FERMION MASSES AND MIXINGS, FLAVOR VIOLATION, AND THE HIGGS BOSON MASS IN SUPERSYMMETRIC UNIFIED FRAMEWORK

By

ABDELHAMID ALBAID

Bachelor of Arts/Science Applied Physics
Jordan University of Science and Technology
Irbid, Amman, Jordan
2001

Master of Arts/Science in Physics
University of Jordan
Amman, Amman, Jordan
2004

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Dissertation Approved:

Kaladi S. Babu

Dissertation Advisor

Flera Rizatdinova

Jacques H. H. Perk

Birne Binegar

Mark Payton

Dean of the Graduate College

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CHAPTER 1

INTRODUCTION

1.1 A Brief Review of the Standard Model (SM)

The standard model (SM) of particle physics is based on the non-abelian gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$. Here the $SU(3)_c$ gauge group describes the theory of strong interaction called quantum chromodynamics (QCD). This type of interactions holds the quarks and the gluons together to form hadrons. Each quark type is called flavor. For example, up, charm and top denoted respectively by u, c and t are three flavors of the up-type quark. Each flavor of quark transforms as the fundamental color triplet of $SU(3)_c$ while the gauge bosons, the gluons, are assigned to the adjoint octet representation of $SU(3)_c$. In this case, we have eight gluons associated with the eight $SU(3)_c$ generators. The $SU(2)_L \times U(1)_Y$ is the gauge group of the Glashow, Weinberg, and Salam model [1] which successfully combines the electromagnetic and weak interactions in one theory called electroweak theory. The total number of generators of $SU(2)_L \times U(1)_Y$ is four. Accordingly, this theory contains four electroweak gauge bosons (three of them conventionally are denoted W_i and the forth one is denoted B). This $SU(2)_L \times U(1)_Y$ symmetry is respected above roughly 100 GeV (the electroweak scale). The electromagnetic interaction arises below the electroweak scale where the electroweak symmetry is broken spontaneously by the Higgs mechanism.

In order to understand how the electroweak symmetry breaking is implemented in the SM, let us first point out that the invariance of the Lagrangian for both quantum electrodynamics (QED) and QCD under local gauge transformations leads respectively to massless photons and gluons. However, this idea can not be applied

	$SU(3)_c \times SU(2)_L \times U(1)_Y$
$L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$,	(1,2,−1)
e_i^c,	(1,1,2)
$Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$,	$(3,2,\frac{1}{3})$
u_i^c,	$(3,1,-\frac{4}{3})$
d_i^c,	$(3,1,\frac{2}{3})$
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$,	(1,2,1)

Table 1.1: The transformation of the lepton (L_i,e^c), quark (Q,u^c,d^c), and Higgs (H) fields under SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

to the weak interaction since the gauge bosons of the weak interaction are massive (of order 90 GeV). One way out of this problem is to consider the situation of a hidden symmetry; the Lagrangian still respects the local gauge symmetry, but picks one of all possible ground states that result from minimizing the potential for a Higgs field as the physical vacuum which breaks the symmetry.

The spontaneous symmetry breaking is implemented by including a doublet of scalar Higgs boson to the SM. The transformations of the quark, lepton and Higgs fields under the SM gauge group are shown Table 1.1. In this Table, all fermion fields are left handed and the generation index i runs from 1 to 3. Let us study the spontaneous symmetry breaking of the gauge group $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ by writing down the Higgs potential for the Higgs field H:

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2, \qquad \mu^2 > 0. \tag{1.1}$$

The above potential is invariant under the SM gauge group. Minimizing the potential $V(H)$, one obtains

$$\langle H \rangle = \langle 0|H|0\rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{1.2}$$

where $v = \mu/\sqrt{\lambda}$. The generator that remains unbroken is $Q = T_3 + \frac{Y}{2}$. Y refers to the electroweak hypercharge. Q is identified as the electric charge. The unbroken charge is easily checked by

$$Q\langle H \rangle = 0. \tag{1.3}$$

The parameter Y needs to be adjusted such that the electric charges of the quarks and the leptons come out right. In general, the broken generators correspond to the gauge bosons that pick up mass, and the unbroken generators correspond to the massless gauge bosons. In this case, there are three broken generators associated with three massive gauge bosons (W^+, W^-, Z^0), and the unbroken charge Q associated with massless gauge boson γ (the electromagnetic field A_μ). The electroweak symmetry breaking scale is around the masses of the gauge bosons (i.e., 100 GeV). We can calculate the masses of electroweak gauge bosons by substituting the vacuum expectation value (VEV) of the Higgs field from Eq.(1.2) into the following gauge invariant kinetic term of the Higgs field:

$$(D_\mu H)(D_\mu H)^\dagger = |\partial_\mu H - \frac{ig}{2}\overrightarrow{\tau}.\overrightarrow{W}_\mu H - \frac{ig'}{2} B_\mu H|^2, \tag{1.4}$$

where the gauge coupling constants g and g' are associated respectively to the gauge groups $SU(2)_L$ and $U(1)_Y$. The masses of the electroweak gauge bosons are then

$$m_W = \frac{ev}{2\sin\theta_W}, \tag{1.5}$$

$$m_Z = \frac{ev}{2\sin\theta_W \cos\theta_W}. \tag{1.6}$$

The gauge coupling constants are parameterized in terms of an angle θ_W (known as the Weinberg angle) defined as follows:

$$\tan\theta_W = \frac{g'}{g}, \tag{1.7}$$

and $e = g\sin\theta_W$. The mass term of fermions cannot be added to the Lagrangian by hand because the left-handed and the right-handed fermions transform differently under $SU(2)_L \times U(1)_Y$. Therefore, one employs the Higgs mechanism that generates mass to the fermions via Yukawa couplings. The Higgs field and its charge conjugate are given respectively by

$$H = \begin{pmatrix} H^+ \\ H_0 \end{pmatrix} \qquad \tilde{H} = i\tau_2 H^* = \begin{pmatrix} H_0^* \\ -H^- \end{pmatrix}. \tag{1.8}$$

The transformation of $\tilde{H}$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$ is (1, 2, -1). We can write the gauge invariant Yukawa couplings as follows:

$$\mathcal{L}_Y = Y_{ij}^d d_i^{cT} H^\dagger Q_j + Y_{ij}^e e_i^{cT} H^\dagger L_j + Y_{ij}^u u_i^{cT} \tilde{H}^\dagger Q_j + h.c., \tag{1.9}$$

where a charge conjugation C is understood to be sandwiched between the fermion fields. As a consequence of spontaneous symmetry breaking, $\mathcal{L}_Y$ leads to mass terms for fermions as follows:

$$\mathcal{L}_Y = D^{cT} M^d D + U^{cT} M^u U + E^{cT} M^e E + h.c., \tag{1.10}$$

where

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad E = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix},$$

$$U^c = \begin{pmatrix} u^c \\ c^c \\ t^c \end{pmatrix}, \quad D^c = \begin{pmatrix} d^c \\ s^c \\ b^c \end{pmatrix}, \quad E^c = \begin{pmatrix} e^c \\ \mu^c \\ \tau^c \end{pmatrix}. \tag{1.11}$$

The mass matrix elements for up-and down-quarks as well as charged leptons are given by

$$M_{ij}^F = \frac{v}{\sqrt{2}} Y_{ij}^F, \quad F = u, d, e. \tag{1.12}$$

Note that in the standard model the right handed neutrino does not exist. Therefore, the neutrinos are massless. The weak eigenstates are not eigenstates of the Hamiltonian. In order to write the Lagrangian in terms of the Hamiltonian eigenstates (i.e mass eigenstates), we need to diagonalize the fermion mass matrices given by Eq.(1.12) by means of bi-unitary transformation given as:

$$V_R^{F^\dagger} M^F V_L^F = M_{diag.}^F, \tag{1.13}$$

where

$$\begin{aligned} M_{diag.}^u &= \mathrm{diag}(m_u, m_c, m_t), \\ M_{diag.}^d &= \mathrm{diag}(m_d, m_s, m_b), \\ M_{diag.}^e &= \mathrm{diag}(m_e, m_\mu, m_\tau). \end{aligned} \tag{1.14}$$

The fermion mass matrices (M^F) are in general neither symmetric nor hermitian. but, $M^{F^\dagger} M^F$ is hermitian and can be diagonalized as follows:

$$V_L^{F^\dagger} M^{F^\dagger} M^F V_L^F = M_{diag.}^{F\dagger} M_{diag.}^F. \tag{1.15}$$

The mass eigenstates (D_0, U_0, E_0, D_0^c, U_0^c, E_0^c) can be written in terms of the weak eigenstates as follows:

$$\begin{aligned} D_0 &= V_L^{d\dagger} D, \quad D_0^c = V_R^{d^T} D^c, \\ U_0 &= V_L^{u\dagger} U, \quad U_0^c = V_R^{u^T} U^c, \\ E_0 &= V_L^{e\dagger} E, \quad E_0^c = V_R^{e^T} D^c. \end{aligned} \tag{1.16}$$

The charged current weak interactions for quarks are given as

$$\begin{aligned} \mathcal{L}_{cc} &= \frac{g}{\sqrt{2}} W^\dagger \overline{U} \gamma_\mu D + h.c. \\ &= \frac{g}{\sqrt{2}} W^\dagger \overline{U}^0 V_{CKM} \gamma_\mu D^0 + h.c. \end{aligned} \tag{1.17}$$

It is clear from the above equation that the charged current $W^{\pm}$ interactions couple to the physical u_j^0 and d_k^0 quarks with a couplings matrix represented by

$$V_{CKM} = V_L^{u\dagger} V_L^d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{1.18}$$

This is called the Cabibbo-Kobayashi-Maskawa mixing matrix [2, 3]. It is a unitary matrix that can be parameterized by three mixing angles and one CP-violation phase:

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1.19}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ and δ is the phase factor responsible for the violation of CP symmetry [3]. All other phases can be removed by field redefinition. It is known experimentally that the CKM mixing angles are small (i.e $s_{13} << s_{23} << s_{12} << 1$). It is convenient to write down an expression for a CP-violation parameter which is phase-convention-independent:

$$\overline{\eta} = -\mathrm{Im}(\mathrm{V_{ud}V_{ub}^*/V_{cd}V_{cb}^*}). \tag{1.20}$$

Unlike the situation in the case of charged current interactions, no flavor mixings exist for neutral current interactions of $SU(2)_L \times U(1)_Y$ at tree level and this has been confirmed to a great accuracy by experiments. However, flavor changing neutral currents (FCNC), which have been measured, but which are strongly suppressed, can be induced by considering higher order corrections. For example, FCNC can be induced in the process $K^0 \leftrightarrow \overline{K^0}$ transition which arises from box diagrams shown in Fig1.1. The calculation on the $K^0 \leftrightarrow \overline{K^0}$ mass difference Δm_k has been done [4], and the result is close to the experimental value of $\Delta m_k = 3.5 \times 10^{-15}$ GeV. This can be considered as a successful prediction of the SM.

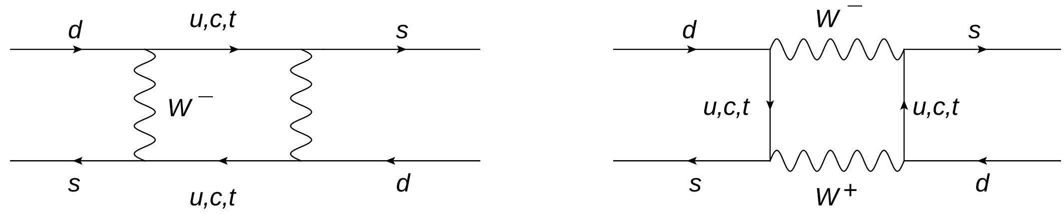


Figure 1.1: Feynman diagrams of $K^0 \leftrightarrow \overline{K^0}$ induced by higher order corrections in the SM.

1.2 Seesaw Mechanism and Leptonic Mixing Matrix

In the previous section, we have seen that SM contains left and right chiral projections for all fermions except the neutrinos. This looks unnatural. Besides, the absence of a right-handed neutrino from Eq.(1.9) leads to massless neutrinos. However, neutrino experiments indicate that the neutrinos have tiny masses. The current experimental values for neutrino masses are [7]

$$\Delta m_{21}^2 = (7.59 \pm 0.20) \times 10^{-5}\text{eV}^2,$$
$$\Delta m_{32}^2 = (2.43 \pm 0.13) \times 10^{-3}\text{eV}^2, \tag{1.21}$$

where $\Delta m_{2,1}^2 = m_2^2 - m_1^2$ and $\Delta m_{3,2}^2 = m_3^2 - m_2^2$. To explain this, let us add to the SM right-handed neutrinos (ν_i^c) corresponding to each charged lepton. The ν_i^c fields transform as (1,1,0) under the SM gauge group. Thus, we can write down the Yukawa couplings for the neutrino sector as follows:

$$\mathcal{L}_Y^\nu = Y_{ij}^\nu \nu_i^{cT} \tilde{H}^\dagger L_j + h.c. \tag{1.22}$$

With a VEV of $\tilde{H}^\dagger$, this gives the following neutrino Dirac mass term

$$\mathcal{L}_Y^\nu = M_D \nu^{cT} \nu + h.c, \tag{1.23}$$

where $(M_D)_{ij} = Y_{ij}^\nu v/\sqrt{2}$. Since the ν^c fields are singlets under the SM gauge symmetry, they can posses a gauge invariant bare mass term (Majorana mass):

$$\mathcal{L}_{bare} = \frac{1}{2} M_R \nu^{cT} \nu^c + h.c. \tag{1.24}$$

We can write the combination of Majorana and Dirac neutrino masses as a matrix for the (ν, ν^c) system as:

$$M_\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}, \tag{1.25}$$

where M_D and M_R are 3×3 matrices. The invariance of the right-handed neutrino mass terms under SM gauge symmetry suggests that they can be above the weak interaction scale. So after integrating out these heavy fields (or equivalently by finding the eigenvalues of the matrix in Eq.(1.25)), the light neutrino masses are suppressed by M_R via:

$$M_\nu = -M_D^T M_R^{-1} M_D, \tag{1.26}$$

where M_D should not exceed about 100 GeV. This idea, known as the seesaw mechanism [5], is an elegant way to explain the smallness of neutrino masses. The light neutrino mass matrix given by Eq.(1.26) can be diagonalized as:

$$V_\nu^T M_\nu V_\nu = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}, \tag{1.27}$$

with $m_{1,2,3}$ being the tiny masses of the three light neutrinos. Now, we can write the leptonic charge current interaction in terms of the mass eigenstates as follows:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}[\overline{e^0}\gamma_\mu V_{PMNS}\nu^0]W^{-\mu} + h.c. \tag{1.28}$$

where $V_{PMNS} = V_L^\dagger V_\nu$ is the leptonic mixing matrix, or the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [6]. In general, the PMNS matrix can be written as

$$V_{PMNS} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix}, \tag{1.29}$$

which can be parameterized in terms of three Euler angles and three phases- one "Dirac phase" and two "Majorana phases". The standard parametrization [7] has $V_{PMNS} = V.P$ where

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{1.30}$$

$$P = \begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}. \tag{1.31}$$

Here $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ which should not be confused with the angles in the quark sector, given in 1.19. The parameters α and β are the Majorana phases, while δ is the Dirac phase. Present constraints on the neutrino mixing angles can be summarized by (2σ error bars quoted)[8]

$$\sin^2\theta_{12} = 0.27 - 0.35, \tag{1.32}$$

$$\sin^2\theta_{23} = 0.39 - 0.63, \tag{1.33}$$

$$\sin^2\theta_{13} \leq 0.040. \tag{1.34}$$

The above data can be well represented by the tri-bimaximal mixing of the form [9]

$$V = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} P, \tag{1.35}$$

which corresponds to $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$ and $\sin^2\theta_{13} = 0$. No information on the Dirac phase δ and on the Majorana phases (β, α) is known at present. There are several thoughts to reproduce the structure in Eq.(1.35). One interesting idea is to employ the discrete flavor symmetry A_4 [10] which will be further discussed in chapter 2.

1.3 Shortcomings of the SM and the Need for New Physics.

The standard model is a trustful theory in the energy range of few 100 GeV. However, things become more obscure beyond the electroweak energy scale. Understanding how nature behaves at higher energy scales might answer many of the standard model's puzzles. For example, the SM has no real explanation of the different strengths of the three gauge couplings associated with the three gauge groups. Also, there is no reason why the fermions transform under the local gauge interactions of the SM in the way shown in Table 1.1, except for the posteriori justification of fitting the data.

Grand unification theory (GUT) provides an understanding of the origin of the three gauge couplings and consequently an understanding of three gauge groups. The GUT idea is described by a unified gauge group which necessitates a single unified gauge coupling. This unified gauge group will be broken at a certain high energy scale (GUT scale) to the SM gauge group. Thus, strong, weak and electromagnetic forces are described in the framework of a single grand unified theory. Moreover, if the unified gauge group is simple, quantization of electric charge will follow automatically because the eigenvalues of the non-abelian group generators are discrete as opposed to the eigenvalues of the abelian $U(1)$ group generator which are continuous. The most popular simple non-abelian groups that are chosen as grand unification groups are $SU(5)$ and $SO(10)$. We will study these GUT groups in details in sections 1.4 and 1.5.

Arbitrary Parameters

The SM has 19 arbitrary parameters. 3 gauge coupling constants (g_s, g, and g' associated respectively with $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$), 9 charged fermion masses, 4 quark mixing parameters, and v, λ (or equivalently to M_z, m_h) and the QCD θ parameter. Besides, if we consider the neutrino sector, there are at least 9 additional parameters: 3 light neutrino masses, 3 mixing angles, and 3 phases (assuming Majorana neutrinos). Thus, the SM has too many arbitrary parameters which are chosen in order to fit the data. On the other hand, GUTs do not contain that many arbitrary parameters. Another advantage of GUT is that the seesaw mechanism can be implemented naturally within $SO(10)$ GUT, since the gauge structure requires the existence of ν^c, as we will see in section 1.5.3.

Grand unification theory describes the three interactions (strong, weak, and electromagnetic) by one gauge coupling constant. However, it is known that these interactions are described by three distinct gauge couplings at low energy ($E \approx 100$ GeV). So the question is how does the grand unification idea reconcile with these three disparate couplings? This question can be answered by the suggestion [11] that the three gauge coupling constants are scale dependent quantities, and if the hypothesis of grand unification holds, the three gauge coupling constants of the SM will meet to a unified value at the GUT scale M_{GUT}. Above the scale M_{GUT} we have one gauge coupling described by a simple unified group. The renormalization group running of the gauge couplings determines the GUT scale. In the SM, however, the gauge couplings come only close to one another forming what is called the GUT triangle as shown in Fig.1.2. This can be fixed by introducing new physics around the TeV scale. The most promising new physics scenario is supersymmetry, which will be further discussed below

Hierarchy Problem

Another problem that needs to be fixed is the hierarchy problem of the SM. This problem occurs because the mass of the Higgs boson receives a quadratically divergent loop correction given by:

$$m^2_{H_{SM}}(phys) \simeq m^2_{H_{SM}} + \frac{c}{16\pi^2}\Lambda^2, \tag{1.36}$$

where $m^2_{H_{SM}}$ is the Higgs mass squared parameter in the Lagrangian and the second term denotes the quadratically divergent loop correction. The cut-off scale Λ is in-

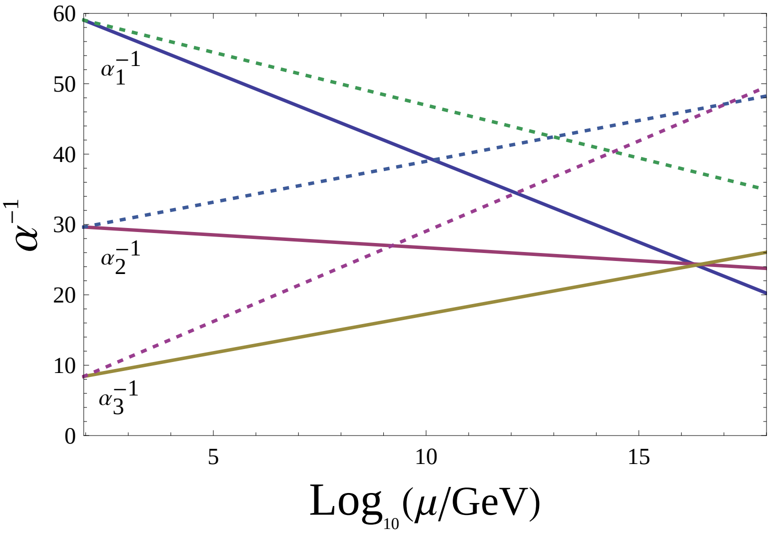


Figure 1.2: The evolution of the inverse gauge couplings α_i^{-1} in the standard model (dashed lines) and in the MSSM (solid lines).

terpreted as the scale at which the SM ceases to be valid. Reasonable values of the energy scale Λ at which the new physics becomes important are chosen such that any extremely fine-tuned cancelation between the two terms on the right-hand side of Eq.(1.36) is avoided. The physical Higgs boson mass $m_{H_{SM}}(phys)$ has to be smaller than a few hundred GeV [12]. Therefore, reasonable values of Λ might be around the TeV scale. A promising scenario that solves the hierarchy problem of the SM and allows the unification of the three gauge coupling constants is supersymmetry (SUSY). In order to avoid extreme fine-tuning, SUSY should exist above an energy scale of order 1 TeV which is being probed at the Large Hadron Collider.

Problems in the Flavor Sector

The SM does not provide an explanation for the existence of three families of fermions, and the observed masses and mixings of the fermions, and the smallness of the quark mixing angles compared to the largeness of the neutrino mixing angles. These problems can be understood either through GUTs and/or by adding a family symmetry.

Some of the features of the fermions such as the three fold replication of fermion generations, mixing properties of the lepton sector—that is two large mixing angles and one small mixing angle—cannot be explained successfully by GUT symmetry alone. So in order to meet these challenges, one may consider the possibility of introducing a flavor symmetry (family symmetry) group which is the symmetry between generations. In this case, the three known generations can be assigned to a representation of the family group. There are many possible candidates for the family symmetry group. Basically, we can divide them into two categories: continuous and discrete groups. The general feature of the global continuous groups is that they lead to undesired Goldstone bosons. On the other hand, it is suggestive to consider discrete non-abelian symmetry because in this case there is no problem with unwanted Goldstone bosons.

Combining grand unification gauge symmetry and family symmetry ($G_{GUT} \times G_{FAM}$) in the framework of supersymmetric theory leads certainly to new physics beyond the SM that solves most of the standard model's puzzles. Many grand unification models with discrete family symmetry have been studied so far [13, 14, 15, 16]. In particular, employing $SO(10) \times A_4$ symmetry may give the tri-bi-maximal mixings structure in Eq.(1.35) [15].

1.3.1 Supersymmetry

Supersymmetry is a symmetry that relates bosons and fermions. It predicts new yet to be discovered superpartner states for each known particle in the SM. The SM particle and its supersymmetric partner belong together to the same supermultiplet which is collectively described in terms of a superfield. In this way a spin-0 boson and a spin-1/2 fermion are described as a chiral superfield and a spin-1 vector boson and a spin-1/2 fermion form a vector superfield. The supersymmetric extension of the SM assumes that all quarks and leptons of the SM are accompanied by their scalar superpartners which are called respectively squarks and sleptons, and the gauge bosons with their fermionic superpartners which are called gauginos. This supersymmetric extension of the SM is called Minimal Supersymmetric Standard Model (MSSM), it is minimal in the sense that it contains the smallest number of new particle states. The SM contains one Higgs doublet field to achieve electroweak symmetry breaking while the MSSM contains two Higgs doublets H_u and H_d which give mass to the up-type and down-type quarks respectively. Their superpartners are called higgsinos. This setup helps in solving the quadratic divergence correction of the Higgs mass due to the fact that the loops involving particles are canceled by the loops involving their superpartners. Another feature in favor of the MSSM is that the gauge couplings unify around 2×10^{16} GeV as shown in Fig 1.2. These features motivate the consideration of supersymmetric GUTs.

Unlike the SM where the baryon and lepton numbers are conserved automatically, there are additional superpotential terms in the case of MSSM that are consistent with $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry, which break the lepton and baryon numbers. These terms are dangerous since the lepton and baryon violating processes are strongly constrained by experiment, especially from proton stability. These unwanted terms can be prohibited by requiring the superpotential to be invariant under R-parity defined by,

$$R = (-1)^{3(B-L)+2s}, \tag{1.37}$$

where s is the spin of the field, and B and L are the baryon and the lepton number respectively. For example $B = 1/3(-1/3)$ for quark (antiquark) superfields, $L = 1(-1)$ for lepton (antilepton) superfields, and zero for the Higgs and gauge superfields.

Supersymmetry Breaking

The supersymmetry algebra tells us that the particle and its superpartner acquire the same mass. However, this is not consistent with experiment since for instance no spin-0 particle has been detected so far with the same mass as the electron. Therefore,

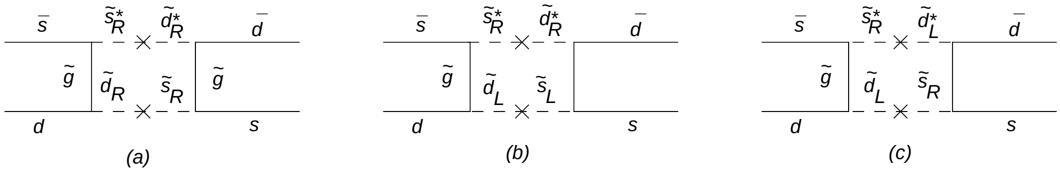


Figure 1.3: These three diagrams contribute to $K^0 \leftrightarrow \overline{K^0}$ mixing in supersymmetric models. They put constraints on the off-diagonal elements of the soft breaking scalar down mass matrix that is indicated by $\times$.

SUSY must be broken somewhere above the energy scale that has been probed so far. SUSY should preferably be broken spontaneously. In other words, the generators of the SUSY does not annihilate the vacuum. Although many models of SUSY breaking have been proposed, there is no complete theory where this is achieved satisfactorily at present. In order to maintain the remarkable cancelation of quadratic divergencies in field theoretical models, SUSY should be broken softly in the effective low energy theory. This can be done by assuming that the outcome of symmetry breaking is extra terms (soft terms), such as additional masses for the scalars. The common philosophy of all the scenarios of SUSY breaking is that SUSY is broken in a "hidden sector" of particles which is decoupled from the visible sector of MSSM particles. The effects of SUSY breaking in the hidden sector are communicated to the visible sector by messengers, resulting in the MSSM soft SUSY breaking terms.

The soft SUSY breaking terms imply flavor mixing. For example, suppose $\tilde{m}_Q^2$ is not diagonal in the soft term $\tilde{d}^\dagger{}_{Li}(m_Q^2)_{ij}\tilde{d}_{Li}$. In this case, the effective Hamiltonian for $K^0 \leftrightarrow \overline{K^0}$ mixing gets contributions from the box diagrams involving squarks and gluinos, such as the ones shown in Fig.1.3. The experimental value of Δm_K puts constraints on the soft SUSY breaking mixing of the three diagrams in Fig1.3. The most striking limit applies to the diagram in Fig1.3(b) [28]:

$$\frac{|Re[\tilde{m}^2_{s^*_R d_R}\tilde{m}^2_{s^*_L d_L}|^{1/2}}{\tilde{m}_q^2} < \frac{\tilde{m}_q \times 10^{-3}}{500 \text{ GeV}}, \tag{1.38}$$

where $\tilde{m}_q$ is the average mass of squarks $\tilde{m}_d$ and $\tilde{m}_s$ and the gluino mass has been assumed equal to the average squark mass. Thus, in order to suppress the off-diagonal entries of $\tilde{m}_Q^2$, we need to assume the masses of the squarks are nearly degenerate. This can be achieved by adding a non-Abelian discrete symmetry group. This can be done either by grouping the first two families into an irreducible doublet [29] or by grouping all three families into an irreducible triplet of the flavor group. For example, the group could be A_4, which is the smallest discrete group that contains a triplet in its irreducible representations.

Another natural solution to the flavor violation problem is obtained by adopting gauge-mediated supersymmetry breaking (GMSB) scenario [59, 60, 61]. In this scenario the supersymmetry breaking is transmitted to the visible sector by SM gauge interactions. In this case the soft masses are generated through loops such that the scalar masses with the same gauge quantum number are automatically degenerate. A model based on the GMSB scenario will be discussed in chapter 4.

1.3.2 Discrete Flavor Symmetry A_4

The non-abelian finite group A_4 is the symmetry group of even permutations of four objects. It has twelve elements and four irreducible representations (irreps): 1, $1'$, $1''$, 3_s, and 3_a with the multiplication rule

$$3 \times 3 = 1 + 1' + 1'' + 3_s + 3_a. \tag{1.39}$$

For example, let (a_1, a_2, a_3), and (b_1, b_2, b_3) transform as triplets under A_4, then the multiplication of 3×3 can be decomposed as

$$a_1 b_1 + a_2 b_2 + a_3 b_3 \sim 1, \tag{1.40}$$

$$a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3 \sim 1', \tag{1.41}$$

$$a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 \sim 1'', \tag{1.42}$$

$$(a_2 b_3 + a_3 b_2, a_3 b_1 + a_1 b_3, a_1 b_2 + a_2 b_1) \sim 3_s, \tag{1.43}$$

$$(a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1) \sim 3_a, \tag{1.44}$$

where $\omega = \exp[2\pi i/3]$. One advantage of the discrete A_4 symmetry is that it is the smallest group that contains a 3-dimensional irrep so that the three generations of the fermions can be accommodated within this triplet. Another advantage is that the FCNC problem might be solved if one considers the combinations of A_4 and SUSY $SO(10)$ GUT. This is due to the fact that the $SO(10)\times A_4$ symmetry allows us to write down one universal mass term for the three generations of sfermions. Consequently, the degeneracy of sfermions is satisfied.

1.4 Minimal SUSY-$SU(5)$

We have pointed out previously that the running behavior of the three gauge couplings with energy scale indicates that they should unify at some point at a high energy scale. This unification of the gauge couplings does not occur exactly in the SM. However, in the case of the MSSM, the unification occurs with impressive precision at $M_{GUT} \approx 2 \times 10^{16}$ GeV. This strongly suggests that MSSM might be remnant of some sort of supersymmetric grand unification theory. Therefore, it is logical to propose a larger gauge group associated with one gauge coupling constant. The first approach of finding a simple gauge group that contains the SM group was the Georgi-Glashow $SU(5)$ model [17]. In this section we will discuss this $SU(5)$ model, its predictions and its experimental implications because it is considered the simplest example of grand unification models and it is a subgroup of $SO(10)$.

1.4.1 $SU(5)$ Matter Fields

The SM gauge group has rank 4. Hence the rank of the grand unification group should be at least 4. There are many possibilities for a rank 4 simple group with one gauge couplings. Among all possibilities, $SU(5)$ is found to be the only choice that meets all the required features: It has complex representation for fermions and it accommodates both integer and fractionally charged fermions. The 15 left-handed SM fermions for one family can be embedded into just two irreps, the antifundamental $\overline{5}_F$ and the two-index antisymmetric tensor 10_F. This can be seen by writing the decomposition of $\overline{5}_F$ and 10_F irreps of $SU(5)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$ as follows:

$$\begin{aligned} \overline{5} &= (\overline{3}, 1, +2/3) \oplus (1, \overline{2}, -1), \\ 10 &= (\overline{3}, 1, -4/3) \oplus (3, 2, +1/3) \oplus (1, 1, +2). \end{aligned} \tag{1.45}$$

Also, this embedding can be depicted in matrix representation as

$$\overline{5} = \begin{pmatrix} d^{c1} \\ d^{c2} \\ d^{c3} \\ e^- \\ \nu \end{pmatrix}, \qquad 10 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ -u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}. \tag{1.46}$$

This assignment is free of chiral anomalies. In the SUSY version of $SU(5)$, these multiplets are promoted to superfields.

1.4.2 Higgs Sectors and Yukawa Couplings in the minimal SUSY-$SU(5)$

In order to test the viability of minimal SUSY $SU(5)$, let us first construct the invariant Yukawa couplings by writing down the $SU(5)$ decomposition of all possible multiplications of the irreps $\overline{5}$ and 10.

$$\overline{5} \times \overline{5} = \overline{10} + 15, \tag{1.47}$$

$$10 \times 10 = \overline{5} + \overline{45} + 50, \tag{1.48}$$

$$\overline{5} \times 10 = 5 + 45. \tag{1.49}$$

It is easy to check that the MSSM superfield Higgs doublet H_u is contained in 5 and 45, and H_d in $\overline{5}$ and $\overline{45}$. Therefore, two quintets 5_H and $\overline{5}_H$ are introduced minimally in the SUSY-minimal $SU(5)$. These two quintets are responsible for breaking $SU(3)_c \times SU(2)_L \times U(1)_Y$ to $SU(3)_c \times U(1)_{em}$. Based on the above analysis, the invariant superpotential that contains only the Yukawa couplings is given as follows:

$$\hat{f} \ni Y^u_{\alpha\beta}\epsilon_{ijklm}10^{ij}_{F\alpha}10^{kl}_{F\beta}5^m_H + Y^d_{\alpha\beta}10^{ij}_{F\alpha}\overline{5}_{Fi\beta}\overline{5}_{Hj}. \tag{1.50}$$

The mass matrices generated by the VEVs of the the $SU(2)_L$ doublets in both $\overline{5}_H$ and 5_H then read

$$M_d = M_L = Y^d\langle\overline{5}_H\rangle, \qquad M_u = Y^u\langle 5_H\rangle. \tag{1.51}$$

Since the first term in Eq.(1.50) contains two identical 10s, the up-quark Yukawa couplings are symmetric in the generation indices, i.e., $M_u = M_u^\top$. Diagonalization of the down quarks and charged leptons mass matrix leads to

$$m_e = m_d \quad m_\mu = m_s \quad m_\tau = m_b. \tag{1.52}$$

Note that the above mass relations are only valid at mass scales where the $SU(5)$ is a good symmetry. But the light fermion masses are observed at low energy scale of order (2-5) GeV. Therefore, the above mass relations should be extrapolated to low energy scale. The results are the following: the first two mass relations in Eq.(1.52) are violated by experiment, while the third one is considered as a successful prediction of minimal SUSY $SU(5)$. One way to correct the bad mass relations for the first and second generations is to employ the 45_H [18]. In this case, the price that we have to pay is including several Higgs multiplets.

It is obvious that the Higgs multiplets $\overline{5}_H$ and 5_H do not break $SU(5)$ to $SU(3)_c \times SU(2)_L \times U(1)_Y$ since they do not contain a SM singlet. The smallest dimensional Higgs representation that contains the SM singlet is the adjoint of $SU(5)$. The adjoint Higgs representation 24_H decomposes under $SU(3)_c \times SU(2)_L \times U(1)_Y$ to

$$24_H = (1,1,0) \oplus (8,1,0) \oplus (1,3,0) \oplus (3,2,-5/6) \oplus (\overline{3},2,+5/6), \tag{1.53}$$

and the (1,1,0) component can acquire a GUT-scale VEV. Equivalently, one can show [19]

$$\langle 24_H \rangle = \sigma \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}. \qquad (1.54)$$

The two Higgs fields 24_H and $\overline{5}_H$ develop hugely different VEVs (i.e., $\langle 24_H \rangle$ of order $M_{GUT} \approx 10^{16}$ GeV and $\langle \overline{5}_H \rangle$ of order $M_W \approx 10^2$ GeV). Consequently, this leads to a huge hierarchy of the gauge symmetry. In non-SUSY model, the parameters at tree level of the Higgs potential should be fine-tuned in order to maintain this huge hierarchy. On the other hand, this fine-tuning gets worse via radiative corrections. However, in the minimal SUSY- $SU(5)$, once the parameters of the Higgs superpotential

$$\hat{f} \ni m_5 \overline{5}_H 5_H + m_{24} Tr[24_H 24_H] + \lambda_1 Tr[24_H 24_H 24_H] + \lambda_2 \overline{5}_H 24_H 5_H \qquad (1.55)$$

are fine-tuned properly at tree level, the SUSY non-renormalization theorem of Grisaru, Rocek and Siegel [67] ensures that it does not get upset by radiative corrections, since according to this theorem these parameters do not receive either finite or infinite corrections.

1.4.3 Gauge Sector of Minimal $SU(5)$

The adjoint representation of $SU(5)$ has the dimension $5^2 - 1 = 24$. Hence, there are 24 gauge bosons associated with $SU(5)$. They decompose under $SU(3)_c \times SU(2)_L \times U(1)_Y$ as given in Eq.(1.53). The gauge bosons of SM are contained within 24 gauge bosons of $SU(5)$ as follows: $(8, 1, 0)$ are $SU(3)_c$ gluons , $(1, 3, 0)$ are the three $SU(2)_L$ vector fields W, and $(1, 1, 0)$ is the $U(1)$ B-field. The remaining 12 gauge bosons, which transform under the SM gauge group as $(3, 2, \frac{5}{3})$ and $(3^*, 2, -\frac{5}{3})$ are called lepto-quark gauge bosons denoted respectively by X and Y. These gauge bosons can be collectively described by a 5×5 matrix form, $A_\mu = A^a \lambda_a/2$, where λ_a are the $SU(5)$ generators (a runs from 1 to 24) and the summation over index a is implied.

As we have discussed before, the Higgs phenomenon can provide masses to the gauge bosons by developing a VEV to the Higgs field. This can be seen by writing down the invariant kinetic term of the Higgs fields as follows

$$\mathcal{L}_{KE} = Tr[(D_\mu 24_H)(D_\mu 24_H)^*]. \tag{1.56}$$

Here the covariant derivative of the adjoint representation 24_H is defined as follows:

$$D_\mu 24_H = \partial_\mu 24_H + ig_5[A_\mu, 24_H], \tag{1.57}$$

where $[A_\mu, 24_H] = A_\mu 24_H - 24_H A_\mu$, and g_5 is the $SU(5)$ gauge coupling. The factor $g_5^2 Tr[A_\mu, \langle 24_H \rangle]^2$ contains the mass term for the gauge bosons. Since 24_H commutes with the generators of the SM gauge group, the gauge bosons of the SM (W_r, B, G^α_β) do not pick up mass, while the X and Y gauge bosons acquire masses according to

$$M_X = M_Y = 5\sqrt{2} g_5 \sigma \tag{1.58}$$

1.5 Minimal SUSY-$SO(10)$

We have seen that the SM fermions can be accommodated within two irreducible representations of the simplest unified model based on $SU(5)$ gauge symmetry. This leads to the unification of the Yukawa couplings of the down quarks and charged leptons. On the other hand, a single 16-dimensional chiral spinor of $SO(10)$ is enough to accommodate all the SM model fermions of one generation. This brings the following benefits: First, the right-handed neutrino is automatically accommodated within the same multiplet. Second, the number of independent parameters of the effective fermion masses and mixing matrices can be reduced considerably. These observations motivate us to consider the $SO(10)$ gauge symmetry.

1.5.1 Matter Fields in $SO(10)$ GUTs

The reducible spinorial representation of $SO(10)$ splits into a pair of spinorial representations 16 and $\overline{16}$ under a chiral projection operator, for details see Ref. [21]. All the femions reside in only one chirality of a $SO(10)$ spinorial representation (i.e, 16-dimensional representation of $SO(10)$). In order to see how the SM fermions can be fitted within a 16-dimensional irrep of $SO(10)$, let us write down its decompositions under $SU(3)_c \times SU(2)_L \times U(1)_Y$:

$$\begin{aligned} 16 &= (3, 2, +1/3) \oplus (1, 2, -1) \oplus (\overline{3}, 1, -4/3) \\ &\oplus (\overline{3}, 1, +2/3) \oplus (1, 1, +2) \oplus (1, 1, 0), \end{aligned} \tag{1.59}$$

where the quantum numbers on the right-hand side (except the last one) are those for the SM fermions (see Table 1), while the last one is the right-handed neutrino. Equivalently, the 16-dimensional irrep of $SO(10)$ can be written in terms of the $SU(5)$ basis as follows:

$$16 = \overline{5} \oplus 10 \oplus 1, \tag{1.60}$$

where the matrix representations of the irreducible representations of $SU(5)$ ($\overline{5}$ and 10) are given in Eq.(1.46). The right-handed neutrino (or equivalently ν^c) is assigned to the singlet of $SU(5)$.

1.5.2 The Higgs Fields and Yukawa Couplings in $SO(10)$ GUTs

The Higgs sector of any realistic $SO(10)$ model should be chosen appropriately in order to satisfy the following requirements. First, the Yukawa couplings should be invariant under $SO(10)$ and compatible with the current data on the quark and the lepton masses and mixings. Second, the Higgs sector should lead to the proper spontaneous symmetry breaking of $SO(10)$ gauge symmetry down to the $SU(3)_c \times SU(2)_L \times U(1)_Y$ of the MSSM. The invariant Yukawa couplings follow from the decomposition of

$$16 \otimes 16 = 10 \oplus 126 \oplus 120. \tag{1.61}$$

Thus, there are three types of $SO(10)$ Higgs multiplets that can give masses to the matter fermions: the 10-dimensional vector representation 10_H, the 126-dimensional 5-index antisymmetric tensor $\overline{126}_H$ and the 120-dimensional three-index antisymmetric tensor 120_H. Then, the most general Yukawa couplings are

$$W_Y = Y_{10}^{\alpha\beta} 16_{F\alpha} 16_{F\beta} 10_H + Y_{120}^{\alpha\beta} 16_{F\alpha} 16_{F\beta} 120_H + Y_{126}^{\alpha\beta} 16_{F\alpha} 16_{F\beta} \overline{126}_H. \tag{1.62}$$

The good feature of the 10-dimensional Higgs multiplet of SUSY-$SO(10)$ is that 10_H contains the SUSY-$SU(5)$ Higgs multiplets 5_H and $\overline{5}_H$ that give masses to the up-type and the down-type quarks respectively. The fermion masses are generated by giving VEVs to the Higgs fields in Eq.(1.62). The fermion masses with Higgs field belonging to the 10-dimensional irrep can be calculated by writing the irreps of $SO(10)$ matter and Higgs fields in terms of $SU(5) \times U_1$ basis as [22]:

$$10 = 5(2) + \overline{5}(-2), \quad 16 = 1(-5) + \overline{5}(3) + 10(-1). \tag{1.63}$$

where the numbers in the bracket are quantum numbers of U_1. Then we construct the invariant combinations of $SU(5) \times U_1$ multiplets as

$$\begin{aligned} Y_{10}^{\alpha\beta} 1_{F\alpha}(-5) \overline{5}_{F\beta i}(3) 5_H^j(2) \;&+\; Y_{10}^{\alpha\beta} \epsilon_{ijklm} 10_{F\alpha}^{ij}(-1) 10_{F\beta}^{kl}(-1) 5_H^m(2) \\ &+\; Y_{10}^{\alpha\beta} \overline{5}_{F\alpha i}(3) 10_{F\beta}^{ij}(-1) \overline{5}_{Hj}(-2). \end{aligned} \tag{1.64}$$

We remind the reader that $\overline{5}_F$ and 10_F are the usual $SU(5)$ representations of Georgi and Glashow given in Eq.(1.46). The first line in Eq.(1.64) shows that the Dirac neutrinos and up-quarks couple with the same Higgs multiplets 5_H while the second line tell us that the charged leptons and down quarks couple with the other Higgs muliplets $\overline{5}_H$. Thus,

$$M_d^{\alpha\beta} = M_e^{\alpha\beta} = Y_{10}^{\alpha\beta} \langle \overline{5}_H \rangle \quad M_u^{\alpha\beta} = M_\nu^{\alpha\beta} = Y_{10}^{\alpha\beta} \langle 5_H \rangle. \tag{1.65}$$

The above fermion mass matrices are symmetric. Since the up and down quark mass matrices in Eq.(1.65) can be diagonalized by the same unitary matrix, the quark mixing matrix is an identity matrix. This can be considered as a zeroth order approximation for the CKM mixing matrix. The 120-dimensional Higgs representation is antisymmetric under the flavor index, however it contributes to mixings between various generations. On the other hand, the 126-dimensional is symmetric under the flavor index and by itself would lead to the following mass relations [21]:

$$M_e = -3Y_{126}v_d^{126} = -3M_d,$$
$$M_\nu = -3Y_\nu v_d^{126} = -3M_u. \quad (1.66)$$

A realistic Higgs spectrum would include, for example, $10_H \oplus \overline{126}_H$. In order to achieve the spontaneous symmetry breaking of $SO(10)$ gauge symmetry down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ a (G_{SM}) of the MSSM, we need to consider all possible Higgs fields that contain G_{SM} singlet in their decomposition under the SM gauge group such as 45_H, 54_H, 210_H and 126_H. Since $SO(10)$ is a rank 5 group, there are many symmetry breaking chains leading to the rank-4 G_{SM}. The most common breaking chains and the Higgs representation that has been used to break the intermediate symmetries at each step are represented in Fig1.4.

In any $SO(10)$ breaking chain, there must be a Higgs multiplet capable to break the considered symmetry down to the subsequent one by giving a VEV to the component that transforms as a singlet under the lower intermediate symmetry group. Being a rank 5 group, there should be at least two Higgs fields to break $SO(10)$ down to the SM. One is needed to break the rank of $SO(10)$ from 5 to 4 while the other breaks the remnant symmetry down to the SM gauge group. There are two simple choices of the Higgs fields that not only break the rank of $SO(10)$ but also give a superlarge mass to the right handed neutrino as shown in section 1.5.3. The choices are an antisymmetric five index tensor $\overline{126}_H$ or a spinor $\overline{16}_H$. In either case, there

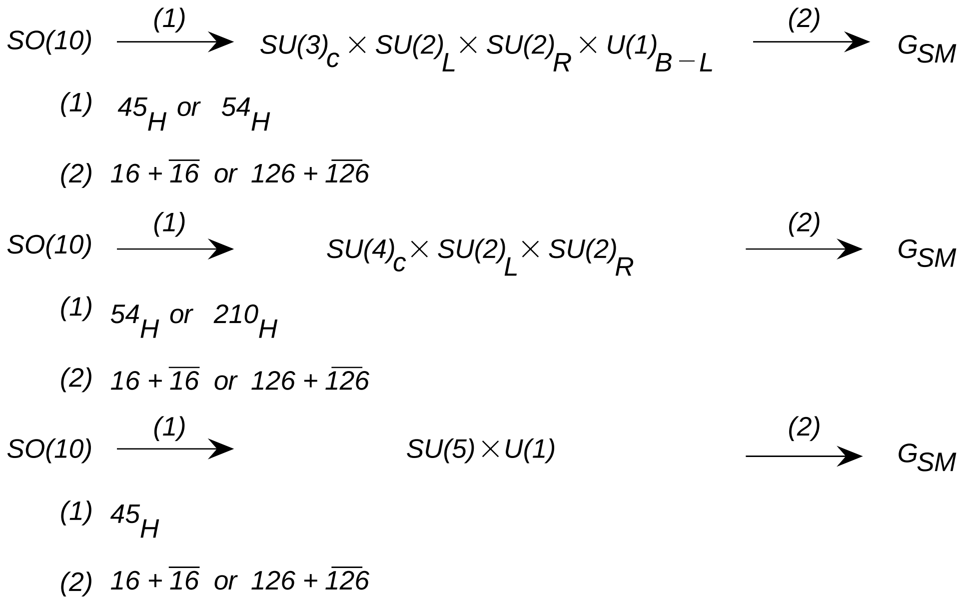


Figure 1.4: The most common breaking chains of $SO(10)$ gauge group to the SM gauge group (G_{SM})

should be a Higgs field in the conjugate representation, 126_H or 16_H, to go along with it, in order to obtain D-term cancelation and consequently maintain the invariance of supersymmetry down to the electroweak scale. Breaking the rank of $SO(10)$ by either 16_H or 126_H leaves $SU(5)$ unbroken because both 16_H and 126_H contain a $SU(5)$ singlet in their decomposition under $SU(5)$ as shown below [22]:

$$\begin{aligned} 126_H &= 1 \oplus \overline{5} \oplus 10 \oplus \overline{15} \oplus 45 \oplus \overline{50}, \\ 16_H &= 1 \oplus \overline{5} \oplus 10. \end{aligned} \tag{1.67}$$

Therefore, a second Higgs field is needed to break $SU(5)$ down to the SM. The appropriate Higgs multiplets of $SO(10)$, that can break $SU(5)$, should contain a 24-dimensional representation with neutral $U(1)$ charge in their $SU(5) \times U(1)$ components (recall that the adjoint of $SU(5)$ (24_H) is used to break $SU(5)$ to G_{321} of SM). For example, the decomposition of the following Higgs multiplets 45_H , 54_H, and 210_H under $SU(5) \times U(1)$ [22]

$$\begin{aligned}
45_H &= 1(0) \oplus 10(4) \oplus \overline{10}(-4) \oplus 24(0), \\
54_H &= 15(4) \oplus \overline{15}(-4) \oplus 24(0), \\
210_H &= 1(0) \oplus 5(-8) \oplus \overline{5}(8) \oplus 10(4) \oplus \overline{10}(-4) \\
&\quad \oplus 24(0) \oplus 40(-4) \oplus \overline{40}(-4) \oplus 75(0) \qquad (1.68)
\end{aligned}$$

makes them capable of breaking $SU(5)$ down to the SM. There are two approaches that have been adopted so far in order to break the $SO(10)$ gauge group to the SM gauge group. One uses large Higgs representations such as 210_H, 126_H, and $\overline{126}_H$ [23]. Although this approach has the advantage that R parity is automatic, the unified gauge coupling diverges in this case just above the GUT scale. On the other hand, the other approach uses only small Higgs representations [24, 25]. This choice of Higgs representations guarantees that the theory is perturbative up to the Planck scale [26] and also has the potential to arise from string theory. Therefore, we shall adopt the simplest breaking scheme; a pair of spinors 16_H and $\overline{16}_H$ is used to break the rank of $SO(10)$ and only one adjoint 45_H is used to break $SU(5)$. The general VEV direction of 45_H required to break $SU(5)$ gauge symmetry is given by [19]

$$\langle 45_H \rangle = \text{diag}(b, b, a, a, a) \otimes i\tau_2. \qquad (1.69)$$

The $\langle 45_H \rangle$ is proportional to the generator of $B-L$ when $b = 0$ and it is proportional to T_{3R} when $a = 0$. The former VEV direction is preferred in the Dimopoulos-Wilczek (DW) [27] mechanism in order to solve the doublet-triplet splitting problem.

1.5.3 Neutrino Masses

The existence of right-handed neutrinos is important to understand the smallness of the neutrino mass as we have seen in the seesaw mechanism in the context of SM. The accommodation of right-handed neutrinos within the 16-dimensional irreps of $SO(10)$

indicates that the seesaw mechanism can be implemented in $SO(10)$ models. In order to see this, let us assume that the only source for the quark and lepton masses is the 10-dimensional Higgs representation of $SO(10)$, causing $M_u = M_\nu$. The following coupling

$$Y_{126} 16_F 16_F \overline{126}_H, \tag{1.70}$$

can be used to generate a Majorana mass term for right-handed neutrinos by giving VEV to the $SU(5)$ singlet component of $\overline{126}_H$, so the combination of the Dirac and Majorana neutrino mass terms are given by

$$\mathcal{L} = \nu^c M_D \nu + \frac{1}{2} M_R \nu^c \nu, \tag{1.71}$$

Here $M_R = Y_{126}\langle 1(\overline{126}_H)\rangle = Y_{126} v_{126}$ and the notation $p(q)$ refers to p of $SU(5)$ contained in q of $SO(10)$. This can be written in a 2×2 mass matrix for the (ν,ν^c) system as given in Eq.(1.25). If we ignore the mixing among generations, the light neutrino masses for the three generations are given by

$$\begin{aligned} m_{\nu_e} &\approx \frac{m_u^2}{M_{R1}}, \\ m_{\nu_\mu} &\approx \frac{m_c^2}{M_{R2}}, \\ m_{\nu_\tau} &\approx \frac{m_t^2}{M_{R3}}, \end{aligned} \tag{1.72}$$

where we have used $M_D = M_u$. The magnitude of the scale $\langle 1(\overline{126}_H)\rangle$ is model-dependent. For example, if the MSSM is a valid symmetry all the way until the GUT scale, then $v_{126} = M_U \approx 2\times 10^{16}$. It is important to point out that the assumption we have made that the fermion masses arise only from 10_H is not good, because it leads to the undesirable relation $m_d/m_s = m_e/m_\mu$. Therefore, we need additional fields, in order to have a realistic $SO(10)$ GUT model.

Another way to give Majorana masses to right-handed neutrinos is by using a bilinear product of $\overline{16}_H$. The relevant interaction is the effective nonrenormalizable interaction $f_{ij}16_i 16_j \overline{16}_H \overline{16}_H/M$ which may arise from integrating out a heavy state with mass M. Several realistic models were published along these lines [30]. By giving a VEV to the component of $\overline{16}$ in the $SU(5)$ singlet direction, the right-handed neutrino mass matrix is generated as follows:

$$M_{R_{ij}} = f_{ij} \frac{\langle \overline{16}_H \rangle^2}{M}. \tag{1.73}$$

If we assume that both $\overline{16}_H$ and 16_H break the rank of $SO(10)$ at the GUT scale, then $\langle \overline{16}_H \rangle \approx 2 \times 10^{16}$ GeV. In order to obtain the heaviest right handed neutrino mass to be of order 2×10^{14} GeV, the mass of the heavy state should be around the Planck scale (2×10^{18} GeV) [31]

One advantage of $\overline{126}_H$ is that it leads to a theory that conserves R parity automatically. This is because $\overline{126}_H$ breaks $B-L$ by two units. Plugging $B-L=2$ back into the R formula in Eq (1.36), one can see that R parity remains invariant even after symmetry breaking. While in the case of $\overline{16}_H$, $B-L$ is broken by one unit, then R parity is not conserved after symmetry breaking. However, the superpotential terms that contain $\overline{16}_H$ and break $B-L$ by one unit can be avoided by imposing a discrete symmetry. Besides, as we mentioned in the previous section, the choice of 16_H and $\overline{16}_H$ is inspired by string theory, and the fact that using small Higgs representations leads to make the unified gauge coupling perturbative up to the Planck scale.

CHAPTER 2

Fermion Masses and Mixings in a Minimal $SO(10) \times A_4$ SUSY GUT

We have seen that the GUT models unify the strong and electroweak interactions into a simple group. The simplest GUT model is based on SU(5) gauge symmetry. The minimal SU(5) model predicts a good mass relation for the third generation (i.e., $m_b^0 = m_\tau^0$ at GUT scale). However, it gives bad prediction for the first and second generation masses (i.e., $m_s^0 = m_\mu^0$, $m_d^0 = m_e^0$ at the GUT scale). In addition, $SU(5)$ does not naturally accommodate the right-handed neutrino. On the other hand, $SO(10)$ models accommodate all chiral fermions of one generation plus a right handed-neutrino within a 16-dimensional irreducible representation (irrep). Also, minimal $SO(10)$ with only 10_H involved in Yukawa couplings leads to the up quark mass matrix being proportional to the down quark mass matrix, so it is considered a good zeroth order approximation for CKM mixings. Models based on $SO(10)$ symmetry, without including any family symmetry, were proposed to explain most of the features of quarks and leptons [32, 33]. However, one is not really fully satisfied with only producing the fermion masses and mixing angles without explaining why we have three generations and without understanding the relation among generations, such as the mass hierarchy and features of the mixing angles. For example, the flavor symmetry A_4 [34] can be employed to explain why the observed neutrino mixing matrix is in very good agreement with the so called tri-bi-maximal (TBM) mixing structure given by Eq(1.35). Thus, it may be important to consider the underlying family symmetry. One of the best candidates for flavor symmetry is the non-Abelian discrete symmetry A_4, for the following reasons. First, it is the smallest group that has a 3-dimensional irrep. Second, SUSY-$SO(10)\times A_4$ symmetry solves the FCNC problem since the scalar fermions, which belong to the 16-irrep of $SO(10)$ and transform as a triplet under A_4, have degenerate masses. Finally, it was shown that the TBM mixing structure for the neutrinos can be obtained by imposing A_4 symmetry [34].

Several models based on the $SO(10)\times A_4$ group have been studied [14, 15, 16]. In these models, large Higgs representations are employed. For example, in Ref.[16], the authors employed a (126_H,3) representation, where the first (second) entry indicates the transformation under $SO(10)$ (A_4), in order to produce the fermion masses and mixing angles for both normal and inverted neutrino mass spectra. Besides employing the large Higgs representation 126_H, the models in Refs.[14, 15] contain more than one adjoint 45_H representation. It has been shown that only one adjoint Higgs field is required to break $SO(10)$ while preserving the gauge coupling unification [35]. Also, using large Higgs representations like 126_H leads to the unified gauge coupling being nonperturbative before the Planck scale, which might be hard to obtain from superstring theory [36]. Therefore, the purpose of this chapter is to construct an $SO(10)\times A_4$ model in which $SO(10)$ is broken to the standard model (SM) group in the minimal breaking scheme. This means using only a spinor-antispinor (16_H,$\overline{16}_H$) to break the rank of $SO(10)$ from five to four, and the right-handed neutrino gets a heavy mass from the antispinor Higgs field ($\overline{16}_H$). Then one adjoint representation 45_H is used to break the group all the way to the SM group. Recently, a numerical analysis for quark and charged lepton masses and mixings based on nonsupersymmetric $SO(10)$ without flavor symmetry was done [33]. The authors did not include the neutrino sector in the numerical fitting. Their result for the atmospheric angle was $\sin\theta_{atm} = 0.89$. However, as this work shows, when the neutrino sector is included, not only is the result a better fit for the atmospheric angle $\sin\theta_{atm} = 0.776$, but the known light neutrino mass differences are also accommodated.

This chapter is organized as follows. In section 2.1, a general structure of the fermion mass matrices for the second and third generations is constructed. Then, based on that structure, the fermion mass hierarchy and relations are explained. In section 2.2, it is shown that introducing several 10-plets of matter fields to the model leads to the doubly lopsided structure which produces large neutrino mixing angles and small quark mixing angles simultaneously [37]. Then, some analytical expressions for quark masses and mixing angles at the GUT scale are derived in a certain approximation on the model parameters. In Sec 2.3, an exact numerical analysis is done to find the outputs at the GUT scale. To get predictions of fermion masses and mixings at low scale, the quark masses and mixings at the GUT scale will be run to the low scale by using renormalization group equations. section 2.4 shows how to get a suitable right-handed neutrino mass structure that gives the correct fits for the atmospheric angle after adding the charged lepton contribution.

2.1 Fermion Mass Structure in $SO(10) \times A_4$ Symmetry

In this section, the renormalizable Yukawa couplings of the SM fermions with the extra spinor-antispinor matter fields are considered as a concrete example of the model. The known matter fields of the SM (quarks and leptons) plus the right handed neutrino are contained in the three spinors (16,3). The ordinary fermions, 16_i, do not couple with 45_H in the minimal $SO(10)$. As a result, some of the predictions of the minimal $SO(10)$ such as $m_\mu = m_s$ and $m_c/m_t = m_s/m_b$ will follow; these are badly broken in nature. Therefore, extra heavy fermion fields must be introduced in order to allow the 45_H to couple directly with the quarks and leptons of the standard model. The transformation of the ordinary fermions and the extra matter fields under A_4 and the additional symmetry $Z_2 \times Z_4 \times Z_2$ are summarized in Table 2.1. Let us consider first the invariant superpotential W_1 under the assigned symmetry that contains the

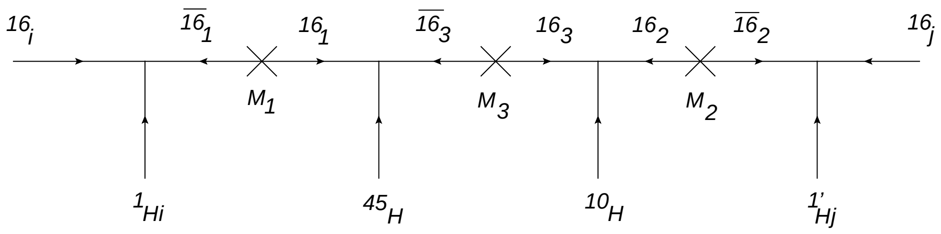


Figure 2.1: This figure shows a diagrammatic representation of the couplings in the superpotential W_1.

coupling of ordinary fermions with the spinor-antispinor matter fields.

$$\begin{aligned} W_1 &= b_1 16_i \overline{16}_1 1_{Hi} + b_2 16_i \overline{16}_2 1'_{Hi} + \Omega 16_1 \overline{16}_3 45_H + a 16_3 16_2 10_H \\ &\quad + M_1 16_1 \overline{16}_1 + M_2 16_2 \overline{16}_2 + M_3 16_3 \overline{16}_3. \end{aligned} \tag{2.1}$$

Table 2.2 summarizes the transformation of the Higgs fields that are needed to achieve a minimum breaking scheme as well as the Higgs singlets that are needed to break the A_4 symmetry. Although in this model, the structure in Eq.(2.1) does not include the Yukawa term $16_i 16_i 10_H$ which is forbidden by the discrete symmetry $Z_2 \times Z_4 \times Z_2$, the ordinary standard model fermions get their masses through their coupling with heavy extra fields. This is similar to how the light neutrinos get their masses through coupling with the heavy right-handed neutrinos in the known see-saw mechanism. The coupling terms in the superpotential W_1 can be represented diagrammatically as shown in Fig.2.1. After integrating out the heavy states, the approximate effective operators can be read from the diagram, i.e.,

$$W_{ij} \approx \sum_{ij} \frac{16_i 16_j \langle 45_H \rangle \langle 10_H \rangle \langle 1_{Hi} \rangle \langle 1'_{Hj} \rangle}{M_1 M_2 M_3}. \tag{2.2}$$

The VEVs of the Higgs fields can be written down in a general form as

$$\langle 45_H \rangle = \Omega Q, \tag{2.3}$$

$$\langle 1_{Hi} \rangle = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}, \tag{2.4}$$

$SO(10)$	16_i	16_1,$\overline{16}_1$	16_2,$\overline{16}_2$	16_3,$\overline{16}_3$	1^c_i
A_4	3	1	1	1	3
$Z_2 \times Z_4 \times Z_2$	+,+,+	+,-,+	-,+,+	+,+,-	+,+,+
$SO(10)$	10_i	$10'_i$	$10''_i$	$10'''_i$	1_i
A_4	3	3	3	3	3
$Z_2 \times Z_4 \times Z_2$	+,i,+	+,$-i$,+	+,i,-	+,$-i$,-	+,$-i$,+

Table 2.1: The transformation of the matter fields under $SO(10)\times A_4$ and $Z_2\times Z_4\times Z_2$.

$$\langle 1'_{Hi} \rangle = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}, \tag{2.5}$$

$$\langle 5(10) \rangle = v_u, \langle \overline{5}(10) \rangle = v_d. \tag{2.6}$$

Here the notation $\langle p(q) \rangle$ refers to a p of SU(5) contained in a q of $SO(10)$. The Q from Eq.(2.3) is a linear combination of $SO(10)$ generators. One can redefine, without loss of generality, the light fermion states as

$$\begin{aligned} 16_1\epsilon_1 + 16_2\epsilon_2 + 16_3\epsilon_3 &= \epsilon 16'_3, \\ 16_1 s_1 + 16_2 s_2 + 16_3 s_3 &= S(16'_2 s_\theta + 16'_3 c_\theta), \end{aligned} \tag{2.7}$$

where $\epsilon = \sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2}$ and $S = \sqrt{s_1^2 + s_2^2 + s_3^2}$. In terms of the redefined light fermion states, after dropping the prime notation and plugging in the VEVs, one gets

$$W_0 \approx \frac{\Omega \epsilon S \langle 10_H \rangle}{M_1 M_2 M_3} (16_3 16_2 Q_{(16_3)} s_\theta + 16_3 16_3 Q_{(16_3)} c_\theta). \tag{2.8}$$

In general, the above effective operator can be written in terms of quark and lepton fields as

$$W_F \approx \frac{\Omega \epsilon S \langle 10_H \rangle}{M_1 M_2 M_3} (F_3 F_2^c Q_F s_\theta + F_3^c F_2 Q_{F^c} s_\theta + F_3 F_3^c (Q_F + Q_{F^c}) c_\theta). \tag{2.9}$$

Here F is a general notation for up quarks (U), neutrinos (N), charged leptons (L), and down quarks (D). The quantity $Q_F(Q_{F^c})$ refers to the assigned charge of the left-handed fermion (charge conjugate of the right-handed fermions) after breaking the $SO(10)$ group down to the SM group. The unbroken charge Q can be written as a linear combination of two generators that commute with $SU(3)_c \times SU(2)_L \times U(1)_Y$ as:

$$Q = 2I_{3R} + \frac{6}{5}\delta(\frac{Y}{2}), \tag{2.10}$$

where I_{3R} is the third generator of $SU(2)_R$ and Y is the hypercharge of the Abelian U(1) group. The charge Q for different quarks and leptons is given by.

$$\begin{aligned} Q_u = Q_d = \frac{1}{5}\delta, \quad Q_{u^c} = -1 - \frac{4}{5}\delta, \quad Q_{d^c} = 1 + \frac{2}{5}\delta, \\ Q_l = Q_\mu = -\frac{3}{5}\delta, \quad Q_{l^c} = 1 + \frac{6}{5}\delta, \quad Q_{\nu^c} = -1. \end{aligned} \tag{2.11}$$

Eq.(2.9) can be expressed in the following matrix form:

$$W_F \approx \begin{pmatrix} F_1^c & F_2^c & F_3^c \end{pmatrix} (\frac{\Omega\epsilon S\langle 10_H\rangle}{M_1 M_2 M_3}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Q_F s_\theta \\ 0 & Q_{F^c} s_\theta & (Q_F + Q_{F^c})c_\theta \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}. \tag{2.12}$$

Some factors that arise from doing the algebra exactly should be included in the above mass matrix as we are going to see later. Finding these factors that we have assumed to be of order one is important in the flavor violation analysis. The first feature of the general mass matrix of the light fermions in Eq.(2.12) is an explanation for the mass hierarchy between the second and third generations in the limit $s_\theta \to 0$. It is remarkable that a relation among generations is related to the vacuum alignment of the A_4 Higgs.

Another feature of the above light fermion mass matrix $m_b^0 = m_\tau^0$ is obtained through $M_{D33} = M_{L33}$, which follows from the relation $Q_{d^c} + Q_d = Q_{l^c} + Q_l$. This relation occurs because both down quarks and charged leptons get their masses from the same Higgs.

A further consequence of the light fermion mass structure is that $m_s^0 \neq m_\mu^0$. This inequality relation follows from $m_\mu^0/m_s^0 = L_{32}L_{23}/D_{32}D_{23} = Q_{l^c}Q_l/Q_{d^c}Q_d$, which

$SO(10)$	10_H	45_H	16_H	$\overline{16}_H$	1_{Hi}	$1'_{Hi}$	$1''_{Hi}$	$1'''_{Hi}$
A_4	1	1	1	1	3	3	3	3
$Z_2 \times Z_4 \times Z_2$	-,+,-	+,-,-	+,$-i$,+	+,$-i$,+	+,-,+	-,+,+	+,+,-	+,i,+

Table 2.2: The transformation of the Higgs fields under $SO(10)\times A_4$ and $Z_2\times Z_4\times Z_2$.

is not necessarily equal to 1. This leads to the following question: What VEV direction should be given to 45_H in order to obtain the Georgi-Jarlskog relation $|m^0_\mu| = 3|m^0_s|$? There are two choices, either $\delta \to 0$ or $\delta \to -1.25$. The former choice gives the unwanted relation $(m^0_c/m^0_t)/(m^0_s/m^0_\tau) \to 1$, while the latter leads to $(m^0_c/m^0_t)/(m^0_s/m^0_\tau) \to 0$. Thus, a good fit for δ should be around -1.25.

2.2 Extension to the First Generation and Doubly Lopsided Structure

In this section, vector 10-plet fermions are added to the model to generate masses and mixings of the first generation. These vector multiplets do not contribute to the up-quark mass matrix since 10-plets do not contain a charge of ($\pm 2/3$). Therefore, the up-quark matrix is still rank 2, and this is consistent with $\frac{m^0_u}{m^0_t} \approx 10^{-5}$ being much smaller than $\frac{m^0_d}{m^0_b} \approx 10^{-3}$ and $\frac{m^0_e}{m^0_\tau} \approx 0.3\times 10^{-5}$. First, I will show how the model leads to the doubly lopsided structure by employing these vector multiplets; then some analytical expressions for masses and mixing angles of fermions at the GUT scale will be derived. Let us first consider the invariant couplings under the assigned symmetry, which can be read from the Feynman diagram in Fig.2.2. The allowed couplings in the superpotential W_2 are

$$W_2 = 16_i 10_i 16_H + M_{10} 10_i 10'_i + h'_{ijk} 10'_i 10'_j 1_{Hk} + h_{ijk} 10_i 10_j 1_{Hk}. \tag{2.13}$$

The important point is that Fig.2.2 gives a flavor-symmetric contribution to the down-quark and charged lepton mass matrices. In order to understand this, recall that the general product of three triplets—(a_1, a_2, a_3), (b_1, b_2, b_3), and (c_1, c_2, c_3)—that

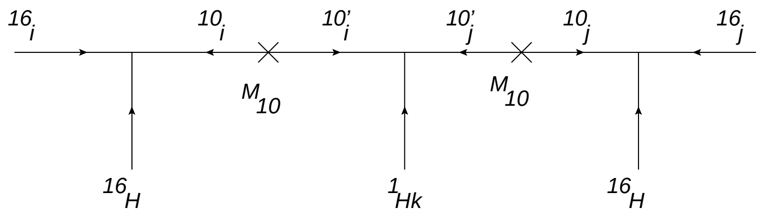


Figure 2.2: This figure leads to the flavor symmetric contribution to the down quarks and charged leptons.

transform as a singlet under A_4 is given by

$$h_1(a_2b_3c_1 + a_3b_1c_2 + a_1b_2c_3) + h_2(a_3b_2c_1 + a_1b_3c_2 + a_2b_1c_3). \tag{2.14}$$

The third term of Eq.(2.13) gives a symmetric contribution since there are two identical 10-plets. The last term in Eq.(2.13) has been ignored by assuming the Yukawa couplings h_{ijk} to be very small. The contribution of Fig.2.2 to the mass matrices of the down quarks and charged leptons, after integrating out the extra vector multiplets is then

$$M_L^s = M_D^s \propto \begin{pmatrix} 0 & c_{12} & c_{13} \\ c_{12} & 0 & c_{23} \\ c_{13} & c_{23} & 0 \end{pmatrix}, \tag{2.15}$$

where c_{12}, c_{13}, and c_{23} are proportional to ϵ_1, ϵ_2, ϵ_3, respectively. To obtain the desired fermion mass structure (the doubly lopsided structure, which is going to be explained later in this section), other couplings need to be included by employing four vector 10-plets plus adding another Higgs singlet $1''_{iH}$ to the model (their transformations under the assigned symmetry are shown in Tables 2.1 and 2.2). The purpose of these couplings is to give a flavor-antisymmetric contribution to the down-quark and charged lepton mass matrices. Since the adjoint of $SO(10)$ (45_H) is an antisymmetric tensor which changes its sign under the interchange $10'_i \leftrightarrow 10'''_i$, one can consider employing the Yukawa coupling $10'''_i 10'_i 45_H$. Also, due to the fact that when we write

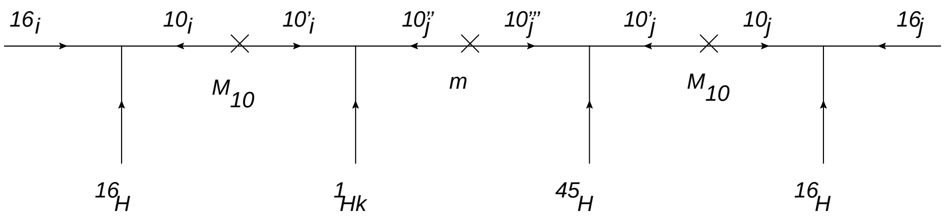


Figure 2.3: This figure leads to the flavor-antisymmetric contribution to the down quarks and charged leptons.

the $SO(10)$-vectors in the SU(5) basis such as $10_i = 5_i + \overline{5}_i$, the charged lepton and down quark contents of 5_i or $\overline{5}_i$ have different chiralities, the structures of matrices M_L and M_D therefore have opposite signs [look at the mass structures in Eqs.(2.18-2.19). It is important to emphasize that the minimum Higgs breaking scheme assumption does not allow us to add another adjoint to the model. Therefore, the same adjoint 45_H Higgs representation that breaks the $SO(10)$ group to the SM group is going to be used. Additional couplings to the previous superpotential can be read from Fig.2.3, i.e.,

$$W_3 = 10'_i 10''_j 1''_{Hk} + m 10''_i 10'''_i + 10'''_i 10'_i 45_H, \tag{2.16}$$

where $\langle 45_H \rangle$ has been defined previously. The VEV of the Higgs singlet $1''_H$ is given below:

$$\langle 1''_H \rangle = \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{pmatrix}. \tag{2.17}$$

After integrating out the heavy states, the following contribution to the M_L and M_D is obtained:

$$M_L^A \propto \begin{pmatrix} 0 & -\delta_3 Q_l & \delta_2 Q_l \\ \delta_3 Q_l & 0 & -\delta_1 Q_l \\ -\delta_2 Q_l & \delta_1 Q_l & 0 \end{pmatrix}, \tag{2.18}$$

$$M_D^A \;\propto\; \begin{pmatrix} 0 & \delta_3 Q_{d^c} & -\delta_2 Q_{d^c} \\ -\delta_3 Q_{d^c} & 0 & \delta_1 Q_{d^c} \\ \delta_2 Q_{d^c} & -\delta_1 Q_{d^c} & 0 \end{pmatrix}, \tag{2.19}$$

where the overall constant has been absorbed in the redefinition of δ_1, δ_2, and δ_3. Equations (2.18-2.19) show that the off-diagonal elements of M_D^A (M_L^A) are proportional to Q_{d^c} (Q_l). This is because $\overline{5}_i(10)$ contains, in its representation, the charge conjugation of a color triplet of the left-handed down quarks d^c_{Li} and the left-handed charged leptons e_{Li}. The full tree-level mass matrices, which are obtained by adding the three superpotentials $W_1 + W_2 + W_3$, have the following forms:

$$M_L \;=\; m_d^0 \begin{pmatrix} 0 & c_{12} + 3\delta_3(\frac{-1+\alpha}{5}) & -\delta_2\alpha + \zeta \\ c_{12} - 3\delta_3(\frac{-1+\alpha}{5}) & 0 & \delta_1\alpha + \beta - 3s(\frac{-1+\alpha}{5}) \\ \zeta - \delta_2\frac{6-\alpha}{5} & \delta_1(\frac{6-\alpha}{5}) + \beta + s(\frac{-1+6\alpha}{5}) & 1 \end{pmatrix}, \tag{2.20}$$

$$M_D \;=\; m_d^0 \begin{pmatrix} 0 & c_{12} + \delta_3(\frac{3+2\alpha}{5}) & -2\delta_2(\frac{3+2\alpha}{5}) + \zeta \\ c_{12} - \delta_3(\frac{3+2\alpha}{5}) & 0 & 2\delta_1(\frac{3+2\alpha}{5}) + \beta + s(\frac{-1+\alpha}{5}) \\ \zeta & s(\frac{3+2\alpha}{5}) + \beta & 1 \end{pmatrix}, \tag{2.21}$$

$$M_U \;=\; m_u^0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (\frac{1-\alpha}{5})s \\ 0 & (\frac{1+4\alpha}{5})s & 1 \end{pmatrix}, \tag{2.22}$$

$$M_N = m_u^0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (\frac{-3+3\alpha}{5})s \\ 0 & s & 1 \end{pmatrix}, \tag{2.23}$$

the convention being used here is the left-handed fermions multiplied from the right. The parameters of the model have been defined as follows:

$$\begin{aligned} \zeta &= c_{13} + \delta_2 Q_{d^c}, \\ \beta &= c_{23} + \delta_1 Q_{d^c}, \\ \delta &= -1 + \alpha, \\ s &= \frac{s_\theta}{(\frac{3}{5}\delta + 1)c_\theta}. \end{aligned} \tag{2.24}$$

The above fermion mass structure has eight parameters. If α goes to zero, the fermion mass matrices in Eqs.(2.20-2.21) go to the SU(5) limit ($m_b^0 = m_\tau^0$, $m_s^0 = m_\mu^0$, $m_d^0 = m_e^0$). To avoid the bad prediction of SU(5) for lighter generations, a good numerical fitting for α should deviate from zero. On the other hand, to keep the good SU(5) prediction for the third generation, the parameter α should satisfy $\alpha << 1$. If δ_1 and δ_2 are of order 1 and the other model parameters are very small ($\beta, \zeta, \alpha, \delta_3, c_{12}, s << \delta_1, \delta_2$), the model leads to the doubly lopsided structure. To see this clearly, let us go to the limit where the small parameters are zero (except s). So the M_D and M_L go to the following form:

$$M_L = M_D^T = m_d^0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (\frac{3s}{5}) \\ -\delta_2 \frac{6}{5} & (\frac{-s}{5}) + \delta_1(\frac{6}{5}) & 1 \end{pmatrix}. \tag{2.25}$$

In diagonalizing M_L of Eq.(2.25), the large off-diagonal elements δ_1 and δ_2 that appear asymmetrically in M_D and M_L must be eliminated from the right by a large left-handed rotation angle θ_{sol} in the 1-2 plane, where $\tan(\theta_{sol}) = -\frac{\delta_2}{\delta_1}$. The next step of diagonalization is to remove the large element $\sigma \approx (\delta_1^2 + \delta_2^2)^{\frac{1}{2}}$ that has been produced after doing the first diagonalization, where the (3,2) element of the matrix in Eq.(2.25) is replaced by σ. This can be done by a rotation acting from the right by a large left-handed angle θ_{23} in the 2-3 plane, where $\tan(\theta_{23}) \approx -\sigma$. On the other hand, there are no corresponding large left-handed rotation angles in diagonalizing M_D since $M_L = M_D^T$. However, the large off-diagonal elements in M_D can be eliminated by large right-handed rotation angles acting from the left on the M_D in Eq.(2.25), while the left-handed rotation angles are small. This explains how the doubly lopsided structure leads to small CKM mixing angles and large neutrino mixing angles simultaneously. If the parameters c_{12}, δ_3, and ζ are zero, analytical expressions can be written down for the ratios of quark and lepton masses of the second and third generations, V_{cb}, and neutrino mixing angles ($\tan\theta_{12}$ and $\tan\theta_{23}$) in terms of δ_1, δ_2, s , α, and β:

$$\begin{aligned}
\frac{m_c^0}{m_t^0} &= \frac{s^2(1-\alpha)(1+4\alpha)}{25}, \\
\frac{m_s^0}{m_b^0} &= \frac{-2(3+2\alpha)(\beta+s(\frac{3+2\alpha}{5}))\sqrt{\delta_1^2+\delta_2^2}}{5(1+\frac{4}{25}(3+2\alpha)^2(\delta_1^2+\delta_2^2))}, \\
\frac{m_\mu^0}{m_\tau^0} &= \frac{\sqrt{(\frac{-3s}{5}(-1+\alpha)+\delta_1\alpha+\beta)^2+\delta_2^2\alpha^2}\sqrt{(\delta_1^2+\delta_2^2)}(6-\alpha)}{5(1+\frac{(6-\alpha)^2}{25}(\delta_1^2+\delta_2^2))}, \\
V_{cb}^D &= \frac{\beta+\frac{s(3+2\alpha)}{5}}{(1+\frac{4}{25}(3+2\alpha)^2(\delta_1^2+\delta_2^2))}, \\
V_{cb}^U &= \frac{-s(1+4\alpha)}{5}, \\
\tan\theta_{12} &= \frac{\delta_2(\frac{6-\alpha}{5})}{\delta_1(\frac{6-\alpha}{5})+s(\frac{-1+6\alpha}{5})+\beta}, \\
\tan\theta_{23} &= -(\frac{6-\alpha}{5})\sqrt{\delta_2^2+\delta_1^2}.
\end{aligned} \tag{2.26}$$

These expressions are derived by using the approximation $\alpha, s, \beta << \delta_1, \delta_2$, and are useful for fitting the data. The best fit for the data is obtained by setting $\tan\theta_{23} = -2$ and $\tan\theta_{12} = 0.68$, which correspond to $\theta_{23} = -63^o$ and $\theta_{12} = 34^o$. The central value of the atmospheric angle is around 45^o. In order to bring 63^o close to the central value, the neutrino sector is required to be included as shown in Sec 2.5. Also, it will be shown that the contribution of the neutrino sector to the solar angle is small.

2.3 Numerical Results

The model can be shown to be concrete by giving numerical values to the parameters of the model, and producing the six mass ratios of quarks and leptons, CKM mixing angles (V_{us}, V_{ub}, and V_{cb}), the CP violation parameter $\eta = -Im(V_{ub}V_{cs}/V_{us}V_{cb})$, and neutrino mixing angles ($\sin\theta_{12}$, and $\sin\theta_{13}$). The ten parameters (δ_1, δ_2, δ_3, α, β, s, ζ, c_{12}, m_d^0, and m_u^0) appearing in Eqs(2.20-2.23) are in general complex. Five phases of the complex parameters can be removed by redefining the phases of the quark and lepton fields. Then, we have ten real parameters and five phases in order to fit the 16 quantities appearing in Table 2.3. However, the best numerical fit is obtained when two parameters (δ_3, c_{12}) are complex while the others are real. If $\delta_1 = -1.302$, $\delta_2 = 1.0142$, $\delta_3 = 0.015 \times e^{4.95i}$, $\alpha = -0.05801$, $s = 0.29$, $\zeta = 0.0105$, $c_{12} = -0.00153e^{1.1126i}$, and $\beta = -0.12303$, the following excellent fit at the GUT scale is obtained : $\frac{m_c^0}{m_t^0} = 0.002717$, $\frac{m_b^0}{m_\tau^0} = 0.958$, $\frac{m_e^0}{m_\mu^0} = 0.00473$, $\frac{m_\mu^0}{m_\tau^0} = 0.0585$, $\frac{m_d^0}{m_e^0} = 3.63$, $\frac{m_s^0}{m_\mu^0} = 0.302$, $\eta = 0.357$, $V_{us} = 0.2264$, $V_{ub} = 0.0037$, $V_{cb} = 0.0362$, $\sin\theta_{12} = 0.569$, and $\sin\theta_{13} = 0.0653$. The above numerical fittings lead to $\sin\theta_{23}^L = 0.904$, which is not close to the central value $\sin\theta_{23}^{atm} = 0.707$. One can see from the superscript L that the mixing angle θ_{23}^L comes only from the charged lepton contribution. To obtain close to the expected atmospheric angle and the correct neutrino mass differences, it is important to include the neutrino sector contribution to the atmospheric angle by finding out a suitable right-handed neutrino structure which respects the assigned symmetry of the model.

In order to compare with experiment, the predicted fermion masses and mixing angles at the low energy scale need to be found. The above numerical values of the fermion masses and mixing angles which are obtained at the GUT scale have been evolved to the low scale in two steps. First, the running from the GUT scale to $M_{SUSY} = 1$ TeV is done by using the two-loop MSSM beta function. The running factors denoted by η_i depend on the value of $\tan\beta$. The known fermion masses and mixing data are best fitted with $\tan\beta = 10$. The running factors for $\tan\beta = 10$ are ($\eta_{s/b}$, $\eta_{\mu/\tau}$, $\eta_{b/\tau}$, $\eta_{c/t}$, η_{cb}= η_{ub})=(0.8736, 0.9968, 0.5207, 0.73986, 0.910335), where $\eta_{i/j} = (m_i^0/m_j^0)/(m_i(1\text{TeV})/\text{m}_\text{j}(1\text{TeV}))$ and $\eta_{cb,ub} = V^0_{cb,ub}/V_{cb,ub}(1\text{TeV})$. The second step is to evolve the fermion masses and mixing angles from $M_{SUSY} = 1$ TeV to the low scale. The renormalization factors η_i that run fermion masses from their respective masses up to the supersymmetric scale $M_{SUSY} = 1$ TeV are computed using three-loop QCD and one-loop QED, or the electroweak renormalization group equation with inputs $\alpha_s(M_Z) = 0.118$, $\alpha(M_Z) = 1/127.9$, and $\sin\theta_w(M_Z) = 0.2315$. The relevant renormalization equations can be found in [38][39]. The results are (η_c, η_b, η_e, η_μ, η_τ, η_t, η_{ub}=η_{cb})=(0.4456, 0.5309, 0.8188, 0.83606, 0.8454, 0.98833, 1.0151).

By using the above renormalization factors, $m_\tau = 1776$ MeV, and $m_t = 172.5$ GeV, the following predictions at the low scale can be obtained: $m_c(m_c) = 1.4$ GeV, $m_b(m_b) = 5.2$ GeV, $m_e(m_e) = 0.511$ MeV, $m_\mu(m_\mu) = 105.6$ MeV, $m_d(2\text{ GeV}) = 7.5$ MeV, $m_s(2\text{ GeV}) = 132$ MeV, $\eta = 0.357$, $V_{us} = 0.2264$, $V_{ub} = 0.004$, $V_{cb} = 0.0392$, $\sin\theta_{12} = 0.569$, and $\sin\theta_{13} = 0.0653$.

Note that the numerical value of m_b is not in perfect agreement with the experimental value $m_b = 4.20^{+0.17}_{-0.07}$ GeV [40]. In order to fix this, the finite gluino and chargino loop corrections [41] are required to be included in the down-type quark masses (m_d, m_s, m_b). The total contributions are denoted as (1+Δ_d), (1+Δ_s), and (1+Δ_b). These corrections are proportional to the supersymmetric particle spectrum: $\Delta_b \approx \tan\beta\left(\frac{2\alpha_3}{3\pi}\frac{\mu M_{\tilde{g}}}{m^2_{\tilde{b}_L}-m^2_{\tilde{b}_R}}\left[f(m^2_{\tilde{b}_L}/M^2_{\tilde{g}})\ -f(m^2_{\tilde{b}_R}/M^2_{\tilde{g}})\right] + \frac{\lambda_t^2}{16\pi^2}\frac{\mu A_t}{m^2_{\tilde{t}_L}-m^2_{\tilde{t}_R}}\left[f(m^2_{\tilde{t}_L}/\mu^2) - f(m^2_{\tilde{t}_R}/\mu^2)\right]\right)$, where $f(x) = \ln(x)/(1-x)$ and the first (second) term refers to the gluino (chargino) correction. Similar expressions exist for Δ_s and Δ_d, but without the chargino contribution and $\tilde{b} \to \tilde{s}, \tilde{d}$. If the chargino loop corrections are negligible and $m_{\tilde{d}}$, $m_{\tilde{s}}$, and $m_{\tilde{b}}$ are degenerate, the equality relation $\Delta_d = \Delta_s = \Delta_b$ is approximately satisfied. In order to get a better fitting for down-type quark masses, let us take $\Delta_d = \Delta_s = \Delta_b = -0.17$, which gives $m_d(2\text{ GeV}) = 6.24$ MeV,

$m_s^0(2\text{ GeV}) = 109.65$ MeV, and $m_b(m_b) = 4.31$ GeV. The comparison of the model predictions and experimental data at the low scale is summarized in Table 2.3, where the quark and charged lepton masses, the CKM mixing angles (V_{ub}, V_{us}, V_{cb}), the neutrino mixing angles ($\sin\theta_{sol}$, $\sin\theta_{atm}$, $\sin\theta_{13}$), and the CP violation parameter (η) are taken from [40]. The masses are all in GeV. Although the model here predicts $m_u(GUT) = 0$, the quantity $\overline{m}_{ud} = (m_u + m_d)/2$ is considered in Table 2.3, where it is assumed that the tiny up quark mass at GUT scale may be generated either by including the coupling $16_i 16_i 10_H$ into the model or by considering higher dimensional operators. If $m_u(2\text{ GeV}) = 2.4$ MeV, the model predictions of the quantities $\overline{m}_{ud}$ and $\frac{m_s}{\overline{m}_{ud}}$, which are well-known from lattice calculations [42], are given in Table 2.3. The asterisks in Table 2.3 indicate that the model predictions of neutrino mixing angles are obtained after including the neutrino sector in section 2.5.

2.4 Right Handed Neutrino Mass Structure

So far, the model gives excellent agreement with the known values for the CKM mixings, the quark masses, the charged lepton masses, the CP violation parameter, and the neutrino mixing angles ($\sin\theta_{12}$ and $\sin\theta_{13}$). However, the whole picture is still not complete and the following question arises. What is the appropriate light neutrino mass matrix ($M_\nu = -M_N^T M_R^{-1} M_N$) that gives not only the correct contribution to the atmospheric angle, but also the correct neutrino mass differences: $\Delta m_{21}^2 = (7.59 \pm 0.2) \times 10^{-5}\text{eV}^2$, $|\Delta m_{32}^2| = (2.43 \pm 0.13) \times 10^{-3}\text{eV}^2$ [37]? In other words, we are looking for a suitable structure of right-handed neutrino mass matrix M_R since M_N is fixed. Recall that the MNS mixing matrix is given by

$$U_{MNS} = U_L^\dagger U_\nu, \qquad (2.27)$$

where U_L and U_ν are the unitary matrices needed to diagonalize the Hermitian lepton matrix $M_L^\dagger M_L$ and the light neutrino matrix M_ν, respectively.

$$M_L^{diag\dagger} M_L^{diag} = U_L^\dagger M_L^\dagger M_L U_L, \quad M_\nu^{diag} = U_\nu^T M_\nu U_\nu, \tag{2.28}$$

where M_ν is assumed to be real and symmetric. The Dirac neutrino mass matrix M_N in Eq. (2.23) has vanishing first row and column, and the same is true for M_ν. So the matrix required to diagonalize M_ν is simply a rotation in the 2-3 plane by an angle θ_ν, while $U_L^\dagger$ is determined numerically from the charged lepton mass matrix. Thus, the mixing matrix of neutrinos is given by

$$U_{MNS} = \begin{pmatrix} -0.14 - 0.81\mathrm{i} & 0.13 + 0.55\mathrm{i} & 0.065 \\ 0.25 + 0.06\mathrm{i} & 0.34 - 0.04\mathrm{i} & 0.90 \\ -0.51 & -0.75 & 0.42 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_\nu & \sin\theta_\nu \\ 0 & -\sin\theta_\nu & \cos\theta_\nu \end{pmatrix}. \tag{2.29}$$

One can conclude that the correct contribution of the neutrino sector to the atmospheric angle is around θ_ν=-20^o. For example, if we take θ_ν=-20^o, the neutrino mixing angles ($\sin\theta_{atm}$, $\sin\theta_{sol}$, $\sin\theta_{13}$) become (0.707, 0.53, 0.21). In order to find the suitable right-handed neutrino mass structure, one can easily prove the inverse of the see-saw relation,

$$M_R = -M_N U_\nu (M_\nu^{diag})^{-1} U_\nu^T M_\nu^T. \tag{2.30}$$

A similar technique was used in Ref [43]. Note that one of the eigenvalues of M_ν is zero (i.e. M_ν^{diag} is singular), so the inverse of M_ν^{diag} does not exist. To overcome this problem, one can generally define M_ν^{diag}=diag(m_1, m_2, m_3), and m_1 will not appear in M_R. By using the numerical result of M_N, θ_ν=-20^o, and m_2/m_3=0.178, the right-handed mass structure can be presented numerically.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.0186 & -0.13 \\ 0 & -0.13 & 1 \end{pmatrix}. \tag{2.31}$$

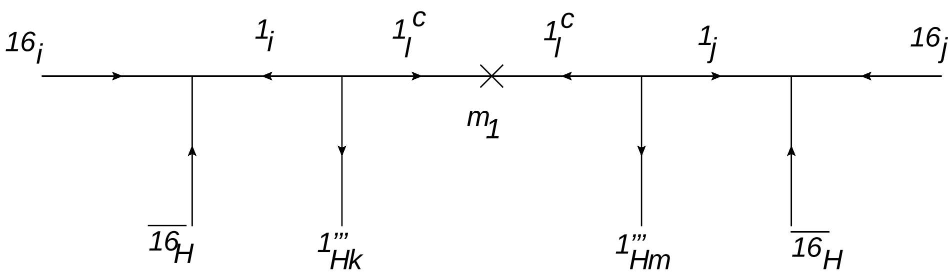


Figure 2.4: This figure leads to the right-handed neutrino mass matrix.

From the above numerical mass matrix, one concludes $(M_R)_{23} \times (M_R)_{23} \approx (M_R)_{22}$, so to a good approximation, the above numerical structure can be represented analytically as follows:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & r^2 & ar \\ 0 & ar & 1 \end{pmatrix}. \tag{2.32}$$

The constant a should not be equal to 1 because then M_R would be singular. Now our mission is to find the Yukawa couplings that respect the symmetry of the model and lead to an analytical structure similar to Eq.(2.32). This can be accomplished by considering the following Yukawa couplings represented by the Feynman diagram in Fig.2.4, i.e.,

$$W_4 = 16_i\overline{16}_H 1_i + h_{ijk} 1_i 1_j^c 1'''_{Hk} + m_1 1_i^c 1_i^c, \tag{2.33}$$

where two fermion singlets 1_i and 1_i^c, which couple with the singlet Higgs $1'''_{iH}$, have been introduced (their transformation under $SO(10)\times A_4$ and the additional symmetry are shown in Tables 2.1-2.2). The product of the three triplets of the second term in Eq. (2.33) that transform as a singlet under A_4 is given by $h_1(N_1N_2^c\alpha_3 + N_2N_3^c\alpha_1 + N_3N_1^c\alpha_2) + h_2(N_1N_3^c\alpha_2 + N_3N_2^c\alpha_1 + N_2N_1^c\alpha_3)$, where α_1, α_2, and α_3 are the VEV's components of $1'''_{iH}$. By assuming $h_1{=}h_2$, Fig.2.4 leads to the desired right handed-neutrino mass structure.

$$M_R = \Lambda \begin{pmatrix} \frac{\alpha_1^2}{\alpha_3^2} & \alpha_1\alpha_2(\frac{-1}{\alpha_3^2} + \frac{2}{\alpha_1^2+\alpha_2^2+\alpha_3^2}) & \frac{-\alpha_1(\alpha_1^2-\alpha_2^2+\alpha_3^2)}{\alpha_3(\alpha_1^2+\alpha_2^2+\alpha_3^2)} \\ \alpha_1\alpha_2(\frac{-1}{\alpha_3^2} + \frac{2}{\alpha_1^2+\alpha_2^2+\alpha_3^2}) & \frac{\alpha_2^2}{\alpha_3^2} & \frac{-\alpha_2(-\alpha_1^2+\alpha_2^2+\alpha_3^2)}{\alpha_3(\alpha_1^2+\alpha_2^2+\alpha_3^2)} \\ \frac{-\alpha_1(\alpha_1^2-\alpha_2^2+\alpha_3^2)}{\alpha_3(\alpha_1^2+\alpha_2^2+\alpha_3^2)} & \frac{-\alpha_2(-\alpha_1^2+\alpha_2^2+\alpha_3^2)}{\alpha_3(\alpha_1^2+\alpha_2^2+\alpha_3^2)} & 1 \end{pmatrix}. \quad (2.34)$$

By comparing the 2-3 block of the above structure with the mass structure in Eq. (2.32), one can see the constant a is equivalent to the quantity $((-\alpha_1^2 + \alpha_2^2 + \alpha_3^2)/(\alpha_1^2 + \alpha_2^2 + \alpha_3^2))$, which is equal to 1 in the limit $\alpha_1 \to 0$. So, let us expand the eigenvalues of the right handed neutrino mass structure in Eq(2.34) around α_1.

$$\begin{aligned} M_{R1} &= 1 + \frac{\alpha_2^2}{\alpha_3^2} + \frac{\alpha_1^2(\alpha_2^4 - 6\alpha_2^2\alpha_3^2 + \alpha_3^4)}{\alpha_3^2(\alpha_2^2 + \alpha_3^2)^2} + \mathcal{O}(\alpha_1^4), \\ M_{R2} &= \frac{4\alpha_1^2\alpha_2^2}{(\alpha_2^2 + \alpha_3^2)^2} - \frac{8\alpha_1^3\alpha_2^3\alpha_3}{(\alpha_2^2 + \alpha_3^2)^{7/2}} + \mathcal{O}(\alpha_1^4), \\ M_{R3} &= \frac{4\alpha_1^2\alpha_2^2}{(\alpha_2^2 + \alpha_3^2)^2} + \frac{8\alpha_1^3\alpha_2^3\alpha_3}{(\alpha_2^2 + \alpha_3^2)^{7/2}} + \mathcal{O}(\alpha_1^4). \end{aligned} \quad (2.35)$$

One can see that two of the right-handed neutrino masses are approximately degenerate for small values of α_1 (i.e. $M_{R2} \approx M_{R3}$). By setting (α_1, α_2, α_3, Λ)=(-0.05, 0.125, 0.994, 8.42×10^{15}), the numerical fit for the neutrino mixing angles, the light neutrino masses, and the right handed-neutrino masses are obtained as follows:

$$\begin{aligned} m_1 &= 0 \text{ eV}, & \sin\theta_{\text{sol}} &= 0.551, & \text{M}_{\text{R1}} &= 8.57 \times 10^{15} \text{ GeV}, \\ m_2 &= 0.01 \text{ eV}, & \sin\theta_{\text{atm}} &= 0.776, & \text{M}_{\text{R2}} &= 1.3 \times 10^{12} \text{ GeV}, \\ m_3 &= 0.056 \text{ eV}, & \sin\theta_{13} &= 0.154, & \text{M}_{\text{R3}} &= 1.28 \times 10^{12} \text{ GeV}. \end{aligned}$$

As can be seen from Table 2.3, the masses and mixing angles of the quarks and leptons after including the neutrino sector are predicted in this model to be within 2σ error bars of their experimental values.

	Model predictions	Experiment	Pull
$m_e(m_e)$	0.511×10^{-3}	0.511×10^{-3}	...
$m_\mu(m_\mu)$	105.6×10^{-3}	105.6×10^{-3}	...
$m_\tau(m_\tau)$	1.776	1.776	...
$\overline{m}_{ud}$	4.32×10^{-3}	$(3.85\pm0.52)\times10^{-3}$	0.9
$m_c(m_c)$	1.4	$1.27^{+0.07}_{-0.11}$	1.85
$m_t(m_t)$	172.5	171.3 ± 2.3	0.52
$\frac{m_s}{\overline{m}_{ud}}$	25.36	27.3 ± 1.5	1.29
$m_s(2Gev)$	109.6×10^{-3}	$105^{+25}_{-35}\times10^{-3}$	0.184
$m_b(m_b)$	4.31	$4.2^{+0.17}_{-0.07}$	0.58
V_{us}	0.2264	0.2255 ± 0.0019	0.473
V_{cb}	39.2×10^{-3}	$(41.2\pm1.1)\times10^{-3}$	1.82
V_{ub}	4.00×10^{-3}	$(3.93\pm0.36)\times10^{-3}$	0.194
η	0.3569	$0.349^{+0.015}_{-0.017}$	0.526
$\sin\theta_{12}^{sol}$	0.551	0.566 ± 0.018	0.83
$\sin\theta_{23}^{atm}$	0.776	0.707 ± 0.108	0.63
$\sin\theta_{13}$	0.154	< 0.22	-

Table 2.3: This Table shows the comparison of the model predictions at low scale and the experimental data.

CHAPTER 3

Flavor Violation in a Minimal $SO(10) \times A_4$ SUSY GUT

Flavor changing neutral current (FCNC) processes impose severe constraints on the soft supersymmetric breaking (SSB) sector of the minimal supersymmetric standard model (MSSM). The simplest way to satisfy the FCNC constraints is to adopt universality in the scalar masses at a high energy scale where the effects of supersymmetry (SUSY) breaking in the hidden sector is communicated to the scalar masses of MSSM via gravitational interactions. For example, in the the minimal supergravity model (mSUGRA) [44] the MSSM is a valid symmetry between the weak scale and grand unification scale (M_{GUT}) at which the universality conditions are assumed to hold. In this case, the leptonic flavor violation (LFV) is not induced. However, in a different class of models studied in Refs [45, 46, 47, 48, 49, 50] the universality of the scalar masses will be broken by radiative corrections. Consequently, FCNC will be induced in these models as discussed below.

If the universality conditions hold at the grand unification scale M_{GUT}, the LFV is induced below GUT scale by radiative corrections in the MSSM with right-handed neutrino [45, 46, 47] or SUSY-$SU(5)$ [48] models. Unfortunately, it is difficult to predict LFV decay rates in these models because the Dirac neutrino Yukawa couplings are arbitrary within MSSM. However, in an $SO(10)$ GUT model, we can predict the LFV decay rates below the GUT scale because the Dirac neutrino couplings are related to the up-type quark Yukawa couplings and are thus fixed.

The FCNC could also be induced above the GUT scale by radiative corrections.

It was shown that as a consequence of the large top Yukawa coupling at the unification scale, SUSY GUTs with universality conditions valid at the scale M^*, where $M_{\rm GUT} < M^* \leq M_{\rm Planck}$, predict lepton flavor violating processes with observable rates [49, 50]. The experimental search for these processes provides a significant test for supersymmetric grand unification theory (SUSY GUT). Both contributions of FCNC that are induced above and below $M_{\rm GUT}$ will be studied in our model.

In this chapter, the flavor violation processes for charged lepton and quark sectors are investigated in the framework of a realistic SUSY GUT model based on the gauge group $SO(10)$ and a discrete non-abelian A_4 flavor symmetry [51]. This model is realistic because it successfully describes the fermion masses, CKM mixings and neutrino mixing angles. This work differs from other studies in several aspects. First, it is different from those based on MSSM with right-handed neutrino masses or SUSY $SU(5)$ in the sense that the Dirac neutrino Yukawa couplings are determined from the fermion masses and mixing fit of the $SO(10) \times A_4$ model. Thus, this model predicts the lepton flavor violation arising from the renormalization group (RG) running from $M_{\rm GUT}$ to the right-handed neutrino mass scales. Second, it is different from those based on SUSY $SO(10)$ studied in [52] in the sense that the FCNC processes are closely tied to fermion masses and mixings. Finally, in the $SO(10) \times A_4$ model flavor violation is induced at the GUT scale at which A_4 symmetry is broken due to large (order one) mixing of the third generation of MSSM fields (ψ_3) with the exotic heavy fields (χ_i, i runs from 1 to 3). This large mixing arises when the A_4 flavor symmetry is broken at the GUT scale. This is different from the case where the flavor violation is induced due to large top Yukawa coupling at the GUT scale [49, 50]. The reason for introducing the exotic heavy fermion fields in our model is to obtain the correct fermion mass relations at the GUT scale as we shall see in section 1. The mass scales of these exotic fields range from 10^{14} GeV to 10^{18} GeV depending on the values of the Yukawa couplings and the scale of A_4 flavor symmetry breaking.

In this chapter we study flavor violation of the hadronic and leptonic processes by calculating the flavor violating scalar fermion mass insertion parameters $(\delta_{AB})_{ij} = \frac{(m^2_{AB})_{ij}}{\tilde{m}^2}$, for $(A,B) = (L,R)$, with $\tilde{m}$ being the average mass of the relevant scalar partner of standard model fermions (sfermions). All the flavor violation sources are included in our calculations. The sfermion mass insertions, $\delta_{LL,RR,LR}$, arise from the large mixing between the ψ_3 and χ_i and the mass insertions, $(\delta^{ij}_{LL})^{RHN}$, arise from RG running from $M_{\rm GUT}$ to the right-handed neutrino mass scales. These scalar mass insertion parameters are analyzed in the framework of our model; then they are compared with their experimental upper bounds. We found that the most stringent constraint on flavor violation comes from the $\mu \to e\gamma$ process. This constraint requires a high degree of degeneracy of the soft masses of MSSM fields and the exotic fields. Therefore, in this model we assume that these soft masses are universal at the scale M^* with $M^* > M_{\rm GUT}$, then we run them down to the GUT scale. The branching ratio $Br(\mu \to e\gamma)$ close to experimental bound (i.e. Br($\mu \to e\gamma$)=1.2×10^{-11}) is obtained when the slepton masses of order 1 TeV , while the Yukawa couplings remain perturbative at the scale M^*. We also found in the framework of our model that once the constraint from $Br(\mu \to e\gamma)$ is satisfied, all the FCNC processes will be automatically consistent with experiments.

This chapter is organized as follows. In section 1, we show how the fermion mass matrices are constructed in $SO(10) \times A_4$ model. In section 2, we discuss the sources of flavor violation by finding the sfermion mass insertion parameters $\delta^{ij}_{LL,RR}$ at the GUT scale at which A_4 symmetry is assumed to be broken as well as below the GUT scale. The results of the $SO(10) \times A_4$ model regarding flavor violation analysis are presented in section 4. Section 5 has our conclusion. The derivation of the light fermion mass matrices and the light neutrino mass matrix after disentangling the exotic fermions is shown in appendix A. In appendix B, we list the renormalization group equations (RGEs) for various SUSY preserving and breaking parameters between $M_{\rm GUT}$ and

M^* relevant for FCNC analysis.

3.1 A Brief Review of Minimal $SO(10) \times A_4$ SUSY GUT

In the $SO(10)$ gauge group, all the quarks and leptons of the SM are naturally accommodated within a 16-dimensional irreducible representation. However, minimal $SO(10)$ (i.e., with only one 10-dimensional Higgs representation) leads to fermion mass relations at the GUT scale, such as $\frac{m_c^0}{m_t^0} = \frac{m_s^0}{m_b^0}$ and $m_\mu^0 = m_s^0$, that are inconsistent with experiment. This can be fixed by introducing exotic $16 + \overline{16}$ fermions and by coupling 16_i with these exotic fields via 45_H, which is used for $SO(10)$ symmetry breaking. The non-abelian discrete A_4 symmetry is chosen in our model because it is the smallest group that has a 3-dimensional representation, so the three generations of SM fields transform as triplet under A_4. Besides, FCNC is not induced in the SUSY-$SO(10) \times A_4$ as long as A_4 symmetry is preserved. However, as we will see later, the breaking of A_4 symmetry at the GUT scale will reintroduce the FCNC via large mixing between the exotic and light fields. Based on the above reasons, a $SO(10) \times A_4$ model is proposed in [51]. In this model, a minimal set of Higgs representations are used to break the $SO(10)$ gauge group to the SM gauge group so the unified gauge coupling remains perturbative all the way to the Planck scale. Employing this minimal Higgs representation and A_4 symmetry, our model successfully accommodates small mixings of the quark sector and large mixings of the neutrino sector in the unified framework as shown summarized below.

The fermion mass matrices of the model proposed in [51] were constructed approximately. In this section, we construct these matrices by doing the algebra exactly and show that the excellent fit for fermion masses and mixings is obtained by slightly modifying the numerical values of the input parameters of Ref.[51]. There are two superpotentials of the model. The first one ($W_{spin.}$) describes the couplings of the standard model fields ($\psi_i(16_i)$, i runs from 1-3) with the exotic heavy spinor-antispinor

$SO(10)$	ψ_i	χ_1,$\overline{\chi}_1$	χ_2,$\overline{\chi}_2$	χ_3,$\overline{\chi}_3$	Z_i^c
A_4	3	1	1	1	3
$Z_2 \times Z_4 \times Z_2$	+,+,+	+,-,+	-,+,+	+,+,-	+,+,+
$SO(10)$	ϕ_i	ϕ'_i	ϕ''_i	ϕ'''_i	Z_i
A_4	3	3	3	3	3
$Z_2 \times Z_4 \times Z_2$	+,i,+	+,$-i$,+	+,i,-	+,$-i$,-	+,$-i$,+

Table 3.1: The transformation of the matter fields under $SO(10)\times A_4$ and $Z_2\times Z_4\times Z_2$.

$SO(10)$	10_H	45_H	16_H	$\overline{16}_H$	1_{Hi}	$1'_{Hi}$	$1''_{Hi}$	$1'''_{Hi}$
A_4	1	1	1	1	3	3	3	3
$Z_2 \times Z_4 \times Z_2$	-,+,-	+,-,-	+,$-i$,+	+,$-i$,+	+,-,+	-,+,+	+,+,-	+,i,+

Table 3.2: The transformation of the Higgs fields under $SO(10)\times A_4$ and $Z_2\times Z_4\times Z_2$.

fields ($\chi_i(16_i)$, $\overline{\chi}_i(\overline{16}_i)$, i runs from 1 to 3), while the second one ($W_{vect.}$) describes the couplings of ψ_i with the exotic 10-vector fields (ϕ_i, ϕ'_i, ϕ''_i, ϕ'''_i, i runs from 1 to 3) as given below:

$$W_{spin.} = b_1\psi_i\overline{\chi}_1 1_{Hi} + b_2\psi_i\overline{\chi}_2 1'_{Hi} + k_1\chi_1\overline{\chi}_3 45_H + a\chi_3\chi_2 10_H + M_\alpha\chi_\alpha\overline{\chi}_\alpha, \quad (3.1)$$

$$W_{vect.} = b_3\psi_i\phi_i 16_H + M_{10}\phi_i\phi'_i + h'_{ijk}\phi'_i\phi'_j 1_{Hk} + h_{ijk}\phi_i\phi_j 1_{Hk} + A_{ijk}\phi'_i\phi''_j 1''_{Hk} + m\phi''_i\phi'''_i + k_2\phi'''_i\phi'_i 45_H. \quad (3.2)$$

The above superpotentials are invariant under A_4 and the additional symmetry $Z_2 \times Z_4 \times Z_2$. The transformations of the matter fields (i.e., the ordinary and exotic fermion fields) and the Higgs fields under the assigned symmetry are given in Table 3.1 and 3.2.

The general fermion mass matrix structure that results from integrating out the exotic heavy spinor-antispinor fields in $W_{spin.}$ is:

$$M_F(spin.) = \left(\frac{aT_1T_2T_3f^2\langle 10_H\rangle}{r_Fr_{F^c}}\right)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Q_Fs_\theta\frac{r_{F^c}}{f} \\ 0 & Q_{F^c}s_\theta\frac{r_F}{f} & (Q_F+Q_{F^c})c_\theta \end{pmatrix}, \tag{3.3}$$

where we have made the following transformation: $\psi_1\epsilon_1 + \psi_2\epsilon_2 + \psi_3\epsilon_3 = \epsilon\psi_3'$ and $\psi_1s_1 + \psi_2s_2 + \psi_3s_3 = S(\psi_2's_\theta + \psi_3'c_\theta)$. Here ϵ_i and s_i are VEV-components of $\langle 1_H\rangle$ and $\langle 1_H'\rangle$ respectively and $s_\theta(c_\theta)$ is $\sin\theta(\cos\theta)$. $f = (1 + T_2^2 + T_1^2(1 + s_\theta^2T_2^2))^{-1/2}$ and $r_F = (1 + Q_F^2T_3^2T_1^2(1 + s_\theta^2T_2^2)f^2)^{1/2}$ are factors that come from doing the algebra exactly (see appendix A). Here $T_1 = \frac{b_1\epsilon}{M_1}$, $T_2 = \frac{b_2S}{M_2}$, $T_3 = \frac{k_1\Omega}{M_3}$ and $Q = 2I_{3R} + \frac{6}{5}\delta(\frac{Y}{2})$ is the unbroken charge that results from breaking $SO(10)$ to the SM gauge group by giving a VEV to 45_H, where $\langle 45_H\rangle = \Omega Q$. The charge Q for different quarks and leptons is given as.

$$\begin{aligned} Q_u = Q_d = \frac{1}{5}\delta, \quad Q_{u^c} = -1 - \frac{4}{5}\delta, \quad Q_{d^c} = 1 + \frac{2}{5}\delta, \\ Q_l = Q_\mu = -\frac{3}{5}\delta, \quad Q_{l^c} = 1 + \frac{6}{5}\delta, \quad Q_{\nu^c} = -1. \end{aligned} \tag{3.4}$$

The above general structure of fermion mass matrix has the following interesting features: (1) The relation $m_b^0 = m_\tau^0$ automatically follows from $Q_d + Q_{d^c} = Q_e + Q_{e^c}$, (2) The hierarchy of the the second and third masses generation is obtained by taking the limit $s_\theta \to 0$, and (3) The approximate Georgi-Jarlskog relation $m_\mu^0 = 3m_s^0$ leads to two possible values for δ, either $\delta \to 0$ or $\delta \to -1.25$, (4) the former possibility is excluded by experiment since it leads to $(m_c^0/m_t^0)/(m_s^0/m_b^0) \to 1$ at the GUT scale, while the latter possibility leads to $(m_c^0/m_t^0)/(m_s^0/m_b^0) \to 0$ which is closer to experiments. Let us define $\delta = 1 + \alpha$. The masses and mixings of the first generation arise from W_{vector}. The full mass matrices arising from W_{spinor} and W_{vector} have the following form:

$$
\begin{aligned}
M_D &= m_d^0 \begin{pmatrix} 0 & (c_{12}+\delta_3(\frac{3+2\alpha}{5}))r_d r_{d^c} & (-2\delta_2(\frac{3+2\alpha}{5})+\zeta)r_{d^c} \\ (c_{12} & 0 & (2\delta_1(\frac{3+2\alpha}{5}) \\ -\delta_3(\frac{3+2\alpha}{5}))r_d r_{d^c} & & +s(\frac{-1+\alpha}{5})+\beta)r_{d^c} \\ \zeta r_d & (s(\frac{3+2\alpha}{5})+\beta)r_d & 1 \\ & & -2(\beta+\frac{3+2\alpha}{5}\delta_1)fc_\theta s_\theta T_2^2 \end{pmatrix}, \\
M_U &= m_u^0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (\frac{1-\alpha}{5})sr_{u^c} \\ 0 & (\frac{1+4\alpha}{5})sr_u & 1 \end{pmatrix}, \qquad (3.5) \\
M_L &= m_d^0 \begin{pmatrix} 0 & (c_{12}+3\delta_3(\frac{-1+\alpha}{5}))r_e r_{e^c} & (-\delta_2\alpha+\zeta)r_{e^c} \\ (c_{12} & 0 & (\delta_1\alpha \\ -3\delta_3(\frac{-1+\alpha}{5}))r_e r_{e^c} & & -3s(\frac{-1+\alpha}{5})+\beta)r_{e^c} \\ (\zeta & (s(\frac{-1+6\alpha}{5})+\delta_1(\frac{6-\alpha}{5}) & 1 \\ -\delta_2\frac{6-\alpha}{5})r_e & +\beta)r_e & -2(\beta+\frac{3+2\alpha}{5}\delta_1)fc_\theta s_\theta T_2^2 \end{pmatrix}, \\
M_N &= m_u^0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (\frac{-3+3\alpha}{5})sr_{\nu^c} \\ 0 & sr_\nu & 1 \end{pmatrix},
\end{aligned}
$$

where the parameters are defined in terms of the Yukawa couplings of the superpotential ($W_{spin.}+W_{vect.}$) and the VEVs of the Higgs fields as shown in appendix A. These matrices are multiplied by left-handed fermions on the right and right-handed fermions on the left. A doubly lopsided structure for the charged lepton and down quark mass matrices of Eq.(3.5) can be obtained by going to the limit $\beta,\zeta,\alpha,\delta_3,c_{12},s \ll 1$ and δ_1,δ_2 are of order one. This doubly lopsided form leads simultaneously to large neutrino mixing angles and to small quark mixing angles. Based only on the above fermion mass matrices in Eq.(3.5), an excellent fit is found for fermion masses (except for the neutrino masses), quark mixing angles and neutrino mixing angles (except the atmospheric angle) by giving the input parameters, appearing in Eq.(3.5), the following numerical values: $\delta_1 = -1.28$, $\delta_2 = 1.01$, $\delta_3 = 0.015 \times e^{4.95i}$, $\alpha = -0.0668$, $s = 0.2897$, $\zeta = 0.0126$, $c_{12} = -0.0011e^{1.124i}$, and $\beta = -0.11218$. The above numerical values lead to $\sin\theta^L_{23} = 0.92$ which is not close to the experimental central value of atmospheric angle $\sin\theta^{atm}_{23} = 0.707$ [7]. This contribution to the atmospheric angle is only from the charged lepton sector. Therefore, the neutrino sector should be included by considering the following superpotential:

$$W_N = b_4\psi_i Z_i \overline{16}_H + h_{ijk} Z_i Z^c_j 1'''_{Hk} + m_1 Z^c_i Z^c_i, \tag{3.6}$$

where two fermion singlets Z_i and Z^c_i that couple with the Higgs singlet $1'''_{Hk}$ have been introduced.

The full neutrino mass matrix is constructed in Appendix B. The Higgs singlet $1'''_{Hk}$ has the VEV-components (α_1, α_2, α_3). The light neutrino mass matrix is obtained by employing the see-saw mechanism. The numerical values ($\alpha_1 = 0.075$, $\alpha_2 = 0.07$, $\alpha_3 = 0.9$, and $\lambda = 0.0465$ eV), where λ is defined in appendix B, lead to not only the correct contribution to the atmospheric angles ($\sin\theta^{atm}_{23} = 0.811$) but also to the correct light neutrino mass differences. The predictions of the fermion masses and mixings are slightly altered by doing the algebra exactly compared to the analysis of Ref.[51]. These predictions and their updated experimental values obtained from [7] are shown in Table 3.3. The right handed-neutrino masses arise from integrating out the exotic fermion singlets Z_i and Z^c_i in Eq.(3.6). The right handed-neutrino mass matrix is

$$M_R = \Lambda \begin{pmatrix} \frac{\alpha_1^2}{\alpha_3^2} & \alpha_1\alpha_2(\frac{-1}{\alpha_3^2} + \frac{2}{\alpha_1^2+\alpha_2^2+\alpha_3^2}) & \frac{-\alpha_1(\alpha_1^2-\alpha_2^2+\alpha_3^2)}{\alpha_3(\alpha_1^2+\alpha_2^2+\alpha_3^2)} \\ \alpha_1\alpha_2(\frac{-1}{\alpha_3^2} + \frac{2}{\alpha_1^2+\alpha_2^2+\alpha_3^2}) & \frac{\alpha_2^2}{\alpha_3^2} & \frac{-\alpha_2(-\alpha_1^2+\alpha_2^2+\alpha_3^2)}{\alpha_3(\alpha_1^2+\alpha_2^2+\alpha_3^2)} \\ \frac{-\alpha_1(\alpha_1^2-\alpha_2^2+\alpha_3^2)}{\alpha_3(\alpha_1^2+\alpha_2^2+\alpha_3^2)} & \frac{-\alpha_2(-\alpha_1^2+\alpha_2^2+\alpha_3^2)}{\alpha_3(\alpha_1^2+\alpha_2^2+\alpha_3^2)} & 1 \end{pmatrix}, \tag{3.7}$$

where $\Lambda = 8.45 \times 10^{15}$ GeV and the right-handed neutrino masses are given by $M_{R1} \approx M_{R2} \approx 1.4 \times 10^{12}$ GeV and $M_{R3} = 8.5 \times 10^{15}$ GeV.

Another interesting feature of this model is that it contains a minimal set of Higgs fields needed to break $SO(10)$ to the SM gauge group. Consequently, the unified gauge coupling remains perturbative all the way up to the Planck scale. This can be understood from the running of the unified gauge coupling with energy scale $\mu > M_{\rm GUT}$ as

$$\frac{1}{\alpha} = \frac{1}{\alpha_G} - \frac{b_G}{2\pi}\log(\frac{\mu}{M_{\rm GUT}}), \tag{3.8}$$

where $\alpha = g^2/(4\pi)$ and $b_G = S(R) - 3C(G)$. Here $C(G)$ is the quadratic Casimir invariant and $S(R)$ is the Dynkin index summed over all chiral multiplets of the model. The unified gauge coupling stays perturbative at the Planck scale (i.e $g(M_P) < \sqrt{2}$) as long as $b_G < 26$. Employing large Higgs representations might lead to $b_G \geq 26$. For example, using $126_H + \overline{126}_H$ gives $b_G = 46$. On the other hand, the $SO(10) \times A_4$-model gives $b_G = 19$ which is consistent with the unified gauge coupling being perturbative till the Planck scale.

We will use the same fit for fermion masses and mixings to calculate the mass insertion parameters $\delta^{ij}_{LL,RR}$, and $\delta^{ij}_{LR,RL}$ in the quark and lepton sectors and consequently investigate the FCNC in this model. The charged lepton and down quark mass matrices in Eq.(3.5) are diagonalized at the GUT scale by bi-unitary transformation:

$$M^{diag.}_{d,l} = V^{\dagger d,l}_R M_{D,L} V^{d,l}_L, \tag{3.9}$$

where $V^{u,d,l}_{R,L}$ are known numerically. Now, we discuss the sources of FCNC in this model.

3.2 Sources of Flavor Violation in $SO(10) \times A_4$ Model

We assume in our flavor violation analysis that A_4 flavor symmetry is preserved above GUT scale and it is only broken at GUT scale. In this case flavor violation is induced at GUT scale where A_4 symmetry is broken. In this section we discuss the flavor violation induced at the GUT scale by studying the sfermion mass insertion parameter $\delta^{ij}_{LL,RR}$ and the chirality flipping mass insertion (A-terms) parameter $\delta^{ij}_{LR,RL}$. We will see that these flavor violation sources arise from large mixing of the light fields with the heavy fields. This large mixing is due to the breaking of A_4 symmetry. In addition, we discuss the induced flavor violation arising below GUT scale through the RG running from $M_{\rm GUT}$ to the right-handed neutrino mass scales.

3.2.1 The Scalar Mass Insertion Parameters

Let us assume the soft supersymmetry breaking terms originate at the messenger scale M^*, where $M_{\rm GUT} < M^* \leq M_{\rm Planck}$. The quadratic soft mass terms of the matter superfields that appear in the superpotential $W_{spin.}$ are

$$-\mathcal{L} = \tilde{m}^2_\psi \psi^\dagger_i \psi_i + \tilde{m}^2_{\chi_i} \chi^\dagger_i \chi_i + \tilde{m}^2_{\overline{\chi}_i} \overline{\chi}^\dagger_i \overline{\chi}_i. \tag{3.10}$$

The MSSM scalar fermions that reside in ψ_i transform as triplets under the non-abelian A_4 symmetry. Since the A_4 symmetry is intact, they have common mass ($\tilde{m}^2_\psi$) at the scale M^*. On the other hand, the exotic fields each of which transforms as singlet under A_4 symmetry have different masses ($\tilde{m}^2_{\chi_i}$, $\tilde{m}^2_{\overline{\chi}_i}$, i runs 1-3) at the scale M^*.

The MSSM scalars remain degenerate above the GUT scale where the A_4 symmetry is broken. In order to find the scalar masses in the fermion mass eigenstates, two transformations are required. The first transformation is needed to block-diagonalize the fermion mass matrix into a light and a heavy blocks as shown in Appendix A. The upper left corner represents the 3×3 light fermions mass matrix. The second transformation is the complete diagonalization of the light fermion mass matrix. Applying the first transformation to the quadratic soft mass terms of Eq.(3.10) by going to the new orthogonal basis (L_2, L_3, H_1, H_2, H_3) as defined in appendix A, the quadratic soft mass matrix of the light states is transformed as follows:

$$\tilde{m}_{\psi}^2 I \rightarrow \tilde{m}_{\psi}^2 I + \delta\tilde{m}_{\psi}^2, \tag{3.11}$$

where,

$$\delta\tilde{m}_{\psi}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & \delta \end{pmatrix}, \tag{3.12}$$

$\epsilon = \frac{f}{r_F} T_2^2 s_\theta (\tilde{m}_{\chi_2}^2 - \tilde{m}_{\psi}^2)$, $\delta = ((\frac{f}{r_F})^2 - 1)\tilde{m}_{\psi}^2 + (\frac{f}{r_F})^2 (\tilde{m}_{\chi_1}^2 T_1^2 + \tilde{m}_{\chi_2}^2 T_2^2 + \tilde{m}_{\chi_3}^2 Q^2 T_1^2 T_3^2)$, and we have safely ignored the terms that contain $s_\theta^2 \ll 1$. It is obvious that the first two generations of the light scalars are almost degenerate because the mixing of the second light generation (L_2) with the heavy states is proportional to $s_\theta \ll 1$. On the other hand, since the mixing of the third light generation (L_3) with the heavy states is of order one, its mass splits from those of the first two generations.

The top Yukawa coupling is given in terms of T_1, T_2, and T_3 as:

$$Y_t = \frac{a f^2 (Q_u + Q_{u^c}) T_1 T_2 T_3}{r_{u^c} r_u}. \tag{3.13}$$

The numerical values of $T_1 = 0.0305$, $T_2 = 2$, $T_3 = 100$ and $a \sim 1.2$ are consistent with the top Yukawa coupling at the GUT scale to be of order $\lambda_t^{GUT} \sim 0.5$ and r_{u,u^c} to be of order one. Plugging these numerical values and $s_\theta = 0.0465$ into the expressions for ϵ and δ gives us:

$$\begin{aligned} (\delta_d, \delta_{d^c}, \delta_e, \delta_{e^c}) &= (0.81, 0.87, 0.88, 0.82)(\tilde{m}_{\chi}^2 - \tilde{m}_{\psi}^2), \\ (\epsilon_d, \epsilon_{d^c}, \epsilon_e, \epsilon_{e^c}) &= (0.061, 0.05, 0.048, 0.06)(\tilde{m}_{\chi}^2 - \tilde{m}_{\psi}^2). \end{aligned} \tag{3.14}$$

Here we have dropped $\tilde{m}_{\chi_1}^2$ terms because their coefficients are negligible. Also, the RGE expressions of $\tilde{m}_{\chi_2}^2$ and $\tilde{m}_{\chi_3}^2$ are the same (see Eq.(B.13)), so we have assumed that $\tilde{m}_{\chi_2}^2 = \tilde{m}_{\chi_3}^2 = \tilde{m}_{\chi}^2$.

The next step is to apply the second transformation by evaluating $V_L^{\dagger d,l} \delta m_{\psi}^2 V_L^{d,l}$ and similarly for $L \rightarrow R$. The unitary matrices $V_L^{d,l}$ are numerically known from the fitting for fermion masses and mixings. So, the mass insertion parameters for charged leptons and down quarks are given respectively by

$$(\delta^{d,e}_{LL,RR})_{ij} \quad = \quad (V^{\dagger d,l}_{L,R}\delta\tilde{m}^2_{d,l}V^{d,l}_{L,R})_{ij}/\tilde{m}^2_{d,l}. \tag{3.15}$$

The above mass insertion analysis without including the superpotential $W_{vect.}$ is good enough because we assumed in our analysis that the mixing of the 10 vector multiplets with the ordinary spinor fields is small.

3.2.2 The Chirality Flipping Mass Insertion (A-terms)

The FV processes are also induced from the off-diagonal entries of the chirality flipping mass matrix $\tilde{M}_{RL}$. The chirality flipping soft terms are divided into two parts $\mathcal{L}_{spin}$ and $\mathcal{L}_{vect}$:

$$\begin{aligned} -\mathcal{L}_{spin} \quad &= \quad \tilde{b}_1 b_1 \tilde{\psi}_i \tilde{\bar{\chi}}_1 1_{Hi} + \tilde{b}_2 b_2 \tilde{\psi}_i \tilde{\bar{\chi}}_2 1'_{Hi} + \tilde{k}_1 k_1 \tilde{\chi}_1 \tilde{\bar{\chi}}_3 45_H \\ &\quad + \tilde{a} a \tilde{\chi}_3 \tilde{\chi}_2 10_H + \tilde{G}_i M_i \tilde{\chi}_i \tilde{\bar{\chi}}_i, \qquad (3.16) \\ -\mathcal{L}_{vect} \quad &= \quad \tilde{b}_3 b_3 \tilde{\psi}_i \tilde{\phi}_i 16_H + \tilde{B}_{10} M_1 0 \tilde{\phi}_i \tilde{\phi}'_i + \tilde{h}'_{ijk} h'_{ijk} \tilde{\phi}'_i \tilde{\phi}'_j 1_{Hk} + \tilde{h}_{ijk} h_{ijk} \tilde{\phi}_i \tilde{\phi}_j 1_{Hk} \\ &\quad + \tilde{A}_{ijk} A_{ijk} \tilde{\phi}'_i \tilde{\phi}''_j 1''_{Hk} + \tilde{g} m \tilde{\phi}''_i \tilde{\phi}'''_i + \tilde{k}_2 k_2 \tilde{\phi}'''_i \tilde{\phi}'_i 45_H. \qquad (3.17) \end{aligned}$$

The fourth term of Eq.(3.16) induces the off-diagonal elements of the chirality flipping mass matrix, if it is written in terms of the new orthogonal basis defined in Eqs.(A.1). This transformation can be represented by

$$\tilde{M}^2_{RL}(spin.) \rightarrow \tilde{a} M_F(spin.), \tag{3.18}$$

where $M_F(spin.)$ is defined in Eq.(3.3). The entire chirality flipping mass matrix in the new orthogonal basis is obtained by including $-\mathcal{L}_{vect}$. The bi-unitary transformations that block-diagonalize the full fermion mass matrix is applied on the entire chirality flipping mass matrix (see Appendix A). Accordingly, the 3×3 quadratic mass matrix ($\tilde{M}^2_{LR}$) associated with the light states is transformed as follows:

$$\tilde{M}^2_{RL} \rightarrow \tilde{a} M_F(spin.) + \tilde{b}_3 M_F(vector), \tag{3.19}$$

where $M_F(vect.) = -mM^{-1}M'$ (see Eq.(A.6)) and we have assumed for simplicity that the soft parameters appearing in Eq.(3.17) are all of the same order. Then, the M^2_{LR} matrix is written in the fermion mass eigenstate basis as:

$$\tilde{M}^2_{RL} \to V^\dagger_R(\tilde{a}M_F(spin.) + \tilde{b}_3 M_F(vect.))V_L. \tag{3.20}$$

It is straightforward to show that the chirality mass insertion parameters are given by:

$$(\delta_{RL})_{ij} = \frac{\tilde{b}_3}{\tilde{m}^2_f} M^{diag.}_{Fi}\delta_{ij} + (\tilde{z}V^\dagger_R M_F(spinor)V_L)_{ij}, \tag{3.21}$$

where $M^{diag.}_F = V^\dagger_R M_F V_L$ and $\tilde{z} = \frac{\tilde{a}-\tilde{b}3}{\tilde{m}^2_f}$. The induced FV arises only from the second term of Eq.(3.21).

3.2.3 Mass Insertion Parameters Induced Below $M_{\rm GUT}$

The Dirac neutrino Yukawa couplings $(Y_N)_{ij}$ induce flavor violating off-diagonal elements in the left-handed slepton mass matrix through the RG running from $M_{\rm GUT}$ to the right-handed neutrino mass scales. The RGEs for MSSM with right-handed neutrinos are given in Ref.[46]. The right-handed neutrinos M_{R_i} are determined in the $SO(10) \times A_4$ model. In this case, the induced mass insertion parameters for left-handed sleptons are given by [50],

$$(\delta^l_{LL})^{RHN}_{ij} = -\frac{3m^2_\psi + \tilde{a}^2}{8m^2_\psi \pi^2} \sum_{k=1}^{3} (Y_N)_{ik}(Y^*_N)_{jk} ln\frac{M_{\rm GUT}}{M_{R_k}}, \tag{3.22}$$

where the matrix Y_N is written in the mass eigenstates of charged leptons and right-handed neutrinos. The total LL contribution for the charged leptons is given by

$$(\delta^l_{LL})^{Tot}_{ij} = (\delta^l_{LL})^{RHN}_{ij} + (\delta^l_{LL})_{ij}. \tag{3.23}$$

3.3 Results

In this section, we investigate the flavor violating processes by calculating the mass insertion parameters δ_{LL}, δ_{RR}, and $\delta_{LR,RL}$, then we compare them with their experimental bounds. These bounds in the quark and lepton sectors were obtained by comparing the hadronic and leptonic flavor changing processes to their experimental values/limits [54, 55]. Eq.(3.12), Eq.(3.14) and Eq.(3.15) are used to calculate $\delta_{LL,RR}$ and Eq.(3.21) is used to calculate $\delta_{LR,RL}$ for both charged leptons and down quarks. The result of mass insertion calculations and their experimental bounds are presented in Table 3.4. In this table, we have defined $\sigma = \frac{\tilde{m}^2_{\chi_2} - \tilde{m}^2_{\psi_i}}{\tilde{m}^2_{\psi_i}}$ and $\tilde{k} = \tilde{z} m_{b,\tau}$

The stringent bounds on leptonic δ_{12}, δ_{13}, and δ_{23} in Table 3.4 come only from the decay rates $l_i \to l_j \gamma$. The experimental bounds on the mass insertion parameters listed in column 3 were obtained by making a scan of m_0 and $M_{1/2}$ over the ranges $m_0 < 380\ GeV$ and $M_{1/2} < 160\ GeV$, where m_0 and $M_{1/2}$ are the scalar universal mass and the gaugino mass respectively [55].

Glancing at Table 3.4, we note that the stringent constraint on leptonic flavor violation arises from δ^l_{12} which corresponds to the decay rate of $\mu \to e\gamma$. On the other hand, there is a weaker constraint that arises from δ^d_{12} on the quark sector. One can do an arrangement such that $\tilde{a} - \tilde{b}_3 = 200$ GeV and $\tilde{m}_f = 800$ GeV (equivalent to $\tilde{k} = 2.6 \times 10^{-4}$) so that all the chirality flipping mass insertions will be within their experimental bounds. This arrangement is possible if the trilinear soft terms vanish at the scale M^*.

Since the stringent constraint comes from the $\mu \to e\gamma$ process, let us discuss the branching ratio of this process in more details. In general, the branching ratio of $l_i \to l_j \gamma$ is given by

$$\frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu_j})} = \frac{48\pi^3 \alpha}{G_F^2} (|A_L^{ij}|^2 + |A_R^{ij}|^2). \tag{3.24}$$

We have used the general expressions for the amplitudes $A^{ij}_{L,R}$ given by Ref.[57] where the contributions from both chargino and the neutralino loops are included. These expressions are written in terms of mass insertion parameters.

The correct suppression of the decay rate $\Gamma(\mu \to e\gamma)$ requires a high degree of degeneracy of the soft mass terms of MSSM fields and the exotic fields. For example,

$\sigma \approx 0.01$, as can be seen from Table 3.4. In order to obtain high degree of degeneracy, let us assume that the SSB terms which are generated at the messenger scale M^* satisfy the universality boundary conditions at the scale M^* given by

$$\begin{aligned} \tilde{m}^2_{\psi_i} &= \tilde{m}^2_{\chi_i} = \tilde{m}^2_{\overline{\chi}_i} = \tilde{m}^2_{10_H} = \tilde{m}^2_{1_H} = \tilde{m}^2_{1'_H} = m_0, \\ M_\lambda &= M_0, \\ \tilde{a} &= \tilde{b}_1 = \tilde{b}_2 = 0, \end{aligned} \tag{3.25}$$

where M_λ is the gaugino mass of $SO(10)$ gauge group. Solving the RGE listed in Appendix C with the boundary conditions given by Eq.(3.25) determines the value of σ. In Table 3.5 we give the branching ratio of the process $\mu \to e\gamma$ predicted by the $SO(10) \times A_4$ model for different choices of the input parameters a, b_1, b_2, $\tilde{m}_\psi$ and $M_{1/2}$ at the GUT scale. The experimental searches have put the upper limit on the branching ratio of $\mu \to e\gamma$ as $\text{Br}(\mu \to \text{e}\gamma) \leq 1.2 \times 10^{-11}$ [56]. Note that $\tilde{m}_\psi$ and $M_{1/2}$ originate respectively from m_0 and M_0 through RGEs. In this Table we consider $\ln \frac{M^*}{M_{\text{GUT}}} = 1$ and $\ln \frac{M^*}{M_{\text{GUT}}} = 4.6$ that correspond respectively to $M^* \approx 3M_{\text{GUT}}$ and $M^* \approx M_{\text{Planck}}$.

Let us analyze the four cases in the Table 3.5. In the cases (I, II and III), the chosen values of the parameters a are consistent with the top Yukawa coupling of order 0.5 at the GUT scale and with the fitting for fermion masses and mixing. On the other hand, the choice of $a = 0.68$ in Case IV is not consistent with the fit. Although the medium slepton masses of order 550 GeV are obtained in Case I, the choice $b_1 = b_2 = 1.9$ corresponds to non-perturbative Yukawa couplings at the scale M^* (i.e. $b_1 = b_2 = 4$ at M^*). In this case, the solutions of the 1-loop RGEs are not trusted since the Yukawa couplings b_1 and b_2 go non-perturbative above the GUT scale. Also, it is important to point out that the flavor violation constraint on $\mu \to e\gamma$ in Case III requires heavy slepton masses (≥ 3 TeV) while it requires slepton masses of order ~ 900 GeV in Case II. In other words, Case II is preferred in our model in the sense that the decay rate of $\mu \to e\gamma$ is close to the experimental limit with a reasonable supersymmetric mass spectrum, so it might be tested in the ongoing MEG experiment[58]. Besides, the Yukawa couplings remain perturbative at the messenger scale M^*. Figure 3.1 shows the allowed values of m_ψ that correspond to the graphs below the x-axis for the cases I and II.

	Predictions	Expt.	Pull
$m_c(m_c)$	1.4	$1.27^{+0.07}_{-0.11}$	1.85
$m_t(m_t)$	172.5	171.3±2.3	0.52
m_s/m_d	19.4	19.5 ± 2.5	0.04
$m_s(2Gev)$	109.6×10^{-3}	$105^{+25}_{-35} \times 10^{-3}$	0.184
$m_b(m_b)$	4.31	$4.2^{+0.17}_{-0.07}$	0.58
V_{us}	0.223	0.2255±0.0019	1.3
V_{cb}	38.9×10^{-3}	$(41.2\pm1.1)\times10^{-3}$	2
V_{ub}	4.00×10^{-3}	$(3.93\pm0.36)\times10^{-3}$	0.7
η	0.319	$0.349^{+0.015}_{-0.017}$	1.7
$m_e(m_e)$	0.511×10^{-3}	0.511×10^{-3}	-
$m_\mu(m_\mu)$	105.6×10^{-3}	105.6×10^{-3}	-
$m_\tau(m_\tau)$	1.776	1.776	-
Δm^2_{21}	$7.69 \times 10^{-3}\text{eV}^2$	$(7.59 \pm 0.2) \times 10^{-3}\text{eV}^2$	0.5
Δm^2_{32}	$2.36 \times 10^{-3}\text{eV}^2$	$(2.43 \pm 0.13) \times 10^{-3}\text{eV}^2$	0.5
$\sin\theta^{sol}_{12}$	0.555	0.566±0.018	0.61
$\sin\theta^{l}_{23}$	0.811	0.707±0.108	0.96
$\sin\theta_{13}$	0.141	< 0.22	

Table 3.3: The fermion masses and mixings and their experimental values. The fermion masses, except the neutrino masses, are in GeV.

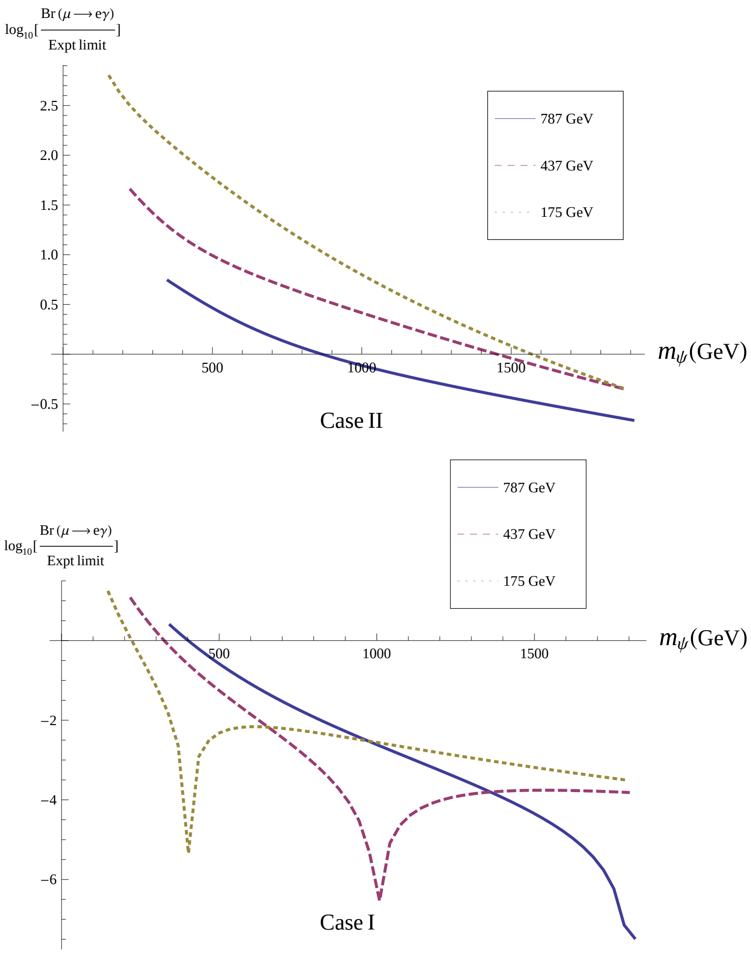


Figure 3.1: The above graphs show the plot of Log of Br($\mu \to e\gamma$) divided by experimental bound (1.2×10^{-11}) versus m_ψ for two cases I and II with $M_{1/2}$=787 GeV, 437 GeV and 175 GeV.

Mass Insertion (δ)	Model Predictions	Exp. Upper Bounds
$(\delta^l_{12})_{LL}$	$0.062\ \sigma + (\delta^l_{12})^{RHN}_{LL}$	6×10^{-4}
$(\delta^l_{12})_{RR}$	$6.1 \times 10^{-4}\ \sigma$	0.09
$(\delta^l_{12})_{RL,LR}$	$(0.084,\ 0.0096)\ \tilde{k}$	10^{-5}
$(\delta^l_{13})_{LL}$	$0.022\ \sigma + (\delta^l_{13})^{RHN}_{LL}$	0.15
$(\delta^l_{13})_{RR}$	$0.028\ \sigma$	-
$(\delta^l_{13})_{RL,LR}$	$(0.0335,\ 0.076)\ \tilde{k}$	0.04
$(\delta^l_{23})_{LL}$	$0.27\ \sigma + (\delta^l_{13})^{RHN}_{LL}$	0.12
$(\delta^l_{23})_{RR}$	$0.034\ \sigma$	-
$(\delta^l_{23})_{RL,LR}$	$(0.055,\ 0.899)\ \tilde{k}$	0.03
$(\delta^d_{12})_{LL}$	$1.9 \times 10^{-4}\ \sigma$	0.014
$(\delta^d_{12})_{RR}$	$0.15\ \sigma$	0.009
$(\delta^d_{12})_{LR,RL}$	$(0.029,\ 0.035)\ \tilde{k}$	9×10^{-5}
$(\delta^d_{13})_{LL}$	$0.014\ \sigma$	0.09
$(\delta^d_{13})_{RR}$	$0.061\ \sigma$	0.07
$(\delta^d_{13})_{LR,RL}$	$(0.173,\ 0.016)\ \tilde{k}$	1.7×10^{-2}
$(\delta^d_{23})_{LL}$	$0.054\ \sigma$	0.16
$(\delta^d_{23})_{RR}$	$0.29\ \sigma$	0.22
$(\delta^d_{23})_{LR,RL}$	$(0.875,\ 0.064)\ \tilde{k}$	(0.006, 0.0045)

Table 3.4: The mass insertion parameters predicted by $SO(10) \times A_4$ model and their experimental upper bounds obtained from [55].

	I	II	III	IV
a	1.14	1.07	1.14	0.62
b_1	1.9	1.5	1.24	1.24
b_2	1.9	1.5	1.24	1.24
$\tilde{m}_{\psi_i}$	542	886	2932	675
$M_{1/2}$	350	787	1924	350
$\mathrm{BR}(\mu \to e\gamma)$	1.4×10^{-13}	1.16×10^{-11}	1.2×10^{-11}	2.2×10^{-12}

Table 3.5: Branching ratio of $\mu \to e\gamma$ for different choices of input parameters at the GUT scale. Cases I and II correspond to $\ln \frac{M^*}{M_{\mathrm{GUT}}} = 1$ and cases III and IV correspond to $\ln \frac{M^*}{M_{\mathrm{GUT}}} = 4.6$. $\tilde{m}_{\psi_i}$ and $M_{1/2}$ are given in GeV

CHAPTER 4

Higgs Boson Mass in Gauge-Mediating Supersymmetry Breaking with Messenger-Matter Mixing

Supersymmetric (SUSY) grand unification theories (GUTs) are promising candidates for physics beyond the standard model (SM). However, supersymmetry is not an exact symmetry at the low-energy scale and it must be broken somehow to be relevant to nature. SUSY can not be broken at tree level since the supertrace theorem leads to non-phenomenological particle spectra. Therefore, it is assumed that SUSY breaking occurs in the hidden sector which has no renormalizable tree level couplings with the observable sector. SUSY breaking is transmitted to the visible sector either via gravitational interactions as inspired by supergravity models (SUGRA)[44], or by SM gauge interactions as in theories with gauge-mediated SUSY breaking (GMSB)[59, 60, 61]. In the first scenario, the soft terms are generated at the Planck scale. In general, these soft terms are not flavor-invariant. The gravity-mediated scenario can only give realistic models if the universality or an approximate alignment between particle and sparticle masses is imposed in order to suppress the flavor violation processes. On the other hand, the universality condition is naturally satisfied in the GMSB where the soft terms are generated at the messenger scale, below the GUT scale, from radiative corrections.

In GMSB theories, messenger fields communicate the SUSY breaking from the hidden sector to the visible sector. In addition to the observable sector, at least one gauge singlet superfield (Z) is needed in order to give mass to the messenger fields and break SUSY by giving vacuum expectation values (VEVs) to its scalar-component ($\langle Z\rangle$) and to its auxiliary F-component ($\langle F_Z\rangle$) respectively. The SUSY breaking factor (i.e. $\langle F_Z\rangle$) that appears in the mass splitting between the fermionic and scalar components of the messenger field is communicated to the MSSM particles through radiative corrections. For example, the gauginos and the scalars of MSSM get their masses at the messenger scale $M_{\rm mess}$ from one-loop and two-loop Feynman diagrams respectively as fellows:

$$M_{\lambda_r} = gN_{\rm mess}\frac{\alpha_r}{4\pi}\Lambda, \tag{4.1}$$

$$\tilde{m}^2 = 2f\sum_{r=1}^{3} N_{\rm mess}C_r^{\tilde{f}}\frac{\alpha_r^2}{(4\pi)^2}\Lambda^2, \tag{4.2}$$

where $N_{\rm mess}$ is called the messenger index. For example, $N_{\rm mess} = 1$ ($N_{\rm mess} = 3$) for messenger fields belong to $5 + \overline{5}$ ($10 + \overline{10}$) of $SU(5)$. Here, $\Lambda = \frac{\langle F_Z\rangle}{\langle Z\rangle}$ is the effective SUSY breaking scale, $C_r^{\tilde{f}}$ are the quadratic Casimir invariants for the scalar fields, and α_r are the gauge coupling constants at the scale $M_{\rm mess}$. These gauge couplings are all equal at the GUT scale. In Eqs.(4.1) and (4.2), f and g are the 1-loop and 2-loop functions whose exact expressions can be found e.g. in Ref.[61]. The universal scalar masses in Eq.(4.2) are obtained when the messenger and matter fields are completely separated. There are additional contributions to universal masses if messenger-matter mixing is allowed.

Two interesting features of GMSB are concluded from Eqs.(4.1) and (4.2). Firstly, the scalar masses are only functions of gauge quantum number so scalar masses with the same gauge quantum number are degenerate. As a result, the supersymmetric flavor problem is solved. Secondly, GMSB is highly predictive since all soft terms at the messenger scale are determined by only two parameters Λ and $N_{\rm mess}$. In order to preserve the successful gauge coupling unification of MSSM, the messenger fields should reside in complete $SU(5)$ multiplets. In this chapter, we consider two cases when the messenger fields belong to $5 + \overline{5}$ and $10 + \overline{10}$ of $SU(5)$. In both cases the perturbative unification is still maintained, as shown in Fig. 4.1.

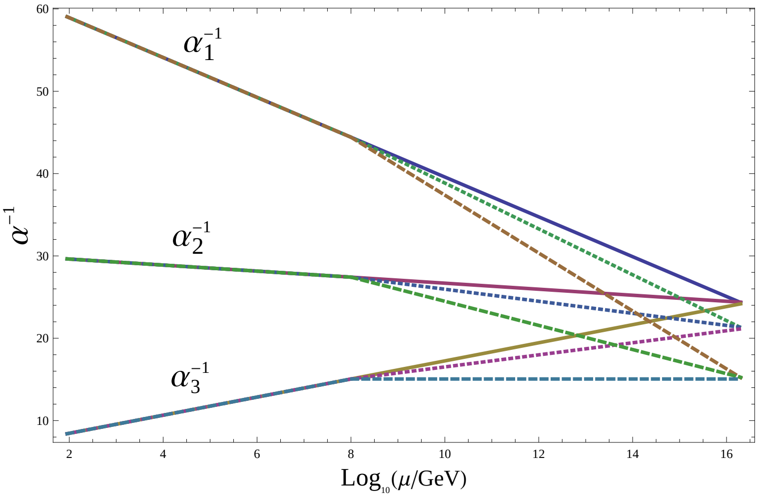


Figure 4.1: The evolutions of the gauge couplings with $M_{\text{mess}} = 10^8$ GeV and $\tan\beta = 10$. Solid lines correspond to MSSM. Dashed lines are for MSSM+$10 + \overline{10}$ and dotted lines are for MSSM+$5 + \overline{5}$.

The complete separation of messenger sector and visible sector is problematic in cosmology because this leads to models possessing stable particles [62]. Besides, messenger-matter couplings are allowed by gauge symmetry and they can only be forbidden by imposing discrete flavor symmetry. If one allows these couplings, additional contributions to the universal scalar mass given by Eq.(4.1) and (4.2) are obtained [63, 64, 65]. These new contributions reintroduce flavor violation either in the leptonic or the quark sector depending on the structure of the messenger fields. In this chapter, we have shown that the induced flavor violation from messenger-matter mixing that occurs mainly with the third generation is still sufficiently suppressed. Another advantage of the messenger-matter mixing—the main result of this chapter—is that it might increase the lightest Higgs mass to value as large as 125 GeV, which is difficult to realize without such mixing.

In order to reproduce the known qualitative features of quark and lepton masses and mixings, we consider the Froggatt-Nielsen mechanism [66]. This mechanism leads to the lopsided structure of down-quark and charged lepton mass matrix. It was shown that in this kind of structure the $\mu \to e\gamma$ decay rate is generally large by adopting gravity mediated SUSY breaking and it is consistent with the experimental limit of Br($\mu \to e\gamma$) only with a heavy SUSY spectrum [67]. On the other hand, the lopsided structure works well in the GMSB regarding the flavor violation processes even with light SUSY spectra as we show in this chapter.

This chapter is organized as fellows: In section 4.1 the Higgs mass bounds are considered in two models. The first is $5+\overline{5}$ model in which the messenger fields belong to the $5+\overline{5}$ representation of $SU(5)$ while the second is $10+\overline{10}$ model in which the messenger fields belong to the $10+\overline{10}$ representation of $SU(5)$. In both models, the messenger-matter couplings (i.e. the exotic couplings) are allowed. We investigate the effect of these couplings on the lightest Higgs mass of MSSM. In section 4.2, we construct the general structure of the superpotential of both models by employing the $U(1)$ flavor symmetry of the Froggatt-Nielsen mechanism as discussed in section 4.2.1. We find that the FCNC processes that are induced by the exotic Yukawa couplings are in agreement with experimental bounds. The Yukawa RGEs between messenger and GUT scales for both models are listed in Appendix C. The soft terms which are induced by the exotic Yukawa couplings are evaluated in Appendix D.

4.1 Higgs Mass Bounds

One of the interesting features of MSSM is setting upper bounds on the lightest Higgs mass. The tree level bound on the lightest Higgs mass equal to M_z has been already excluded by the LEP2 lower bound $m_h > 114.4$ [68]. However, radiative corrections push this mass above the LEP2 bound. The leading 1- and 2- loop contributions to the CP-even Higgs boson mass in the MSSM are given by [70, 71]

$$\begin{aligned} m_h^2 &= M_z^2 \cos^2 2\beta (1 - \frac{3}{8\pi^2}\frac{m_t^2}{v^2} t) \\ &+ \frac{3}{4\pi^2}\frac{m_t^4}{v^2}[\frac{1}{2}\chi_t + t + \frac{1}{16\pi^2}(\frac{3}{2}\frac{m_t^2}{v^2} - 32\pi\alpha_3)(\chi_t t + t^2)], \end{aligned} \quad (4.3)$$

where $v^2 = v_d^2 + v_u^2$,

$$t = \log(\frac{M_s^2}{M_t^2}), \;\; \chi_t = \frac{2\tilde{A}_t^2}{M_s^2}(1 - \frac{\tilde{A}_t^2}{12M_s^2}). \tag{4.4}$$

Here the scale M_s has been defined in terms of the stop mass eigenvalues as

$$M_s^2 = m_{\tilde{t_1}} m_{\tilde{t_2}}, \tag{4.5}$$

$\tilde{A}_t = A_t - \mu \cot\beta$, where A_t denotes the stop left and stop right soft mixing parameter.

The upper bound on the lightest Higgs mass depends crucially on the soft supersymmetry breaking terms. For example, the upper bound of around 125 GeV corresponds to the maximal mixing condition, $\tilde{A}_t = \sqrt{6}M_s$. Since there are restrictions on these soft terms from GMSB, it will be interesting to study the effect of these restrictions on the lightest CP-even Higgs mass. In the following subsections we will investigate the effect of allowing messenger-matter couplings on the soft terms of MSSM and consequently on the lightest CP-even Higgs mass. In the ordinary GMSB (i.e. without messenger-matter mixing), both A-terms and the soft breaking parameter B vanish at the messenger scale. However, B can be induced in the process of running. By using the following equations that result from minimizing the Higgs potential,

$$\frac{M_z^2}{2} = -\mu^2 - \frac{m_{H_u}^2 \tan^2\beta - m_{H_d}^2}{\tan^2\beta - 1}, \tag{4.6}$$

$$\sin 2\beta = \frac{2B\mu}{2\mu^2 + m_{H_u}^2 + m_{H_d}^2}, \tag{4.7}$$

one can solve for the parameters $\tan\beta$ and μ. Then $\tan\beta$ turns out to be large (around 35-45) when the messenger scale is close to the effective SUSY breaking scale Λ. On the other hand, by allowing messenger-matter couplings B is induced significantly at low energy scale. This can be understood from the following RGE for the parameter B:

$$\frac{dB}{dt} = \frac{1}{2\pi}(3\alpha_t A_t + 3\alpha_2 M_2 + \frac{3}{5}\alpha_1 M_1), \tag{4.8}$$

λ'_0	m_h(GeV)	$\Lambda(10^5\text{GeV})$	$M(10^{13}\text{GeV})$	$\tilde{m}_{t_1}$(GeV)	$\tilde{m}_{t_2}$(GeV)
0	117	2	1.78	1634	2012
0.8	118	2	10	1590	1857
1.2	119	2	10	1065	2788

Table 4.1: We show the values of the minimal GMSB input parameters, Λ, λ_{ex} and M_{mess} that lead to the highest m_h values at $\tan\beta = 10$.

where $\alpha_t = \frac{\lambda_t}{4\pi}$ and λ_t is the top Yukawa coupling. Since A_t does not vanish in the presence of messenger-matter mixing as shown in Eqs.(4.13) and (4.21), the first term of Eq.(4.8) that pushes B to large values becomes more significant than in the case when A_t is zero. This leads to small $\tan\beta$. For example in the $10 + \overline{10}$ model, the range $1.64 \leq \tan\beta \leq 7$ corresponds to 10^5 GeV $\leq M_{\text{mess}} \leq 10^{14}$ GeV.

In the subsequent analysis, we will give the scalar mass spectrum that leads to the highest m_h for two cases. The first case is to assume a non-vanishing B is somehow generated at the messenger scale such that $\tan\beta = 10$ is obtained by using Eqs.(4.6) and(4.7). The potential solution to the μ problem based on flavor symmetries was suggested by Ref. [72]. The authors of Ref. [72] gave an example of $B\mu \sim \mu^2$ that leads to unconstrained values on $\tan\beta$ by introducing three singlets that are charged under $U(1)$ flavor symmetry. The second case is having a vanishing B at the messenger scale as predicted by both $5 + \overline{5}$ and $10 + \overline{10}$ models. In this case $\tan\beta$ is determined by Eqs.(4.6) and (4.7) where B at low energy scales is obtained by solving the RGE with the boundary condition of vanishing B at the messenger scale.

4.1.1 Higgs Mass Bounds in the $5 + \overline{5}$ Model

The messenger fields belonging to $5 + \overline{5}$ of $SU(5)$ decompose to down-quark singlets d^c_m and $\overline{d^c}_m$, and to lepton doublets L_m and $\overline{L}_m$. The additional contributions to the

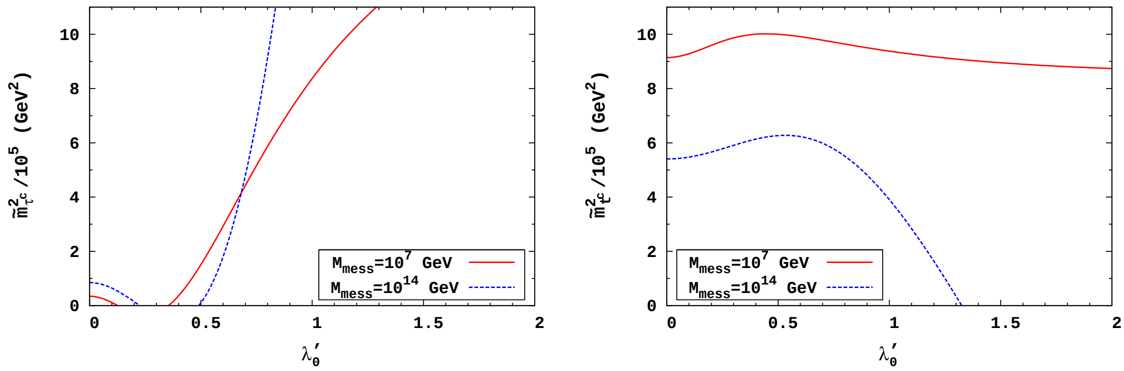


Figure 4.2: The left graph is $\tilde{m}^2_{\tau^c}$ versus λ'_0 at the scale M_{mess} for two different messenger scales. The right graph is $\tilde{m}^2_{t^c}$ versus λ'_0 at the low energy scale for two different messenger scales.

MSSM superpotential due to messenger-matter couplings is

$$W_{5+\bar{5}} \;=\; f_d \overline{d^c}_m d^c_m Z + \lambda'_b Q_3 d^c_m H_d + f_e \overline{L}_m L_m Z + \lambda'_{\tau^c} L_m e^c_3 H_d. \tag{4.9}$$

We assume the messenger fields couple only with the third generation of MSSM. We will show later that the superpotential $W_{5+\bar{5}}$ can be obtained by imposing the $U(1)$ flavor symmetry of the Froggatt-Nielsen mechanism. Also, we have assumed that the exotic Yukawa couplings λ'_b and λ'_{τ^c} (f_d and f_e) are obtained from one unified coupling $\lambda'_0(f_0)$ at the GUT scale by solving the RGEs listed in the Appendix C.1 between the messenger scale and the GUT scale.

In the universal case (i.e. without including messenger-matter couplings), the scalar masses are obtained by employing Eqs.(4.1) and (4.2), while the trilinear soft terms (A-terms) vanish at the scale M_{mess}. There are new contributions to the universal scalar masses and A-terms in the presence of messenger matter couplings. It was shown [63] that the messenger-matter couplings induce negative one-loop contributions to the supersymmetry-breaking masses. However, these one-loop contributions can be safely ignored in the limit of $F/M^2_{\text{mess}} \leq g_3/4\pi$, as we will assume in this chapter. On the other hand, these couplings induce dominant two-loop contributions to the quadratic soft terms and one-loop contributions to the A-terms. The expressions for supersymmetry-breaking terms induced by messenger-matter mixing were derived in [64, 65]. The Yukawa couplings λ'_b and λ'_{τ^c} cause splitting on the masses of squark doublet (Q_3) and the right handed selectron singlet (e_3^c) respectively. In order to find this splitting, we will employ the general expression in Ref.[65]. In addition to the universal masses, the mass shifts $\delta\tilde{m}^2_{Q_3}$, $\delta\tilde{m}^2_{e_3^c}$ and $\delta\tilde{m}^2_{Hd}$ due to the messenger-matter couplings at the messenger scale are given as follows (see Appendix D.1):

$$\delta\tilde{m}^2_{Q_3} = \frac{\alpha'_b\Lambda^2}{8\pi^2}\left(3\alpha'_b + \frac{1}{2}\alpha'_{\tau^c} - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{7}{30}\alpha_1\right), \tag{4.10}$$

$$\delta\tilde{m}^2_{e_3^c} = \frac{2\alpha'_{\tau^c}\Lambda^2}{8\pi^2}\left(2\alpha'_{\tau^c} + \frac{3}{2}\alpha'_b - \frac{3}{2}\alpha_2 - \frac{9}{10}\alpha_1\right), \tag{4.11}$$

$$\delta\tilde{m}^2_{H_d} = \frac{\delta\tilde{m}^2_{e_3^c}}{2} + 3\delta\tilde{m}^2_{Q_3} + \frac{3\Lambda^2\alpha'_b\alpha_t}{16\pi^2}, \tag{4.12}$$

and the A-terms generated by messenger-matter couplings at the messenger scale are

$$\delta A_t = -\frac{1}{4\pi}\alpha'_b\Lambda, \tag{4.13}$$

$$\delta A_b = -\left(\frac{4\alpha'_b + \alpha'_{\tau^c}}{4\pi}\right)\Lambda, \tag{4.14}$$

$$\delta A_\tau = -\left(\frac{3\alpha'_b + 3\alpha'_{\tau^c}}{4\pi}\right)\Lambda, \tag{4.15}$$

where $\alpha'_b = \frac{\lambda'^2_b}{4\pi}$, and $\alpha'_{\tau^c} = \frac{\lambda^2_{\tau^c}}{4\pi}$. Since λ'_b and λ'_{τ^c} originate from one unified coupling λ'_0 as shown in the left graph of Fig.4.3, the scalar mass spectra depend on λ'_0, the messenger scale $M_{\rm mess}$, and the effective SUSY breaking scale Λ. In order to prevent negative squared mass generated at the scale $M_{\rm mess}$, some ranges of λ'_0 are excluded. These ranges depend on the value of $M_{\rm mess}$. The lower value of $M_{\rm mess}$ is taken to be around 10^7 GeV to ensure $F/M^2_{\rm mess} \leq g_3/4\pi$, so the one-loop contribution to the scalar masses from messenger-matter mixing is ignored. The upper bound $M_{\rm mess} < 10^{14}$ GeV arises from demanding that the gravity mediated contributions, proportional to $\langle F_Z\rangle/M_P$, amount at most to 0.1 percent of the gauge mediated contributions. The left graph of Fig.4.2 shows the interval $0.1 < \lambda'_0 < 0.5$ that leads to negative $\tilde{m}^2_{\tau_R}$ at the scale $M_{\rm mess}$ is roughly applicable to all values of $M_{\rm mess}$.

Below the scale M_{mess}, the theory is just the MSSM. Therefore, we have solved the one-loop RGEs of MSSM at the supersymetry breaking scale with the boundary conditions at the scale M_{mess} given by Eqs.(4.1) and (4.2) and Eqs.(4.10)-(4.15). The soft breaking mass squared $m^2_{H_u}$ is driven to negative values at low energy scale leading to the electroweak symmetry breaking. In order to avoid driving $\tilde{m}^2_{t_R}$ to negative values at low energy scale, otherwise the color charge will be broken, a region of λ'_0 is forbidden. For example, the region of $\lambda'_0 > 1.3$ for $M_{\text{mess}} = 10^{14}$ GeV is forbidden as shown in the right graph of Fig.4.2. In that forbidden case $\tilde{m}^2_{t_R}$ is driven to negative values because of the term that contains the top Yukawa couplings in the RGE of right-handed stop mass. This term increases with larger exotic Yukawa coupling.

All the soft terms at the messenger scale are fully determined by three parameters: λ'_0, Λ and M_{mess}. Consequently, the lightest Higgs mass is also determined by these three parameters. As we discussed previously, the maximal mixing condition — $\tilde{A}_t = \sqrt{6}M_s$— gives the upper bound on the lightest Higgs mass of MSSM. It is not possible to realize this maximal condition in GMSB without messenger-matter mixing because A_t vanishes at the scale M_{mess} and the induced value at low energy scale through RGEs is not sufficient. On the other hand, allowing messenger matter couplings generates A_t as shown in Eq.(4.13). This leads to an enhancement of the Higgs mass. By allowing these parameters to be in the respective ranges 4×10^4 GeV $< \Lambda < 2 \times 10^5$ GeV, 10^7 GeV $< M_{\text{mess}} < 10^{14}$ GeV and $0 < \lambda'_0 < 2$, we report the numerical values of these parameters that give rise to the highest m_h value in Table 4.1. In this Table, we exclude values of λ'_0 that give negative values for $\tilde{m}^2_{\tau_R}$ and $\tilde{m}^2_{t_R}$. The lightest Higgs mass around 117 GeV is obtained in the $5+\overline{5}$ model without messenger-matter mixing and a small enhancement of the Higgs mass is obtained in the presence of messenger-matter mixing as shown in Table 4.1. However, large enhancement of the lightest Higgs mass is obtained when the messenger fields belong to $10+\overline{10}$ in the presence of messenger-matter mixing as shown in the next subsection.

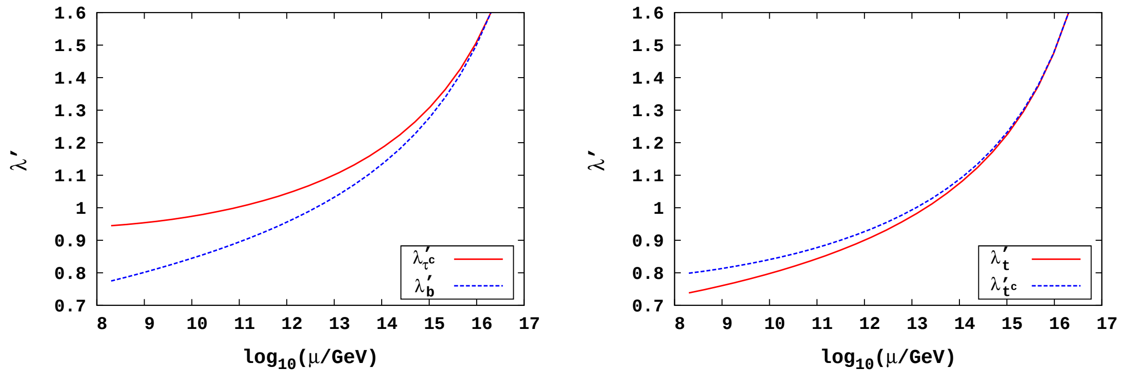


Figure 4.3: The left (right) graph shows the running of two exotic Yukawa couplings from the GUT scale $M_{\rm GUT} = 2 \times 10^{16}$ GeV to the messenger scale $M_{\rm mess} = 10^8$ GeV for the $5 + \overline{5}$ $(10 + \overline{10})$ model where the unified Yukawa coupling is taken to be $\lambda'_0 = 1.6$.

4.1.2 Higgs Mass Bounds in the $10 + \overline{10}$ Model

In this subsection we have messenger fields belonging to $10 + \overline{10}$ of $SU(5)$. This decomposes in terms of MSSM multiplets as:

$$10 + \overline{10} = (Q + \overline{Q}) + (u^c + \overline{u^c}) + (e^c + \overline{e^c}). \tag{4.16}$$

We have assumed the messenger fields only couple with the third generation of MSSM fields. In this case the MSSM superpotential has the additional contribution

$$\begin{aligned} W_{10+\overline{10}} &= \lambda'_{t^c} Q_3 u^c_m H_u + \lambda'_t Q_m u^c_3 H_u + \lambda'_m Q_m u^c_m H_u \\ &+ f_{e^c} \overline{e^c}_m e^c_m Z + f_{u^c} \overline{u^c}_m u^c_m Z + f_Q \overline{Q}_m Q_m Z. \end{aligned} \tag{4.17}$$

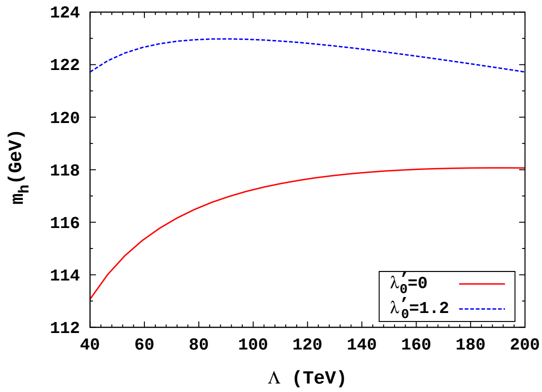
Although the coupling $Q_m d^c H_d$ is allowed by gauge symmetry, we have not included it in the above superpotential because it is suppressed by the small expansion parameter ϵ as we will see later. We have assumed that the Yukawa couplings λ'_{t^c} and λ'_t are equal to one unified coupling λ'_0 at the GUT scale as shown in the right graph of Fig.4.3. The three Yukawa couplings f_{e^c}, f_Q and f_{u^c} are equal to f_0 at the GUT scale as well. In other words, the six Yukawa couplings appearing in the superpotential $W_{10+\overline{10}}$ are reduced to three (λ'_0, f_0 and λ'_{m0}) at the GUT scale. These six Yukawa couplings are obtained from the unified ones by solving the RGEs given in Appendix C.2.

The exotic Yukawa couplings λ'_{t^c}, λ'_t and λ'_m generate 2-loop (1-loop) scalar masses (A-terms) at the scale $M_{\rm mess}$ as shown in Appendix D.2. So, the universal scalar masses given by Eqs.(4.1) and (4.2), substituting $N_{\rm mess} = 3$, have additional contributions at the scale $M_{\rm mess}$ given by

$$\begin{aligned}
\delta\tilde{m}^2_{Q_3} &= \frac{\Lambda^2}{8\pi^2}\Big(\alpha'_{t^c}(3\alpha'_{t^c} + \frac{3}{2}\alpha'_t + \frac{5}{2}\alpha'_m - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1) \\
&- \alpha_t(\frac{5}{2}\alpha'_t + \frac{3}{2}\alpha'_m)\Big), \qquad (4.18) \\
\delta\tilde{m}^2_{u^c_3} &= \frac{2\Lambda^2}{8\pi^2}\Big(\alpha'_t(3\alpha'_t + \frac{3}{2}\alpha'_{t^c} + 2\alpha'_m - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1) \\
&- \alpha_t(2\alpha'_{t^c} + \frac{3}{2}\alpha'_m)\Big), \qquad (4.19) \\
\delta\tilde{m}^2_{H_u} &= \frac{3\Lambda^2}{8\pi^2}\Big(\alpha'_{t^c}(3\alpha'_{t^c} + \frac{3}{2}\alpha'_t + \frac{5}{2}\alpha'_m - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1) \\
&+ \alpha'_t(3\alpha'_t + \frac{3}{2}\alpha'_{t^c} + 2\alpha'_m - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1) \\
&+ \alpha'_m(3\alpha'_m + 2\alpha'_t + \frac{5}{2}\alpha'_{t^c} - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1)\Big), \qquad (4.20) \\
\delta A_t &= -\left(\frac{5\alpha'_t + 4\alpha'_{t^c} + 3\alpha'_m}{4\pi}\right)\Lambda, \qquad (4.21) \\
\delta A_b &= -\frac{\alpha'_{t^c}}{4\pi}\Lambda, \qquad (4.22)
\end{aligned}$$

where $\alpha'_{t^c} = \frac{\lambda'^2_{t^c}}{4\pi}$, $\alpha'_t = \frac{\lambda'^2_t}{4\pi}$, and $\alpha'_m = \frac{\lambda'^2_m}{4\pi}$. The interesting feature of the $10+\overline{10}$ model is that A_t is generated sufficiently at the scale $M_{\rm mess}$. Consequently, we are able to obtain the maximal mixing condition (i.e. $\frac{A_t}{Ms} = \sqrt{6}$) that leads to the upper Higgs mass limit of the MSSM.

In order to find the Higgs mass and the other scalar mass spectra, we solved the MSSM RGEs numerically from the messenger scale to the low scale. The scalar mass spectra depend on the four parameters Λ, $M_{\rm mess}$, λ'_0 and λ'_{m0}. We report the values of three of these parameters Λ, $M_{\rm mess}$, and λ'_0 for $\lambda'_{m0} = 0$ and $\lambda'_{m0} = 1.6$ that lead to



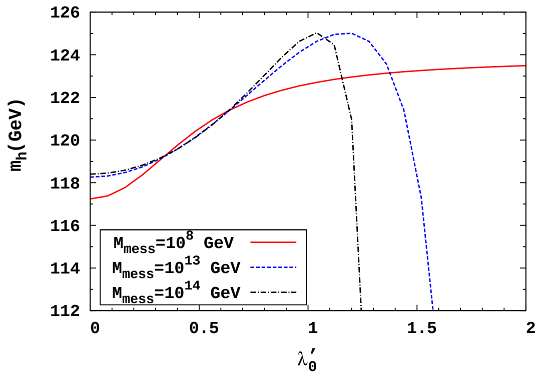


Figure 4.4: The left graph is a plot of m_h versus Λ for $\lambda'_0 = 0$ and $\lambda'_0 = 1.2$. The right graph is m_h versus λ'_0 for different messenger scales at $\Lambda = 10^5$ GeV.

the highest m_h in Table 4.3 and Table 4.4 respectively. In the case of $\lambda'_{m0} = 1.6$, the Higgs mass can be up to 125 GeV for $\lambda'_0 = 0.4$. Let us take $\lambda'_{m0} = 0$ for simplicity. In the case of messenger fields belonging to $10 + \overline{10}$ without messenger-matter mixing, the Higgs mass limit 119 GeV corresponds to around 3.5 TeV for the lightest stop mass. However, in the presence of messenger-matter mixing, we can obtain a Higgs mass limit up to 125 GeV corresponding to around 1 TeV for the lightest stop mass as can be seen from Table 4.3.

The left graph of Fig.4.4 shows the lightest Higgs mass with messenger-matter mixing is enhanced about 10 GeV compared to the case without messenger-matter mixing for low values of Λ (i.e. around $\Lambda = 4 \times 10^4$ GeV) and it is enhanced around 6 GeV for larger Λ. The low values of Λ correspond to $500 - 600$ GeV of the lightest stop mass which might be accessible to LHC. The right graph of Fig.4.4 shows a constraint on the values of the exotic Yukawa coupling λ'_0 when the messenger scale is above $\sim 10^{13}$ GeV. This constraint arises from the stop mass turning negative at low energy scale.

The range of messenger scale $M_{\rm mess} \leq 3 \times 10^8$ GeV is preferred by cosmology because this corresponds to gravitino mass less than $\sim$ 1 keV [69]. Therefore, we

Name		$10+\overline{10}$	$10+\overline{10}$	$5+\overline{5}$
Inputs	M_{mess}	10^8	4×10^5	10^8
	N_{mess}	3	3	1
	$\Lambda(10^5\text{GeV})$	0.3	0.3	0.95
	$\tan\beta$	10	5.6	11.6
	λ'_0	1.2	1.2	1.2
Higgs:	m_h	121	117.7	114.6
	m_H^0	675	675	1107
	m_A	675	674	1107
	$m_{H^\pm}$	679	678	1110
Gluino:	$\tilde{m}_g$	852	852	899
Neutralinos:	m_{χ_1}	121	127	128
	m_{χ_2}	234	245	248
	m_{χ_3}	667	658	706
	m_{χ_4}	675	668	713
Charginos:	χ_1^+	236	233	250
	χ_2^+	676	667	738
Squarks:	$\tilde{m}_{u_L,c_L}$	810	787	1120
	$\tilde{m}_{u_R,c_R}$	786	765	1071
	$\tilde{m}_{d_L,s_L}$	810	787	1121
	$\tilde{m}_{d_R,s_R}$	782	763	1064
	$\tilde{m}_{b_L}$	692	682	997
	$\tilde{m}_{b_R}$	780	763	1045
	$\tilde{m}_{t_L}$	692	682	997
	$\tilde{m}_{t_R}$	518	531	890
Sleptons:	$\tilde{m}_{e_L,\mu_L}$	224	201	371
	$\tilde{m}_{\nu_{eL},\nu_{\mu L}}$	224	201	371
	$\tilde{m}_{e_R,\mu_R}$	168	150	182
	$\tilde{m}_{\tau_L}$	224	201	352
	$\tilde{m}_{\tau_R}$	167	150	1014

Table 4.2: The spectra corresponding to $10+\overline{10}$ model and $5+\overline{5}$ model. All the masses are in GeV.

λ'_0	m_h(GeV)	$\Lambda(10^5$GeV)	$M_{\rm mess}(10^8$GeV)	$\tilde{m}_{t_1}$(GeV)	$\tilde{m}_{t_2}$(GeV)	A_t/M_s
0	119	1.6	3.16×10^5	3590	4145	-0.86
0.4	120	1.36	1	2756	3289	-1.1
0.8	123	0.912	10^5	1553	2143	-1.55
1.2	125	0.784	17782	1088	1751	-1.95
1.6	125	0.784	1778	1066	1743	-2
2	125	0.784	177	1138	1762	-1.93

Table 4.3: We show the values of the GMSB input parameters, Λ, λ'_0 and $M_{\rm mess}$ that lead to the highest m_h values. These values correspond to $\lambda'_{m0} = 0$ and $\tan\beta = 10$.

find that the lightest Higgs mass up to 123.5 GeV can be obtained at $M_{\rm mess} = 10^8$ GeV. We give the spectra for both the $10 + \overline{10}$ and $5 + \overline{5}$ models in Table 4.2. In this Table, the given values of $\tan\beta = 5.6$ and $\tan\beta = 11.6$ for the $10 + \overline{10}$ model and the $5 + \overline{5}$ model respectively are obtained from Eqs.(4.6) and (4.7) where B vanishes at the scale $M_{\rm mess}$, while the given value $\tan\beta = 10$ for the $10 + \overline{10}$ model is an arbitrary choice.

4.2 Flavor Violation

GMSB has the interesting feature that FCNC processes are naturally suppressed in agreement with experimental bounds. This suppression is due to the degeneracy of scalar masses at the messenger scale. This degeneracy is broken when the messenger-matter coupling is allowed. As we have seen previously, the third generation of the scalar masses splits from the other two. Consequently, the flavor violating off-diagonal elements of quadratic scalar matrix are introduced in the fermion mass eigenstate basis.

In this section we will investigate the flavor violation processes of the charged

λ'_0	m_h(GeV)	$\Lambda(10^5$GeV)	$M_{\rm mess}(10^{11}$GeV)	$\tilde{m}_{t_1}$(GeV)	$\tilde{m}_{t_2}$(GeV)	A_t/M_s
0	123	0.97	178	1344	2163	-1.8
0.4	125	0.91	316	1046	1969	-2.1
0.8	125	0.848	56	960	1831	-2.3
1.2	125	0.848	10	997	1834	-2.3
1.6	125	0.784	1.78	1005	1716	-2.3
2	125	0.784	1	1007	1717	-2.3

Table 4.4: We show the values of the GMSB input parameters, Λ, λ'_0 and $M_{\rm mess}$ that lead to the highest m_h values. These values correspond to $\lambda'_{m0} = 1.6$ and $\tan\beta = 10$.

leptons and down quarks for both $5+\overline{5}$ and $10+\overline{10}$ models through the mass insertion parameters given by

$$(\delta^{d,l}_{LL,RR})_{ij} = (U^{\dagger d,l}_{L,R}, \tilde{m}^2_{LL,RR} U^{d,l}_{L,R})_{ij}/\tilde{m}^2_{d,l}, \quad (4.23)$$

$$(\delta^{d,l}_{LR,RL})_{ij} = (U^{\dagger d,l}_{R,L} \tilde{m}^2_{LR,RL} U^{d,l}_{L,R})_{ij}/\tilde{m}^2_{d,l}, \quad (4.24)$$

where $\tilde{m}^2_{d,l}$ is the average of the diagonal entries of the quadratic scalar mass matrix for the down quarks and charged leptons, $U^{\dagger d,l}_{L,R}$ are the bi-unitary transformations needed to diagonalize the down quark and charged lepton mass matrix and the matrix $\tilde{m}^2_{LR,RL}$ is related to trilinear soft terms (A-terms).

4.2.1 Flavour Violation in $5+\overline{5}$ Model

Let us first show how to obtain the superpotential $W_{5+\overline{5}}$ in Eq.(4.9) by imposing $U(1)$ flavor symmetry. The hierarchy in the masses and mixings of the quarks and leptons can be understood by employing the Froggatt-Nielsen mechanism. In this approach, $U(1)$ flavor symmetry is assumed. This flavor symmetry is broken at high scale, M^*, by giving a VEV to a scalar field "S", usually SM singlet. The fermion mass matrix in the effective theory below M^* appears as a power expansion in the parameter $\epsilon = \frac{\langle S \rangle}{M^*}$.

SU(5)	10_1	10_2	10_3	$\overline{5}_1$	$\overline{5}_2$,$\overline{5}_3$	5_u,$\overline{5}_d$	S	5_m	$\overline{5}_m$	Z
$U(1)$	4	2	0	p+1	p	0	-1	$-\alpha$	0	α

Table 4.5: The $U(1)$ charge assignments to the messenger, MSSM, Z and S fields.

The superfields of MSSM can be accommodated into three copies of $\overline{5} + 10$ of $SU(5)$. The two Higgs doublets H_u and H_d reside respectively in 5_u and $\overline{5}_d$ of $SU(5)$. The $U(1)$ charge assignments for all superfields are shown in Table 4.5. The new superpotential invariant under the $U(1)$ flavor symmetry at the messenger scale is

$$\begin{aligned} W_{5+\overline{5}} &= (k_m d^c_m + k_1 d^c_1 \epsilon^{1+p} + k_2 d^c_2 \epsilon^p + k_3 d^c_3 \epsilon^p)\overline{d^c}_m Z \\ &+ (k'_m L_m + k'_1 L_1 \epsilon^{1+p} + k'_2 L_2 \epsilon^p + k'_3 L_3 \epsilon^p)\overline{L}_m Z \\ &+ (\lambda_1 Q_1 \epsilon^4 + \lambda_2 Q_2 \epsilon^2 + \lambda_3 Q_3) d^c_m H_d \\ &+ L_m(\lambda'_1 e^c_1 \epsilon^4 + \lambda'_2 e^c_2 \epsilon^2 + \lambda'_3 e^c_3) H_d. \end{aligned} \tag{4.25}$$

Without loss of generality, one can redefine the combinations of fields in the parenthesis as:

$$f_d d'^c_m = k_m d^c_m + k_1 d^c_1 \epsilon^{1+p} + k_2 d^c_2 \epsilon^p + k_3 d^c_3 \epsilon^p, \tag{4.26}$$

$$f_e L'^c_m = k'_m L_m + k'_1 L_1 \epsilon^{1+p} + k'_2 L_2 \epsilon^p + k'_3 L_3 \epsilon^p, \tag{4.27}$$

$$\lambda'_b Q'_3 = \lambda_1 Q_1 \epsilon^4 + \lambda_2 Q_2 \epsilon^2 + \lambda_3 Q_3, \tag{4.28}$$

$$\lambda'_{\tau^c} e^{c'}_3 = \lambda'_1 e^c_1 \epsilon^4 + \lambda'_2 e^c_2 \epsilon^2 + \lambda'_3 e^c_3. \tag{4.29}$$

Dropping the prime notation on the superfields we obtain $W_{5+\overline{5}}$ in Eq.(4.9). The Yukawa coupling interactions of the superpotential $W_{5+\overline{5}}$ are

$$\mathcal{L}_Y = f_d \overline{Q}_m Q_m Z + \lambda'_b Q_3 d^c_m H_d + f_e \overline{L}_m L_m Z + \lambda'_{\tau^c} L_m e^c_3 H_d. \tag{4.30}$$

In order to decouple the fermionic part of the messenger superfield, we redefine the fermionic fields in Eq.(4.30) as $f'_d \overline{d'^c}_m = f_d \overline{d^c}_m \langle Z \rangle + \lambda'_b d_3 v_d$ and $f'_e \overline{e'}_m = f_e \overline{e}_m \langle Z \rangle +$

$\lambda'_{\tau^c} e_3^c v_d$, where $v_d = \langle H_d \rangle$. Accordingly, the fermionic mass matrix can be written as:

$$M_d = \begin{pmatrix} Y_{11}^d v_d \epsilon^{5+p} & Y_{12}^d v_d \epsilon^{3+p} & Y_{13}^d v_d \epsilon^{1+p} & \eta_1 \\ Y_{21}^d v_d \epsilon^{4+p} & Y_{22}^d v_d \epsilon^{2+p} & Y_{23}^d v_d \epsilon^{p} & \eta_2 \\ Y_{31}^d v_d \epsilon^{4+p} & Y_{32}^d v_d \epsilon^{2+p} & Y_{33}^d v_d \epsilon^{p} & \eta_3 \\ 0 & 0 & 0 & f_m^d \langle Z \rangle \end{pmatrix}, \tag{4.31}$$

$$M_e = \begin{pmatrix} Y_{11}^e v_d \epsilon^{5+p} & Y_{12}^e v_d \epsilon^{4+p} & Y_{13}^e v_d \epsilon^{4+p} & 0 \\ Y_{21}^e v_d \epsilon^{3+p} & Y_{22}^e v_d \epsilon^{2+p} & Y_{23}^e v_d \epsilon^{2+p} & 0 \\ Y_{31}^e v_d \epsilon^{1+p} & Y_{32}^e v_d \epsilon^{p} & Y_{33}^e v_d \epsilon^{p} & 0 \\ \eta_1 & \eta_2 & \eta_3 & f_m^e \langle Z \rangle \end{pmatrix}. \tag{4.32}$$

The off-diagonal block elements η are negligible because they are of order $\sim \frac{v_d}{\langle Z \rangle}$. The Yukawa couplings $Y_{ij}^{d,e}$ in the above matrices are taken to be of order one. The convention being used here is the above matrices are multiplied from right by left-handed fermions and from left by right-handed fermions. The above charged lepton and down quark mass matrices have lopsided structure that lead to an order one atmospheric angle and to small quark mixing, V_{cb}, simultaneously. The upper 3×3 block of the light fermion mass matrices for both down-quarks and charged leptons can be diagonalized by bi-unitary transformations:

$$U_L^e = U_R^d \sim \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & \omega & \omega \\ \epsilon & \omega & \omega \end{pmatrix}, \tag{4.33}$$

$$U_R^e = U_L^d \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & -\epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{4.34}$$

where $U_{L,R}^e$ and $U_{L,R}^d$ are used to diagonalize the charged lepton and down quark mass matrices respectively and ω is an order one parameter.

Since the messenger superfields couple with left-handed down quark and right-handed charged lepton superfields, the flavor violating off-diagonal elements are only

SU(5)	10_1	10_2	10_3	$\overline{5}_1$	$\overline{5}_2$,$\overline{5}_3$	5_u,$\overline{5}_d$	S	10_m	$\overline{10}_m$	Z
$U(1)$	4	2	0	1+p	p	0	-1	0	-α	α

Table 4.6: The $U(1)$ charge assignments to the $10+\overline{10}$ messenger, MSSM, Z and S superfields.

induced in the quadratic scalar mass matrices for the left-handed down quarks and right-handed charged leptons. These matrices are given in Appendix D.1. By using Eqs.(4.23) and (4.24), where the unitary transformations are given in Eqs.(4.33) and (4.34), we present the mass insertion parameters as power expansions in ϵ in Table 4.7. The experimental bounds of the mass insertion parameters δ_{LL}, δ_{RR} and $\delta_{LR,RL}$ that are presented in the table were obtained by comparing the hadronic and leptonic flavor changing processes to their experimental values [73, 57]. We used the branching-ratio expressions of the decay rates $l_i \to l_j\gamma$ given in [57] in order to find the experimental upper bounds on the leptonic mass insertion parameters that is consistent with the spectra presented in Table 4.2. The numerical values of $\kappa^{d,l} = \frac{m_{b,\tau}A_{d,l}}{\tilde{m}^2_{d,\tau}}$ are given in Table 4.7. These numerical values are based on the spectra given in Table 4.2. We can see from Table 4.7 that the $5+\overline{5}$ model is safe from flavor violation problems as long as $p \geq 2$.

4.2.2 Flavour Violation in $10+\overline{10}$ Model

The new superpotential when the messenger fields belong to $10+\overline{10}$ of $SU(5)$ is given by

$$
\begin{aligned}
W_{10+\overline{10}} &= Q_m(\lambda'_m u^c_m + \lambda'_u u^c_1\epsilon^4 + \lambda'_c u^c_2\epsilon^2 + \lambda'_t u^c_3)H_u + u^c_m(\lambda'_{u^c}Q_1\epsilon^4 \\
&+ \lambda'_{c^c}Q_2\epsilon^2 + \lambda'_{t^c}Q_3)H_u + Q_m(\lambda_{d1}d^c_1\epsilon^{1+p} + \lambda_{d2}d^c_2\epsilon^p \\
&+ \lambda_{d3}d^c_3\epsilon^p)H_d + e^c_m(\lambda_{e1}L_1\epsilon^{1+p} + \lambda_{e2}L_2\epsilon^p + \lambda_{e_3}L_3\epsilon^p)H_d \\
&+ \overline{Q}_m(k_{Qm}Q_m + k_{Q_1}Q_1\epsilon^4 + k_{Q_2}Q_2\epsilon^2 + k_{Q_3}Q_3)Z + \overline{u^c}_m(k_{um}u^c_m + k_{u_1}u^c_1\epsilon^4
\end{aligned}
$$

$$+ \quad k_{u_2} u_2^c \epsilon^2 + k_{u_3} u_3^c) Z + \overline{e^c}_m (k_{em} e_m^c + k_{e_1} e_1^c \epsilon^4 + k_{e_2} e_2^c \epsilon^2 + k_{e_3} e_3^c) Z. \qquad (4.35)$$

The $U(1)$ charge assignments for the messenger, MSSM, S, and Z are given in Table 4.6. One can redefine the linear combination of the fields inside the last five parentheses of Eq.(4.35). This redefinition simplifies the superpotential $W_{10+\overline{10}}$ as follows:

$$\begin{aligned} W_{10+\overline{10}} &= (\lambda'_{u^c} \epsilon^4 Q_1 + \lambda'_{c^c} \epsilon^2 Q_2 + \lambda'_{t^c} Q_3) u_m^c H_u + Q_m (\lambda'_u \epsilon^4 u_1^c + \lambda'_c \epsilon^2 u_2^c \\ &+ \lambda'_t u_3^c) H_u + \lambda'_m Q_m u_m^c H_u + \lambda'_b \epsilon^p Q_m d_3^c H_d + \lambda'_\tau \epsilon^p L_3 e_m^c H_d \\ &+ f_{e^c} \overline{e^c}_m e_m^c Z + f_{u^c} \overline{u^c}_m u_m^c Z + f_Q \overline{Q}_m Q_m Z. \end{aligned} \qquad (4.36)$$

In the scalar mass analysis, the Yukawa couplings suppressed by the expansion parameter ϵ are ignored in the superpotential $W_{10+\overline{10}}$ given by Eq.(4.17). However, we keep them in the flavor violation analysis. In the $10 + \overline{10}$ model, the flavor violating off-diagonal elements are induced in the scalar matrices of the left-handed down quarks, right-hand down quarks, and left-handed charged leptons. These matrices are evaluated in Appendix D.2. Using Eqs.(4.23) and (4.24) and the unitary transformation given in Eqs.(4.33) and (4.34), the mass insertion parameters for the $10 + \overline{10}$ model are listed in Table 4.7. The stringent constraint comes from the $\mu \rightarrow e\gamma$ decay as shown in Table 4.7. The inequality $p \geq 1$ should be satisfied in order to suppress the $\mu \rightarrow e\gamma$ decay process. This justifies why we have ignored such couplings $Q_m d^c H_d$ and $e_m^c L H_d$ in the scalar mass spectrum analysis.

Mass Insertion (δ)	$5+\overline{5}$	$10+\overline{10}$	Exp. Bounds
$(\delta^l_{12})_{LL}$	-	ϵ^{4p+1}	0.00028
$(\delta^l_{12})_{RR}$	ϵ^6	-	0.0004
$(\delta^l_{12})_{RL,LR}$	$\kappa^l_5(\epsilon^{p+4},\, \epsilon^{p+3})$	$\kappa^l_{10}\ \epsilon^{3p+1}$	1.3×10^{-6}
$(\delta^l_{13})_{LL}$	-	ϵ^{4p+1}	0.026
$(\delta^l_{13})_{RR}$	ϵ^4	-	0.04
$(\delta^l_{13})_{RL,LR}$	$\kappa^l_5(\epsilon^{p+4},\, \epsilon^{p+1})$	$\kappa^l_{10}\ \epsilon^{3p+1}$	0.002
$(\delta^l_{23})_{LL}$	-	ϵ^{4p}	0.02
$(\delta^l_{23})_{RR}$	ϵ^2	-	0.03
$(\delta^l_{23})_{RL,LR}$	$\kappa^l_5(\epsilon^{p+2},\, \epsilon^{p})$	$\kappa^l_{10}\ \epsilon^{3p}$	0.0015
$\left(\sqrt{\mathrm{Re}(\delta^{\mathrm{d}}_{12})^2_{\mathrm{LL}}}, \sqrt{\mathrm{Im}(\delta^{\mathrm{d}}_{12})^2_{\mathrm{LL}}}\right)$	ϵ^6	ϵ^6	(0.065, 0.0052)
$\left(\sqrt{(\mathrm{Re}(\delta^{\mathrm{d}}_{12})^2_{\mathrm{RR}})}, \sqrt{(\mathrm{Im}(\delta^{\mathrm{d}}_{12})^2_{\mathrm{RR}})}\right)$	-	ϵ^{1+4p}	(0.065, 0.0052)
$\left(\sqrt{\mathrm{Re}(\delta^{\mathrm{d}}_{12})^2_{\mathrm{LR}}}, \sqrt{(\mathrm{Im}(\delta^{\mathrm{d}}_{12})^2)_{\mathrm{LR}}}\right)$	$\kappa^d_5\epsilon^{4+p}$	$\kappa^d_{10}\epsilon^{1+3p}$	$(0.007,\, 5.2\times 10^{-5})$
$\left(\sqrt{\mathrm{Re}(\delta^{\mathrm{d}}_{12})^2_{\mathrm{LR}}}, \sqrt{(\mathrm{Im}(\delta^{\mathrm{d}}_{12})^2)_{\mathrm{LR}}}\right)$	$\kappa^d_5\epsilon^{3+p}$	$\kappa^d_{10}\epsilon^{1+3p}$	$(0.007,\, 5.2\times 10^{-5})$
$\sqrt{\mathrm{Re}(\delta^{\mathrm{d}}_{12})_{\mathrm{LL}}(\delta^{\mathrm{d}}_{12})_{\mathrm{RR}}}$	-	$\epsilon^{3.5+2p}$	0.00453
$\sqrt{\mathrm{Im}(\delta^{\mathrm{d}}_{12})_{\mathrm{LL}}(\delta^{\mathrm{d}}_{12})_{\mathrm{RR}}}$	-	$\epsilon^{3.5+2p}$	0.00057
$(\mathrm{Re}\delta^{\mathrm{d}}_{13}, \mathrm{Im}\delta^{\mathrm{d}}_{13})_{\mathrm{LL}}$	ϵ^4	ϵ^4	(0.238, 0.51)
$(\mathrm{Re}\delta^{\mathrm{d}}_{13}, \mathrm{Im}\delta^{\mathrm{d}}_{13})_{\mathrm{RR}}$	-	ϵ^{1+4p}	(0.238, 0.51)
$(\mathrm{Re}\delta^{\mathrm{d}}_{13}, \mathrm{Im}\delta^{\mathrm{d}}_{13})_{\mathrm{LR,RL}}$	$\kappa^d_5(\epsilon^{4+p},\, \epsilon^{1+p})$	$\kappa^d_{10}\epsilon^{1+3p}$	(0.0557, 0.125)
$(\delta^d_{23})_{LL}$	ϵ^2	ϵ^2	1.19
$(\delta^d_{23})_{RR}$	-	ϵ^{1+4p}	1.19
$(\delta^d_{23})_{LR,RL}$	$\kappa^d_5(\epsilon^{p+2},\, \epsilon^{p})$	$\kappa^d_{10}(\epsilon^2,\, 1)$	0.04

Table 4.7: The calculated mass insertion parameters for the $5+\overline{5}$ and $10+\overline{10}$ models and their experimental upper bounds. The numerical values of κ's are $\kappa^d_5 = 0.0066$, $\kappa^l_5 = 0.032$, $\kappa^d_{10} = 0.0028$ and $\kappa^l_{10} = 0.0025$.

CHAPTER 5

CONCLUSION

In spite of the impressive success of the standard model in producing most of the observed low energy data, it leaves many unanswered fundamental questions. Therefore, we need to go beyond the standard model. Grand unification theory is a more symmetrical theory than the standard model, it combines the standard model interactions (electroweak and strong interactions) into one simple gauge group that has one gauge coupling constant. In addition, since one family of fermions is now grouped into a larger representation of the GUT symmetry, fewer Yukawa couplings are obtained in the GUT model. The minimal-$SO(10)$ gauge group has several advantages over the minimal-$SU(5)$ such as: (1) One family of the standard model fermions plus the right-handed neutrino are unified into one 16-dimensional irreducible representation of the $SO(10)$. This is in contrast to two irreducible representations of the $SU(5)$ ($\overline{5}$, 10) are required to unify one family of fermions (except the right-handed neutrino). (2) Since the right-handed neutrino is automatically accommodated in the 16-dimentional irreducible representation of $SO(10)$, which is not the case in $SU(5)$, the seesaw mechanism is implemented naturally in the $SO(10)$. (3) Minimal $SO(10)$ model has less free parameters than the $SU(5)$ model.

The three gauge couplings do not unify at high energy scale in the SM. However, if the supersymmetric extension of the standard model (MSSM) is used instead of the standard model, not only the unification of the three gauge coupling constants is obtained, but also the gauge hierarchy problem is solved. The price we pay by supersymmetrizing the theory is increasing the number of free parameters and getting new sources of flavor violation. These SUSY-shortcomings might be solved if we know the origin of supersymmetry breaking. There are two main proposed scenarios for supersymmetry breaking. Gravity-mediated and gauge-mediated supersymmetry breaking. The gravity mediated supersymmetry breaking scenario gives only realistic models if the scalar mass universality condition is assumed at M^*. Consequently, the FCNC problem is solved. This universality boundary condition can be arranged by employing flavor symmetry such as the non-abelian discrete A_4 symmetry. On the other hand, the universality condition is naturally satisfied in the gauge-mediated supersymmetry breaking scenario. Besides, models with gauge-mediated supersymmetry breaking are highly predictive. In this study, both scenarios are considered in the unification framework.

The $SO(10) \times A_4$ model is the first SUSY grand unification model based on the gauge symmetry SO(10) with the discrete family symmetry A_4 leading to the doubly lopsided structure for lepton and down quark mass matrices. This structure successfully accommodates the largeness of the neutrino mixing angles and the smallness of the CKM mixing angles. A few works on $SO(10) \times A_4$ have recently been published, but what makes this work unique is the assumption of using the minimal set of Higgs fields that break SO(10) to the SM group. This assumption acts as an important guide for searching for good models. The possibilities of renormalizable Yukawa interactions for quarks and leptons are very limited because the minimum Higgs breaking scheme is imposed and the superpotential must respect the assigned symmetry of the model. Based on that, a general mass structure for the heavy SM fermion generations has been obtained which explains the following features: (1) $m_b^0 \approx m_\tau^0$, (2) $\frac{m_\mu^0}{m_s^0} = 3$, (3) $\frac{m_c^0}{m_t^0} << \frac{m_s^0}{m_b^0}$. It is important to mention that another work [32] obtained the same mass structure for heavy fermions. In that work, the authors did not employ the flavor symmetry but showed that the hierarchy between the second and third generations can be understood by choosing a specific direction of $\langle 45_H \rangle$. Also, they employed another adjoint Higgs field $\langle 45_H \rangle$ to include the first family in their model. On the other hand, in the $SO(10) \times A_4$ model, the above features of heavy fermions have been obtained by picking a specific direction of $\langle 45_H \rangle$, but the hierarchy between the three generations can be understood in the framework of A_4-symmetry. Without adding another adjoint to the model, the first family is successfully included in the model and excellent predictions are obtained. For fitting purposes, some approximate analytical expressions given in Eq.(2.26) are derived for mass ratios and mixing angles of the quarks and the leptons by combining the Yukawa couplings represented by the three Feynman diagrams in Figs. (2.1)-(2.3). However, exact numerical fitting at the low scale was done. Without including the neutrino sector, the model predictions at the low scale for the masses and the mixing angles (except the atmospheric angle) of the quarks and the charged leptons, as well as the CP violation parameter, are in excellent agreement with data (i.e. within 2σ). The atmospheric angle needs to be corrected by considering the neutrino sector. The symmetry of the model succeeds in producing the appropriate right-handed neutrino structure that gives not only the correct contribution to the atmospheric angle, but also the correct neutrino mass differences. The neutrino contribution to the solar angle is negligible.

I also investigated flavor violating processes that arises below and above the GUT scale in the $SO(10) \times A_4$ model in chapter 3. Above the GUT scale, I study how flavor violation gets linked with the fitting of fermion masses and mixing through the factors T_1, T_2, and T_3. The requirement of top Yukawa coupling being ~ 0.5 necessitates some of these factors to be large. Consequently, this corresponds to an order one mixing of the light fields with the exotic heavy fields. In this case, flavor violation is reintroduced at the GUT scale when A_4 symmetry is broken. The stringent constraint on the $\mu \to e\gamma$ decay rate requires a high degree of degeneracy of the soft quadratic masses of the exotic heavy fields and the light fields. Therefore, all the quadratic soft masses are assumed to be universal at the scale $M^* \sim 3M_{GUT}$. Flavor violation is also induced below the GUT scale in the presence of right handed neutrinos through the RG running from M_{GUT} to the right handed neutrino mass scales. This FV source is predicted by the $SO(10) \times A_4$ model because the Dirac neutrino Yukawa couplings are determined from the fermion masses and mixing fitting. Combining all sources of FV, we found that the choice of the slepton masses of order 1 TeV is associated with a $\mu \rightarrow e\gamma$ decay rate close to the current experimental bound. This choice is consistent with the correct fitting for the fermion masses and mixing as well as with the Yukawa couplings being perturbative at M^*.

In the last part of this thesis, I present the work done in collaboration with my advisor Prof. K. S. Babu. In this work we have constructed the superpotential for the $5 + \overline{5}$ and $10 + \overline{10}$ models by employing $U(1)$ flavor symmetry of the Froggatt-Nielsen mechanism. The assigned symmetry for both models allows messenger-matter couplings. These couplings enhance the lightest Higgs mass of the MSSM. We have shown by allowing the messenger-matter mixing in the $10+\overline{10}$ model that the lightest Higgs mass can be increased up to 125 GeV with the lightest stop mass around 1 TeV. The value of 125 GeV is the upper limit allowed by the leading 1 and 2-loop order corrections to the lightest CP even Higgs boson of the MSSM when $M_t^{pole} = 175$ GeV. We also found, consistent with cosmology preference, that the lightest Higgs mass can go up to 121 GeV with the scalar mass spectra below 1 TeV. Introducing messenger-matter couplings in the $10 + \overline{10}$ model has also the advantage of obtaining all the scalar mass spectrum below 1 TeV with $m_h \sim 118$ even at the messenger scale close to the effective SUSY breaking scale Λ. This advantage is not available in the ordinary GMSB when the messenger scale close to Λ. These results are consistent with the gauge coupling being perturbative and unified at the the GUT scale and with the exotic Yukawa couplings being unified at the GUT scale as well as with the FCNC processes being suppressed in agreement with experimental bounds.

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APPENDIX A

Diagonalization of Fermion Mass Matrix

A.1 Derivation of the Light Fermion Mass Matrix

In order to block-diagonalize the mass matrix of $W_{spin.}$ of Eq(3.1), we define the new orthogonal basis as $Y = UX$, where Y(X) is the column matrix that contains the new(old) eigenstates and U is the 5×5 orthogonal matrix (i.e $U^T U = UU^T = I$). These matrices are given by:

$$\begin{pmatrix} L_2 \\ L_3 \\ H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} -N_1 & 0 & 0 & N_1 s_\theta T_2 & 0 \\ \frac{f N_1 c_\theta s_\theta T_2^2}{r_F} & -\frac{f}{N_1 r_F} & \frac{f T_1}{N_1 r_F} & \frac{f N_1 c_\theta T_2}{r_F} & -\frac{f Q_F T_1 T_3}{N_1 r_F} \\ 0 & 0 & G_F Q_F T_3 & 0 & G_F \\ N_2 s_\theta T_2 & N_2 c_\theta T_2 & 0 & N_2 & 0 \\ \frac{f N_2 c_\theta s_\theta T_2^2 T_1}{G_F r_F} & -\frac{f N_2 T_1}{N_1^2 G_F r_F} & -\frac{f G_F}{N_2 r_F} & \frac{f N_2 c_\theta T_2 T_1}{G_F r_F} & \frac{f Q_F G_F T_3}{N_2 r_F} \end{pmatrix} \begin{pmatrix} \psi_2 \\ \psi_3 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}, \quad \text{(A.1)}$$

where $N_1 = 1/\sqrt{1 + T_2^2 s_\theta^2}$, $N_2 = 1/\sqrt{1 + T_2^2}$, $G_F = 1/\sqrt{1 + T_3^2 Q_F^2}$, $f = (1+T_2^2+T_1^2(1+s_\theta^2 T_2^2))^{-1/2}$, and $r_F = \sqrt{(1 + Q_F^2 T_3^2 T_1^2 (1 + s_\theta^2 T_2^2) f^2)}$. The parameters appearing in the above matrix are assumed to be real. Define e_i, E_i, $\overline{E^c}_i$, g_i, g_i', g_i'', and g_i''' to be the charge (-1) leptons in the ψ_i, χ_i, $\overline{\chi}_i$, ϕ_i, ϕ_i', ϕ_i'', and ϕ_i''', respectively; and define e_i^c, E_i^c, $\overline{E}_i$, g_i^c, $g_i'^c$, $g_i''^c$, and $g_i'''^c$ to be the charge $(+1)$ antileptons in the same representations. By writing the old eigenstates appearing in the superpotential $(W_{spin} + W_{vect})$ of Eqs.(3.1) and (3.2) in terms of the new ones, and restricting attention to the electron-type leptons, one gets a 21×21 mass matrix:

$$W_{mass} = \begin{pmatrix} e_i^c & E_\alpha^c & \overline{E}_\alpha & g_i^c & g_i^{\prime c} & g_i^{\prime\prime c} & g_i^{\prime\prime\prime c} \end{pmatrix} \begin{pmatrix} m_0 & m \\ M' & M \end{pmatrix} \begin{pmatrix} e_i \\ E_\alpha \\ \overline{E^c}_\alpha \\ g_i \\ g_i' \\ g_i'' \\ g_i''' \end{pmatrix}, \tag{A.2}$$

where,

$$m_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{afv_d Q_e s_\theta T_1 T_2 T_3}{r_e} \\ 0 & -\frac{afv_d Q_{e^c} s_\theta T_1 T_2 T_3}{r_{e^c}} & -\frac{af^2 v_d (Q_e + Q_{e^c}) c_\theta T_1 T_2 T_3}{r_e r_{e^c}} \end{pmatrix}.$$

The matrices M', m and M can be written in the compact form as

$$M' = \begin{pmatrix} M'_{11} \\ 0 \end{pmatrix}, \tag{A.3}$$

$$m^T = \begin{pmatrix} m_{11} \\ 0 \end{pmatrix}, \tag{A.4}$$

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{13} \end{pmatrix}, \tag{A.5}$$

where

$$M'_{11} = \begin{pmatrix} 0 & aN_1 v_d G_{e^c} s_\theta T_2 & \frac{afN_1 v_d G_{e^c} c_\theta T_2}{r_e} \\ 0 & 0 & -\frac{afN_2 v_d Q_e T_1 T_3}{N_1 r_e} \\ 0 & \frac{afN_1 v_d G_{e^c} Q_{e^c} s_\theta T_2 T_3}{N_2 r_{e^c}} & \frac{af^2 v_d c_\theta \left(N_1^2 G_{e^c}^2 Q_{e^c} - N_2^2 Q_e T_1^2\right) T_2 T_3}{N_1 N_2 G_{e^c} r_e r_{e^c}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ b_3 v_1 & 0 & 0 \\ 0 & -b_3 N_1 v_1 & \frac{b_3 f N_1 v_1 s_\theta c_\theta T_2^2}{r_e} \\ 0 & 0 & -\frac{b_3 f v_1}{N_1 r_e} \end{pmatrix},$$

$$m_{11} = \begin{pmatrix} 0 & aN_1 v_d G_e s_\theta T_2 & \frac{afN_1 v_d c_\theta G_e T_2}{r_{e^c}} \\ 0 & 0 & -\frac{afN_2 v_d Q_{e^c} T_1 T_3}{N_1 r_{e^c}} \\ 0 & \frac{afN_1 v_d G_e Q_e s_\theta T_2 T_3}{N_2 r_e} & \frac{af^2 v_d c_\theta \left(N_1^2 G_e^2 Q_e - N_2^2 Q_{e^c} T_1^2\right) T_2 T_3}{N_1 N_2 G_e r_e r_{e^c}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ b_3 v_5 & 0 & 0 \\ 0 & -b_3 N_1 v_5 & \frac{b_3 f N_1 v_5 c_\theta s_\theta T_2^2}{r_{e^c}} \\ 0 & 0 & -\frac{b_3 f v_5}{N_1 r_{e^c}} \end{pmatrix},$$

$$
M_{11} = \begin{pmatrix}
0 & aN_2 v_d G_{e^c} & \frac{afN_2 v_d G_{e^c} c_\theta T_1 T_2}{G_e r_e} & M_1 G_{e^c} Q_{e^c} T_3 \\
aN_2 v_d G_e & 0 & \frac{af v_d G_e Q_e T_3}{r_e} & M_1 N_2 c_\theta T_1 T_2 \\
\frac{afN_2 v_d G_e c_\theta T_1 T_2}{G_{e^c} r_{e^c}} & \frac{af v_d G_{e^c} Q_{e^c} T_3}{r_{e^c}} & \frac{af^2 v_d \left(G_e^2 Q_e + G_{e^c}^2 Q_{e^c}\right) c_\theta T_1 T_2 T_3}{G_e G_{e^c} r_e r_{e^c}} & -\frac{fM_1\left(N_1^2 G_{e^c}^2 + N_2^2 T_1^2\right)}{N_1^2 N_2 G_{e^c} r_{e^c}} \\
M_1 G_e Q_e T_3 & M_1 N_2 c_\theta T_1 T_2 & -\frac{fM_1\left(N_1^2 G_e^2 + N_2^2 T_1^2\right)}{N_1^2 N_2 G_e r_e} & 0 \\
0 & \frac{M_2}{N_2} & 0 & 0 \\
\frac{M_3}{G_e} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & b_3 N_2 v_1 s_\theta T_2 & \frac{b_3 f N_2 v_1 s_\theta c_\theta T_1 T_2^2}{G_e r_e} & 0 \\
0 & b_3 N_2 v_1 c_\theta T_2 & -\frac{b_3 f N_2 v_1 T_1}{N_1^2 G_e r_e} & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},
$$

$$M_{12} = \begin{pmatrix} 0 & \frac{M_3}{G_{e^c}} & 0 & 0 & 0 & 0 & 0 \\ \frac{M_2}{N_2} & 0 & 0 & b_3 N_2 v_5 s_\theta T_2 & b_3 N_2 v_5 c_\theta T_2 & 0 & 0 \\ 0 & 0 & 0 & \frac{b_3 f N_2 v_5 s_\theta c_\theta T_1 T_2^2}{G_{e^c} r_{e^c}} & -\frac{b_3 f N_2 v_5 T_1}{N_1^2 G_{e^c} r_{e^c}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{10} & 0 & 0 & 0 & h\epsilon_3 \\ 0 & 0 & 0 & M_{10} & 0 & h\epsilon_3 & 0 \\ 0 & 0 & 0 & 0 & M_{10} & h\epsilon_2 & h\epsilon_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_2\gamma_3 \\ 0 & 0 & 0 & 0 & 0 & A_1\gamma_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_2\gamma_2 & A_1\gamma_1 \\ 0 & 0 & 0 & 0 & 0 & k_2\Omega Q_e & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_2\Omega Q_e \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

and

$$M_{13} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ h\epsilon_2 & 0 & A_1\gamma_3 & A_2\gamma_2 & -k_2\Omega Q_e & 0 & 0 \\ h\epsilon_1 & A_2\gamma_3 & 0 & A_1\gamma_1 & 0 & -k_2\Omega Q_e & 0 \\ 0 & A_1\gamma_2 & A_2\gamma_1 0 & 0 & 0 & -k_2\Omega Q_e & \\ A_1\gamma_2 & 0 & 0 & 0 & m & 0 & 0 \\ A_2\gamma_1 & 0 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m \\ 0 & m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 & 0 \\ k_2\Omega Q_e & 0 & 0 & m & 0 & 0 & 0 \end{pmatrix}.$$

Here $v_1 = \langle 1(16_H)\rangle$, $v_5 = \langle \overline{5}(16_H)\rangle$, $v_d = \langle \overline{5}(10_H)\rangle$, $s_\theta \equiv \sin\theta$ and $c_\theta \equiv \cos\theta$. The above 21×21 mass matrix may be block-diagonalized as follows [32]:

$$U_R \begin{pmatrix} m_0 & m \\ M' & M \end{pmatrix} U_L^\dagger = \begin{pmatrix} (m_0 - mM^{-1}M')(1+y^\dagger y)^{-1/2} & 0 \\ 0 & (MM^\dagger + M'M'^\dagger) \end{pmatrix}, \qquad (A.6)$$

where

$$U_R = \begin{pmatrix} I & (m_0 M'^\dagger + mM^\dagger)(MM^\dagger + M'M'^\dagger)^{-1} \\ (MM^\dagger + M'M'^\dagger)^{-1}(m_0^\dagger M' + m^\dagger M) & I \end{pmatrix}, \qquad (A.7)$$

and

$$U_L = \begin{pmatrix} (1+y^\dagger y)^{-1/2} & 0 \\ 0 & (MM^\dagger + M'M'^\dagger)^{-1/2} \end{pmatrix} \begin{pmatrix} I & -y^\dagger \\ M' & M \end{pmatrix}. \qquad (A.8)$$

Here $y = M^{-1}M'$. Terms of order $(M_{Weak}/M_{GUT})^2$ have been dropped. Then the 3×3 light fermion mass matrix of charged leptons in Eq.(3.5) is obtained by applying the relation in the left upper block of the matrix in the Eq.(A.6), where the factor $(1+y^\dagger y)^{-1/2}$ is close to identity for small mixing between the ψ_i and the 10-plet vectors. Similarly, one can obtain the down-type quark mass matrix. The parameters appearing in Eq.(3.5) are defined as follows:

$$\zeta = c_{13} + \delta_2 \frac{3+2\alpha}{5}, \tag{A.9}$$

$$\beta = c_{23} - \delta_1 \frac{3+2\alpha}{5}, \tag{A.10}$$

$$s = \frac{5 s_\theta}{f(2+3\alpha)c_\theta}, \tag{A.11}$$

$$c_{12} = \frac{b_3^2 h N_1 v_1 v_5 \epsilon_3}{a f^2 v_d c_\theta M_{10}^2 (Q_e + Q_{e^c}) T_1 T_2 T_3}, \tag{A.12}$$

$$\delta_3 = \frac{(A_1 - A_2) b_3^2 k_2 N_1 v_1 v_5 \gamma_3 \Omega}{a f^2 m v_d c_\theta M_{10}^2 \left(Q_e + Q_{e^c}\right) T_1 T_2 T_3}, \tag{A.13}$$

$$c_{13} = \frac{b_3^2 h v_1 v_5 \left(\epsilon_2 - N_1^2 \epsilon_3 c_\theta s_\theta T_2^2\right)}{a f N_1 v_d c_\theta M_{10}^2 \left(Q_e + Q_{e^c}\right) T_1 T_2 T_3}, \tag{A.14}$$

$$\delta_2 = \frac{(A_1 - A_2) b_3^2 k_2 v_1 v_5 \Omega \left(\gamma_2 + \mathrm{N}_1^2 \gamma_3 \mathrm{c}_\theta \mathrm{s}_\theta \mathrm{T}_2^2\right)}{a f m N_1 v_d c_\theta M_{10}^2 \left(Q_e + Q_{e^c}\right) T_1 T_2 T_3}, \tag{A.15}$$

$$c_{23} = \frac{-b_3^2 h v_1 v_5 \epsilon_1}{a f v_d c_\theta M_{10}^2 (Q_e + Q_{e^c}) T_1 T_2 T_3}, \tag{A.16}$$

$$\delta_1 = \frac{(-A_1 + A_2) b_3^2 k_2 v_1 v_5 \gamma_1 \Omega}{a f m v_d c_\theta M_{10}^2 \left(Q_e + Q_{e^c}\right) T_1 T_2 T_3}. \tag{A.17}$$

The above parameters are written in terms of the Yukawa couplings and the VEVs of the Higgs fields appearing in the superpotentials W_{spin} and $W_{vect.}$ in Eqs.(3.1) and (3.2). The parameters γ_1, γ_2 and γ_3 appearing in the above Eqs.(A.13), (A.15) and (A.17) are the VEV components of the Higgs singlet $1''_H$.

A.2 Light Neutrino Mass Matrix

The neutrino mass matrix can be obtained from the superpotentials given by Eqs.(3.1) and (3.6). For simplicity, the contribution from the superpotential $W_{vect.}$ in Eq.(3.2) is ignored by assuming the coupling of the ordinary spinor fields 16_i with the vector multiplets is small. Define the right- and left-handed neutrinos, denoted respectively by (ν_i^c and ν_i), residing in ψ_i. Similarly, $\nu_{\chi_i}^c$ and $\nu_{\chi_i}(\overline{\nu^c}_{\overline{\chi}_i}$ and $\overline{\nu}_{\overline{\chi}_i})$ reside in $\chi_i(\overline{\chi}_i)$, where i runs from 1 to 3. Including the six singlets denoted by Z_i and Z_i^c, one can construct 24×24 mass matrix written in the following compact form

$$W_{mass} = N^T \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} N, \tag{A.18}$$

where

$$N^T = \begin{pmatrix} \nu_i & \nu_{\chi_i} & \overline{\nu^c}_{\overline{\chi}_i} & \nu_i^c & \nu_{\chi_i}^c & \overline{\nu}_{\overline{\chi}_i} & Z_i & Z_i^c \end{pmatrix}, \tag{A.19}$$

and

$$M_D^T = \begin{pmatrix} 0 \\ C \\ 0 \end{pmatrix}, \tag{A.20}$$

where

$$C = \begin{pmatrix} 0 & 0 & -\frac{afv_u Q_\nu s_\theta T_1 T_2 T_3}{r_\nu} \\ 0 & -\frac{afv_u Q_{\nu^c} s_\theta T_1 T_2 T_3}{r_{\nu^c}} & -\frac{af^2 v_u c_\theta (Q_\nu + Q_{\nu^c}) T_1 T_2 T_3}{r_\nu r_{\nu^c}} \\ 0 & aN_1 v_u G_{\nu^c} s_\theta T_2 & \frac{afN_1 v_u c_\theta G_{\nu^c} T_2}{r_\nu} \\ 0 & 0 & -\frac{afN_2 v_u Q_\nu T_1 T_3}{N_1 r_\nu} \\ 0 & \frac{afN_1 v_u G_{\nu^c} Q_{\nu^c} s_\theta T_2 T_3}{N_2 r_{\nu^c}} & \frac{af^2 v_u c_\theta \left(N_1^2 G_{\nu^c}^2 Q_{\nu^c} - N_2^2 Q_\nu T_1^2\right) T_2 T_3}{N_1 N_2 G_{\nu^c} r_\nu r_{\nu^c}} \end{pmatrix}.$$

The matrix M_R can be written in the compact form

$$M_R = \begin{pmatrix} M_{R11} & M_{R12} & M_{R13} & M_{R14} \end{pmatrix}, \tag{A.21}$$

where the matrices M_{R11}, M_{R12}, M_{R13}, and M_{R14} are given respectively by

$$
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
aN_1 v_u G_\nu s_\theta T_2 & 0 & \frac{afN_1 v_u G_\nu Q_\nu s_\theta T_2 T_3}{N_2 r_\nu} & 0 \\
\frac{afN_1 v_u c_\theta G_\nu T_2}{r_{\nu^c}} & -\frac{afN_2 v_u Q_{\nu^c} T_1 T_3}{N_1 r_{\nu^c}} & \frac{af^2 v_u c_\theta \left(N_1^2 G_\nu^2 Q_\nu - N_2^2 Q_{\nu^c} T_1^2\right) T_2 T_3}{N_1 N_2 G_\nu r_\nu r_{\nu^c}} & 0 \\
0 & aN_2 v_u G_{\nu^c} & \frac{afN_2 v_u c_\theta G_{\nu^c} T_1 T_2}{G_\nu r_\nu} & M_1 G_{\nu^c} Q_{\nu^c} T_3 \\
aN_2 v_u G_\nu & 0 & \frac{af v_u G_\nu Q_\nu T_3}{r_\nu} & M_1 N_2 c_\theta T_1 T_2 \\
\frac{afN_2 v_u c_\theta G_\nu T_1 T_2}{G_{\nu^c} r_{\nu^c}} & \frac{af v_u G_{\nu^c} Q_{\nu^c} T_3}{r_{\nu^c}} & \frac{af^2 v_u c_\theta \left(G_\nu^2 Q_\nu + G_{\nu^c}^2 Q_{\nu^c}\right) T_1 T_2 T_3}{G_\nu G_{\nu^c} r_\nu r_{\nu^c}} & -\frac{fM_1\left(N_1^2 G_{\nu^c}^2 + N_2^2 T_1^2\right)}{N_1^2 N_2 G_{\nu^c} r_{\nu^c}} \\
M_1 G_\nu Q_\nu T_3 & M_1 N_2 c_\theta T_1 T_2 & -\frac{fM_1\left(N_1^2 G_\nu^2 + N_2^2 T_1^2\right)}{N_1^2 N_2 G_\nu r_\nu} & 0 \\
0 & \frac{M_2}{N_2} & 0 & 0 \\
\frac{M_3}{G_\nu} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
\begin{pmatrix}
0 & 0 & 0 & aN_1 v_u G_\nu s_\theta T_2 & \frac{afN_1 v_u c_\theta G_\nu T_2}{r_{\nu^c}} & 0 \\
0 & 0 & 0 & 0 & -\frac{afN_2 v_u Q_{\nu^c} T_1 T_3}{N_1 r_{\nu^c}} & aN_2 v_u G_{\nu^c} \\
0 & 0 & 0 & \frac{afN_1 v_u G_\nu Q_\nu s_\theta T_2 T_3}{N_2 r_\nu} & \frac{af^2 v_u c_\theta \left(N_1^2 G_\nu^2 Q_\nu - N_2^2 Q_{\nu^c} T_1^2\right) T_2 T_3}{N_1 N_2 G_\nu r_\nu r_{\nu^c}} & \frac{afN_2 v_u c_\theta G_{\nu^c} T_1 T_2}{G_\nu r_\nu} \\
0 & 0 & 0 & 0 & 0 & M_1 G_{\nu^c} Q_{\nu^c} T_3 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{M_3}{G_{\nu^c}} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{M_3}{G_{\nu^c}} & 0 & 0 & 0 & 0 \\
\frac{M_2}{N_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & vb_4 & 0 & 0 & 0 \\
0 & 0 & 0 & -N_1 v b_4 & \frac{fN_1 v b_4 c_\theta s_\theta T_2^2}{r_\nu} & 0 \\
0 & 0 & 0 & 0 & -\frac{f v b_4}{N_1 r_\nu} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
\begin{pmatrix}
aN_2v_uG_\nu & \frac{afN_2v_uc_\theta G_\nu T_1T_2}{G_{\nu^c}r_{\nu^c}} & M_1G_\nu Q_\nu T_3 & 0 & \frac{M_3}{G_\nu} & 0 \\
0 & \frac{afv_uG_{\nu^c}Q_{\nu^c}T_3}{r_{\nu^c}} & M_1N_2c_\theta T_1T_2 & \frac{M_2}{N_2} & 0 & 0 \\
\frac{afv_uG_\nu Q_\nu T_3}{r_\nu} & \frac{af^2v_uc_\theta\left(G_\nu^2Q_\nu+G_{\nu^c}^2Q_{\nu^c}\right)T_1T_2T_3}{G_\nu G_{\nu^c}r_\nu r_{\nu^c}} & -\frac{fM_1\left(N_1^2G_\nu^2+N_2^2T_1^2\right)}{N_1^2N_2G_\nu r_\nu} & 0 & 0 & 0 \\
M_1N_2c_\theta T_1T_2 & -\frac{fM_1\left(N_1^2G_{\nu^c}^2+N_2^2T_1^2\right)}{N_1^2N_2G_{\nu^c}r_{\nu^c}} & 0 & 0 & 0 & 0 \\
\frac{M_2}{N_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & vb_4 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
N_2vb_4s_\theta T_2 & \frac{fN_2vb_4c_\theta s_\theta T_1T_2^2}{G_\nu r_\nu} & 0 & 0 & 0 & 0 \\
N_2vb_4c_\theta T_2 & -\frac{fN_2vb_4T_1}{N_1^2G_\nu r_\nu} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c\alpha_3 \\
0 & 0 & 0 & 0 & 0 & c\alpha_2
\end{pmatrix},
$$

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-N_1 v b_4 & 0 & 0 & 0 & 0 \\
\frac{f N_1 v b_4 c_\theta s_\theta T_2^2}{r_\nu} & -\frac{f v b_4}{N_1 r_\nu} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
N_2 v b_4 s_\theta T_2 & N_2 v b_4 c_\theta T_2 & 0 & 0 & 0 \\
\frac{f N_2 v b_4 c_\theta s_\theta T_1 T_2^2}{G_\nu r_\nu} & -\frac{f N_2 v b_4 T_1}{N_1^2 G_\nu r_\nu} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c\alpha_3 & c\alpha_2 \\
0 & 0 & c\alpha_3 & 0 & c\alpha_1 \\
0 & 0 & c\alpha_2 & c\alpha_1 & 0 \\
c\alpha_3 & c\alpha_2 & m_1 & 0 & 0 \\
0 & c\alpha_1 & 0 & m_1 & 0 \\
c\alpha_1 & 0 & 0 & 0 & m_1
\end{pmatrix}. \tag{A.22}
$$

The light neutrino mass matrix is given by the seesaw formula as fellows

$$
M_\nu \;=\; M_D M_R^{-1} M_D^T = \lambda \begin{pmatrix} 0 & 0 & 0 \\ 0 & \kappa & \eta \\ 0 & \eta & 1 \end{pmatrix}, \tag{A.23}
$$

where

$$
\begin{aligned}
\lambda \;&=\; \frac{\Lambda a^2 c^2 f^2 v_d^2 T_1^2 T_2^2 \left(\left(\alpha_1^2+\alpha_3^2\right) Q_\nu^2 r_{\nu^c}^2 s_\theta^2 + 2N_1^2 \alpha_2 \alpha_3 c_\theta Q_\nu r_{\nu^c} s_\theta \left(\left(Q_\nu + Q_{\nu^c}\right) r_\nu\right.\right.}{m_1 N_1^2 v^2 b_4^2 r_\nu^2 r_{\nu^c}^2} \\
&+\; \frac{\left.\left. Q_\nu r_{\nu^c} s_\theta^2 T_2^2\right) + N_1^4 \left(\alpha_1^2+\alpha_2^2\right) c_\theta^2 \left(\left(Q_\nu + Q_{\nu^c}\right) r_\nu + Q_\nu r_{\nu^c} s_\theta^2 T_2^2\right)^2\right) T_3^2}{m_1 N_1^2 v^2 b_4^2 r_\nu^2 r_{\nu^c}^2}, \\
\eta \;&=\; \frac{N_1^2 Q_{\nu^c} r_\nu^2 s_\theta \left(\alpha_2 \alpha_3 Q_\nu r_{\nu^c} s_\theta + N_1^2 \left(\alpha_1^2+\alpha_2^2\right) c_\theta \left(\left(Q_\nu + Q_{\nu^c}\right) r_\nu + Q_\nu r_{\nu^c} s_\theta^2 T_2^2\right)\right)}{f(A+B)},
\end{aligned}
$$

$$\kappa = \frac{N_1^4 \left(\alpha_1^2 + \alpha_2^2\right) Q_{\nu^c}^2 r_\nu^4 s_\theta^2}{f^2(A+B)}. \tag{A.24}$$

Here the numerical values of α_1, α_2, α_3 and λ are given in section 2, and we have defined

$$\begin{aligned}
A &= \left(\alpha_1^2 + \alpha_3^2\right) Q_e^2 r_{e^c}^2 s_\theta^2 + 2N_1^2 \alpha_2 \alpha_3 c_\theta Q_e r_{e^c} s_\theta \left(\left(Q_e + Q_{e^c}\right) r_e + Q_e r_{e^c} s_\theta^2 T_2^2\right), \\
B &= N_1^4 \left(\alpha_1^2 + \alpha_2^2\right) c_\theta^2 \left(\left(Q_e + Q_{e^c}\right) r_e + Q_e r_{e^c} s_\theta^2 T_2^2\right).
\end{aligned}$$

APPENDIX B

RGE from the Scale M^* to the GUT Scale in the $SO(10) \times A_4$ Model

Neglecting all the couplings in the superpotential W_{vector}, since they do not contribute to the top Yukawa coupling, we present only the RGEs that are needed to find the parameter σ at the GUT scale. The one-loop RGE's of the unified gauge (g_G) coupling, the couplings appearing in W_{spinor}, and the trilinear soft terms associated with W_{spinor} between the scale M^* and GUT scale are

$$16\pi^2 \frac{dg_G}{dt} = 19 g_G^3, \tag{B.1}$$
$$16\pi^2 \frac{db_1}{dt} = b_1(20b_1^2 + b_2^2 - 45g_G^2), \tag{B.2}$$
$$16\pi^2 \frac{db_2}{dt} = b_2(20b_2^2 + b_1^2 - 45g_G^2), \tag{B.3}$$
$$16\pi^2 \frac{da}{dt} = a(18a^2 - \frac{63}{2}g_G^2), \tag{B.4}$$
$$16\pi^2 \frac{d\tilde{b}_1}{dt} = 2(20b_1^2\tilde{b}_1 + b_2^2\tilde{b}_2 + 45g_G^2 M_\lambda), \tag{B.5}$$
$$16\pi^2 \frac{d\tilde{b}_2}{dt} = 2(20b_2^2\tilde{b}_2 + b_1^2\tilde{b}_1 + 45g_G^2 M_\lambda), \tag{B.6}$$
$$16\pi^2 \frac{d\tilde{a}}{dt} = 28\tilde{a}a^2 + 63g_G^2 M_\lambda. \tag{B.7}$$

The RGE's soft mass terms for the fields appearing in W_{spinor} are given below:

$$\begin{aligned} 16\pi^2 \frac{d\tilde{m}^2_{\psi_i}}{dt} &= 2b_1^2(\tilde{m}^2_{\psi_i} + \tilde{m}^2_{\overline{\chi}_1} + \tilde{m}^2_{1_{H_i}} + \tilde{b}_1^2) \\ &+ 2b_2^2(\tilde{m}^2_{\psi_i} + \tilde{m}^2_{\overline{\chi}_2} + \tilde{m}^2_{1'_{H_i}} + \tilde{b}_2^2) - 45g_G^2 M_\lambda^2, \end{aligned} \tag{B.8}$$
$$16\pi^2 \frac{d\tilde{m}^2_{\overline{\chi}_1}}{dt} = 6b_1^2(\tilde{m}^2_{\psi_i} + \tilde{m}^2_{\overline{\chi}_1} + \tilde{m}^2_{1_{H_i}} + \tilde{b}_1^2) - 45g_G^2 M_\lambda^2, \tag{B.9}$$
$$16\pi^2 \frac{d\tilde{m}^2_{\overline{\chi}_2}}{dt} = 6b_2^2(\tilde{m}^2_{\psi_i} + \tilde{m}^2_{\overline{\chi}_2} + \tilde{m}^2_{1'_{H_i}} + \tilde{b}_2^2) - 45g_G^2 M_\lambda^2, \tag{B.10}$$
$$16\pi^2 \frac{d\tilde{m}^2_{1_{H_i}}}{dt} = 32b_1^2(\tilde{m}^2_{\psi_i} + \tilde{m}^2_{\overline{\chi}_1} + \tilde{m}^2_{1_{H_i}} + \tilde{b}_1^2), \tag{B.11}$$

$$16\pi^2 \frac{d\tilde{m}^2_{1'_{H_i}}}{dt} = 32 b_2^2 (\tilde{m}^2_{\psi_i} + \tilde{m}^2_{\overline{\chi}_2} + \tilde{m}^2_{1'_{H_i}} + \tilde{b}_2^2), \quad \text{(B.12)}$$

$$16\pi^2 \frac{d\tilde{m}^2_{\chi_{2,3}}}{dt} = 10 a^2 (\tilde{m}^2_{\chi_2} + \tilde{m}^2_{\chi_3} + \tilde{m}^2_{10_H} + \tilde{a}^2) - 45 g_G^2 M_\lambda^2, \quad \text{(B.13)}$$

$$16\pi^2 \frac{d\tilde{m}^2_{10_H}}{dt} = 16 a^2 (\tilde{m}^2_{\chi_2} + \tilde{m}^2_{\chi_3} + \tilde{m}^2_{10_H} + \tilde{a}^2) - 36 g_G^2 M_\lambda^2. \quad \text{(B.14)}$$

Here $\tilde{m}^2_{1_{H_i}}$, $\tilde{m}^2_{1'_{H_i}}$ and $\tilde{m}^2_{10_H}$ are the quadratic soft masses for the Higgs superfields appearing in W_{spin} defined in Eq.(3.1) and the quadratic soft masses $\tilde{m}^2_{\psi_i}$, $\tilde{m}^2_{\overline{\chi}_{1,2}}$, and $\tilde{m}^2_{\chi_{1,2}}$ are defined in Eq.(3.10).

APPENDIX C

Yukawa Couplings RGEs

C.1 MSSM with $5 + \overline{5}$ Messenger Fields

Here we derive the ordinary Yukawa couplings and the exotic Yukawa couplings appearing in Eq(4.30) between the messenger scale and the GUT scale:

$$\begin{aligned}
\frac{dg_3^2}{dt} &= \frac{-g_3^4}{4\pi^2}, \\
\frac{dg_2^2}{dt} &= \frac{g_2^4}{4\pi^2}, \\
\frac{dg_1^2}{dt} &= \frac{19g_1^4}{20\pi^2}, \\
\frac{d\lambda_t^2}{dt} &= \frac{\lambda_t^2}{8\pi^2}[6\lambda_t^2 + \lambda_b^2 + \lambda_b'^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2], \\
\frac{d\lambda_b^2}{dt} &= \frac{\lambda_b^2}{8\pi^2}[6\lambda_b^2 + \lambda_t^2 + \lambda_\tau^2 + \lambda_{\tau^c}'^2 + 4\lambda_b'^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2], \\
\frac{d\lambda_\tau^2}{dt} &= \frac{\lambda_\tau^2}{8\pi^2}[4\lambda_\tau^2 + 3\lambda_b^2 + 3\lambda_{\tau^c}'^2 + 3\lambda_b'^2 - 3g_2^2 - \frac{9}{5}g_1^2], \\
\frac{d\lambda_b'^2}{dt} &= \frac{\lambda_b'^2}{8\pi^2}[6\lambda_b'^2 + 4\lambda_b^2 + \lambda_{\tau^c}'^2 + \lambda_t^2 + \lambda_\tau^2 + f_d^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2], \\
\frac{d\lambda_{\tau^c}'^2}{dt} &= \frac{\lambda_{\tau^c}'^2}{8\pi^2}[4\lambda_{\tau^c}'^2 + 3\lambda_b^2 + 3\lambda_b'^2 + 3\lambda_\tau^2 + f_e^2 - 3g_2^2 - \frac{9}{5}g_1^2], \\
\frac{df_d^2}{dt} &= \frac{f_d^2}{8\pi^2}[5f_d^2 + 2f_e^2 + 2\lambda_b'^2 - \frac{16}{3}g_3^2 - \frac{4}{15}g_1^2], \\
\frac{df_e^2}{dt} &= \frac{f_e^2}{8\pi^2}[4f_e^2 + 3f_d^2 + \lambda_{\tau^c}'^2 - 3g_2^2 - \frac{3}{5}g_1^2],
\end{aligned}$$

C.2 MSSM with $10+\overline{10}$ Messenger Fields

Here we write only the RGEs for Yukawa couplings that are not suppressed by the parameter ϵ between two scales, the messenger and the GUT scale.

$$
\begin{aligned}
\frac{dg_3^2}{dt} &= 0, \\
\frac{dg_2^2}{dt} &= \frac{g_2^4}{4\pi^2}, \\
\frac{dg_1^2}{dt} &= \frac{3g_1^4}{5\pi^2}, \\
\frac{d\lambda_t^2}{dt} &= \frac{\lambda_t^2}{8\pi^2}[6\lambda_t^2+\lambda_b^2+4\lambda_{t^c}'^2+5\lambda_t'^2+3\lambda_m'^2-\frac{16}{3}g_3^2-3g_2^2-\frac{13}{15}g_1^2], \\
\frac{d\lambda_b^2}{dt} &= \frac{\lambda_b^2}{8\pi^2}[6\lambda_b^2+\lambda_t^2+\lambda_\tau^2+\lambda_{t^c}'^2-\frac{16}{3}g_3^2-3g_2^2-\frac{7}{15}g_1^2], \\
\frac{d\lambda_\tau^2}{dt} &= \frac{\lambda_\tau^2}{8\pi^2}[4\lambda_\tau^2+3\lambda_b^2-3g_2^2-\frac{9}{5}g_1^2], \\
\frac{d\lambda_m'^2}{dt} &= \frac{\lambda_m'^2}{8\pi^2}[6\lambda_m'^2+4\lambda_t'^2+5\lambda_{t^c}'^2+3\lambda_t^2+f_Q^2+f_{u^c}^2-\frac{16}{3}g_3^2-3g_2^2-\frac{13}{15}g_1^2], \\
\frac{df_{e^c}^2}{dt} &= \frac{f_{e^c}^2}{8\pi^2}[3f_{e^c}^2+6f_Q^2+3f_{u^c}^2-\frac{16}{3}g_3^2-3g_2^2-\frac{12}{5}g_1^2], \\
\frac{df_{u^c}^2}{dt} &= \frac{f_{u^c}^2}{8\pi^2}[3f_{u^c}^2+6f_Q^2+f_{e^c}^2+2\lambda_{t^c}'^2+2\lambda_m'^2-\frac{16}{3}g_3^2-\frac{16}{15}g_1^2], \\
\frac{df_Q^2}{dt} &= \frac{f_Q^2}{8\pi^2}[8f_Q^2+3f_{u^c}^2+f_{e^c}^2+\lambda_t'^2+\lambda_m'^2-\frac{16}{3}g_3^2-3g_2^2-\frac{1}{15}g_1^2], \\
\frac{d\lambda_t'^2}{dt} &= \frac{\lambda_t'^2}{8\pi^2}[6\lambda_t'^2+3\lambda_{t^c}'^2+5\lambda_t^2+4\lambda_m'^2+f_Q^2-\frac{16}{3}g_3^2-3g_2^2-\frac{13}{15}g_1^2], \\
\frac{d\lambda_{t^c}'^2}{dt} &= \frac{\lambda_{t^c}'^2}{8\pi^2}[6\lambda_{t^c}'^2+3\lambda_t'^2+4\lambda_t^2+5\lambda_m'^2+\lambda_b^2+f_{u^c}^2-\frac{16}{3}g_3^2-3g_2^2-\frac{13}{15}g_1^2].
\end{aligned}
$$

APPENDIX D

Generated Scalar Masses due to Messenger-Matter Mixing

In this Appendix we will present the generated soft mass terms due to messenger-matter mixing by employing the following expressions: [65]

$$\delta\tilde{m}^2_Q(M_{\rm mess}) = -\frac{1}{4}\{\sum_\lambda(\frac{d\Delta\gamma}{d\lambda}\beta_>[\lambda] - \frac{d\gamma_<}{d\lambda}\Delta\beta[\lambda]) + [\gamma_>,\gamma_<]\}\Lambda^2, \tag{D.1}$$

$$\delta A_{abc}(M_{\rm mess}) = \frac{1}{2}(\lambda_{a'bc}\Delta\gamma_a^{a'} + \lambda_{ab'c}\Delta\gamma_b^{b'} + \lambda_{abc'}\Delta\gamma_c^{c'})\Lambda, \tag{D.2}$$

for both $5+\overline{5}$ and $10+\overline{10}$ model. Where the sum is over the ordinary and exotic Yukawa couplings, $\Delta\beta[\lambda(M_{\rm mess})] = \beta_>[\lambda(M_{\rm mess})] - \beta_<[\lambda(M_{\rm mess})]$, and $\Delta\gamma(M_{\rm mess}) = \gamma_>(M_{\rm mess}) - \gamma_<(M_{\rm mess})$. Here $\gamma_>(\gamma_<)$ is the anomalous dimension above (below) $M_{\rm mess}$ and $\beta[\lambda]$ is the beta function for Yukawa coupling λ.

D.1 $5+\overline{5}$ Model

Let us write the $\Delta\gamma(M_{\rm mess})$ for the quark doublet Q_3, right-handed electron e^c_3 and down Higgs doublet as follows:

$$\Delta\gamma_{Q_{33}}(M_{\rm mess}) = -\frac{\lambda'^2_b}{8\pi^2}, \tag{D.3}$$

$$\Delta\gamma_{e^c_{33}}(M_{\rm mess}) = -2\frac{\lambda'^2_{\tau^c}}{8\pi^2}, \tag{D.4}$$

$$\Delta\gamma_{H_d}(M_{\rm mess}) = -\frac{3\lambda'^2_b + \lambda'^2_{\tau^c}}{8\pi^2}. \tag{D.5}$$

The anomalous dimensions for left-handed down quarks and right-handed electrons below $M_{\rm mess}$ are given respectively by

$$\gamma_{Q_{ij}<}(M_{\rm mess}) = -\frac{Y^u_{ki}Y^u_{kj} + Y^d_{ki}Y^d_{kj} - (8/3)g_3^2 - (3/2)g_2^2 - (1/30)g_1^2}{8\pi^2}, \tag{D.6}$$

$$\gamma_{e^c_{ij}<}(M_{\rm mess}) = -\frac{2Y^e_{ik}Y^e_{jk} - (3/10)g_1^2}{8\pi^2}, \tag{D.7}$$

where

$$Y^u = \begin{pmatrix} Y^u_{11}\epsilon^8 & Y^u_{12}\epsilon^6 & Y^u_{13}\epsilon^4 \\ Y^u_{21}\epsilon^6 & Y^u_{22}\epsilon^4 & Y^u_{23}\epsilon^2 \\ Y^u_{31}\epsilon^4 & Y^u_{32}\epsilon^2 & Y^u_{33} \end{pmatrix}, \tag{D.8}$$

$$Y^d = \epsilon^p \begin{pmatrix} Y^d_{11}\epsilon^5 & Y^d_{12}\epsilon^3 & Y^d_{13}\epsilon \\ Y^d_{12}\epsilon^4 & Y^d_{22}\epsilon^2 & Y^d_{23} \\ Y^d_{13}\epsilon^4 & Y^d_{23}\epsilon^2 & Y^d_{33} \end{pmatrix}, \tag{D.9}$$

$$Y^e = \epsilon^p \begin{pmatrix} Y^e_{11}\epsilon^5 & Y^e_{12}\epsilon^4 & Y^e_{13}\epsilon^4 \\ Y^e_{12}\epsilon^3 & Y^e_{22}\epsilon^2 & Y^e_{23}\epsilon^2 \\ Y^e_{13}\epsilon & Y^e_{23} & Y^e_{33}, \end{pmatrix}, \tag{D.10}$$

By keeping only the leading term of the expansion parameter ϵ, we calculate

$$\Delta\beta_{Y^u_{33}}(M_{\text{mess}}) = \frac{Y^u_{33}}{16\pi^2}\lambda_b'^2, \tag{D.11}$$

$$\Delta\beta_{Y^e_{12,22,13,23}}(M_{\text{mess}}) = \frac{Y^e_{12,22,13,23}}{16\pi^2}(\lambda_{\tau^c}'^2 + 3\lambda_b'^2), \tag{D.12}$$

$$\Delta\beta_{Y^e_{32,33}}(M_{\text{mess}}) = 3\frac{Y^e_{32,33}}{16\pi^2}(\lambda_{\tau^c}'^2 + \lambda_b'^2). \tag{D.13}$$

The beta-functions for λ_b' and λ_{τ^c}' above M_{mess} are given respectively by

$$\beta_{\lambda_b'>}(M_{\text{mess}}) = \frac{\lambda_b'}{16\pi^2}(6\lambda_b'^2 + \lambda_e'^2 + (Y^u_{33})^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2), \tag{D.14}$$

$$\beta_{\lambda_{\tau^c}'>}(M_{\text{mess}}) = \frac{\lambda_{\tau^c}'}{16\pi^2}(4\lambda_{\tau^c}'^2 + 3\lambda_b'^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2). \tag{D.15}$$

Note that $[\gamma_>, \gamma_<] = [\Delta\gamma, \gamma_<]$. Plugging Eqs.(D.3-D.15) into Eqs.(D.1,D.2) and keeping only the leading expansion parameter ϵ we obtain

$$\delta\tilde{m}^2_{e^c} \sim \delta\tilde{m}^2_{e^c_3} \begin{pmatrix} \epsilon^{8+2p} & \epsilon^{6+2p} & \epsilon^{4+2p} \\ \epsilon^{6+2p} & \epsilon^{4+2p} & \epsilon^{2+2p} \\ \epsilon^{4+2p} & \epsilon^{2+2p} & 1 \end{pmatrix}, \tag{D.16}$$

$$\delta A_e \sim \frac{\Lambda\epsilon^p}{(16\pi^2)} \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^1 & (3\lambda_b'^2 + \lambda_{\tau^c}^2) & 3(\lambda_b'^2 + \lambda_{\tau^c}^2) \end{pmatrix}, \tag{D.17}$$

$$\delta A_d \sim \delta A_b \epsilon^p \begin{pmatrix} \epsilon^3 & \epsilon & \epsilon \\ \epsilon^4 & \epsilon^2 & 1 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{D.18}$$

$$\delta \tilde{m}_Q^2 \sim \delta \tilde{m}_{Q_3}^2 \begin{pmatrix} 0 & 0 & \epsilon^4 \\ 0 & 0 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{D.19}$$

$$\delta A_t = \frac{\Lambda^2}{2(16\pi^2)^2} Y_{33}^u \lambda_b'^2, \tag{D.20}$$

where $\delta \tilde{m}_{e_3^c}^2$, $\delta \tilde{m}_{Q_3}^2$ and δA_b are given respectively by Eq. (4.11), Eq. (4.10) and Eq. (4.14).

D.2 $10 + \overline{10}$ Model

By looking at the superpotential $W_{10+\overline{10}}$ in Eq(4.35), we can write $\Delta\gamma_Q$, $\Delta\gamma_{u^c}$ and $\Delta\gamma_{H_u}$ as

$$\Delta\gamma_Q(M_{\rm mess}) = \frac{-1}{8\pi^2} \begin{pmatrix} \lambda_{u^c}'^2 \epsilon^8 & \lambda'_{u^c}\lambda'_{c^c}\epsilon^6 & \lambda'_{u^c}\lambda'_{t^c}\epsilon^4 \\ \lambda'_{u^c}\lambda'_{c^c}\epsilon^6 & \lambda_{c^c}'^2\epsilon^4 & \lambda'_{t^c}\lambda'_{c^c}\epsilon^2 \\ \lambda'_{u^c}\lambda'_{t^c}\epsilon^4 & \lambda'_{t^c}\lambda'_{c^c}\epsilon^2 & \lambda_{t^c}'^2 \end{pmatrix}, \tag{D.21}$$

$$\Delta\gamma_{u^c}(M_{\rm mess}) = \frac{-1}{8\pi^2} \begin{pmatrix} 2\lambda_u'^2 \epsilon^8 & 2\lambda'_u\lambda'_c\epsilon^6 & 2\lambda'_u\lambda'_t\epsilon^4 \\ 2\lambda'_u\lambda'_c\epsilon^6 & 2\lambda_c'^2\epsilon^4 & 2\lambda'_t\lambda'_c\epsilon^2 \\ 2\lambda'_u\lambda'_t\epsilon^4 & 2\lambda'_t\lambda'_c\epsilon^2 & 2\lambda_t'^2 \end{pmatrix}, \tag{D.22}$$

$$\Delta\gamma_{H_u}(M_{\rm mess}) = \frac{-3(\lambda_t'^2 + \lambda_{t^c}'^2 + \lambda_m'^2)}{8\pi^2}. \tag{D.23}$$

The beta-functions for the exotic Yukawa couplings appearing in the above matrices above messenger scale are

$$\beta_{\lambda'_{u^c,c^c}>}(M_{\rm mess}) = \frac{\lambda'_{u^c,c^c}}{16\pi^2}(5\lambda_m'^2 + 3\lambda_{t^c}'^2 + 3\lambda_t'^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2) \tag{D.24}$$

$$\beta_{\lambda'_{t^c}>}(M_{\rm mess}) = \frac{\lambda'_{t^c}}{16\pi^2}(5\lambda'^2_m + 6\lambda'^2_{t^c} + 3\lambda'^2_t + 4(Y^u_{33})^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2), \tag{D.25}$$

$$\beta_{\lambda_{u,c}>}(M_{\rm mess}) = \frac{\lambda_{u,c}}{16\pi^2}(4\lambda'^2_m + 3\lambda'^2_{t^c} + 3\lambda'^2_t - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2), \tag{D.26}$$

$$\beta_{\lambda'_t>}(M_{\rm mess}) = \frac{\lambda'_t}{16\pi^2}(4\lambda'^2_m + 6\lambda'^2_t + 3\lambda'^2_{t^c} + 5(Y^u_{33})^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2). \tag{D.27}$$

The anomalous dimensions $\gamma_{Q<}$ are given by Eq. (D.6) and for right-handed up quarks they are given by

$$\gamma_{u^c_{ij}<}(M_{\rm mess}) = -\frac{2Y^u_{ik}Y^u_{jk} - (16/6)g_3^2 - (8/15)g_1^2}{8\pi^2}, \tag{D.28}$$

and

$$\Delta\beta_{Y^u_{13,23}}(M_{\rm mess}) = \frac{Y^u_{13,23}}{16\pi^2}(3\lambda'^2_m + 4\lambda'^2_{t^c} + 3\lambda'^2_t), \tag{D.29}$$

$$\Delta\beta_{Y^u_{31,32}}(M_{\rm mess}) = \frac{Y^u_{31,32}}{16\pi^2}(3\lambda'^2_m + 3\lambda'^2_{t^c} + 5\lambda'^2_t), \tag{D.30}$$

$$\Delta\beta_{Y^u_{33}}(M_{\rm mess}) = \frac{Y^u_{33}}{16\pi^2}(3\lambda'^2_m + 4\lambda'^2_{t^c} + 5\lambda'^2_t). \tag{D.31}$$

Using Eqs.(D.1,D.2), we obtain

$$\delta\tilde{m}^2_Q \sim \delta\tilde{m}^2_{Q_3}\begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{D.32}$$

$$\delta\tilde{m}^2_{u^c} \sim \delta\tilde{m}^2_{u^c_3}\begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{D.33}$$

$$\delta A_u \sim \delta\tilde{A}_t\begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{D.34}$$

$$\delta A_d \sim \delta\tilde{A}_b \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{D.35}$$

where $\delta\tilde{m}^2_{Q_3}$, $\delta\tilde{m}^2_{u_3^c}$, and $\delta\tilde{A}_t$, and $\delta\tilde{A}_b$ are given respectively by Eqs.(4.18, 4.19, 4.21, 4.22).

The coupling $\lambda\epsilon^p\overline{5}10_m\overline{5}_d$ induces scalar quadratic masses for both right-handed down quarks and left-handed charged leptons and trilinear soft terms (A_d,A_e). These generated soft terms are obtained by following the same previous steps as:

$$\delta\tilde{m}^2_e \sim \delta\tilde{m}^2_{d^c} \sim \frac{\Lambda^2}{2(16\pi^2)^2} \begin{pmatrix} \epsilon^{2+4p} & \epsilon^{1+4p} & \epsilon^{1+4p} \\ \epsilon^{1+4p} & \epsilon^{4p} & \epsilon^{4p} \\ \epsilon^{1+4p} & \epsilon^{4p} & \epsilon^{4p} \end{pmatrix}, \tag{D.36}$$

$$\delta A_e \sim \frac{\Lambda}{2(16\pi^2)} \begin{pmatrix} \epsilon^{5+3p} & \epsilon^{4+3p} & \epsilon^{4+3p} \\ \epsilon^{3+3p} & \epsilon^{2+3p} & \epsilon^{2+3p} \\ \epsilon^{1+3p} & \epsilon^{3p} & \epsilon^{3p} \end{pmatrix}, \tag{D.37}$$

$$\delta A_d \sim \frac{\Lambda}{2(16\pi^2)} \begin{pmatrix} \epsilon^{5+3p} & \epsilon^{3+3p} & \epsilon^{1+3p} \\ \epsilon^{4+3p} & \epsilon^{2+3p} & \epsilon^{3p} \\ \epsilon^{4+3p} & \epsilon^{2+3p} & \epsilon^{3p} \end{pmatrix}. \tag{D.38}$$

VITA

Abdelhamid Albaid

Candidate for the Degree of

Doctor of Philosophy

Dissertation: FERMION MASSES AND MIXINGS, FLAVOR VIOLATION, AND THE HIGGS BOSON MASS IN SUPERSYMMETRIC UNIFIED FRAMEWORK

Major Field: Physics

Biographical:

Personal Data: Born in Amman, Jordan on January 12, 1979.

Education:

Received the B.S. degree from Jordan University of Science and Technology, Irbid, Jordan, 2001, Applied Physics

Received the M.S. degree from The University of Jordan, Amman, Jordan, 2004, Physics

Completed the requirements for the degree of Doctor of Philosophy with a major in Physics at Oklahoma State University in July, 2011.

Experience:

Intermediate algebra instructor at Northern Oklahoma College

College algebra instructor at Northern Oklahoma College

Chemistry tutor, Northern Oklahoma College

Recitation Teaching Assistant, Department of Physics, Oklahoma State University

Laboratory Teaching Assistant, Department of Physics, Oklahoma State University.

Name: Abdelhamid Albaid Date of Degree: July, 2011

Institution: Oklahoma State University Location: Stillwater, Oklahoma

Title of Study: FERMION MASSES AND MIXINGS, FLAVOR VIOLATION, AND THE HIGGS BOSON MASS IN SUPERSYMMETRIC UNIFIED FRAMEWORK

Pages in Study: 121 Candidate for the Degree of Doctor of Philosophy

Major Field: Physics

In spite of the impressive success of the standard model (SM) in explaining a wide variety of low energy experiments, many fundamental questions remain unanswered. Combining grand unification symmetry and flavor symmetry in the supersymmetric framework leads to interesting new physics that might solve many of the puzzles of the SM. I construct a supersymmetric grand unification model based on the gauge symmetry $SO(10)$ and the non-abelian discrete symmetry A_4. In the framework of this model the small mixings of the quark sector and the large mixings of the lepton sector are successfully accommodated by employing exotic heavy fermion fields with superheavy masses. An excellent fit to the quark and lepton masses and mixings as well as to the CP violation parameter is obtained. Flavor violating processes in the quark and lepton sectors within this realistic supersymmetric $SO(10) \times A_4$ model are also investigated. I find that flavor violation is induced at grand unification scale as a consequence of the large mixing of the light fermion fields and the exotic heavy fields. The stringent experimental constraint from $\mu \to e\gamma$ decay rate requires a high degree of degeneracy of the supersymmetry breaking soft scalar masses of the exotic heavy fields and supersymmetric scalar partners of the light fermion fields. The choice of slepton masses of order 1 TeV is found to be consistent with the constraints from branching ratio of $\mu \to e\gamma$ and with all other flavor changing neutral current (FCNC) processes being sufficiently suppressed.

In a related project, we study the effect of allowing messenger-matter mixing in a class of models with gauge-mediated supersymmetry breaking in the unification framework. We find that the maximal mixing condition that leads to the upper limit of the lightest Higgs mass ($\sim$ 125 GeV) of MSSM can be obtained in models with the messenger fields belonging to $10+\overline{10}$ representations of $SU(5)$ gauge symmetry. Consistent with the cosmological preference for the messenger scale of $\leq 3 \times 10^8$ GeV, the lightest Higgs mass of order 121 GeV is obtained, along with all superparticle masses below 1 TeV. Our results are also consistent with the gauge and exotic Yukawa couplings being perturbative and unified at the GUT scale as well as with all FCNC processes being suppressed in agreement with experimental bounds.

ADVISOR'S APPROVAL: ______________________________