expansionary period in an economy with households facing low idiosyncratic risk, households' holdings of illiquid assets, delivering higher returns, increase. This results in a rise in the share of constrained households, which dampens aggregate savings responses.

The relative intensity of the three effects determines the response of aggregate savings and the current account to business cycle fluctuations. The quantitative contributions of these effects were assessed through the lens of a two-asset small open economy HANK model calibrated to two economies: a relatively wealthy country with relatively large insurance

1 Kaplan, Moll and Violante (2018) and a burgeoning literature afterwards.

against idiosyncratic risk on the one hand, and a relatively wealth-poor country with high idiosyncratic risk on the other.

Results show that compared to a representative agent model (RANK), savings of heterogeneous agents are smaller in the low-risk economy and larger in the high-risk one. In the low idiosyncratic risk economy precautionary savings are small, and extensive margin together with procyclical changes in the share of borrowing-constrained households dampen aggregate savings responses. On the contrary, in the high-risk economy the precautionary savings are larger, and the distributional effect contributes positively to the current account dynamics, leading to increased savings responses in HANK compared to RANK. The relative contribution of the precautionary savings and distributional effects is obtained through the novel decomposition, which shows the opposite cumulative contributions of the distributional effects in the high- and low-risk countries. The magnitudes of the distributional effects are similar to the contributions of the precautionary savings (5–10%).

As a result of these three opposing effects, the aggregate consumption volatility, in the presence of borrowing constraints, is a U-shaped function of the level of idiosyncratic risk. At the low level of risk, when the risk is increased, precautionary savings increase and the distributional effect starts to shift towards the amplification of savings responses to business cycles, compensating for the extensive margin. Hence, the aggregate consumption volatility decreases. Once the risk becomes as large as in the high-risk economy, the precautionary effect dominates and the aggregate consumption volatility rises. Calibrated values of the idiosyncratic risk in the two economies reveal that the wealthy country is on the left side of the parabola, while the wealth-poor country is on the right side of the U-shaped curve.

Wealth and idiosyncratic risk also impact the countries' gross positions vis-à-vis the rest of the world. High idiosyncratic risk endogenously generates a large demand for liquid assets in response to a positive TFP shock, which a financial intermediary passes on to the rest of the world. In a wealthy country, in an expansion, households invest in high-return illiquid assets and even liquidate liquid holdings. Consequently, in a boom, the wealthy country holds a long position in illiquid assets and a short position in liquid assets while the wealth-poor country holds the opposite position.

A quantitative HANK model utilized to obtain results features heterogeneous households facing Bewley-type idiosyncratic labor income shocks. Households can save in liquid and illiquid assets. They are heterogeneous in income and wealth, but homogeneous in terms of their consumption basket of foreign and domestically produced goods. A sophisticated financial intermediary aggregates households' savings and invests them in domestic and foreign liquid and illiquid assets. Firms face capital adjustment costs, which are crucial for current account dynamics (see Mendoza ,1991), and nominal rigidities. The latter is also important in open economies, as sticky prices may affect the pass-through of shocks to the real exchange rate.[2] Model impulse response functions are analyzed in response to a 1% TFP shock.

The micro-moments used to calibrate the small open economy HANK model are computed using two household-level surveys. The US Panel Study of Income Dynamics (PSID) is widely used to calibrate the HANK models. For the wealth-poor and high idiosyncratic risk country, I use the Russia Longitudinal Monitoring Survey (RLMS) since it contains rich information on income, consumption, and wealth for a large representative sample of households. Another advantage of this survey compared to other household-level surveys in high idiosyncratic risk countries is that it has been running for over 20 years, and features a panel component that can be used to estimate households' earnings process. The limitation is that, as in PSID, information about households at the very top of the wealth distribution is limited.

Turning to the empirical support of the model results, I utilize panel components of both surveys that allow for an empirical estimation of the endogenous changes in the wealth distribution of households in response to TFP shocks, as well as for a comparison to analogous changes obtained in the quantitative model. Using local projections, I estimate the probability of being borrowing-constrained for households in a range of income and wealth percentiles. I show that in the high-risk country, households' probability of being borrowing-constrained, even at the top of the income and wealth distributions, decreases by 0.2% in response to a positive TFP shock that increases households' income by 1%. In contrast, for households in the low-risk country, the same probability rises by 0.4%. This leads to a countercyclical

2 See Aguiar, Amador and Arellano (2023) and Acharya, Challe and Dogra (2020) for the study of optimal policies in heterogeneous-agent models.

evolution of the share of borrowing-constrained households in the high-risk country which amplifies aggregate savings responses and boosts current account, and to a procyclical evolution of the share in the country with a low level of idiosyncratic risk. For gross flows this means that households' demand for liquid assets is procyclical in the wealth-poor high-risk country but countercyclical in the wealth-rich low-risk country. These empirical dynamics are well aligned with the dynamics in the quantitative model.

Empirical results also show that the changes in the shares of borrowing-constrained households for both countries are mainly driven by households at the top of the income distribution. Since this group's share of unconstrained households is also larger, mainly savings of these households impact current account dynamics. These novel empirical results highlight the importance of income and asset distribution for the dynamics of capital flows.

The paper is organized as follows. Below, I briefly discuss the related literature. Section 2 describes the data, with Section 3 presenting empirical responses obtained using household-level data and local projections. Section 4 outlines a quantitative HANK model and presents results. Section 5 concludes.

Related literature. Comparably few papers study international macroeconomic questions in an environment with heterogeneous households. The work most closely related to this paper is de Ferra, Romei and Mitman (2021) . The authors also study how household heterogeneity affects the current account. However, the theoretical focus is on financial liberalization and the depth of financial markets rather than on the intertemporal approach to the current account studied in this paper. On the empirical side, de Ferra et al. (2021) use aggregate statistics from a large sample of countries, whereas this paper uses household-level data from two distinct economies. In this respect, the empirical analysis presented in this paper can be seen as complementary. Other small open economy HANK papers study sudden stops: de Ferra, Mitman and Romei (2020) ; foreign monetary policy shocks: Guo, Ottonello and Perez (2020) , and Zhou (2022) ; and exchange rate shocks: Auclert, Rognlie, Souchier and Straub (2021b) .

On the international macroeconomics literature front, the paper extends the intertemporal approach to the current account extensively analyzed in Obstfeld and Rogoff (1996) . Implications of the intertemporal approach to the current account were tested in Nason and Rogers (2006) and many others. Most tests failed, and the framework was extended along numerous dimensions. Mendoza, Quadrini and Ríos-Rull (2009) , and Angeletos and Panousi (2011) use extensions related closest to this paper. However, both papers do not analyze endogenous changes in the wealth distribution along the cycle and do not feature assets of different degrees of liquidity that are crucial in modeling heterogeneous marginal propensities to consume and, thus, determining aggregate savings. Finally, this paper's focus is on TFP shocks rather than on financial liberalization.

Regarding the heterogeneous households literature, this paper is most closely related to Auclert, Bardóczy, Rognlie and Straub (2021a) . This paper's model is an open economy extension of the Auclert et al. (2021a) two-asset HANK model, and is also solved in discrete time using the sequence-space Jacobian method. The comparison between HANK and RANK/two-agent (TANK) models that is conducted in this paper is also studied in closed economies in, among others, Bayer, Born and Luetticke (2020) , Bhandari, Evans, Golosov and Sargent (2017) , and Werning (2015) .[3]

The results of this paper regarding the liquid asset demand of a wealth-poor country vis-à-vis the rest of the world are also related to the strand of literature that studies the demand for safe assets. The larger demand for safe assets from emerging and developing economies was extensively documented in Gourinchas and Rey (2007) . Among recent papers, Kekre and Lenel (2021) study demand for safe dollar bonds in a model in which risk-bearing capacity in the US is greater than that in the rest of the world. Conversely, this paper studies wealth and idiosyncratic risk endogenously forming household portfolios.

2. Data

This section describes data from two household-level surveys, PSID and RLMS, used throughout this paper. These surveys are used in the empirical analysis in Section 3 and for model calibration to a wealthy low idiosyncratic risk and a wealth-poor high idiosyncratic risk

3 A heterogeneous agent framework is incorporated into general equilibrium models in McKay and Reis (2016) , Kaplan et al. (2018) , Bayer, Lütticke, Pham-Dao and Tjaden (2019) , Auclert and Rognlie (2018) , Auclert, Rognlie and Straub (2018) , Faia, Kudlyak and Shabalina (2021) and others.

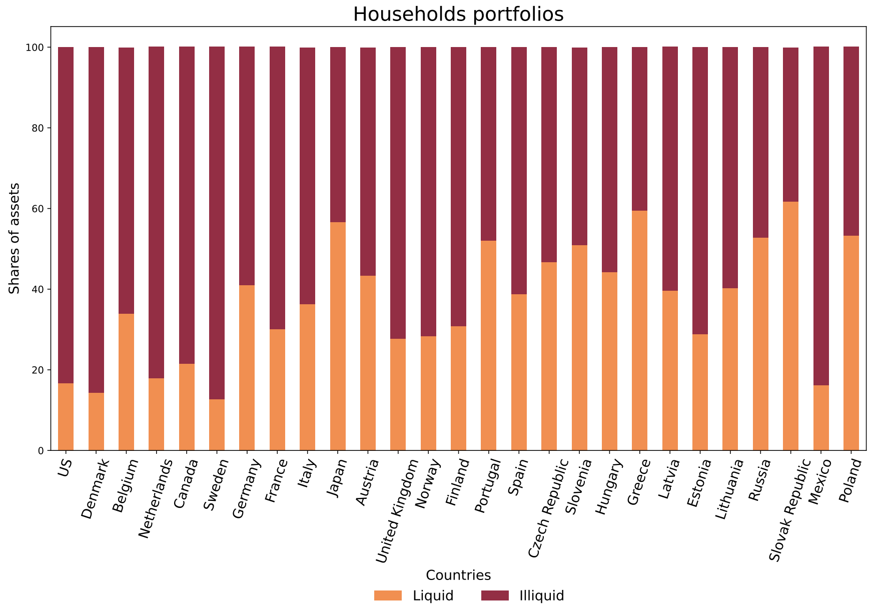


Figure 1: Household Portfolios. The figure shows the financial portfolios of households across OECD countries (excluding Luxemburg and Ireland). Liquid assets consist of currency and deposits, debt securities and loans. Illiquid assets consist of equity and investment fund shares, insurance, pensions, standardized guarantee schemes, and other accounts receivable. Portfolios are shown for the year 2019. Countries are arranged, from left to right, by wealth levels in 2019 from richest to poorest.

economies in Section 4. To place the countries for which the two surveys are used in a wider geography, Figure 1 shows a noisy but discernibly upward-sloping trend in the share of liquid assets in the financial portfolios of households in a sample of OECD countries sorted according to their wealth levels from richest to poorest. The two countries, the US and Russia, represent limiting cases in terms of liquid portfolio shares of the households and wealth levels among the OECD countries.

Figure 1 also shows that differences in idiosyncratic risk, which translate into larger share of liquid assets in the households portfolios, are also usually associated with differences in wealth, with institutions and economic policies likely driving the two. Therefore, throughout the paper, for brevity, a wealthy low idiosyncratic risk economy will be referred to as a low-risk country, and a wealth-poor high idiosyncratic risk economy as a high-risk country.

2.1. Description of PSID and RLMS Surveys

2.1.1. PSID Survey

The description of the PSID for the US economy is kept brief, as it is widely used in the literature. A wealth variable, which has been continuously reported in PSID only since the 1999 wave, is needed to calculate the shares of borrowing-constrained households (see Section 2.2 for details). Thus, the sample period is limited to from 1999 to the most recent wave, 2019. The data cleaning procedure and identification of borrowing-constrained households follows Kaplan, Violante and Weidner (2014) . For computation of taxes, NBER's TAXSIM 32 program is employed. A comparison of per capita labor income from survey data and aggregate statistics[4] alongside the shares of borrowing-constrained households across income and wealth percentiles is presented in Appendix B.

2.1.2. RLMS Survey

Since 2000, this survey has been conducted annually. This paper uses data up until the most recent wave, which covers the year 2019.[5] The RLMS survey asks around 5000 households in each wave in detail about their consumption and income.[6]

The RLMS data is cleaned following Aguiar and Bils (2015) , see Appendix A for details. All restrictions imposed jointly exclude 20% of the data, suggesting a high percentage of reliable responses.[7]

Comparing aggregate income/consumption data obtained from the RLMS survey to data presented in aggregate statistics suggests that the survey lacks households at the top of the income distribution (see Appendix B). However, the difference between survey and aggregate statistics is comparable to the difference between survey and aggregate statistics for the US (see Appendix B), which suggests that both surveys lack households at the top of the income/wealth distribution to a roughly equal extent.

4 National Income and Product Accounts (NIPA) is used as a source of aggregate data.
5 There are four waves prior 2000: 1994, 1995, 1996, 1998, however, continuous time series can only be constructed from 2000.
6 The dataset for each wave is paired with a constructed variables dataset provided on the RMLS website.
7 Table 9 in Appendix A shows how many observations were excluded by each restriction. In the PSID 30% of the responses were excluded.

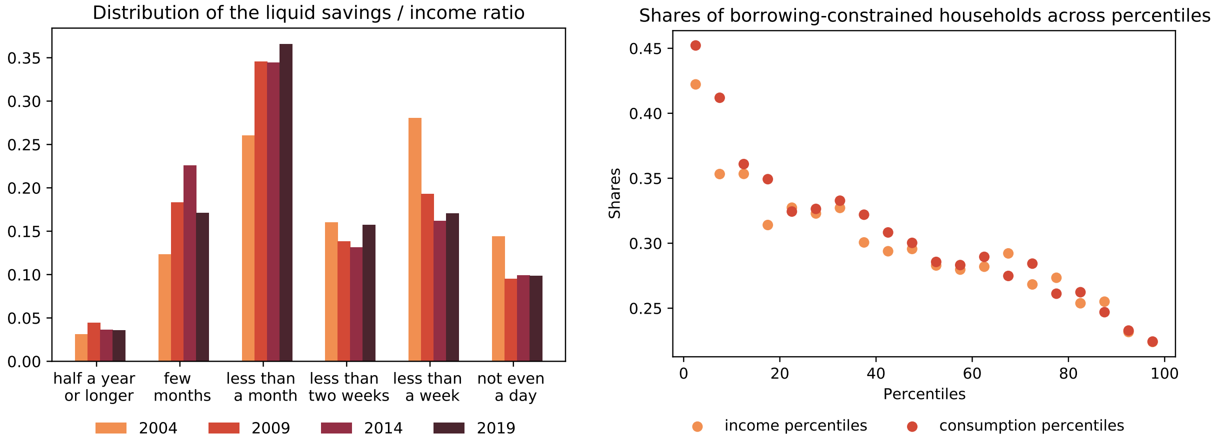
2.2. Identification of borrowing-constrained households

In Kaplan et al. (2014) borrowing-constrained households are defined as those with liquid wealth equal to a week or less of earnings. The share of borrowing-constrained (or hand-to-mouth) households obtained by the authors relying on the Survey of Consumer Finances (SCF) dataset varies from 17.5% to 35%. Aguiar and Bils (2020) using the same definition of borrowing-constrained households but PSID data identify 40% of households to be borrowing-constrained. For the sample considered in this paper, the corresponding mean share of borrowing-constrained households amounts to 38%.[8]

RLMS data contains limited information about the liquid assets of households. However, the survey features the following question: "Imagine an unpleasant situation in which all members of your family lost their sources of income. How long do you think your family would be able to live at your present level - in other words, without decreasing your expenditures - without any income? Consider only your savings, not selling any of your possessions." This question, in the following referred to as question F12_A, allows for the computation of the same ratio Kaplan et al. (2014) use for the identification of the borrowing-constrained households, i.e. the amount of liquid assets of the households as a ratio of consumption (which is equal to income for constrained households). Surveyed households are presented with six options to answer this question, ranging from "not even a day " to "half a year or longer". The distribution of household responses to question F12_A is presented in Figure 2a. More responses are concentrated around low levels of liquid assets, with sizable increases after economic crises in Russia in 2008 and 2014.

For comparability with PSID and the Kaplan et al. (2014) methodology households are considered borrowing-constrained when they answered "not even a day" or "less than a week" in response to question F12_A (i.e. 33% of the households on average). The shares of borrowing-constrained households across income and consumption percentiles calculated using RLMS are plotted in Figure 2b. These shares are larger for the bottom percentiles.

8 Calculated using family weights.



(a) Shares of households across liquidity bins **(b) Shares of borrowing-constrained households**

Figure 2: Shares of Borrowing-Constrained Households in RLMS. The left-hand side graph shows shares of households that give answers specified on the X-axis to question F12_A in the RLMS (about the period for which a household can sustain their consumption level using only liquid savings, i.e. without receiving income or selling assets). Shares are presented for 2004, 2009, 2014, and 2019 years (bars from left to right correspond to the years in ascending order). The right-hand side figure shows shares of borrowing-constrained households across income and consumption percentiles. Borrowing-constrained households answer "less than a week" or "not even a day" to question F12_A in the RLMS.

2.3. Comparison of the Surveys' Statistics

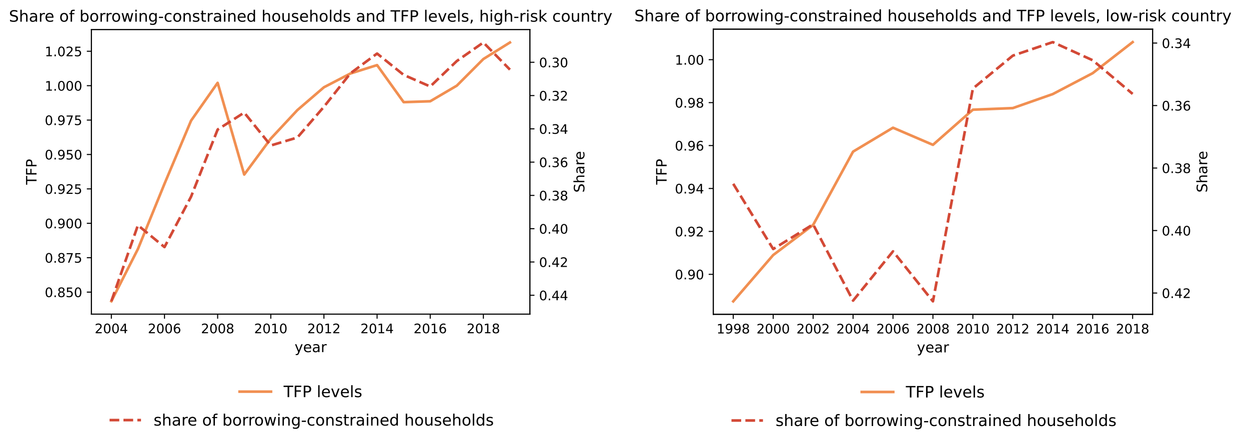
Households in the PSID are substantially wealthier compared to the RLMS data, see Table 1 for household income and wealth distributions in both countries.[9] The share of borrowing-constrained households among the income-rich is smaller than among the income-poor in both countries. In the PSID, there are more wealthy yet borrowing-constrained households.

Higher levels of idiosyncratic risk (see Section 4.2 for the calibration of the income process in the two economies) paired with lower levels of wealth lead to the accumulation of liquid assets during economic expansions in the high-risk economy. Meanwhile, households in the low-risk country largely save in illiquid assets during boom periods, as those deliver higher returns. This result, obtained empirically in Section 3 and in the model, is supported by the suggestive evidence from aggregate statistics. Figure 3a and Figure 3b show that the aggregate share of borrowing-constrained households evolves far more countercyclically in the high-risk country than in the low-risk country (the right axis is inverted on the graphs). A TFP surge in the first ten years of the RLMS sample was associated with a decrease

9 Average exchange rate between the Russian ruble and the US dollar is such that in 2003 is 30.7 so that 33.6 Russian 2003 rubles are equal to one 2006 US dollar.

Table 1: Sample Statistics Across Income and Wealth Percentiles. The table presents sample statistics calculated using PSID and RLMS data. The sample was divided into four subsamples based on the medians of the income and wealth distributions (in RLMS, instead of the wealth median, a dummy indicating whether a household has illiquid wealth was used). The shares are shown in percentages. Income and wealth variables show mean real income and mean real wealth per group in 2006 dollars. Income is presented in hundreds of thousands, and wealth is presented in hundreds of millions. The statistics are averaged across sample years.

Variables	bottom 50% income, bottom 50% wealth	top 50% income, bottom 50% wealth	bottom 50% income, top 50% wealth	top 50% income, top 50% wealth
	PSID			
share of borrowing-constrained hhs	66%	51%	24%	20%
share of poor borrowing-constrained hhs	47%	19%	0%	0%
share of wealthy borrowing-constrained hhs	19%	32%	24%	20%
income	0.26	0.75	0.31	1.05
labor income	0.26	0.73	0.30	1.00
asset income	0.001	0.01	0.007	0.05
wealth	-0.08	-0.09	3.69	2.09
liquid wealth	-0.08	-0.10	1.89	0.72
illiquid wealth	0.004	0.009	1.80	1.37
	RLMS			
share of borrowing-constrained hhs	42%	33%	34%	28%
share of poor borrowing-constrained hhs	32%	27%	0%	0%
share of wealthy borrowing-constrained hhs	10%	6%	34%	28%
income $\cdot 10$	0.21	0.70	0.23	0.71
illiquid wealth $\cdot 10$	-0.001	-0.008	0.16	0.24



(a) **RLMS** (b) **PSID**

Figure 3: Cyclicality of the Shares of Borrowing-Constrained Households. The figures show the cyclicality of the shares of borrowing-constrained households. TFP is presented on the left Y-axis. Shares are presented on the right inverted Y-axis. Data for TFP levels is obtained from Feenstra, Inklaar and Timmer (2015) . Shares were calculated using RLMS and PSID household-level surveys. The X-axis shows time in years.

in the share of borrowing-constrained households from 45% to around 30%, while in the US, before and after the Great Recession, TFP levels increased in tandem with that share. The financial crisis in the US was associated with a considerable decrease in the number of borrowing-constrained households due to a marked decrease in the number of wealthy hand-to-mouth that liquidated their illiquid assets (see Appendix B for correlations with real per capita GDP instead of TFP levels).

In terms of income and wealth inequalities, both economies are similarly unequal, see Figure 9a as well as the ratios of mean incomes of the top 50% compared to the bottom 50% of the income distributions in Table 1.

3. Empirical Responses to TFP Shocks

The empirical analysis presented in this section shows impulse responses of the shares of borrowing-constrained households to TFP shocks. It is crucial to obtain responses of the shares empirically, since the direction of the change is ambiguous. The empirical responses adequately match those in the model this paper introduces, as shown in Section 4.2. Responses of the shares are obtained using local projections with externally obtained TFP shocks controlling for income and wealth percentiles of households' respective distributions. The general equilibrium effects of savings on the current account are studied in the quantitative HANK model in Section 4.

3.1. Identification of the TFP Shocks

In this empirical analysis externally constructed series of TFP shocks are used. TFP series are taken from Feenstra et al. (2015) [10] for both countries. Additionally, for the US, the TFP series constructed by J. Fernald[11] is used as a robustness check. The first series is constructed using the Törnqvist quantity index of factor endowments, whereas the second series relies on utilization-adjusted Solow residuals constructed from sectoral data.

For all TFP series, Dickey-Fuller unit root tests for all widely used levels of significance

10 Penn World Table 10.0.

11 https://www.johnfernald.net/TFP, accessed on 22.06.22.

do not reject the null hypothesis of the presence of the unit root. Therefore, as in Fernald (2014) , TFP shocks are obtained using the first differences of TFP levels.

3.2. Empirical Specification

Similar to Cloyne, Ferreira and Surico (2020) , and Coibion, Gorodnichenko, Kueng and Silvia (2017) , Jordà (2005) 's local projections are used to characterize the effects of aggregate shocks on households' behavior. The outcome of interest is how households in different income and wealth percentiles respond to changes in their income induced by TFP shocks. Therefore, a two-stage regression model is employed. In the first stage, for each group of households (groups are constructed based on the households' positions in the income and wealth distributions), changes in their income are regressed on TFP shocks and a vector of household demographics, see Equation (1). In the second stage, Equation (2), the dummy of being borrowing-constrained is regressed on its lags (to control for trends before the shocks), the instrumented variable, and the households' demographics:

$$y_{i,t} = \nu_i + B^j(L)y_{i,t-1} + C^j(L)\hat{z}_{i,t} + \beta_1^j X_{i,t} + \varepsilon_{i,t} \tag{1}$$

$$z_{i,t} = \mu_i + D^j(L)g_t + H^j(L)g_t \cdot X_{i,t} + \beta_2^j X_{i,t} + \eta_{i,t} \tag{2}$$

where $y_{i,t}$ is household i's dummy for being borrowing-constrained at time t, fixed effects are captured in ν_i and μ_i, z represents changes in the logarithm of household real income, X are controls,[12] and g_t are TFP shocks. Each regression is run separately for each of the four groups indexed by j, obtained by bisecting income and wealth distributions at the respective medians.[13] $B(L), C(L), D(L)$, and $H(L)$ are lag polynomials. Appendix B presents responses for real liquid and real illiquid holdings for the PSID sample, as well as consumption responses for the RLMS sample. These responses provide additional insights and support the empirical results discussed below.

12 Changes in age and age squared of the household head, and changes in the number of family members.

13 The first group is those in the bottom 50% of the income distribution and the bottom 50% of the wealth distribution, another group is those in the top 50% of the income distribution and the bottom 50% of the wealth distribution, etc.

Sargan-Hansen tests indicate that the instruments are valid. LM tests for the relevance of the instruments indicate that the instruments are relevant where responses are significant. Relevance of the instrument does, however, decrease in further periods of local projections.

In the robustness checks, Arellano-Bond estimates are compared to fixed-effects regression estimates, and high-dimensional fixed effects are compared to standard fixed effects estimates. By including and excluding variables I also experiment with lags of the dependent variable in the second stage, lags of the dependent variable and time trend in the first stage, and interaction terms (household demographics with TFP shocks) in both stages.

Additionally, for the US, different series are used for TFP shocks (see Section 3.1 for the description of the TFP shocks). Finally, reduced form estimates where y is directly regressed on TFP shocks are also obtained. Results are qualitatively robust to all of these alterations.

3.3. Empirical Results

Empirical impulse responses for the shares of the borrowing-constrained households in high-risk and low-risk countries are presented in Figure 4. Cumulative responses of different variables are shown in Table 2. In Appendix B.2 impulse responses for the liquid and illiquid assets using PSID, for consumption using RLMS, and the first-stage impulse responses in both countries are displayed.

Figure 4 showcases stark differences in the behavior of income-rich households in the high-risk and the low-risk countries: while in the low-risk country, the number of borrowing-constrained households in the income-rich bracket increases in response to a positive TFP shock, the share of constrained households among the income-rich in the high-risk country decreases. Households in the low-risk country largely increase savings in illiquid assets, as they deliver higher returns (see also liquid and illiquid asset responses in Appendix B.2). In the high-risk country, even households at the top of the income and wealth distributions save in the form of liquid assets in response to positive TFP shocks. They also have large marginal propensities to consume (MPCs), see Table 2 and Appendix B.2, in response to income shocks induced by increases in TFP.[14]

14 Calculated MPCs should not be seen as general MPCs, but rather as MPCs only for the income shocks

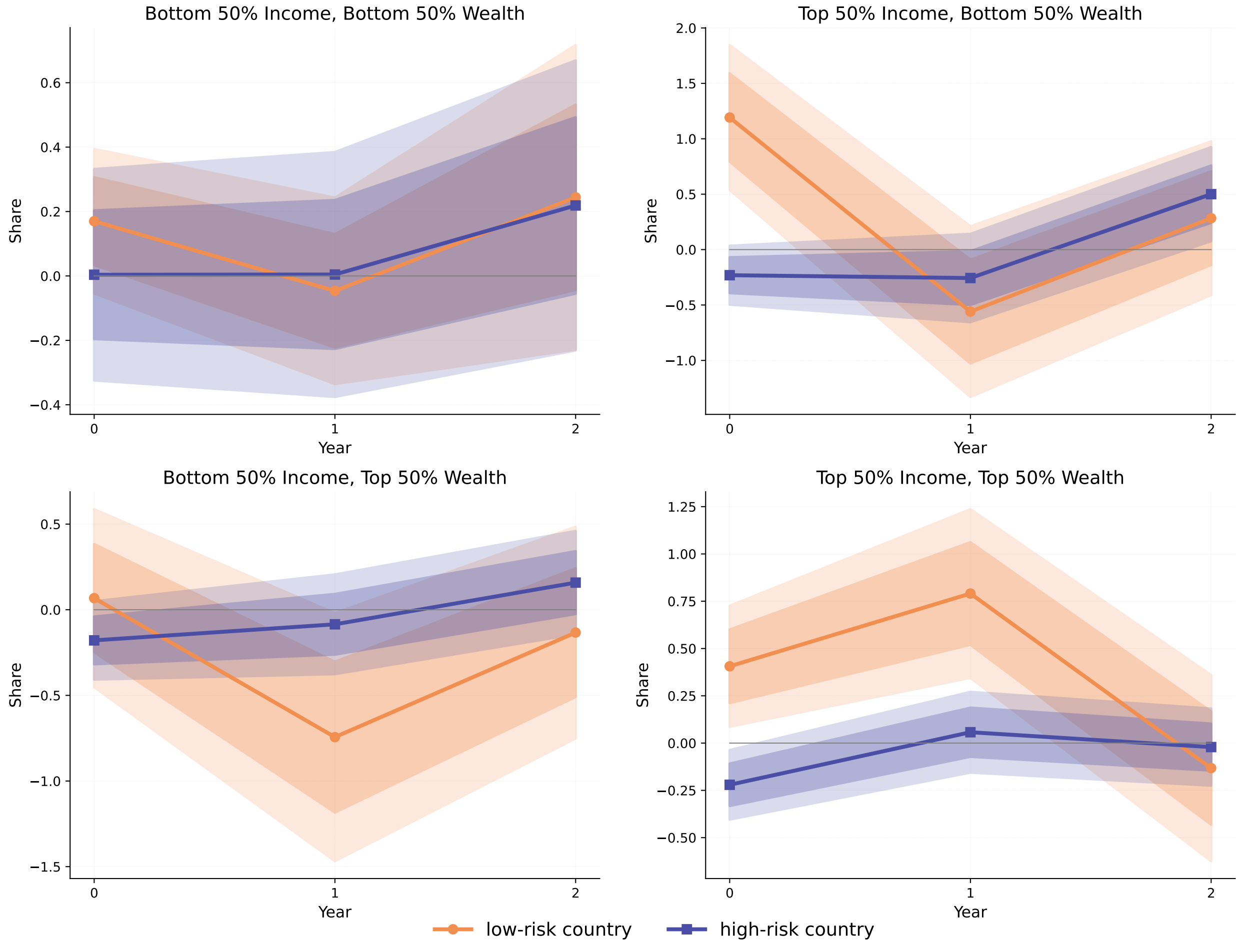


Figure 4: Empirical Impulse Responses of Probabilities of Being Borrowing-Constrained to Income Shocks. The figure shows impulse response functions (IRFs) for four groups of households (income poor/rich, wealth poor/rich). The lines correspond to the mean IRFs (orange for the low-risk country and violet for high-risk country), inner shaded areas show one standard deviation around the mean responses, outer shaded areas 90% confidence intervals. The X-axis shows time in years. The probability of becoming borrowing-constrained is shown in first differences.

Table 2: Empirical Cumulative Impulse Responses. The table shows cumulative impulse responses obtained from the regression model presented in eq. (1) and eq. (2). Cumulative responses are obtained by summing up responses for the first four years after the shock. 90% confidence intervals are shown in brackets.

Variable, Country	**Cumulative IRFs with 90% CI**			
	Income Poor, Wealth Poor	Income Rich, Wealth Poor	Income Poor, Wealth Rich	Income Rich, Wealth Rich
	Probability of being borrowing-constrained			
p(borrowing-constrained), low-risk country	0.16 (-0.23 0.56)	-0.27 (-0.88 0.34)	-0.23 (-0.84 0.38)	0.58 (0.03 1.13)
p(borrowing-constrained), high-risk country	0.08 (-0.23 0.39)	-0.24 (-0.51 0.03)	-0.01 (-0.29 0.27)	-0.26 (-0.41 -0.10)
	Other Variables			
liquid, low-risk country	-0.23 (-2.82 2.36)	-0.42 (-3.25 2.41)	1.98 (-1.03 4.98)	-1.80 (-4.90 1.30)
illiquid, low-risk country	-0.60 (-1.86 0.65)	5.14 (0.46 9.83)	-0.29 (-3.14 2.56)	-1.78 (-5.14 1.60)
consumption, high-risk country	-0.17 (-0.73 0.39)	0.52 (0.18 0.86)	0.53 (-0.09 1.15)	0.92 (0.63 1.22)
	First Stage			
income, low-risk country	-0.01 (-0.03 0.02)	0.09 (0.06 0.11)	-0.06 (-0.10 -0.02)	0.05 (0.03 0.07)
income, high-risk country	-0.01 (-0.03 0.02)	0.02 (-0.003 0.03)	-0.01 (-0.03 0.01)	0.02 (0.01 0.04)

Therefore, not only is the share of unconstrained households (who save) large among the income-rich, but changes in the share of borrowing-constrained households also take place in this group of households. Hence, the income-rich are the ones that have the largest impact on the current account dynamics.

On the other hand, the responses of income-poor households are similar in both countries. Among this group, the share of borrowing-constrained households is large, and does not vary with the business cycle. Thus, these households only impact current account dynamics through general equilibrium effects.

Wealth-rich and income-poor households consume all additional income, presenting a classic example of the wealthy hand-to-mouth households. This can be seen from insignificant responses of probabilities in both countries. For this group of households TFP shock entails a huge consumption increase in the high-risk country, and insignificant responses of liquid and illiquid assets in the low-risk country.

Summary. Empirical results presented in this section show that savings responses to TFP shocks are heterogeneous. In the high-risk country, households have high MPCs, save little, and do so in liquid assets. In contrast, in the low-risk country, wealthy households largely save in the form of illiquid assets. In the low-risk country, the largest increase in

induced by TFP shocks. Nevertheless, the magnitudes are comparable with estimates for Peru's economy in Hong (2023) .

liquid savings in response to a positive TFP shock is obtained for income- and wealth-poor households.

The implications for the current account are straightforward, as it directly depends on the amount of savings. In addition, subject to the supply of liquid and illiquid assets in the economy, the savings responses of households form the demand for both types of assets vis-à-vis the rest of the world.

4. Quantitative HANK model

The model this paper builds on is a discrete-time two-asset HANK model developed by Auclert et al. (2021a) . Their model is extended to a small open economy framework and calibrated to the high-risk and low-risk economies.

The model features heterogeneous households that face idiosyncratic income risk and save in the form of liquid and illiquid assets. The two-asset structure is needed to disentangle demand for different savings forms and compare its dynamics across countries. In addition, two assets are essential to reproduce large average marginal propensities to consume, as shown in Kaplan and Violante (2022) . Households are homogeneous in the consumption basket of home- and foreign-produced goods. On the supply side, monopolistically competitive firms face capital adjustment costs, which are important for current account dynamics (Mendoza ,1991). Nominal rigidities are included as sticky prices may affect the pass-through of shocks to the real exchange rate. A debt-elastic interest rate ensures determinacy. The fiscal authority follows a balanced-budget policy.

4.1. Model Description

4.1.1. Households

Idiosyncratic risk. At the beginning of each period t households experience an idiosyncratic productivity shock. The shock follows an n_e-state Markov process with a transition matrix $P(e_{t+1}|e_t)$ and stationary distribution $\pi(e)$.

Labor income. Real after-tax labor income ξ_t of a household at time t is a function of the idiosyncratic productivity state e_t, real wage w_t, labor hours n_t, and tax rate τ_t:

$$\xi_t = (1 - \tau_t) e_t w_t n_t \tag{3}$$

Bellman equation. In each period t each household decides on consumption c_t and savings in illiquid assets a_t and liquid assets b_t. A representative household head chooses labor hours, n_t. The state in period t is (e_t, a_{t-1}, b_{t-1}) where e_t is the idiosyncratic productivity shock. Each household maximizes the following value function, given the state:

$$\begin{aligned} V(e_t, a_{t-1}, b_{t-1}) &= \max_{c_t, a_t, b_t} u(c_t, n_t) + \beta E_t V(e_{t+1}, a_t, b_t) \\ \text{s.t.} \quad & c_t + a_t + b_t = \xi_t + (1 + r_t^a) a_{t-1} + (1 + r_t^b) b_{t-1} - \Phi(a_t, a_{t-1}) \\ & a_t \geq 0, \quad b_t \geq \underline{b} \end{aligned} \tag{4}$$

where ξ_t is labor income, and r_t^a and r_t^b are interest rates on illiquid and liquid assets, respectively. $\Phi(.)$ is the convex portfolio adjustment cost defined below, and β the time discount factor. The instantaneous utility function takes the standard separable form $u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{n^{1+\rho}}{1+\rho}$. For liquid assets, the condition $b_t \geq \underline{b}$ acts as a borrowing constraint; illiquid assets can be only positive. The value function depends on the idiosyncratic income shock, e_t, through ξ_t. Portfolio adjustment costs take the following functional form:

$$\Phi(a_t, a_{t-1}) = \frac{\chi_1}{\chi_2} \left| \frac{a_t - (1 + r_t^a) a_{t-1}}{(1 + r_t^a) a_{t-1} + \chi_0} \right|^{\chi_2} [(1 + r_t^a) a_{t-1} + \chi_0] \tag{5}$$

with $\chi_0 > 0$, $\chi_1 > 0$ and $\chi_2 > 1$. Note that $\Phi(a_t, a_{t-1})$ is bounded, differentiable [15] and convex in both arguments.

Labor hours are chosen by the household head and are determined through the following intratemporal first-order condition:

$$\varphi n_t^\rho = \int u_c(e_t, a_{t-1}, b_{t-1}) \frac{\partial \xi_t}{\partial n_t} dD(e_t, a_{t-1}, b_{t-1}) \tag{6}$$

15 Except at $a_t = (1 + r_t^a) a_{t-1}$.

The solution to the optimization problem are policy functions $c(e_t, a_{t-1}, b_{t-1})$, $a(e_t, a_{t-1}, b_{t-1})$ and $b(e_t, a_{t-1}, b_{t-1})$ that depend on the interest rate and tax path $\left\{r_s^a, r_s^b, \tau_s\right\}_{s\geq t}$, taken as given by households.

Home and Foreign Goods. The consumption good is a composite of home- and foreign-produced goods with a CES aggregation technology that is homogeneous across households. Therefore, aggregate demand for home- and foreign-produced goods can be linked to aggregate consumption of households in the following way

$$C_{H,t} = (1-\gamma)\left(\frac{p_{H,t}}{p_t}\right)^{-\eta} C_t \tag{7}$$

$$C_{F,t} = \gamma\left(\frac{p_{F,t}}{p_t}\right)^{-\eta} C_t \tag{8}$$

$$C_{H,t}^* = \left(\frac{p_{H,t}}{p_{F,t}}\right)^{-\eta_f} Y_{F,t}^* \tag{9}$$

where γ governs the home bias. $C_{H,t}$ and $C_{F,t}$ are consumption of home- and foreign-produced goods respectively. The price of home-produced goods is $p_{H,t}$ and p_t the price level in the home economy. Finally, η is the elasticity of substitution between those goods. Foreign households have a downward-sloping demand for the country's exports, $C_{H,t}^*$, driven by the foreign country's income, $Y_{F,t}^*$. The slope of this demand is governed by η_f. The supply of the foreign good is assumed to be completely elastic at a fixed price normalized to 1 in the foreign currency. The law of one price holds at the goods level, thus, $p_{F,t} = \mathcal{E}_t$ with $\mathcal{E}_t$ being the nominal exchange rate.

Household aggregates. Aggregate policy functions for consumption and asset holdings are obtained by integrating individual policy functions weighted by the measure of households in state e_t that own assets in sets A_- and B_- at the start of date t, which is given by $D(e_t, A_-, B_-,) = Pr(e = e_t, a_{t-1} \in A_-, b_{t-1} \in B_-)$.

$$\mathcal{A}_t(r_t^a, r_t^b, \tau_t, N_t, w_t) = \int a(e_t, a_{t-1}, b_{t-1}) dD(e_t, a_{t-1}, b_{t-1}) \tag{10}$$

$$\mathcal{B}_t(r_t^a, r_t^b, \tau_t, N_t, w_t) = \int b(e_t, a_{t-1}, b_{t-1}) dD(e_t, a_{t-1}, b_{t-1}) \tag{11}$$

$$\mathcal{C}_t(r_t^a, r_t^b, \tau_t, N_t, w_t) = \int c(e_t, a_{t-1}, b_{t-1}) dD(e_t, a_{t-1}, b_{t-1}) \tag{12}$$

4.1.2. Firms: Labor Demand, Capital Demand, and Pricing Decisions

Each intermediate-good firm s produces output by combining capital and labor inputs in a Cobb-Douglas production function:

$$y_{s,t} = z_t k_{s,t-1}^{\nu} N_{s,t}^{1-\nu} \tag{13}$$

where $y_{s,t}$ is the intermediate output, z_t total factor productivity, $k_{s,t-1}$ and $N_{s,t}$ are capital and labor inputs, respectively, and ν is the capital share.

Each firm chooses capital and labor demands, $k_{s,t}$ and $N_{s,t}$, the price of their good's variety, $p_{s,t}$, and investment, $I_{s,t}$, to maximize the sum of future discounted real profits, which recursively reads as follows:

$$J_{s,t}\left(k_{s,t-1}\right) = \max_{p_{s,t}, k_{s,t}, I_{s,t}, N_{s,t}} \left\{ \frac{p_{s,t}}{p_t} y_{s,t} - w_t N_{s,t} - I_{s,t} - \frac{\eta}{2\kappa} \ln(1+\pi_{s,t})^2 Y_t + \frac{E_t J_{s,t+1}(k_{s,t})}{1+r_{t+1}} \right\} \tag{14}$$

$$\text{s.t.} \qquad k_{s,t} = (1-\delta)k_{s,t-1} + I_{s,t} - \frac{1}{2\delta\varepsilon_I}\left(\frac{k_{s,t}-k_{s,t-1}}{k_{s,t-1}}\right)^2 k_{s,t-1} \tag{15}$$

$$p_{s,t} = \left(\frac{Y_t}{y_{s,t}}\right)^{\frac{1}{\eta}} p_{H,t} \tag{16}$$

$$y_{s,t} = z_{s,t} k_{s,t-1}^{\nu} N_{s,t}^{1-\nu} \tag{17}$$

where equation (15) is the capital accumulation equation, δ is the depreciation rate of capital, and $\frac{\eta}{2\kappa}\ln(1+\pi_{s,t})^2 Y_t$ quadratic price adjustment costs. Firms face quadratic adjustment costs on physical capital, with a standard functional form of $\frac{1}{2\delta\varepsilon_I}\left(\frac{k_{s,t}-k_{s,t-1}}{k_{s,t-1}}\right)^2 k_{s,t-1}$, which leads to a variable price of capital.

Defining $q_{s,t}$ as the Lagrange multiplier on the capital accumulation equation (hence a shadow price of capital), the firm's first-order condition for capital is:

$$(1+r_{t+1})q_{s,t} = E_t\left(\nu z_{t+1}\left(\frac{N_{s,t+1}}{k_{s,t}}\right)^{1-\nu} mc_{s,t+1} - \right. \tag{18}$$
$$\left. - \left[\frac{k_{s,t+1}}{k_{s,t}} - (1-\delta) + \frac{1}{2\delta\varepsilon_I}\left(\frac{k_{s,t+1}-k_{s,t}}{k_{s,t}}\right)^2\right] + \frac{k_{s,t+1}}{k_{s,t}} q_{s,t+1}\right)$$

where $mc_{s,t+1}$ is the Lagrange multiplier on the production constraint and represents real marginal costs. The first-order conditions for investment and labor are

$$q_{s,t} = 1 + \frac{1}{\delta\varepsilon_I}\left(\frac{k_{s,t}-k_{s,t-1}}{k_{s,t-1}}\right) \tag{19}$$

$$w_t = (1-\nu)\frac{y_{s,t}}{N_{s,t}} mc_{s,t} \tag{20}$$

The first-order condition with respect to prices leads to the Phillips curve:

$$\log(1+\pi_{s,t}) = \kappa(mc_{s,t} - \frac{1}{\mu_p}\frac{p_{s,t}}{p_t}) + \frac{1}{1+r_{t+1}} E_t \log(1+\pi_{s,t+1})\frac{Y_{t+1}}{Y_t} \tag{21}$$

4.1.3. Asset Returns

Let r_t denote the ex-post return on government bonds. Let v_t be the aggregate price of equity and d_t aggregate dividends. The real return on equity is $\frac{d_{t+1}+v_{t+1}}{v_t}$. The no-arbitrage condition is given by: $v_t = \frac{E_t d_{s,t+1} + E_t v_{s,t+1}}{1+r_{t+1}}$. As in Kaplan et al. (2018) bonds are liquid, and equities are illiquid. A financial intermediary collects all liquid and illiquid savings of households and invests them in government bonds, domestic equities, and foreign liquid and illiquid assets.

The demand for foreign liquid and illiquid assets is determined by the difference between demand for and supply of each type of asset in the home economy:

$$(1+r_t^a) = \left(\frac{v_{t-1}}{\mathcal{A}_{t-1}}\right)\frac{d_t+v_t}{v_{t-1}} + \left(1-\frac{v_{t-1}}{\mathcal{A}_{t-1}}\right)(1+r_t^*)\frac{q_t}{q_{t-1}} \tag{22}$$

$$(1+r_t^b) = \left(\frac{B^g}{\mathcal{B}_{t-1}}\right)(1+r_t-\psi) + \left(1-\frac{B^g}{\mathcal{B}_{t-1}}\right)(1+r_t^*-\psi)\frac{q_t}{q_{t-1}} \tag{23}$$

where q_t is the real exchange rate defined as $q_t \equiv \frac{\mathcal{E}_t}{p_t}$. The liquidity premium is equal at home and abroad. The government does not issue new debt and pays interest on a constant debt level B^g.

The uncovered interest rate parity ties the nominal rate in the economy to the world interest rate i^* through movements in the nominal exchange rate $\mathcal{E}$ and a debt-elastic risk premium which depends on a parameter ϕ and net foreign assets NFA_t (NFA).

$$1+i_t = (1+i^*)\frac{E_t\mathcal{E}_{t+1}}{\mathcal{E}_t} - \phi(exp(-NFA_t)-1) \tag{24}$$

Net foreign assets are the cumulative sum of trade balances, which is written in the recursive form as follows:

$$NX_t = \frac{p_{H,t}}{p_t}C_{H,t}^* - \frac{p_{F,t}}{p_t}C_{F,t} \tag{25}$$

$$NFA_t = (1+\tilde{r}_t)NFA_{t-1} + NX_t \tag{26}$$

where $1+\tilde{r}_t = \frac{\mathcal{A}_{t-1}-v_{t-1}}{NFA_{t-1}}(1+r_t^*)\frac{q_t}{q_{t-1}} + \frac{\mathcal{B}_{t-1}-B^g}{NFA_{t-1}}(1+r_t^*-\psi)\frac{q_t}{q_{t-1}}$ is the return on the net foreign asset position, i.e. the weighted sum of returns on liquid and illiquid foreign assets. The current account is determined according to its balance of payment definition $CA_t = NX_t + \tilde{r}_t NFA_{t-1}$.

4.1.4. Monetary and Fiscal Policy

Fiscal policy follows a balanced-budget policy:

$$\tau_t w_t N_t = r_t B^g + G \tag{27}$$

Monetary policy follows a simple Taylor-type rule:

$$i_t = \bar{r} + \phi_\pi \pi_{H,t} \tag{28}$$

where i_t is the monetary policy interest rate, ϕ_π is the weight on domestic inflation $\pi_{H,t}$. The natural interest rate is given by $\bar{r}$, which is equal to the steady state real interest rate. The exchange rate is assumed to be flexible. Finally, the Fisher equation reads as $1 + r_t = \frac{1+i_{t-1}}{1+\pi_t}$.

4.1.5. Market Clearing: Labor, Goods and Asset Markets

Aggregate goods supply is equal to aggregate demand, hence:

$$Y_t - (\Phi_t + \psi_t^p + \psi_t^k) = \mathcal{C}_t + G_t + I_t + NX_t, \tag{29}$$

where aggeragte consumption is obtained through the households' income and wealth distribution D_t, defined above, ψ_t^p and ψ_t^k are total price and capital adjustment costs.[16] Asset market clearing implies

$$\mathcal{A}_t + \mathcal{B}_t = v_t + B^g + NFA_t, \tag{30}$$

where aggregation is once more obtained through distribution D_t. Labor demand equals labor supply.

4.1.6. Equilibrium

Definition 1: Competitive Equilibrium

A Competitive Equilibrium of the economy satisfies the following definition:

- The sequence $[c_t, a_t, b_t]_{t=0}^{\infty}$ solves households' consumption-saving decisions, given the distribution of idiosyncratic shocks, $P(e_{t+1}|e_t)$, and the sequence of prices r_t^a, r_t^b, r_t, w_t.
- Aggregate household consumption and asset holdings are equal to the product of individual optimal policy functions and the distribution of households across assets.

16 Dependence of the costs on the risk premium from UIP and dynamics of the liquidity premium due to exchange rate fluctuations are suppressed for clarity.

- Firms choose labor demand $N_{s,t}$, prices $p_{s,t}$ and capital inputs $k_{s,t}$ in order to optimize their real discounted profits, given in eq. (14).
- Monetary policy determines the short-term interest rate according to eq. (28). Fiscal policy follows a balanced budget rule as in eq. (27).
- No arbitrage conditions between all types of assets and the law of one price at the goods level are satisfied.
- Market clearing and aggregate resource constraints are satisfied.

4.2. Model Calibration

The model is calibrated for the low-risk country (US) following Auclert et al. (2021a) , and Kekre and Lenel (2021) for the open economy parameters. Calibration for the high-risk country (Russia) is primarily based on Malakhovskaya and Minabutdinov (2014) and Semko (2013) . Wealth targets were calculated using RLMS and household savings data from the national Central Bank's database. The model is calibrated at quarterly frequency for both economies. Table 3 and Table 7 summarize parameter values for the household model blocks. Table 4 and Table 8 summarize parameter values for the rest of the respective economy. Details on the computation of wealth targets and on estimation of the labor income process for the high-risk country can be found in Appendix A.

Model parameters for the two countries mainly differ in their values for the respective labor income process, wealth levels, and capital adjustment costs parameters. While households in the low-risk economy are much wealthier, households in the high-risk country hold relatively more liquid wealth and face higher labor income risk. Firms in the high-risk country face a larger capital depreciation rate, but smaller capital and price adjustment costs. The latter leads to more volatile investment dynamics. Additionally, the labor income tax rate is lower and the elasticity of substitution between home- and foreign-produced goods in the high-risk country is also lesser (see Devarajan, Go and Robinson (2023) for comprehensive estimation of Armington elastisities for developed and developing countries).

Labor income process for the high-risk country. Individual-level data from RLMS

17 htm = share of borrowing-constrained households.

Table 3: **Low-risk Country: Household Parameter Values, Description and Source**

Parameter	Description	Value and source
β	Time discount factor	0.977, target interest rate
χ_0	Portfolio adj. cost pivot	0.25, Auclert et al. (2021a)
χ_1	Portfolio adj. cost scale	10.11, target $\mathcal{B}_h = 1.04Y$
χ_2	Portfolio adj. cost curvature	1.985, target htm= 0.38[17]
σ	EIS	0.5, Auclert et al. (2021a)
ρ	Inverse Frisch elasticity	1, Auclert et al. (2021a)
ρ_z	Autocorrelation of earnings	0.966, Auclert et al. (2021a)
σ_z	Cross-sectional std of log earnings	0.92, Auclert et al. (2021a)
φ	Disutility parameter	1.96 (target $N = 1$)

Table 4: **Low-risk Country: Parameter Values, Description and Source**

Parameter	Description	Value and source
Asset Markets		
r	Real interest rate	0.0125, Auclert et al. (2021a)
ψ	Liquidity premium	0.005, Auclert et al. (2021a)
p/Y	Steady state price of equity	11.2, target $p + Bg = 14Y$
Production		
δ	Depreciation rate	0.02, Auclert et al. (2021a)
ε_I	Capital adj. costs parameter	4, Auclert et al. (2021a)
K/Y	Steady state capital-output ratio	10, Auclert et al. (2021a)
Monetary and Fiscal Policy		
ϕ	Coefficient on inflation in Taylor rule	1.5, Auclert et al. (2021a)
ϕ_y	Coefficient on output gap in Taylor rule	0, Auclert et al. (2021a)
τ	Tax rate	0.36, target $G/Y = 0.2$
B^g	Bond supply	2.8, Auclert et al. (2021a)
κ	Slope of the Phillips curve	0.1, Auclert et al. (2021a)
Open economy		
γ	Home bias	0.4, Kekre and Lenel (2021)
$\eta = \eta_f$	Elasticity of substitution for home vs. foreign goods	1.5, Kekre and Lenel (2021)

Table 5: Income Process Estimation Fit for the High-risk Country. The table shows empirical moments calculated using RLMS data and model moments obtained using estimated process parameters.

Moment	Data	Model
Standard deviation: annual log earnings	2.48	2.37
Standard deviation: 1-year change	2.81	2.75
Standard deviation: 5-year change	3.31	3.35
Frac. 1-year change < 10%	0.24	0.03
Frac. 1-year change < 20%	0.38	0.06
Frac. 1-year change < 50%	0.54	0.15

for 2000 to 2019 is used to estimate the labor income process for the high-risk country. The data is cleaned following Guvenen, Karahan, Ozkan and Song (2021) .[18] The functional form of the income process in Auclert et al. (2021a) 's discrete-time HANK model for the US is a simple AR(1) process with normally distributed errors. Therefore, for consistency, I chose the same functional form for the labor income process in the high-risk country. As targets in the Generalized Method of Moments (GMM) estimation the same moments as in Kaplan et al. (2018) are selected, with two exceptions:[19] Kurtosis is not targeted as excess kurtosis in an AR(1) process with normally distributed errors is never positive. Also, standard deviations are chosen as target moments instead of variances, so that the levels of moments are closer to one other, allowing for the identity weighting matrix's use in GMM estimation.[20] For alternative choices of targeting moments and weighting matrices see Table 11. The main trade-off in the estimation is to match large variances and, at the same time, a large mass concentrated close to zero with a process that has normally distributed errors. The estimated process has a persistence of 0.65 and an unconditional standard deviation equal to 2.15, as shown in Table 7. Table 5 shows that the estimated process produces moments close to those in the data. Lower persistence and higher standard deviation of the income process were also obtained in the estimates of the income process for another high-risk economy, China (Yu and Zhu ,2013 and Fan, Song and Wang ,2010).

18 For more details see Appendix A.

19 Since the Auclert et al. (2021a) model is a discrete time version of Kaplan et al. (2018) , the origin of income process estimates is Kaplan et al. (2018) .

20 Targeted variances in Kaplan et al. (2018) are of the same order of magnitude as targeted fractions. Thus no issue arises with different levels when targeting variances.

21 Elasticities of the share of borrowing-constrained households to income were calculated as a ratio of IRFs: the IRF of the share to a 1% TFP shock with persistence 0.96 over the IRF of households' income. TFP

Table 6: Model Fit: Income and Asset Distributions, Asset Elasticities - Data vs. Model Statistics. Model statistics are computed at the steady state, except for the elasticities. Income statistics from the model include after-tax labor income and capital gains. Data on income, liquid, illiquid assets, and shares of borrowing-constrained households are obtained from the US Survey of Consumer Finance and RLMS. Liquid assets consist of transaction accounts, directly held bonds, directly held stocks, and credit card balances. Illiquid assets consist of certificates of deposits, savings bonds, cash value of insurance, other managed assets, retirement accounts, stock holdings, and primary residence net of mortgage home loans. Data on Gini coefficients is from the World Inequality Database and Kaplan et al. (2018) . Elasticities of the shares of borrowing-constrained households to income are derived from local projections (values in the brackets show the responses of the aggregated shares in the first and the second years, see Section 3.3). Data on bonds and equity elasticities are computed based on balance of payment statistics (Sources: BEA and CBR external sector statistics).

Statistics	Data, low-risk country	Model, low-risk country	Data, high-risk country	Model, high-risk country
Wealth distribution				
Mean Liquid Assets/GDP	0.26	0.26	0.41	0.41
Mean Illiquid/GDP	2.92	3.24	0.92	0.92
Share of borrowing-constrained hhs	0.38	0.38	0.33	0.36
Gini coefficients				
Income	0.52	0.41	0.58	0.65
Liquid assets	0.98	0.75	-	0.66
Illiquid assets	0.81	0.52	0.67	0.44
Elasticities[21]				
Share of borrowing-constrained to income	{0.41, 0.02}	{0.03, 0.03}	{-0.10, -0.04}	{-0.08, -0.16}
Corr(Bonds / GDP, GDP)	-0.08	-0.08	0.21	0.58
Corr(Equities / GDP, GDP)	0.06	0.23	0.21	-0.72

Table 7: High-risk Country: Household Parameter Values, Description and Source

Parameter	Description	Value and source
β	Time discount factor	0.926, target interest rate
χ_0	Portfolio adj. cost pivot	0.25, Auclert et al. (2021a)
χ_1	Portfolio adj. cost scale	0.54, target $\mathcal{B}_h = 1.64Y$
χ_2	Portfolio adj. cost curvature	2.4, target htm=0.33[22]
σ	EIS	0.5, Semko (2013)
ρ	Inverse Frisch elasticity	1, Semko (2013)
ρ_z	Autocorrelation of earnings	0.65, estimated with RLMS
σ_z	Cross-sectional std of log earnings	2.15, estimated with RLMS
φ	Disutility parameter	1.71 (target $N = 1$)

Table 8: High-risk Country: Parameter Values, Description and Source

Parameter	Description	Value and source
Asset Markets		
r	Real interest rate	0.0125, Auclert et al. (2021a)
ψ	Liquidity premium	0.005, Auclert et al. (2021a)
p/Y	Steady state price of equity	5.04 target $p + Bg = 5.32Y$
Production		
δ	Depreciation rate	0.025, Malakhovskaya and Minabutdinov (2014)
ε_I	Capital adj. costs parameter	12.05, Malakhovskaya and Minabutdinov (2014)
K/Y	Steady state capital-output ratio	9.26, target $\alpha = 0.33$ Malakhovskaya and Minabutdinov (2014)
Monetary and Fiscal Policy		
ϕ	Coefficient on inflation in Taylor rule	1.5, Auclert et al. (2021a)
ϕ_y	Coefficient on output gap in Taylor rule	0, Auclert et al. (2021a)
τ	Tax rate	0.29, target $G/Y = 0.2$
B^g	Bond supply	0.28, calculated from total government bonds supply
κ	Slope of the Phillips curve	0.38, Malakhovskaya and Minabutdinov (2014)
Open economy		
γ	Home bias	0.26, Malakhovskaya and Minabutdinov (2014)
$\eta = \eta_f$	Elasticity of substitution for home vs. foreign goods	0.67, Semko (2013)

To assess the fit of the fully calibrated HANK models for each of the two economies, Table 6 compares key distributional moments calculated at the model's respective steady states and in the data. The model reproduces mean liquid and illiquid asset holdings, shares of borrowing-constrained households, and income Gini coefficients. Like numerous other HANK models wealth concentration is underestimated at the top. The model also performs well in matching elasticities of the share of borrowing-constrained households to their income,

persistence is taken from the data. Bonds to GDP and equity to GDP elasticities were averaged across 50 simulations. Breaking down debt instruments into short- and long-term components reveals that both debt-to-GDP elasticities are negative for the US, i.e. in booms, equity holdings increase, holdings of all types of debt decrease. In contrast, in Russia, the short-term debt component is procyclical, but the long-term component is countercyclical.

22 Targeting both the steady state htm and the elasticity of htm to income.

the empirical counterpart of which was obtained in Section 3. Note that while calibration parameters are set to match the wealth distribution moments in Table 6, the Gini coefficients and the elasticities are untargeted moments.

On the international front, the model elasticities of equity and debt components of the financial account to GDP, too, match empirical data except for the demand for equities in the high-risk country which is more countercyclical in the model than in the data.

Solution of the model is obtained using sequence-space Jacobian method introduced in Auclert et al. (2021a) . This method linearizes the model to first order in aggregates and features aggregate certainty equivalence, therefore the impulse responses to MIT shocks are the same as those of the full stochastic model. As shown in Boppart, Krusell and Mitman (2018) non-linearities in aggregate shocks are usually small.

4.3. Quantitative Results

The paper's primary focus is on the intertemporal saving decisions of heterogeneous households and their impact on international net and gross capital flows. This is assessed, for the low- and high-risk countries, respectively, via impulse response functions of the calibrated HANK model to a 1% TFP shock.

In the following, the mechanism through which heterogeneous marginal propensities to save impact aggregate savings and the current account is discussed, as well as a comparison of the responses in the two respective economies. Then, the responses are compared to the representative agent framework, a decomposition of the three effects is shown, and finally, a U-shaped pattern of the aggregate consumption volatility is presented.

4.3.1. Comparison between Countries

Figure 5 displays the impulse responses for selected model variables in response to a 1% TFP shock that follows an AR(1) process. A comparison between responses for the two modelled economies reveals that, in line with empirical results, households in a low-risk country mostly save in the form of illiquid assets (first row, second panel). In contrast, relative to the low-risk country, households in the high-risk economy increase their liquid

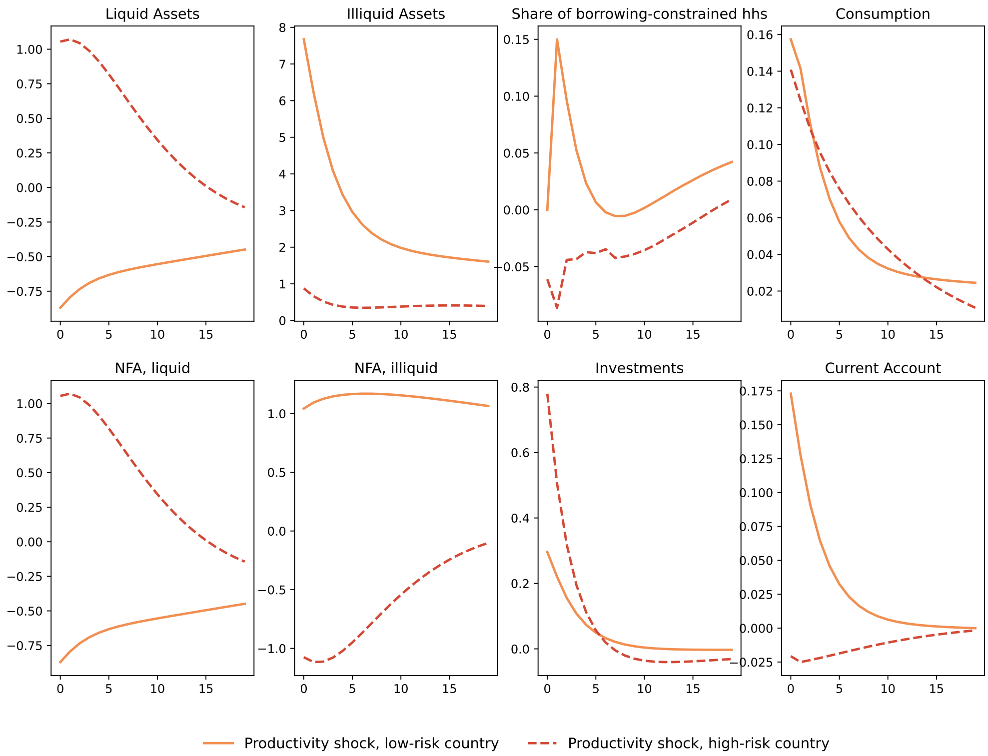


Figure 5: Impulse Responses to a TFP Shock: Country Comparison. The figure shows the percentage deviations of the specified variables from the steady state in response to a TFP shock. The shock is a 1% increase in total factor productivity; it follows an AR(1) process with persistence parameter 0.7, such that the shock mostly dissipates by the 13th quarter. The figure shows the impulse responses for the model calibrated to the low-risk economy (solid orange lines) and to the high-risk economy (dashed red lines). The X-axis shows time in quarters.

holdings in response to a TFP shock (first row, first panel). The two reasons for such stark differences in the savings behavior in the two economies are wealth and idiosyncratic risk. With a CRRA utility function, a lower level of wealth in the dynamic optimization leads to less risk-taking behavior of high-risk economy households. Higher labor income risk in the high-risk economy furthermore induces larger precautionary savings.

As a result of the savings behavior of the households, the model calibrated to the low-risk country produces an increase in the share of borrowing-constrained households in response to a positive TFP shock. Matching the empirical results, the same share in the model calibrated to the high-risk country falls due to the households' formation of the liquid buffers (first row third panel, for the empirical responses see Section 3).

A large demand for liquid assets in the high-risk country through the financial intermediary is translated into a long position in liquid assets in the expansionary period (second row, first panel). In contrast, in an expansion, the low-risk country borrows liquid assets from the rest of the world and invests in illiquid assets (second row, second panel).

Investment responses are primarily driven by parameters related to the intermediate-good firm's problem: depreciation rate, capital adjustment costs, and elasticity of export demand. Following larger savings responses in the high-risk country compared to the low-risk country (which puts more downward pressure on the real interest rate) and higher volatility of investment due to the calibration parameters, the investment increase in the high-risk country is also larger. This in turn leads to a countercyclical response of the current account in the high-risk economy.

Nominal rigidities are not important for the presented results, but they allow a better match of the share of borrowing-constrained households' responses to the empirical ones (see Figure 23). Without nominal rigidities equity prices increase on impact by less (see Figure 24). Thus, households' demand for illiquid assets is weaker, which leads to a smaller increase in the share of borrowing-constrained households in the low-risk country.

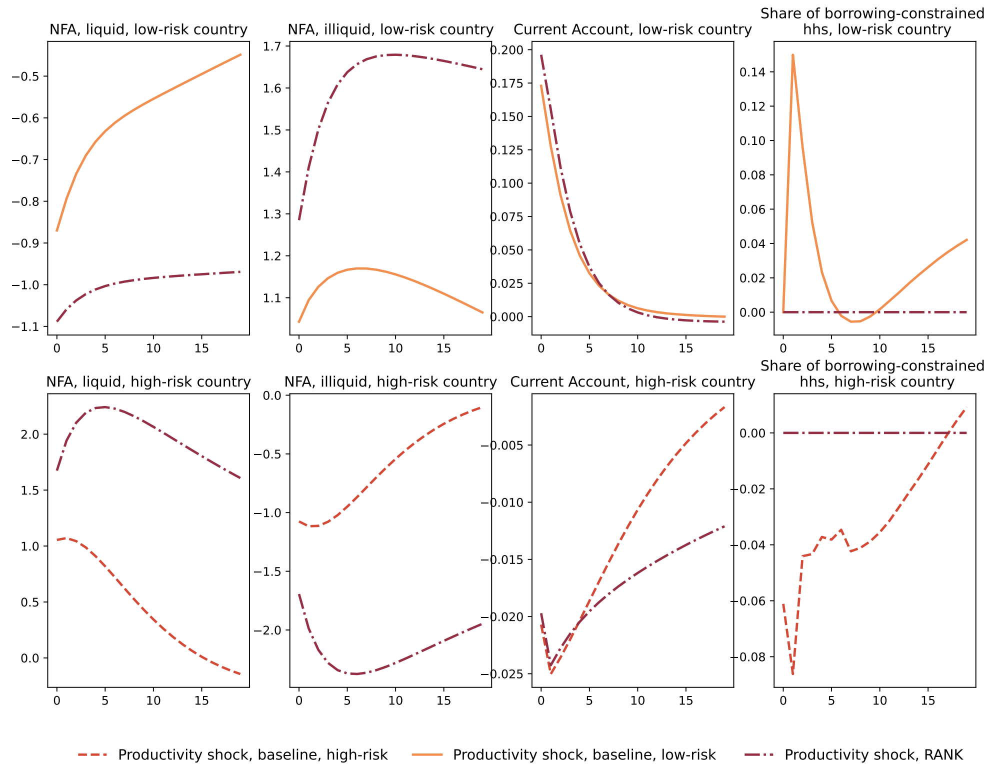


Figure 6: Impulse Responses to a TFP Shock: HANK and RANK Comparison. The figure shows the percentage deviations of the specified variables from the steady state in response to a TFP shock. The shock is a 1% increase in total factor productivity; it follows an AR(1) process with persistence parameter 0.7, such that the shock mostly dissipates by the 13th quarter. The figure shows the impulse responses for the baseline HANK model (solid orange lines for the low-risk country and dashed red lines for the high-risk country), and a RANK model (dash-dotted burgundy lines for both countries). The X-axis shows time in quarters.

4.3.2. Comparison with RANK and Current Account Decomposition

The impulse response functions in the HANK model are compared to IRFs in a RANK model in Figure 6. While in the low-risk economy current account is less procyclical/more countercyclical in a HANK model than in a RANK, in the high-risk country the opposite result holds. This is due to larger precautionary savings in the high-risk country and amplification of the savings responses coming from the distributional effect (decrease in the share of borrowing-constrained households). At the same time, the extensive margin and the dampening of the distributional effect dominate the precautionary savings effect in the low-risk country.

To understand the quantitative contribution of the three effects: the presence of borrowing-constrained households, the precautionary savings, and the distributional effect, Figure 7 shows the relative contribution of the effects in the current account dynamics. The presence of borrowing-constrained households results in a dampening of aggregate savings and more countercyclical current accounts. While precautionary savings in both countries positively contribute to the dynamics of the current account (with a larger quantitative contribution of the precautionary savings in the high-risk country), the distributional effect leads to a more procyclical current account in the high-risk country and a more countercyclical current account in the low-risk economy. The magnitude of the distributional effect is similar to the magnitude of the precautionary savings effect in both countries.[23]

For the analytical derivations of the contribution of the three effects to the current account dynamics see Appendix D. There, in an analytical heterogeneous agent model based on Debortoli and Galí (2017) framework, the current account is explicitly written as a function of the representative agent part (intertemporal consumption smoothing) and the three additional effects coming from household heterogeneity.

Distribution of income. As shown in Appendix D the distribution of additional income between constrained and unconstrained households matters for the quantitative results. Figure 28 shows that in expansions with government spending being procyclical (as compared to being fixed as in the baseline), more income is distributed to constrained households, as they rely more on asset income than unconstrained households (in this scenario taxes are higher in expansions due to the balanced budget of the government). This leads to a decrease in aggregate savings responses to TFP shocks.

4.3.3. Consumption Volatility

Lastly, the U-shaped aggregate consumption volatility shows that at the low level of idiosyncratic risk the extensive margin dominates, savings responses to business cycles are dampened, and the consumption volatility is large. Once the risk increases the precautionary savings increase, and the distributional effect shifts towards the amplification of the savings responses to business cycles, compensating for the extensive margin. As a result, consumption

23 The precautionary savings effect is obtained as a difference between the baseline response and the response where $e_t = 1$ for all households following the TFP shock. The distributional effect is obtained using current account log-linearization and is equal to $-S\frac{\lambda}{1-\lambda}\tilde{\lambda}_t$ where λ is the share of borrowing-constrained households, $\tilde{}$ stands for the log-deviations from the steady state and S is the steady-state value of savings. The effect due to the presence of borrowing-constrained households is the difference between HANK and RANK responses, ignoring also the other two effects.

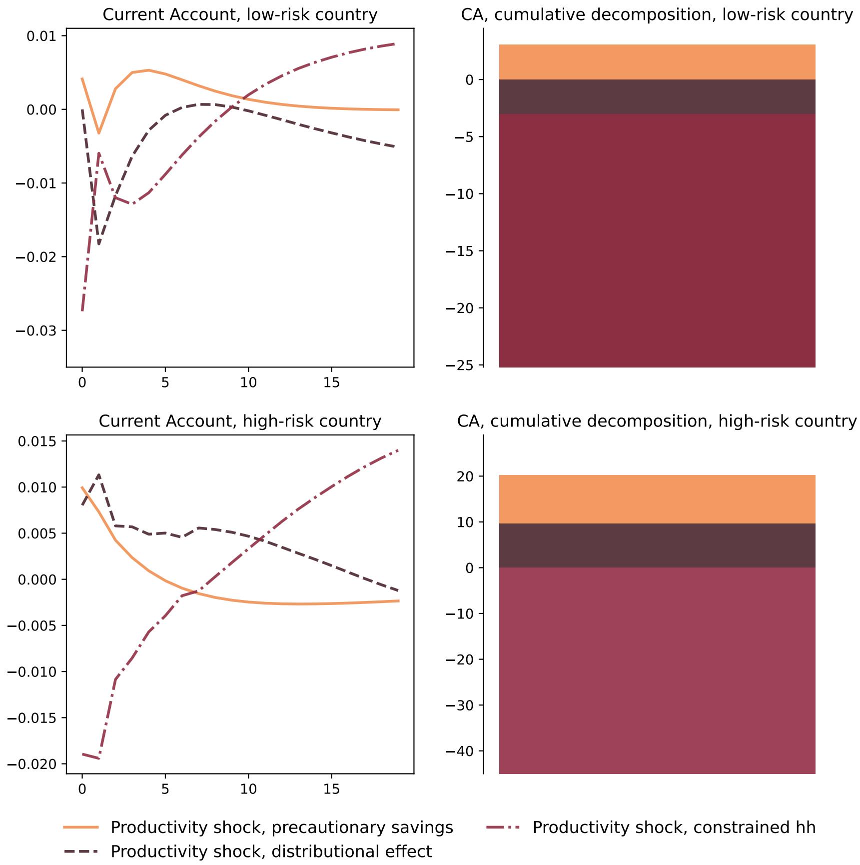


Figure 7: Decomposition of the Dynamics of the Current Account. The figure shows the contribution of the three effects in current account dynamics in response to a TFP shock. The contributions are shown in percentage points in the left graphs, and in percentages in the right graphs of the figure. The shock is a 1% increase in total factor productivity; it follows an AR(1) process with persistence parameter 0.7, such that the shock mostly dissipates by the 13th quarter. The left graphs show the dynamics of the three effects and the right graphs show the cumulative contributions. All effects are calculated as a difference between the model's response with an effect and without it. The contribution of precautionary savings is shown in orange, the distributional effects are shown in brown, and the contribution of the presence of borrowing-constrained households is shown in burgundy. The X-axis on the left graphs shows time in quarters.

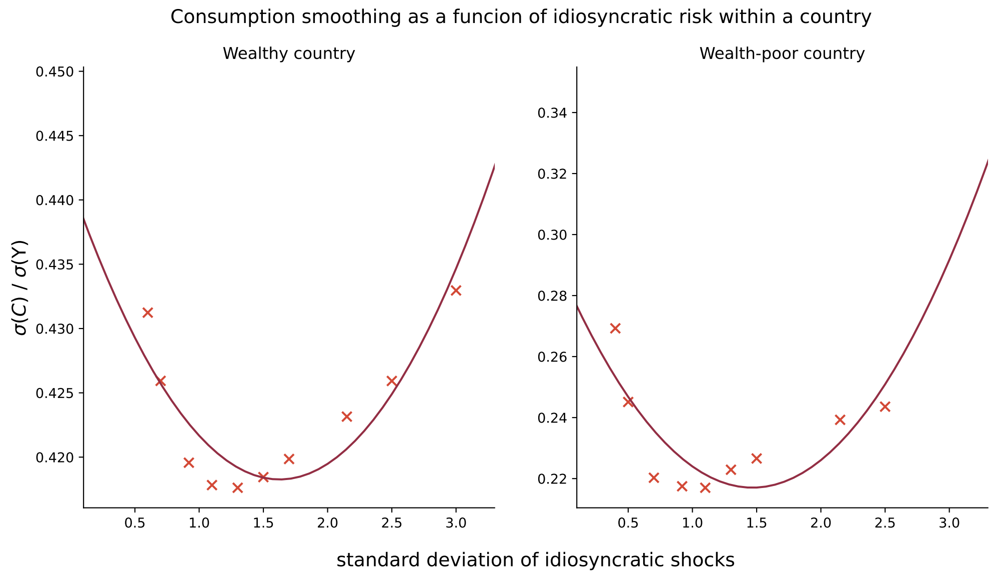


Figure 8: Consumption volatility as a function of the level of idiosyncratic risk. The aggregate consumption standard deviations are shown as a ratio to the standard deviations of GDP. Zero standard deviation of risk corresponds to RANK with borrowing constraints.

volatility decreases. With the further increase in idiosyncratic risk precautionary savings rise and dominate the extensive margin, leading to increased aggregate consumption volatility.

For the consumption and gross position volatilities as a function of risk and wealth see Appendix B.2. The aggregate consumption volatility decreases as a function of wealth due to an increased ability of the households to smooth shocks. The volatility of liquid NFA increases with risk. Fixing the portfolio adjustment cost parameter χ_1 as compared to fixing the ratio of liquid to illiquid assets shifts the trough of the U-shaped consumption volatility function (see Figure 25).[24] In the closed economy version of the model calibrated to the US, consumption volatility is higher for all levels of idiosyncratic risk due to the absence of international risk sharing, but it experiences the same U-shaped pattern.

5. Conclusion

This paper evaluates the role of heterogeneous marginal propensities to save in shaping net and gross international capital flows. This assessment is done by extending the intertemporal approach to the current account to include heterogeneous households differing in their levels

24 In Figure 8 the standard deviation of risk is changed in the calibration, keeping the amount of liquid and illiquid wealth as in the baseline calibration. Instead in Figure 25 χ_1 is fixed, but then the ratio of liquid to illiquid assets varies with the standard deviation of idiosyncratic risk.

of wealth and facing aggregate and idiosyncratic risks. Three novel effects are revealed: (i) borrowing-constrained households do not directly impact current account since they have a low ability to smooth their consumption; (ii) the savings of unconstrained households are determined by their levels of wealth and the amount of idiosyncratic risk in the economy. Additionally, (iii) levels of idiosyncratic risk and wealth determine endogenous changes in the distribution of the households in response to shocks, which amplify savings responses when the wealth level is low and idiosyncratic risk high. In the reverse case, responses are dampened.

An important finding is that idiosyncratic risk and wealth affect countries' international portfolios. This finding has broad implications for understanding the role of within-country risk and endogenous changes in investors' risk aversion following changes in wealth in the formation of the gross capital flows between countries.

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A. Data Appendix

Data Cleaning, RLMS. I use only households who are complete income reporters, whose real income is larger than 0 and smaller than 99.9 percentile of the distribution of income.[25] As households report not only income and consumption but also savings, I use only those responses where deviations from the budget constraint are smaller than half of the income. Also, I use only those responses where expenditure on each category is smaller than half of the total expenditures (except for durables).

Variable Construction. In constructing income and wealth variables using PSID dataset I follow Kaplan et al. (2014) . Labor income is obtained by subtracting asset income from total family income. Asset income includes income from rent, income from dividends, income from earned interest, income from trust funds and royalties, asset part of the business income and income from farming (of the head and wife). Liquid assets are obtained by subtracting credit card charges, student loans, medical or legal bills, and loans from relatives from money in checking or savings accounts, money market funds, certificates of deposit, government savings bonds, or Treasury bills, not including assets held in employer-based pensions or IRA's. Illiquid assets are obtained by subtracting housing loans from value of the real estate owned, money in IRA and other illiquid assets owned (bonds, life insurance, etc.).

Those who had intermittent "headship", appeared only once, whose income grows more than 500%, falls by more than 80%, or is below \$100, who have missing observations on race, education, or state of residence were dropped.

25 Aguiar and Bils (2015) use 5% and 95% of the income distribution. However, as is explained later, the survey lacks high-income households, thus, I have cut only the most unreliable observations.

Table 9: Data Cleaning. The table shows how many responses were excluded by each restriction. The first row reports the original sample size. The final row represents the sample used in the analysis.

Restriction	Number of responses left	Percentage of responses left
Total number of responses	138 371	100%
Complete income reporters	129 394	94%
Income > 0	128 477	93%
Income < 99.9 percentile of income distribution	128 348	93%
(Exp + Savings - Income) / Income < 0.5 Income	121 730	88%
Exp. each category (except durables) < 0.5 Total Exp.	109 012	79%

In RLMS total income is taken from constructed variables file. It includes wages, pensions, stipends, alimony, insurance payments, subsidies, transfers, income from rent, income from farming, etc. Illiquid assets is the value of the housing owned less credits and net loans. In Russia retirement accounts are not accessable. Non-durable consumption includes fridge, washer, freezer, microwave, TV, hairdryer, VCR, DVD player, computers, camera, mp3 player, cars, trucks, motorcycles, boats and lawn mowers. Total consumption is also taken from constructed variables file.

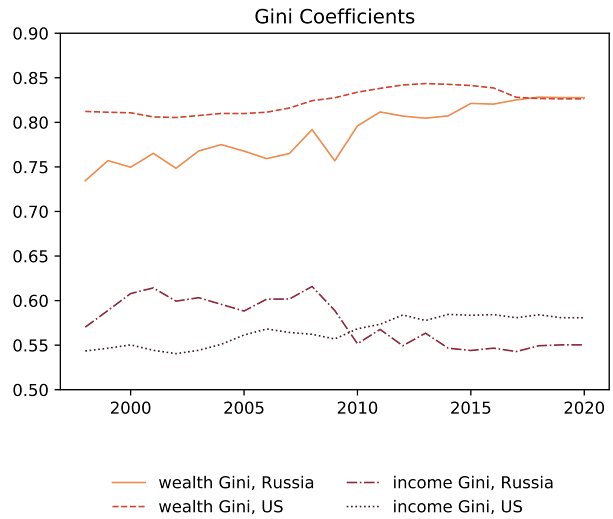
Estimation of the Income Process for High-risk Country. I use only males from 25 to 60 years old who have earnings above the minimum income threshold, equivalent to earnings from one quarter of full-time work (13 weeks at 40 hours per week) at half of the legal minimum wage in year t. Additionally, I drop observations that report less than 270 russian rubles in 2003 prices (equivalent to \$1500 used by Guvenen et al. (2021)). I also drop individuals who are observed in less than three waves or those who do not report race, age or education. Education is adjusted as in Guvenen et al. (2021) .

Following the survey design two alternative variables are used as wages for robustness: 1) the last 12-month average wage in the primary job 2) the sum of last month's wages in primary and secondary jobs, bonuses, and income received as self-employed. Moments of wage residuals obtained from the regression of wage logarithm on age, age squared, race and education variables are used as data moments. Simulated moments of the AR(1) process with normally distributed residuals are used as theoretical moments.

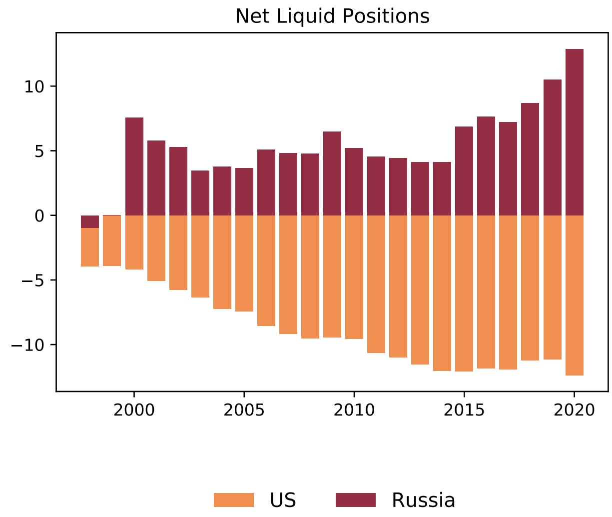
Wealth Moments in the High-risk Country. All data needed to calculate wealth ratios is only available since 2018. Therefore, all ratios are calculated as averages for the years 2018 and 2019. The volume of government bonds in circulation is taken from the website of the Russian Ministry of Finance (see OFZ volume there). Amount of liquid and illiquid holdings of households is calculated using RLMS and Household Savings database of the russian central bank. Following Kaplan et al. (2018) liquid holdings consist of deposits, bonds, money in brokers' accounts and cash taken from Household Savings database. Illiquid assets are calculated as a sum of net housing and net durables, calculated using average net housing and net durables from RLMS, and the value of equity holdings taken from the Household Savings database.

B. Additional Graphs

B.1. Data summary graphs

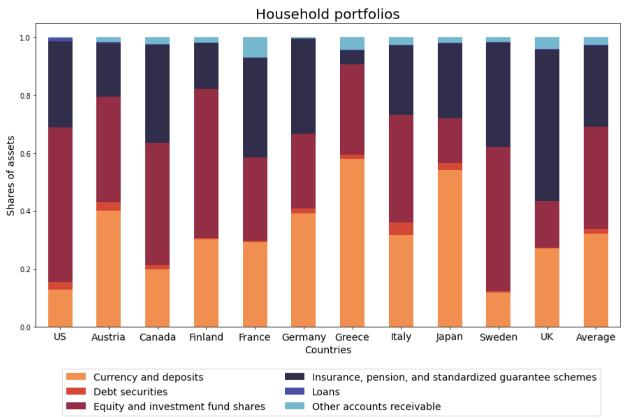


(a) **Inequality**

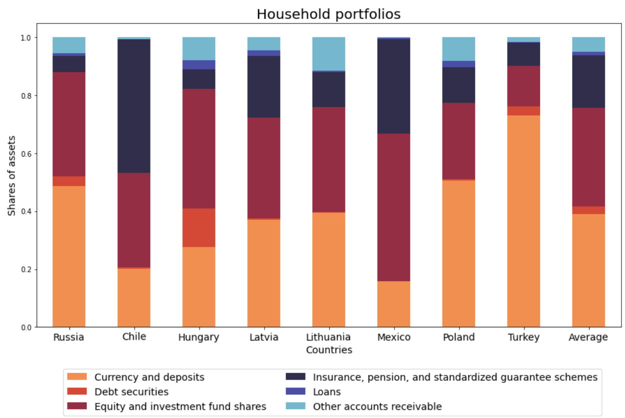


(b) **Liquid Assets Positions**

Figure 9: Gini Coefficients and Liquid Positions. The left-hand side figure shows Gini coefficients for the US and Russia. Data is obtained from World Inequality Database. The right-hand side figure shows net liquid asset holdings as a percentage share of GDP for Russia and for the US. Net liquid holdings are calculated as a sum of Official Reserves (minus Gold), Portfolio Assets and Other Assets, minus Portfolio Debt and Other Liabilities. Data is obtained from Lane and Milesi-Ferretti (2017) . X-axis shows time in years.

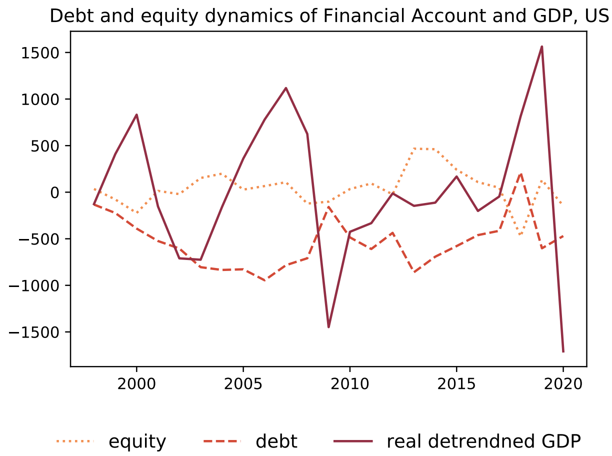


(a) **Developed countries**

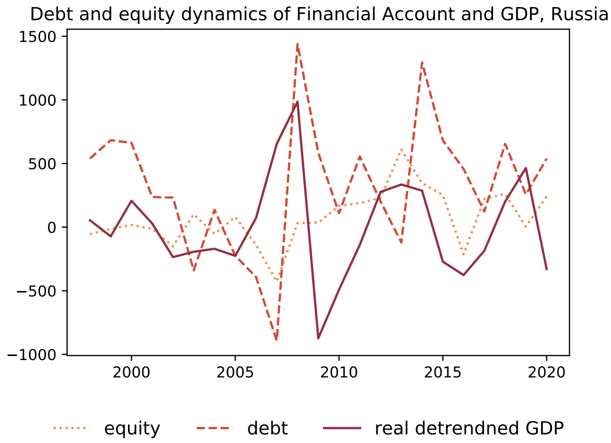


(b) **Emerging countries**

Figure 10: Household portfolios. The left-hand side graph shows household portfolios for the developed countries and the right-hand side shows portfolios of the emerging countries. Each asset is presented as a share from the total portfolio. X-axis shows countries. Data is taken from OECD and Russian Central Bank database. Portfolios are presented for the last available year (2021).

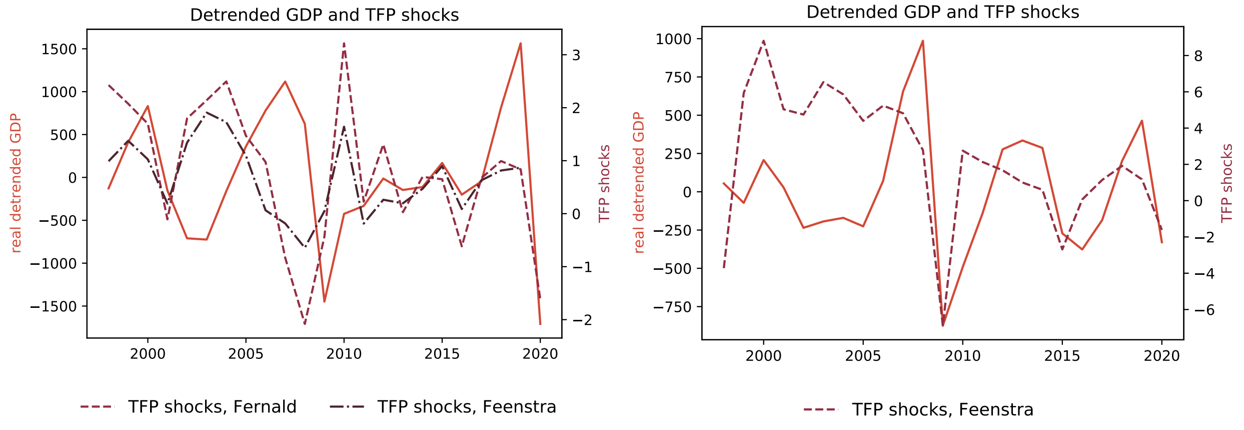


(a) **Low-risk Country**



(b) **High-risk Country**

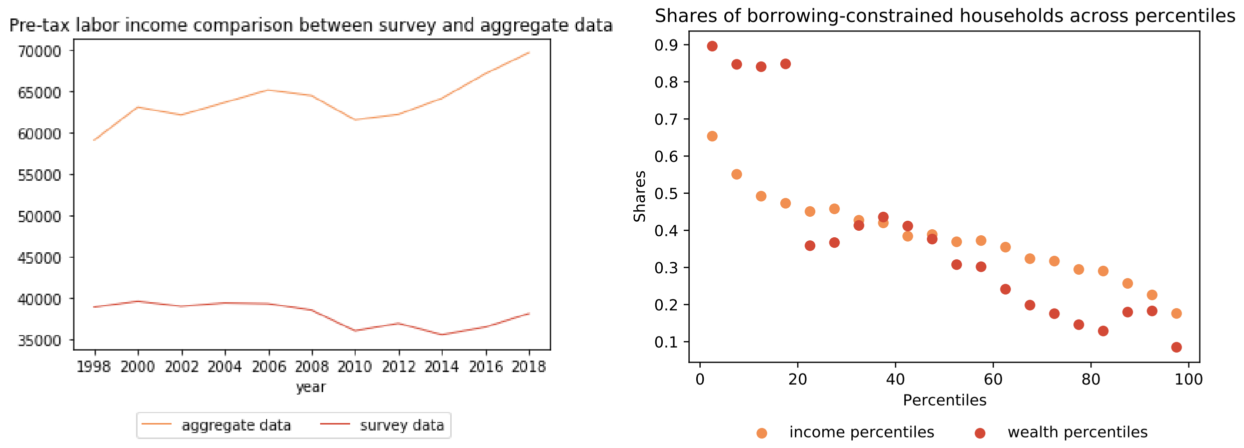
Figure 11: Cyclicality of equity and debt. The left-hand side graph shows series for the US and the right-hand side graph shows time series for Russia. Equity and debt are aggregated across foreign direct investment, portfolio investment and other investment. HP filter was used to detrend GDP series. GDP deflator was used to convert nominal to real values. Series for the US are presented in billions of 2012 US dollars. Series for Russia are presented in billions of 2003 rubles. X-axis shows time in years. Data for equity and debt is taken from the IMF database (BPM6). GDP data is taken from Rosstat and FRED.



(a) **Low-risk Country**

(b) **High-risk Country**

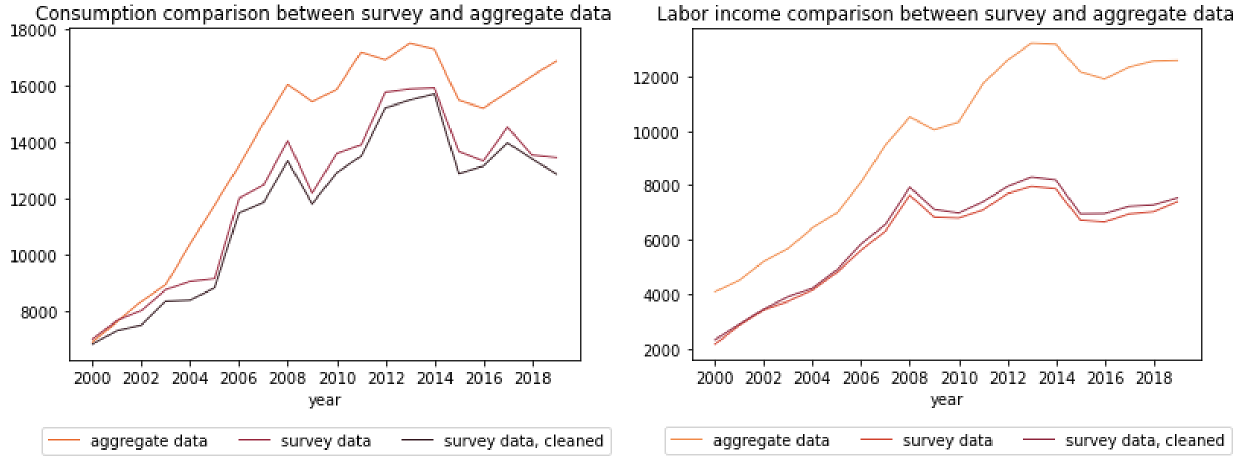
Figure 12: Detrended GDP vs. TFP series. The left-hand side graph shows series for the US and the right-hand side graph shows time series for Russia. HP filter was used to detrend GDP series. GDP deflator was used to convert nominal to real values. TFP series was taken from Penn World Table 10.0. Series for the US are presented in billions of 2012 US dollars. Series for Russia are presented in billions of 2003 rubles. X-axis shows time in years. GDP data is taken from Rosstat and FRED.



(a) **Dynamics of labor income**

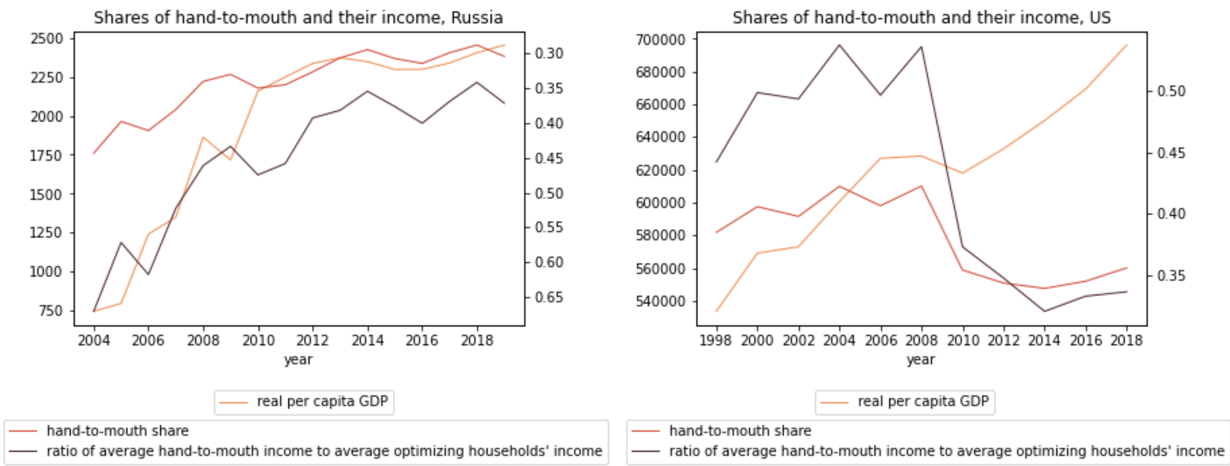
(b) **Shares of borrowing-constrained households**

Figure 13: Labor Income and Share of Borrowing-constrained Households. The left-hand side figure shows a comparison between per household real labor income constructed from PSID and taken from NIPA. The right-hand side figure shows shares of borrowing-constrained households across income and wealth percentiles in the US averaged across 1998-2018 years.



(a) **Consumption** (b) **Labor Income**

Figure 14: Comparison Between Aggregate Statistics and Survey Data. The figures show consumption and labor income per household calculated from aggregate statistics (Rosstat) and from survey data (RLMS). All variables are real 2003 rubles and expressed in per month values. X-axis shows the time in years.



(a) **High-risk Country** (b) **Low-risk Country**

Figure 15: Cyclicality of borrowing-constrained shares. The figures show shares of borrowing-constrained households (λ) and ratios of average income of borrowing-constrained households and optimizing households. GDP is presented on the left Y-axis. Shares are presented on the right and inversed Y-axis. Data for real per capita GDP was from Rosstat (Russia, 2003 rubles) and FRED (US, 2012 dollars). Shares were calculated using RLMS (Russia) and PSID (US) households' surveys. X-axis shows years.

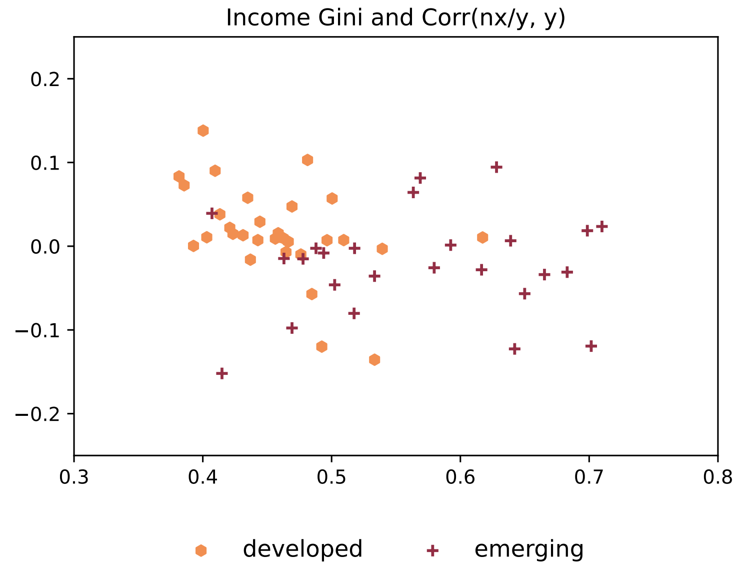


Figure 16: Income Inequality and Cyclicality of Net Export The figure shows income Gini coefficients (obtained from the World Inequality Database) and cyclicality of the net export (calculated based on the data from the IMF database, GDP was detrended using HP filter) for a sample of countries. For each country the inequality and cyclicality was obtained for the same periods of time, with the starting year not earlier than 1980 and ending year not later than 2020. The exact time span for each country depends on the data availability.

B.2. Estimation graphs

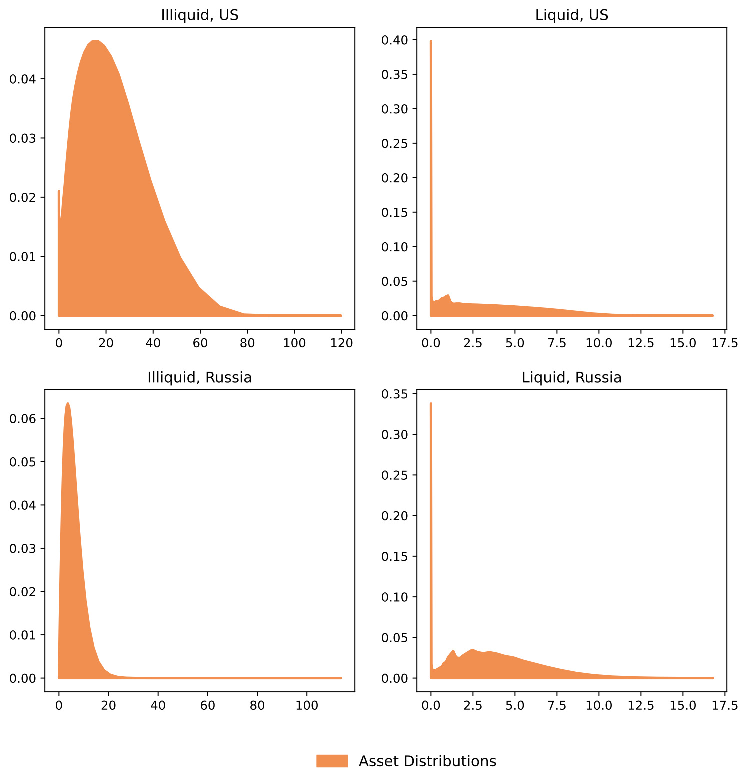


Figure 17: Distribution of Assets in the HANK models calibrated to the Low-risk and High-risk Economies. The figure shows steady state liquid and illiquid distributions.

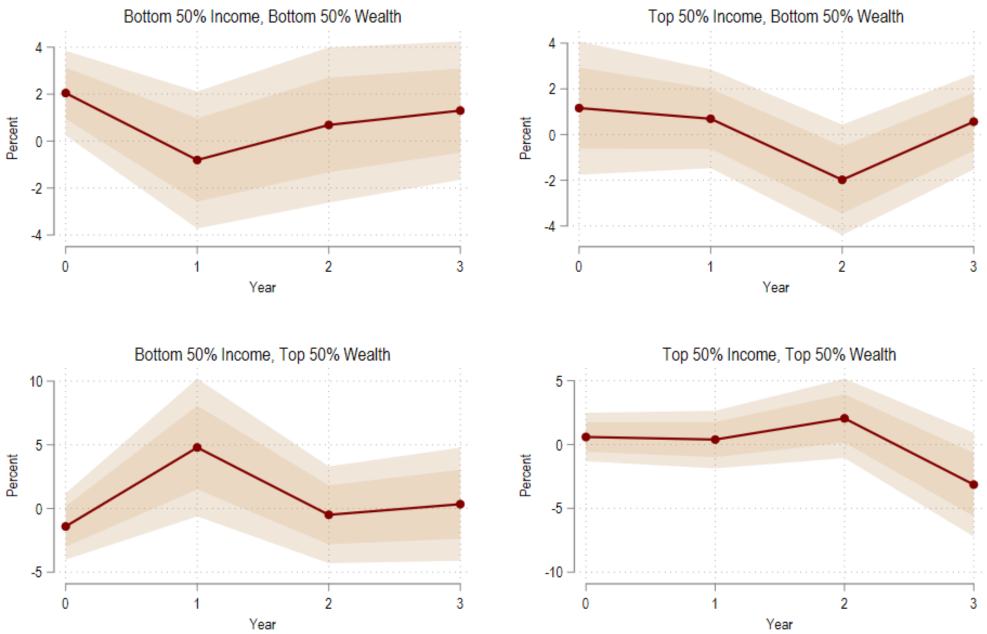


Figure 18: Empirical Impulse Responses of Liquid Assets to Income Shocks, Low-risk Country. The figures show impulse-responses for four groups of households (income poor/rich, wealth poor/rich). Red lines correspond to mean IRFs, inner shaded areas show one standard deviation around the mean responses, outer shaded areas show 90% confidence intervals. X-axis shows years. Liquid Assets are shown in log-differences.

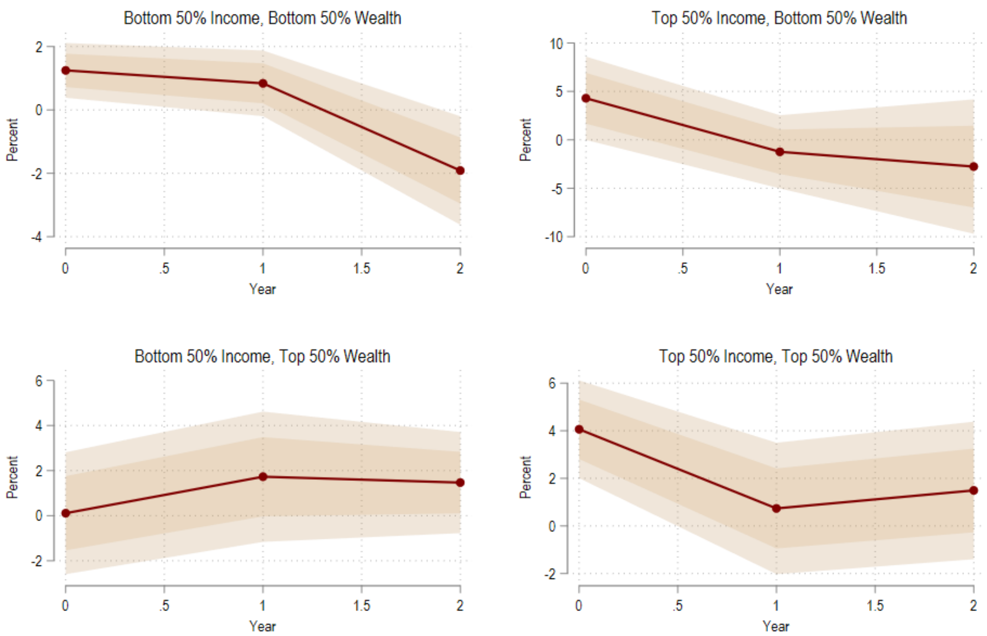


Figure 19: Empirical Impulse Responses of Illiquid Assets to Income Shocks, Low-risk Country. The figures show impulse-responses for four groups of households (income poor/rich, wealth poor/rich). Red lines correspond to mean IRFs, inner shaded areas show one standard deviation around the mean responses, outer shaded areas show 90% confidence intervals. X-axis shows years. Illiquid Assets are shown in log-differences.

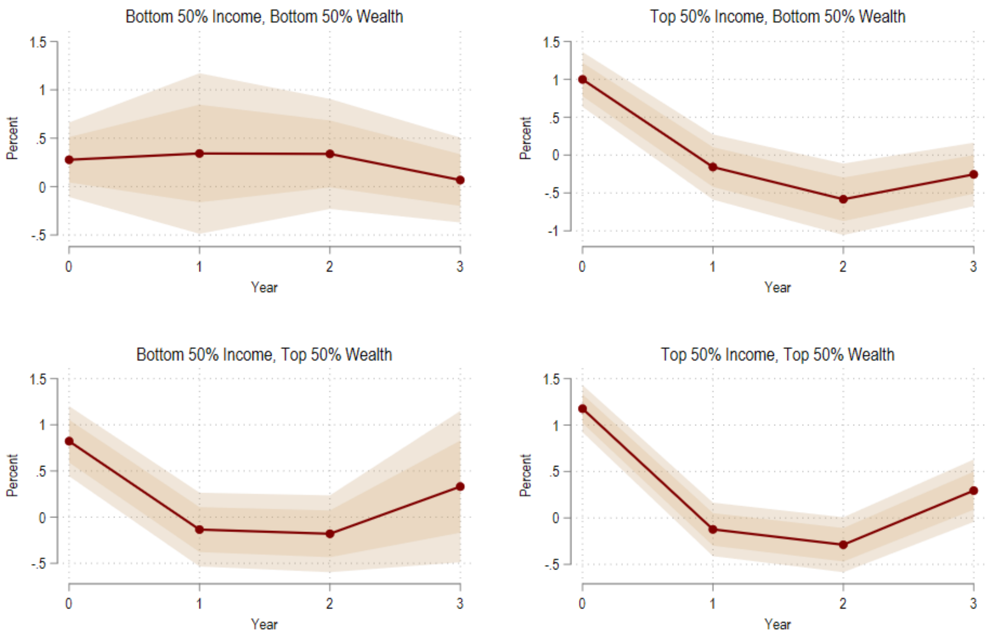


Figure 20: Empirical Impulse Responses of Consumption to Income Shocks, High-risk Country. The figures show impulse-responses for four groups of households (income poor/rich, wealth poor/rich). Red lines correspond to mean IRFs, inner shaded areas show one standard deviation around the mean responses, outer shaded areas show 90% confidence intervals. X-axis shows years. Consumption is shown in log-differences.

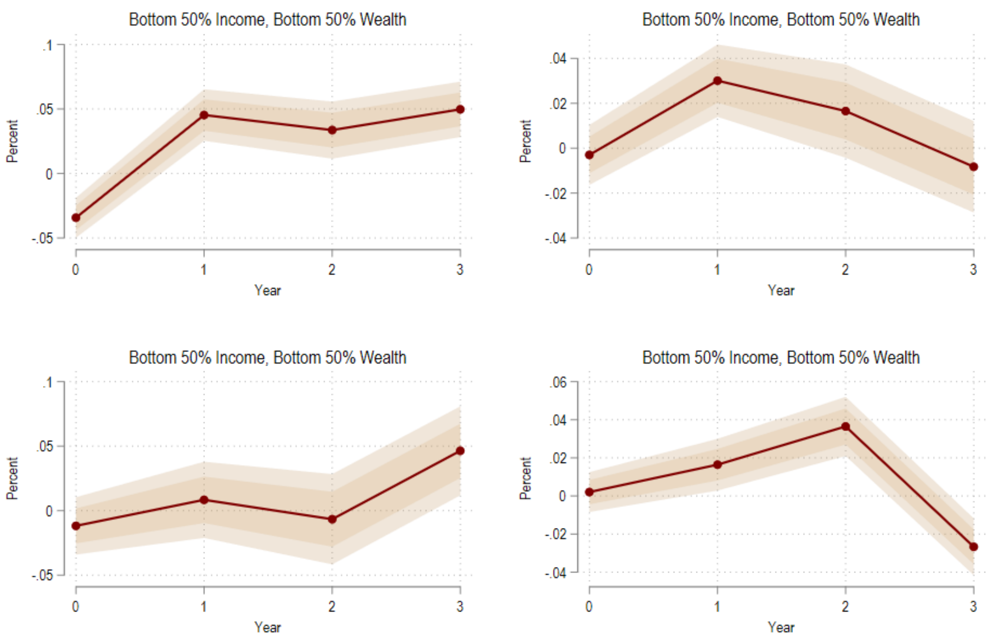


Figure 21: Empirical Impulse Responses of Income to TFP Shocks, Low-risk Country. The figures show impulse-responses for four groups of households (income poor/rich, wealth poor/rich). Red lines correspond to mean IRFs, inner shaded areas show one standard deviation around the mean responses, outer shaded areas show 90% confidence intervals. X-axis shows years. Income is shown in log-differences.

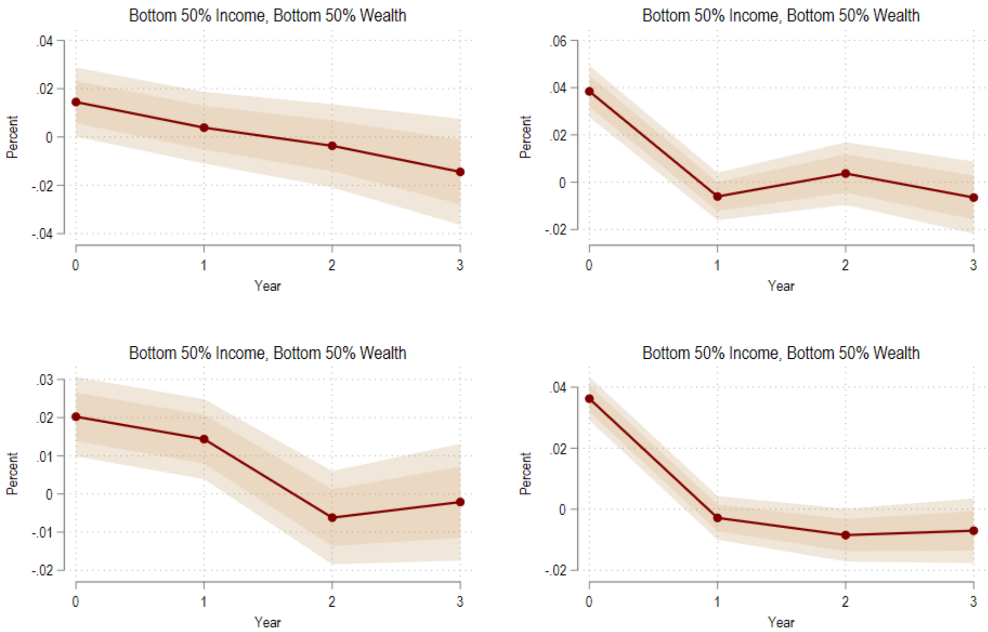


Figure 22: Empirical Impulse Responses of Income to TFP Shocks, High-risk Country. The figures show impulse-responses for four groups of households (income poor/rich, wealth poor/rich). Red lines correspond to mean IRFs, inner shaded areas show one standard deviation around the mean responses, outer shaded areas show 90% confidence intervals. X-axis shows years. Income is shown in log-differences.

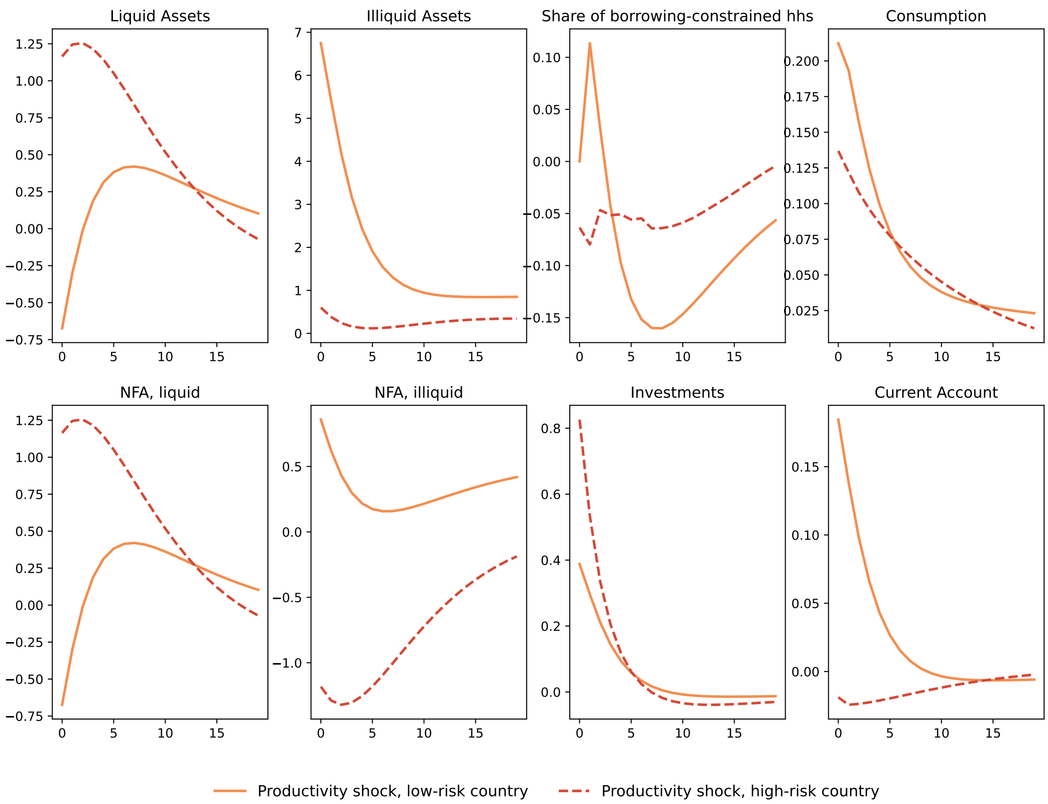


Figure 23: Impulse Responses to a TFP Shock, RBC Model.

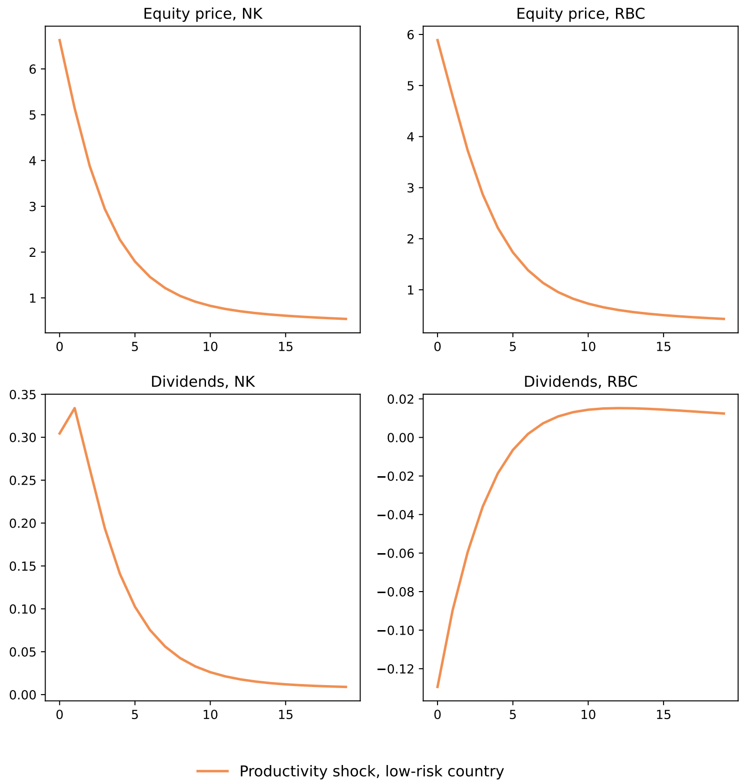


Figure 24: Impulse Responses of Equity Price and Dividends to a TFP Shock in RBC vs. NK Model.

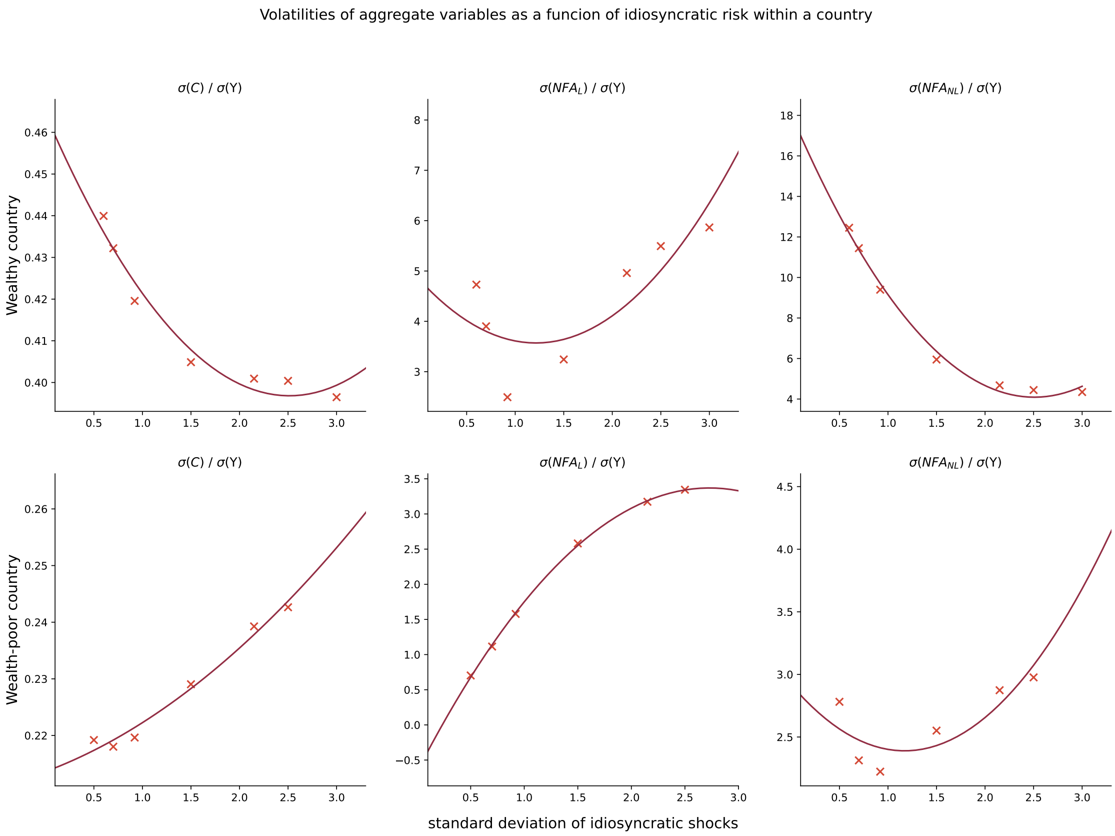


Figure 25: Volatilities of Aggregate Consumption and Gross International Positions as a Function of Idiosyncratic Risk with Untargetted Value of Liquid Assets.

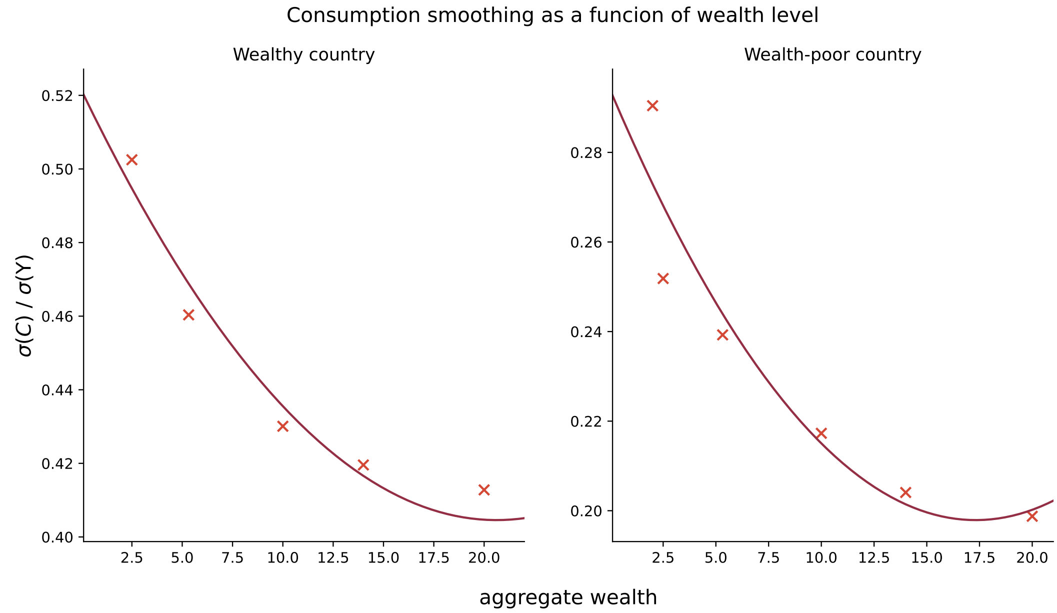


Figure 26: Aggregate Consumption Volatility as a Function of Wealth.

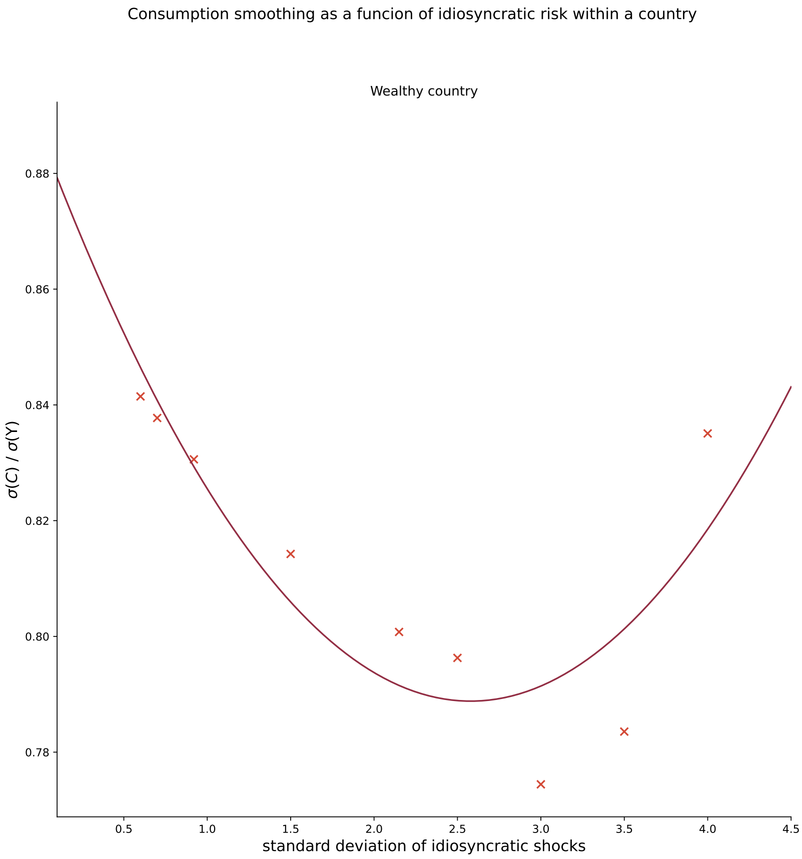


Figure 27: Aggregate Consumption Volatility as a Function of Idiosyncratic Risk in the Closed Economy Model Calibrated to the US.

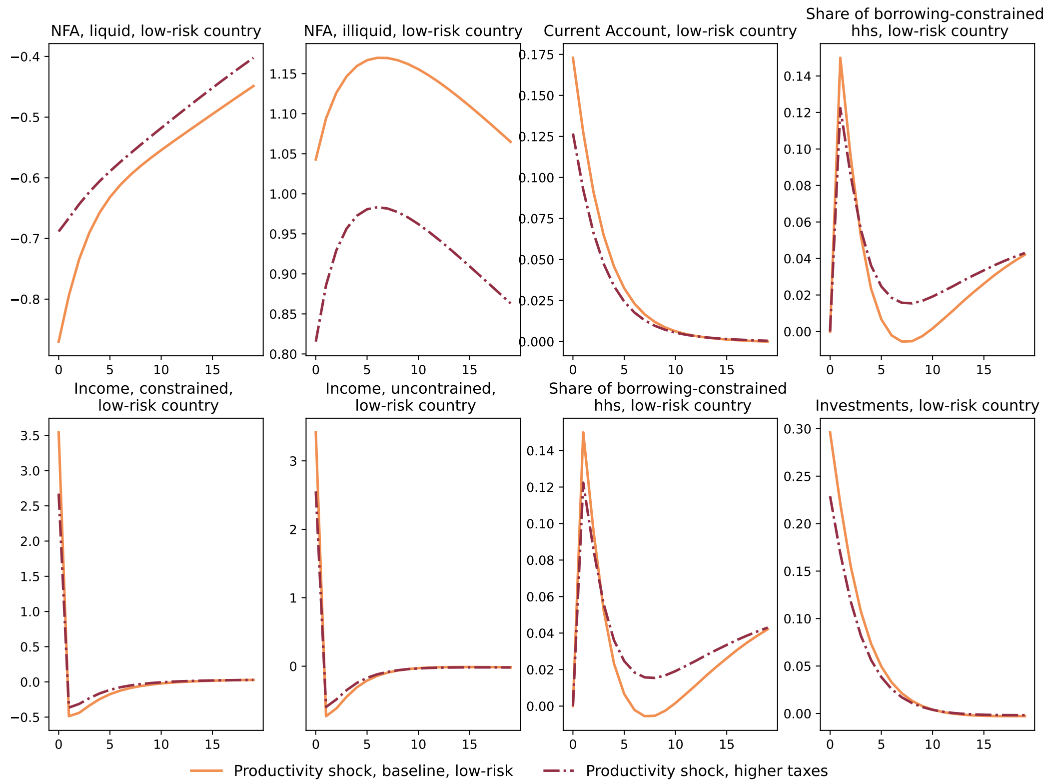


Figure 28: Impulse Responses to a TFP Shock, Higher Taxes.

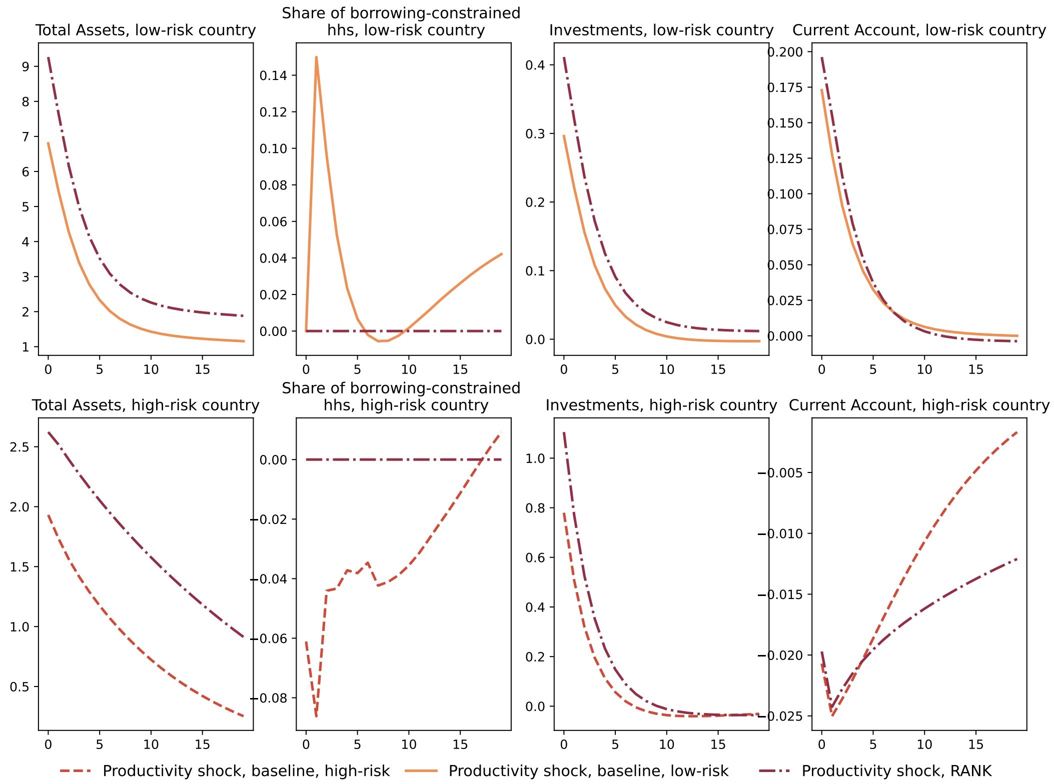


Figure 29: Impulse Responses to a TFP Shock, HANK vs. RANK, Additional Variables.

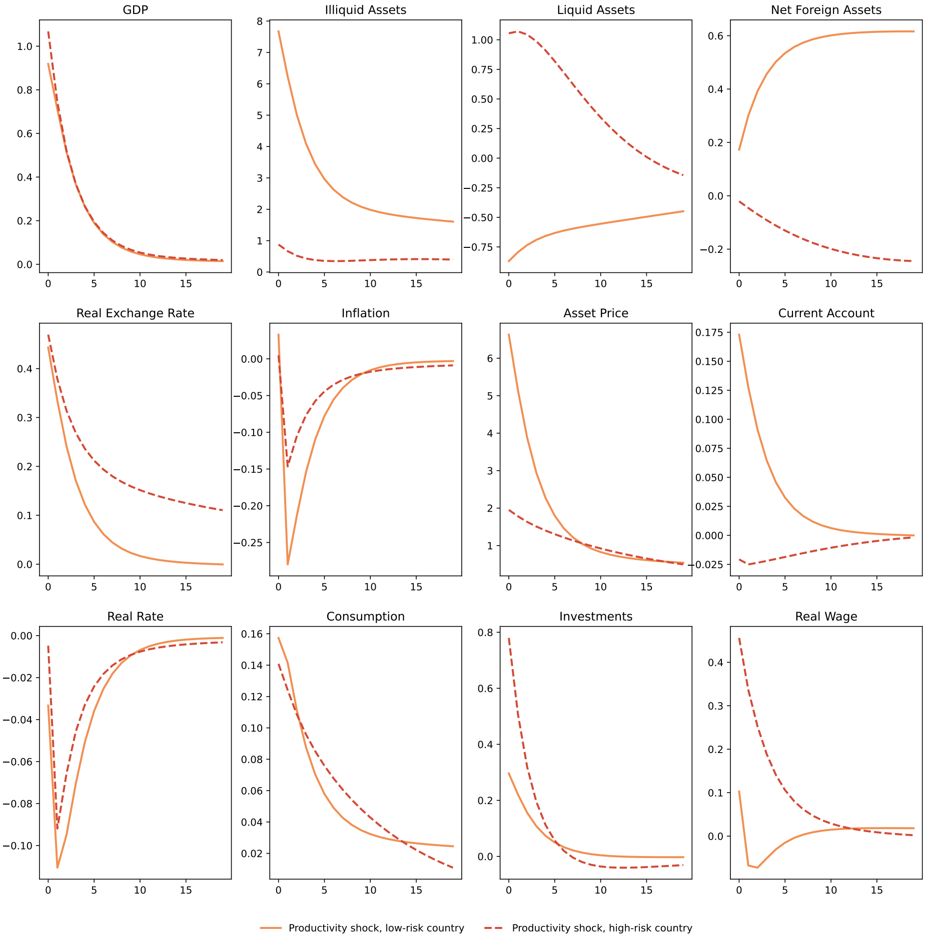


Figure 30: Impulse Responses to a TFP Shock, Country Comparison, Additional Variables.

C. Additional Tables

Table 10: Income distributions. The table shows empirical averaged across sample years income distributions for the high-risk and the low-risk countries with income variable normalized to have the same sample mean.

	High-risk Country	**Low-risk Country**
mean	0.63	0.63
std	0.52	0.59
skewness	3.1	10.0
kurtosis	22.2	279.6
1%	0.06	0.07
5%	0.13	0.13
10%	0.18	0.18
25%	0.29	0.30
50%	0.50	0.51
75%	0.80	0.80
90%	1.20	1.16
95%	1.53	1.47
99%	2.53	2.44

Table 11: Earning Process Estimation, High-risk Country. The table shows estimation parameters for different specifications when the wage variable is the last 12-month average wage in the primary job.

Parameter	**Target Only Variances,** $W = I$	**Target Variances,** $W = I$	**Target Std,** $W = I$	**Target Variances,** W **rescaled by means**	**Target Variances, two-step GMM**
Persistence	0.542	0.54	0.579	0.678	0.9
Standard deviation	2.14	2.15	2.09	1.78	2.0

D. Derivations of Current Account with Heterogeneous Households

In this section I derive current account expression that accounts for household heterogeneity. The main focus of these derivations is to understand how the presence of borrowing-constrained households affects foreign borrowing and lending. I follow Debortoli and Galí (2017) analytical framework where heterogeneity is captured by three variables, two of which are particularly important for the borrowing-constrained households. The framework is more flexible than TANK models as it also captures precautionary savings motives. Other analytical HANK models, being insightful in many dimensions, are less suitable for this paper as they either focus on precautionary savings like Acharya and Dogra (2020) or approximate models around zero liquidity steady state (see Bilbiie ,2021). The derivations are presented, firstly, for the two-period perfect foresight model to show insights in a simple framework. Then the model is extended to an infinite horizon economy. Furthermore, finally, I show how incorporating uncertainty affects the results.

D.1. Two-period model

The economy is populated by a set $\mathcal{U} \subset [0,1]$ of atomistic optimizing households. Each optimizing household i maximizes two-period CRRA utility function $U(c_1^i, c_2^i) = u(c_1^i) + \beta u(c_2^i)$ with $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ in a perfect foresight environment subject to individual budget constraint $c_1^i + \frac{c_2^i}{R} = y_1^i + \frac{y_2^i}{R}$, where R represents gross rate of return in the world capital market on date 1, which could not be affected by the small open economy. The solution to this problem is an individual Euler equation

$$(c_1^i)^{-\sigma} = \beta R (c_2^i)^{-\sigma} \tag{D.1}$$

Let $c_t^o \equiv \frac{1}{1-\lambda_t} \int_{s \in \mathcal{U}} c_t^i di$ denote average consumption of optimizing households which constitute share $1-\lambda_t$ in the economy in period t. Borrowing-constrained households, accordingly, constitute share λ_t in the economy and consume their income $y_t^h = \gamma_t y_t^o$, which is related to the average income of the optimizing households $y_t^o = \frac{1}{1-\lambda_t} \int_{s \in \mathcal{U}} y_t^i di$ by the variable γ_t. In principal, borrowing-constrained households could be heterogeneous in income too. However, that will not affect the results for the current account.

Production function has a standard Cobb-Douglas form $Y_t = AK_t^{\alpha}$ where A denotes constant total factor productivity and α stands for the capital share in production. All households supply labor inelastically, and without loss of generality, the labor supply is normalized to 1. In the first period economy is endowed with capital stock K_0. Production in the second period depends on the (dis)investments in the first period I_1, and, for simplicity, capital has zero depreciation rate such that $K_1 = K_0 + I_1$. After production in the first period, the households eat all capital stock, thus $I_2 = -K_1$.

There is no government policy, therefore, aggregate resource constraint is

$$(1-\lambda_1)c_1^o + \lambda_1\gamma_1 y_1^o + I_1 + \frac{(1-\lambda_2)c_2^o + \lambda_2\gamma_2 y_2^o - (K_0 + I_1)}{R} = AK_0^\alpha + \frac{A(K_0+I_1)^\alpha}{R} \tag{D.2}$$

where consumption of the borrowing-constrained households is substituted by their income and K_1 is substituted from the capital accumulation equation. Aggregating eq. (D.1) across optimizing households delivers

$$c_1^o = \frac{1}{1-\lambda_1}\int_{s\in\mathcal{U}} c_1^i di = \frac{1}{1-\lambda_1}\int_{s\in\mathcal{U}} (\beta R)^{-\frac{1}{\sigma}} c_2^i di = (\beta R)^{-\frac{1}{\sigma}} \frac{1-\lambda_2}{1-\lambda_1} c_2^o \tag{D.3}$$

which relates the average consumption of optimizing households across periods. Notice, that if $\beta = R$ in this setting aggregate consumption of optimizing households, i.e. $(1-\lambda_t)c_t^o$, is equalized across periods.

Let the investment decision be given to a representative firm that optimizes her discounted sum of profits and rebates them as dividends to households (the distribution of profits across households is governed by γ_t). Then her maximization problem is solved by equalizing the marginal return on capital to the net world interest rate.

$$\max_{I_1} d = AK_0^\alpha - I_1 + \frac{A(K_0+I_1)^\alpha}{R} + \frac{K_0+I_1}{R} \tag{D.4}$$

$$R - 1 = \alpha A(K_0+I_1)^{\alpha-1} \tag{D.5}$$

Substituting eq. (D.3) and eq. (D.5) into the eq. (D.2) the equation could be solved for aggregate consumption of optimizing households $(1-\lambda_1)c_1^o$

$$(1-\lambda_1)c_1^o = \frac{1}{1+\frac{(\beta R)^{\frac{1}{\sigma}}}{R}}\left[(Y_1^*-I_1^*)\left(1-\frac{\lambda_1\gamma_1}{1-\lambda_1(1-\gamma_1)}\right)+\frac{1}{R}(Y_2^*+K_1^*)\left(1-\frac{\lambda_2\gamma_2}{1-\lambda_2(1-\gamma_2)}\right)\right] \tag{D.6}$$

where for the sake of intuitive representation, optimal investments I_1^* are not substituted with their value from eq. (D.5) (the same applies to optimal outputs Y_1^* and Y_2^*).

Competitive equilibrium. From eq. (D.6) we infer that aggregate consumption of optimizing households depends on their lifetime income which is the total income available for consumption less the share that is consumed by the borrowing-constrained households (these households constitute share λ_t, γ_t governs the share of the their income out of total households' income).

The current account follows from the solution for optimal consumption.

$$\begin{aligned} CA_1 &= Y_1 - I_1 - (1-\lambda_1)c_1^o - \lambda_1 y_1^h = \\ &= \frac{\tilde{\beta}}{1+\tilde{\beta}}\left(1-\frac{\lambda_1\gamma_1}{1-\lambda_1(1-\gamma_1)}\right)(Y_1^*-I_1^*) - \frac{1}{1+\tilde{\beta}}\frac{1}{R}\left(1-\frac{\lambda_2\gamma_2}{1-\lambda_2(1-\gamma_2)}\right)(Y_2^*+K_1^*) \end{aligned} \tag{D.7}$$

where $\tilde{\beta} = \frac{(\beta R)^{\frac{1}{\sigma}}}{R}$. Equation (D.7) shows that the current account depends only on the income of optimizing households. In contrast, the income of borrowing-constrained households does not play any role in current account determination. There is a parallel of this result with the result that the production of non-tradable goods does not enter current account expression. The larger the share of borrowing-constrained households λ_t and the share of income that goes into their pockets γ_t, the less is the share of output that plays a role in the determination of between countries' capital flows.[26]

In this setting λ_t and γ_t are exogenous but in the quantitative model they evolve endogenously.

26 $\frac{\partial CA_1}{\partial \lambda_1} = -\frac{\tilde{\beta}}{1+\tilde{\beta}}(Y^*-I^*)\frac{\gamma_1}{(1-\lambda_1(1-\gamma_1))^2}$ and $\frac{\partial CA_1}{\partial \gamma_1} = -\frac{\tilde{\beta}}{1+\tilde{\beta}}(Y^*-I^*)\frac{\lambda_1(1-\lambda_1)}{(1-\lambda_1(1-\gamma_1))^2}$.

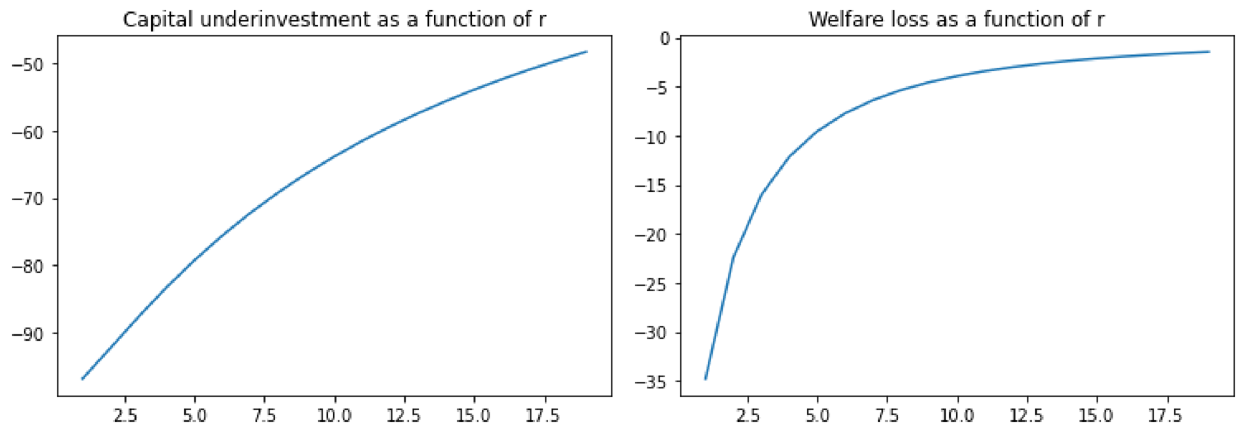
Household head for optimizing households. Suppose that investment decision is in the hands of optimizing agents. For them to make this decision, I have to impose a household head for optimizing households such that aggregate resource constraint becomes his budget constraint. The result is that this household head (who now chooses the level of consumption and investment as if he is a representative household but who realizes the presence of borrowing-constrained households) underinvests compared to the competitive equilibrium case. He anticipates that some of the next period's income will go to borrowing-constrained households. On the other hand, the cost of investment encounter only optimizing households. This reduces his incentives to invest, and he charges a premium on his investments. F.O.C. for investments transforms into

$$\alpha A(K_0 + I_1)^{\alpha-1} = \frac{R}{1 - \frac{\lambda_2\gamma_2}{1-\lambda_2(1-\gamma_2)}} - 1 \tag{D.8}$$

This underinvestment could be quite substantial (see fig. 31, for example, for the rate of return at 2.5% household head chooses the level of capital which is 90% less than the capital level in competitive equilibrium case). Welfare loss from this decision induces per period consumption which is 19% less than in the competitive equilibrium case. These results, however, are for the two-period model, thus, quantitative magnitudes should not be taken too seriously but should be instead considered indicative. This could also be a source of differential marginal products across countries.

D.2. Multi-period model with uncertainty

Moving away from perfect foresight two-period model to the infinite-horizon model with uncertainty delivers atomistic individuals' Euler equation that has an expectation in front of future consumption. That makes it impossible to obtain average optimizing agents' consumption in the same simple form as it was in a perfect foresight setting, but Debortoli and Galí (2017) setting allows to obtain average optimizing agents' consumption in a compact form (see eq. (D.10) in absolute and eq. (D.11) in log-linearized terms).



(a) Capital underinvestment in two-period model **(b) Welfare losses from underinvestment in two-period model**

Figure 31: Capital Underinvestment and Welfare Losses. The left-hand side figure shows in percentage points how much less household head invests compared to the competitive equilibrium case ($\frac{K_1^{hh}-K_1^{CE}}{K_1^{CE}}$) as a function of interest rate. The right-hand side figure shows welfare losses calculated as percentage loss in per period consumption of optimizing households. The X-axis shows the interest rate in percentage points. Calibration: $\lambda_2 = 0.1$, $\gamma_2 = 0.8$, $\alpha = 0.33$, $Y_1 = 1$.

$$(c_t^i)^{-\sigma} = \beta E_t {c_{t+1}^i}^{-\sigma} R_{t+1} \tag{D.9}$$

$$(c_t^o)^{-\sigma} = \beta E_t (c_{t+1}^o)^{-\sigma} R_{t+1} \left(\frac{1 - E_t\lambda_{t+1}}{1-\lambda_t}\right)^{-\sigma} E_t\theta_{t+1} \tag{D.10}$$

$$\hat{c}_t^o = E_t\hat{c}_{t+1}^o + \frac{\lambda}{1-\lambda}(\hat{\lambda}_t - E_t\hat{\lambda}_{t+1}) - \frac{1}{\sigma}E_t\hat{\theta}_{t+1} - \frac{1}{\sigma}E_t\hat{R}_{t+1} \tag{D.11}$$

where $E_t\theta_{t+1} = \frac{\left(\int c_t^i di\right)^{-\sigma}}{\int (c_t^i)^{-\sigma} di} \frac{E_t R_{t+1} \int (c_{t+1}^i)^{-\sigma} di}{E_t R_{t+1}\left(\frac{1-E_t\lambda_{t+1}}{1-\lambda_{t+1}} \int c_{t+1}^i di\right)^{-\sigma}}$ and captures dispersion of consumption among optimizing households and thus is a measure of precautionary savings.[27] The ˆ denotes log-deviations from the steady state.

The log-linearized aggregate consumption could be directly obtained from the aggregate consumption definition and using the distribution rule of income governed by γ.

$$\hat{c}_t = \frac{1-\lambda}{1-\lambda+\lambda\gamma}\hat{c}_t^o + \frac{\lambda\gamma}{1-\lambda+\lambda\gamma}\hat{no}_t + \frac{\lambda\gamma(1-\lambda)}{(1-\lambda+\lambda\gamma)^2}\hat{\gamma}_t - \frac{(1-\lambda)\lambda(1-\gamma)}{(1-\lambda+\lambda\gamma)^2}\hat{\lambda}_t \tag{D.12}$$

27 Definition of θ differs from Debortoli and Galí (2017) to separate λ from θ.

The aggregate resource constraint could be log-linearized following Campbell and Mankiw (1989) log-linearization technique, which delivers

$$\hat{ca}_t^* \equiv \hat{no}_t - \hat{c}_t = -\sum_{s=t}^{\infty}\left(\frac{1}{R}\right)^{s-t+1}(\Delta\hat{no}_{s+1} - \Delta\hat{c}_{s+1}) \tag{D.13}$$

where $NO_t = Y_t - I_t - G_t$. Plugging the log-linearization of aggregate consumption into log-linearized resource constraint delivers current account expression for the infinite-horizon model with uncertainty.

$$\hat{ca}_t^* = -\sum_{s=t}^{\infty}\left(\frac{1}{R}\right)^{s-t+1}(1 - \frac{\lambda\gamma}{1-\lambda+\lambda\gamma})E_t\Delta\hat{no}_{s+1} + \frac{1-\lambda}{1-\lambda+\lambda\gamma}\frac{1}{\sigma}\sum_{s=t}^{\infty}\left(\frac{1}{R}\right)^{s-t+1}E_t\left(\hat{\theta}_{s+1} + \hat{R}_{s+1}\right)$$
$$+ \frac{\lambda\gamma}{(1-\lambda+\lambda\gamma)^2}\sum_{s=t}^{\infty}\left(\frac{1}{R}\right)^{s-t+1}E_t(\Delta\hat{\lambda}_{s+1} + (1-\lambda)\Delta\hat{\gamma}_{s+1}) \tag{D.14}$$

The first term is the same as in a representative agent framework, except only the share of income that goes to the optimizing agents should be taken into account (as in the previous models). The second term represents precautionary savings and responses to changes in the world interest rate. The larger the precautionary savings, the larger the current account. The last term shows how a decrease in the share of borrowing-constrained households $\Delta\hat{\lambda}_{s+1}$ or the share of income that borrowing-constrained households get $\Delta\hat{\gamma}_{s+1}$ additionally decrease current account. This can amplify or dampen current account responses: future expected increase/decrease in $\Delta\hat{\lambda}_{s+1}$ and $\Delta\hat{\gamma}_{s+1}$ increases/decreases today's current account.

D.3. Additional log-linearizations

Log-linearization of the aggregate consumption equation

Using market clearing conditions for goods and assets markets and aggregate resource constraint for households

$$NO_t = Y_t - G_t - I_t = C_t + NX_t = C_t + B_t - (1+r_t)B_{t-1} \tag{D.15}$$

$$A_t = K_t + B_t \tag{D.16}$$

$$C_t + A_t - A_{t-1} = (1-\lambda_t)y_t^o + \lambda_t y_t^h \tag{D.17}$$

assuming no depreciation of capital the relation between net output and the income of borrowing-constrained households could be obtained

$$r_t B_{t-1} + NO_t + I_t = (1-\lambda_t+\lambda_t\gamma_t)y_t^o = \frac{1}{\gamma_t}(1-\lambda_t+\lambda_t\gamma_t)y_t^h \tag{D.18}$$

Steady state relation of consumption of optimizing households and aggregate consumption (using $NO = C$, given $CA = 0$ in the steady state) are

$$C = (1-\lambda)c^o + \lambda y^h = (1-\lambda)c^o + \frac{\lambda\gamma}{1-\lambda+\lambda\gamma}C \tag{D.19}$$

$$c^o = \frac{1}{1-\lambda+\lambda\gamma}C \tag{D.20}$$

Below all letters without time subscripts denote steady state values, and all letters with time subscripts denote deviations from the steady state.

$$\begin{aligned} &C(1+c_t) = (1-\lambda-\lambda\lambda_t)c^o(1+c_t^o) + \lambda(1+\lambda_t)y^h(1+y_t^h) = \\ &(1-\lambda-\lambda\lambda_t)c^o(1+c_t^o) + NO(1+no_t)\frac{\lambda\gamma}{1-\lambda+\lambda\gamma}(1+\frac{1}{1-\lambda+\lambda\gamma}(\lambda_t+(1-\lambda)\gamma_t)) \end{aligned} \tag{D.21}$$

Therefore,

$$c_t = \frac{1-\lambda}{1-\lambda+\lambda\gamma}c_t^o + \frac{\lambda\gamma}{1-\lambda+\lambda\gamma}no_t + \frac{\lambda\gamma(1-\lambda)}{(1-\lambda+\lambda\gamma)^2}\gamma_t - \frac{(1-\lambda)\lambda(1-\gamma)}{(1-\lambda+\lambda\gamma)^2}\lambda_t \tag{D.22}$$

Log-linearization of the aggregate resource constraint

Equation for log-linearization

$$\sum_{s=t}^{\infty}\left(\frac{1}{R}\right)^{s-t}C_s = RB_t + \sum_{s=t}^{\infty}\left(\frac{1}{R}\right)^{s-t}NO_s \tag{D.23}$$

Let $\Phi_s = \sum_{s=t}^{\infty}\left(\frac{1}{R}\right)^{s-t}C_s$, $B_s = RB_t$, $\Psi_s = \sum_{s=t}^{\infty}\left(\frac{1}{R}\right)^{s-t}NO_s$. Then equation $\Phi_s = B_s + \Psi_s$ could be rewritten in log-terms (without time subscripts are steady state values, small letters denote logarithms of variables)

$$\begin{aligned} ln(1-\frac{B_s}{\Phi_s}) = ln(1-\frac{B}{\Phi}) - \left(1-\frac{1}{1-\frac{B}{\Phi}}\right)ln(\frac{B}{\Phi}) + \left(1-\frac{1}{1-\frac{B}{\Phi}}\right)ln(\frac{B_s}{\Phi_s}) = \\ = k_o + (1-\frac{1}{\Omega_0})(b_s-\phi_s) = \psi_s - \phi_s \end{aligned} \tag{D.24}$$

Relation between Φ_s and C_s which is $\frac{\Phi_{s+1}}{\Phi_s} = (1-\frac{C_s}{\Phi_s})R$ could be also written is log-terms

$$\begin{aligned} \phi_{s+1}-\phi_s = r + ln(1-\frac{C}{\Phi}) - \left(1-\frac{1}{1-\frac{C}{\Phi}}\right)ln(\frac{C}{\Phi}) + \left(1-\frac{1}{1-\frac{C}{\Phi}}\right)ln(\frac{C_s}{\Phi_s}) = \\ = r + k_1 + (1-\frac{1}{\Omega_1})(c_s-\phi_s) \end{aligned} \tag{D.25}$$

Using

$$\phi_{s+1}-\phi_s = \Delta c_{s+1} + (c_s-\phi_s) - (c_{s+1}-\phi_{s+1}) \tag{D.26}$$

eq. (D.25) delivers

$$c_s - \phi_s = -\sum_{j=1}^{\infty}\Omega_1^j \Delta c_{s+j} + \sum_{j=1}^{\infty}\Omega_1^j (r+k_1) \tag{D.27}$$

Analogously

$$no_s - \psi_s = -\sum_{j=1}^{\infty} \Omega_2^j \Delta no_{s+j} + \sum_{j=1}^{\infty} \Omega_2^j (r + k_2) \tag{D.28}$$

Plugging eq. (D.27) and eq. (D.28) into eq. (D.24) and using $B = 0$ thus $\Omega_0 = 1$ and $\Omega_1 = 1 - \frac{C}{\Phi} = 1 - \frac{C}{C\frac{1}{1-\frac{1}{1+r}}} = \frac{1}{1+r} = \Omega_2$, and therefore $k_1 = k_2$ delivers

$$k_0 - no_s + c_s = \sum_{j=1}^{\infty} \left(\frac{1}{1+r} \right)^j (\Delta no_{s+j} - \Delta c_{s+j}) \tag{D.29}$$

in log-terms (subtracting logarithms of steady state values delivers an equation in log-deviations used in the main derivations).