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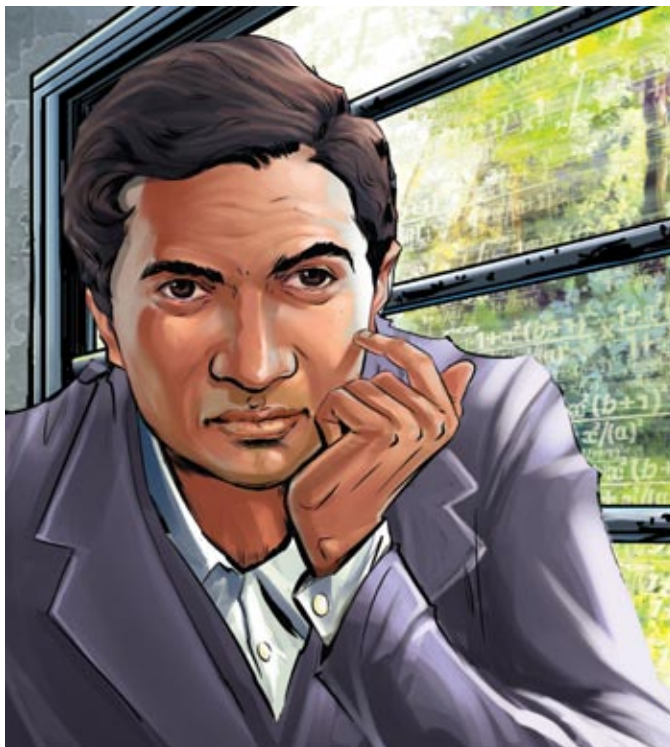


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SRINIVASA RAMANUJAN:

THE PRIDE OF INDIAN MATHEMATICS

The Life of Ramanujan Part 2

Dr. James Philip

Introduction

Named after Srinivasa Ramanujan, the number 1729 is known as the Ramanujan Number or Hardy-Ramanujan Number and it has several interesting properties.

Ramanujan had a passion for numbers and their properties. According to Littlewood, every positive integer was one of Ramanujan's personal friends. The following anecdote described by G.H. Hardy about the Ramanujan number illustrates his felicity with numbers:

I remember going to see him once when he was lying ill in Putney. I had ridden in taxi-cab No. 1729, and remarked that the number seemed to me rather a dull one, and that I hoped that it was not an unfavourable omen. "No," he replied, "it is a very interesting number; it is the

smallest number expressible as a sum of two cubes in two different ways."

The Special Property of the Ramanujan Number

The Ramanujan Number has the special property that it can be expressed as the sum of two cubes (of positive integers) in two different ways:

That is, $1729 = 12^3 + 1^3 = 10^3 + 9^3$.

This property was first recognised by Ramanujan. However, it is not the only number with this property. In fact, it is the first number of a more general family of numbers known as Taxicab numbers of category 2. 1729 is the smallest such number.

Some other numbers of the type are:

$$4104 = 16^3 + 2^3 = 15^3 + 9^3$$

$$13832 = 24^3 + 2^3 = 20^3 + 18^3$$

$$20683 = 27^3 + 10^3 = 24^3 + 19^3$$

$$32832 = 32^3 + 4^3 = 30^3 + 18^3$$

$$39312 = 34^3 + 2^3 = 33^3 + 15^3$$

$$40033 = 34^3 + 9^3 = 33^3 + 16^3$$

More about 1729

Apart from the above, we can observe several other properties of the Ramanujan Number. Some of these are given below:

1. It is the largest natural number with the following property: Take the sum of the digits of a number. This sum multiplied by its reverse gives back the original number.
i.e., $1+7+2+9=19$, and $19 \times 91 = 1729$.
The other numbers of the type are 1458, 81 and 1.
2. 1729th decimal digit holds significance in the decimal representation of the transcendental number e. From 1729th digit you can get the first occurrence of all ten digits consecutively and they are 0719425863.
3. The two parts of the number 17 and 29 are prime numbers.
4. It is the product of three alternate prime numbers: i.e., $1729 = 7 \times 13 \times 19$.
5. The sum of digits ($1+7+2+9 = 19$) is a factor of the Ramanujan number.
6. We have, $1729 = 12^3 + 1^3 = 10^3 + 9^3$. Both $12+1$ and $10+9$ are factors of 1729.
7. It may be written as the difference of two squares in at least 4 ways:
 $73^2 - 60^2$, $55^2 - 36^2$,
 $127^2 - 120^2$, $865^2 - 864^2$.

8. It can be written as the sum of cubes of 4 numbers: $1^3 + 6^3 + 8^3 + 10^3$.
9. It can be written as the sum of cubes of 6 numbers: $1^3 + 3^3 + 4^3 + 5^3 + 8^3 + 10^3$.
10. Any number between 1 and 100 can be represented using digits of the Ramanujan Number and applying standard mathematical operations:

E.g.: $1 = 1+7+2-9$; $2 = 1 + [(7+2)/9]$;
 $20 = \sqrt{17^2} + \sqrt{9}$ $100 = 1+2+97$

Conclusion

Every number is interesting in itself, but Ramanujan Number stands out due to several unique properties. It was a genius like Ramanujan who started appreciating such properties and so it is called Ramanujan Number. The Ramanujan Number is one of a more general type of numbers called Taxicab Numbers which we discuss in Part III.