TIFR/TH/23-3

The Higgs Boson and its Physics — an Overview

Sreerup Raychaudhuri
Department of Theoretical Physics, Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400 005, India.
E-mail: sreerup@theory.tifr.res.in

April 18, 2023

Abstract

The Higgs boson plays a central role in the Standard Model, as well as in theories which go beyond it. This article is therefore divided into two parts. The first takes s historical approach and shows how the mass problem entered weak interaction theory from the beginning and how it was solved by invoking the Higgs boson. This is followed by a construction of the Glashow-Salam-Weinberg model, again stressing the role played by the Higgs doublet. This part culminates in the Higgs boson discovery of 2012. The second part first discusses the shortcomings of the Standard Model and then touches upon the major theories which try to improve upon it, mostly with profound consequences on the Higgs sector. This is followed up by short descriptions of a number of popular extensions of the Higgs sector, and culminates in a brief introduction to effective field theory approaches to studying the Higgs sector.

Invited article to appear in a Special Issue of the Indian Journal of Physics

Part 1 : Roads to the Higgs boson

The 2012 discovery of the Higgs boson created a worldwide sensation and was followed a year later by the award of a much-belated Nobel Prize to two of the seven scientists who had originally worked out the basic theory of this elusive particle in 1964. While it was widely reported that the search for the Higgs boson was the culmination of an effort which had lasted the 48 years from 1964 to 2012, the real journey had started much earlier. In fact, it started in 1934 when Enrico Fermi made a facile assumption in an effort to explain beta decay. The Higgs boson discovery was, therefore, the culmination of a 78 year-long series of developments.

In order to appreciate the proper significance of the Higgs boson as the keystone, as it were, of the electroweak theory, it is necessary to retrace this long path and see how the current formalism was arrived at. The first part of this article, therefore, does just this. It starts with the Fermi theory of weak interactions and explains successive developments in a part-historical part-pedagogic manner and ends with the Higgs boson discovery.

However, the Higgs boson, now that it is known to exist, throws up almost as many questions as it answers. Is it an elementary particle or a composite? Are there more scalars in the theory? Can we understand why at high energies, it behaves as if tachyonic? All of this has led to intense speculation about the nature and properties of this particle, and suggestions that it is just a part of some additional structures in the electroweak sector – the tip of the iceberg, as it were. The second part of this article will discuss these ideas in some detail.

This article will deal mostly with theoretical and phenomenological aspects of the Higgs story. The purely experimental aspects are undoubtedly fascinating, but they call for a different article and a different author with the appropriate expertise.

The titles of the different sections invoke different evolutionary stages in the historical development of Western art. This is not whimsical but a serious attempt to draw parallels with parallel fields of human creativity, of which science forms just one, even though it is expressed in a different form.

1 Beginnings: Gauge Theory and Electroweak Unification

The concept of a *symmetry*, though understood in geometry from ancient times, and intuitively used by Newton and his followers, finds its precise expression in the variational formulation of classical mechanics. A symmetry is really a *no-go theorem*, for it means that we cannot physically distinguish between two different configurations of the dynamical variables describing a system, i.e. the action remains invariant. It was Emmy Nöther who famously discovered in 1918 that there is an intimate connection between symmetries of the action and conserved quantities [1], but the fact that there is an intimate connection between symmetries and the fundamental forces of Nature was really an epiphany central to Einstein's 1915 theory of general relativity [2]. The *equivalence principle*, the foundation on which general relativity is based, is simply the statement that general coordinate invariance – a symmetry – becomes manifest in experiments as a gravitational force [3]. General coordinate invariance is, of course, just a no-go theorem for attaining an absolute frame of reference à la Newton. Soon, Hermann Weyl [4] discovered another symmetry of the free Einstein equations, viz., under dilatations, to which he gave the name 'gauge' symmetry.

The advent of quantum mechanics [5] in 1925-26 led to a similar idea being applied by V. Fock and, independently, by F. London [6] in the context of the Schrödinger equation. The force, in this case, turned out to be another well-known one, viz., electromagnetism. The name given by Weyl to a dilatation, i.e., gauge transformation, was now re-purposed to describe a phase transformation. In modern parlance, it can be a combination of the two. However, since the core idea invokes special relativity, it is better expressed in a fully relativistic formalism.

We start, therefore, with the Lorentz-invariant action for a free complex scalar field

$$S \stackrel{\rm def}{=} \int d^4x \left[\partial^\mu \varphi^*(x)\ \partial_\mu \varphi(x) + M^2 \varphi^*(x)\varphi(x)\right] \tag{1}$$

and note that it is invariant under a phase transformation

$$\varphi(x) \to \varphi'(x) = e^{-ie\alpha}\ \varphi(x) \tag{2}$$

if α is independent of spacetime coordinates x, but not otherwise. Here e is a book-keeping parameter, which can be different for different fields. However, a spacetime-independent α would mean the same phase transformation at all points in spacetime — which would require a signal to propagate instantaneously. Relativity forbids this and therefore, demands that α should be spacetime-dependent, i.e. $\alpha = \alpha(x)$. On the other hand, the equation of motion, which arises from this action is

$$\left(\Box + M^2\right)\ \varphi(x) = 0 \tag{3}$$

indicating that α is the phase of the scalar wavefunction, when we make the transition from a field theory to a single-particle equation. In quantum mechanics, the probability interpretation tells us that the phase of this wavefunction does not affect any physics, and this means that there must exist a symmetry under phase transformations — another no-go theorem that the phase cannot be measured. Translated to the field theory we have a paradox, viz., if α is local, then this phase invariance is lost. This may be compared to the loss of local Lorentz invariance of the spacetime derivative ∂_μ in a curved space, and it has the same solution as in general relativity, viz., to add a field of force $A_\mu(x)$ which will nullify the effect and restore the symmetry. Thus, we rewrite the action as

$$S \stackrel{\text{def}}{=} \int d^4x \left[D^\mu\varphi^*(x)\ D_\mu\varphi(x) + M^2\varphi^*(x)\varphi(x)\right] \tag{4}$$

where $D_\mu = \partial_\mu + iA_\mu(x)$ is called a (gauge) covariant derivative and we require that the transformations

$$\varphi(x) \to \varphi'(x) = e^{-i\alpha}\ \varphi(x) \qquad\qquad A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\alpha(x) \tag{5}$$

occur simultaneously. Taken together, we refer to this as a local *gauge transformstion.* The new field $A_\mu(x)$ – which is known as the *gauge field* – must have its own dynamics, which involves the construct $\partial_\mu A_\nu(x)$. The minimal Lorentz and gauge invariant construct with this object is $F^{\mu\nu}F_{\mu\nu}$, where $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$. Adding this to the action, we obtain

$$S \stackrel{\text{def}}{=} \int d^4x \left[D^\mu\varphi^*(x)\ D_\mu\varphi(x) + M^2\varphi^*(x)\varphi(x) - \frac{1}{4}\ F^{\mu\nu}(x)\, F_{\mu\nu}(x)\right] \tag{6}$$

from which the equations of motion become

$$\begin{aligned} \left(D_\mu + iA_\mu\right)^2 \varphi(x) &= 0 \\ \partial_\mu F^{\mu\nu} &= J^\nu(x) \end{aligned} \tag{7}$$

where

$$J_\mu(x) \stackrel{\text{def}}{=} i\left[\varphi^*(x)\ \partial_\mu\ \varphi(x) - \varphi(x)\ \partial_\mu\ \varphi^*(x)\right] \tag{8}$$

is a conserved Nöther current corresponding to the phase symmetry. These correspond so perfectly to the equations of a charged scalar $\varphi(x)$ in an electromagnetic field $A_\mu(x)$ that it is quite natural to identify the gauge field A_μ with the force field of Maxwellian electrodynamics. This is the essence of Fock and London's understanding of the electromagnetic field as a consequence of the phase symmetry of quantum mechanics.

We note that the addition of a mass term

$$S_m \stackrel{\text{def}}{=} \int d^4x\ M_A^2\, A^\mu(x)A_\mu(x) \tag{9}$$

would result in a mass M_A for the quanta of the electromagnetic field, i.e. the photon. However, such a term is clearly not gauge invariant and hence the photon must perforce remain massless — as indeed, it is.

It is somewhat mind-boggling to think that all electromagnetic phenomena – from lightning bolts in the sky to the tiny currents in our brains – arise from an inability to measure the phase of so tenuous a thing as a wavefunction [1]. However, this is no more surprising than that a universal force like gravity, felt so acutely on the Earth's surface, should stem from our inability to detect the state of acceleration of an observer. A more pedantic statement would be that our mathematical understanding of forces in terms of fields of force lend themselves to an understanding in terms of symmetry [7].

The discovery of the Dirac equation in 1928 [8] immediately led to a reformulation of the above theory for a charged fermion field, and this led to a description of *quantum elctrodynamics*, or QED, which has the classical action

$$S_{\rm QED} \stackrel{\rm def}{=} \int d^4x \left[i\overline{\psi}(x)\gamma^\mu D_\mu \psi(x) - m\overline{\psi}(x)\psi(x) - \frac{1}{4}\, F^{\mu\nu}(x)\, F_{\mu\nu}(x) \right] \tag{10}$$

where $D_\mu = \partial_\mu + ieQ_e A_\mu$. This has local gauge invariance under

$$\psi(x) \to \psi'(x) = e^{-ieQ_e\alpha}\,\psi(x) \qquad\qquad A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu \alpha(x) \tag{11}$$

where e is the proton charge and $Q_e = -1$. This also leads to the conservation of the Nöther current

$$J_\mu(x) \stackrel{\rm def}{=} e\overline{\psi}(x)\gamma_\mu\psi(x) \qquad\qquad \partial_\mu J^\mu(x) = 0 \tag{12}$$

which is nothing but the electromagnetic current familiar to us in everyday life. It is this formulation which initiated all the triumphs of the 1930s and 1940s in quantum electrodynamics.

Almost simultaneously with the invention of quantum field theory [9], however came the realisation that there are two more fundamental fields of force. Chadwick's discovery of the neutron in 1932 [10] inspired Heisenberg to postulate the existence of the *strong* (nuclear) force [11], which was soon formulated as a field theory by Yukawa [12]. But also, simultaneously, came the first understanding of weak interactions. As far back as 1899, Rutherford had identified alpha and beta particles as separate components of radioactive emanations, with greatly different penetrating power [13]. It was, however, Becquerel who identified the beta rays as high-energy electrons [14]. The fact that they have energies around an MeV was a clear proof that they did not come from among the atomic orbitals which have binding energies of a few tens of keV at most. However, the question of how high energy electrons can remain bound inside the positively-charged nucleus remained a mystery till the advent of quantum mechanics. It was Ambartzumian and Iwanenko who proved [15], using the Uncertainty Principle, that electrons bound inside nuclei must have energies close to a GeV[2], whereas the beta particles have energies an order of magnitude less. It was clear, therefore, that

[1] It certainly failed to convince Einstein himself, whose original idea it stemmed from. This led to Einstein attacking the very basis of phase invariance, viz., the probability interpretation of quantum mechanics, coining the dictum 'God does not play dice'. But we now have plenty of evidence that, in fact, He does.

[2] This is a textbook exercise today.

the beta rays must be produced *at the instant* of interaction. The extremely long lives of nuclides undergoing beta decay made it clear that this interaction was not the strong interaction.

The situation was clarified by the neutron discovery. The mass difference between a neutron and a proton is just about 1.2 MeV, and therefore, the beta decay process could be interpreted as a neutron decaying into a proton and an electron. Pauli (1930) then postulated an additional massless, neutral particle – the neutrino – to ensure energy-momentum conservation [16]. This novel object would be a participant only in the *weak* (nuclear) interaction, which must be the cause of beta decay processes of the form

$$ {}_1n^0 \to {}_1p^+ + e^- + \bar{\nu}_e \tag{13} $$

where we have written $\bar{\nu}_e$ instead of just ν with the benefit of hindsight. The ingredients were all in place now for Fermi to put together his heuristic theory of beta decay [17], with the famous current-current form of the interaction[3], i.e.

$$ S_\beta = \int d^4x \; G_W \; J^\mu_{(h)}(x) \, J^{(\ell)}_\mu(x) \tag{14} $$

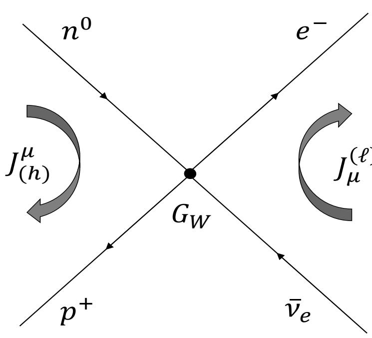
where the *hadronic* current is

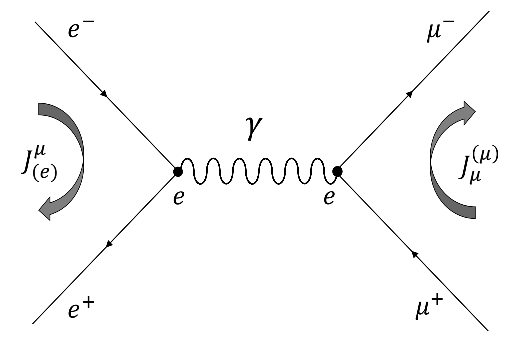
$$ J^\mu_{(h)}(x) \stackrel{\text{def}}{=} \overline{\psi}_p(x)\gamma^\mu\psi_n(x) \tag{15} $$

and the *leptonic* current is

$$ J^{(\ell)}_\mu(x) \stackrel{\text{def}}{=} \overline{\psi}_e(x)\gamma_\mu\psi_\nu(x) \tag{16} $$

while G_W is a (dimensionful) coupling constant.





This is a classic four-fermion interaction, and was clearly inspired [20] by the form of the effective interaction for an electromagnetic scattering of the type, say, $e^+e^- \to \mu^+\mu^-$, which takes the form

$$ \begin{aligned} \mathcal{L}_{\text{eff}} &= e^2 \, J^\mu_{(\text{e})} \frac{g_{\mu\nu}}{E^2_{\text{cm}}} J^\nu_{(\mu)} \qquad (17) \\ &= \frac{e^2}{E^2_{\text{cm}}} \, \overline{\psi}_e(x)\gamma^\mu\psi_e(x) \; \overline{\psi}_\mu(x)\gamma^\mu\psi_\mu(x) \end{aligned} $$

In QED the effective coupling e^2/E^2_{cm} is energy-dependent. Whatever was the reasoning which led Fermi to set this as to a constant for weak interactions eventually proved to be a very deep insight — and, as a matter of fact, it lies at the very core of the theme of this article. The choice of vector currents, instead of any other Lorentz covariant structures, is more understandable as inspired by the example of QED, but it proved to be an equally brilliant choice[4].

[3]It is worth mentioning that Fermi had submitted his paper to *Nature*, which had rejected it '*because it contained speculations too remote from reality to be of interest to the reader*' [18].

[4]In fact, when eventually published, Fermi's work was considered less remarkable for its form of the interaction

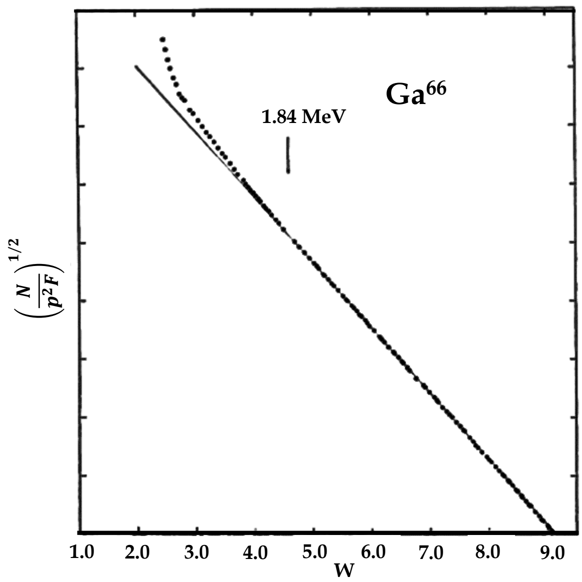


Figure 1: Fermie-Kurie plot of β decay of Ga-66. Reproduced from Ref. [23] (1963).

With this new interaction, Fermi was able to predict the energy distribution of the beta particles. His original graphs were qualitative, but soon they were found to stand up extremely well to experimental measurements [22]. A later comparison [23] is shown in Figure 1 which shows a Fermi-Kurie plot in radioactive Gallium-66, where the horizontal axis is W, the energy of the beta particle, and the vertical axis is the quantity $(N/p^2F)^{1/2}$, where N is the number of beta particles with momentum p and F is the so-called Fermi factor.

The (solid) straight line is the prediction of the Fermi theory and it may be seen that the dots, which are the experimental data, correspond very closely with the theoretical line. Deviations at low energies may be attributed to scattering of the beta particles in the solid material. Apart from this, the excellent straight line formed by the data points is a strong vindication of Fermi's theory of beta decay. Moreover, the fact that the line remains straight as it intersects the horizontal axis is a proof that the neutrino mass is zero — at least so far as this experiment goes. In fact the best upper bounds on neutrino mass do come from beta-decay data[5].

It later turned out that the Fermi theory with vector-vector interactions cannot explain the energy plots in many cases of beta decay, prompting Gamow and Teller to add an axial vector-axial vector current to the Fermi interaction [25]. Ever since, beta decay processes are classified in textbooks as as *Fermi transitions* and *Gamow-Teller transitions.* It was also not long before the weak interaction was found to be *universal*, i.e. a similar four-fermion current- current interaction could correctly predict the muon lifetime with the same value of G_W as required for beta decay [26]. The Fermi

than for making a connection between the interaction action and the decay rate, which he later formulated as the Golden Rule (see E. Fermi, *Nuclear Physics* U. of Chicago Press (1950)). The groundwork for this, however, had already been done by Dirac [21]

[5]Though we now know that neutrinos have tiny but nonzero masses, no beta decay experiment has ever reached the sensitivity to see a deviation in the Fermi-Kurie line close to its intersection with the horizontal axis. The latest results are reported by [24].

theory remains, to this day, the best explanation of the low energy behaviour of weakly-decaying particles.

Trouble arises, however, when we go to higher energies. The reason is not difficult to understand. If we go back to the QED process $e^+e^- \to \mu^+\mu^-$, we can calculate the total cross-section, neglecting fermion masses, as [27]

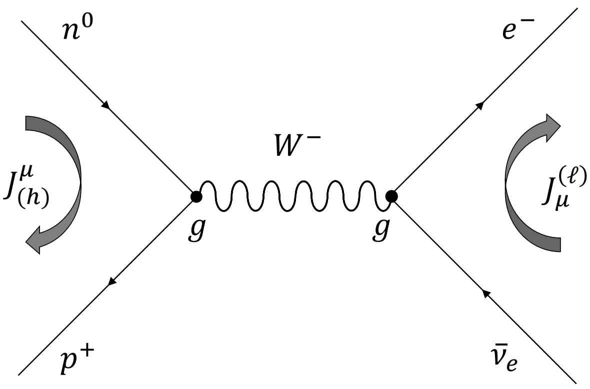
$$\sigma_{\rm QED} = \frac{1}{12\pi}\left(\frac{e^2}{E_{\rm cm}}\right)^2 \tag{18}$$

which arises (see above) when we have an effective coupling constant $e^2/E_{\rm cm}^2$. If we replace this, for a weak interaction between massless fermions, by G_W, then we should get

$$\sigma_{\rm weak} = \frac{e^4}{12\pi}\frac{1}{E_{\rm cm}^2} \times \left(\frac{G_W}{e^2/E_{\rm cm}^2}\right)^2 = \frac{G_W^2}{12\pi}E_{\rm cm}^2 \tag{19}$$

Unlike the QED case, this grows without limit as $E_{\rm cm}$ grows, and will sooner or later render the S-matrix non-unitary[6]. Since this is not acceptable in any quantum theory, we must conclude that the Fermi theory breaks down at higher energies. On the other hand, making G_W energy-dependent à la QED would lead to large deviations in the Fermi-Kurie plot.

The above paradox is nicely solved by the so-called *intermediate vector boson* (IVB) hypothesis [28]. For, if the massless photon γ is replaced by a massive vector boson W in the $e^+e^- \to \mu^+\mu^-$ process, we will get a diagram of the form



where the overall factor in the cross-section will be replaced by

$$\left(\frac{e^2}{E_{\rm cm}^2}\right)^2 \to \left(\frac{g^2}{E_{\rm cm}^2 - M_W^2}\right)^2 \tag{20}$$

where M_W is the mass of the W boson and g replaces e as a weak coupling constant. Now, obviously, in the low- and high- energy limits, we will have

$$\left(\frac{g^2}{E_{\rm cm}^2 - M_W^2}\right)^2 \to \begin{cases} \left(g^2/M_W^2\right)^2 & \text{when } E_{\rm cm} \ll M_W \\ \left(g^2/E_{\rm cm}^2\right)^2 & \text{when } E_{\rm cm} \gg M_W \end{cases} \tag{21}$$

The low energy limit is a constant $G_W^2 = \left(g^2/M_W^2\right)^2$ — exactly as guessed by Fermi — and in

[6] In technical language, it will violate perturbative unitarity.

the high energy limit we get the suppression factor $E_{\rm cm}^{-4}$, which is sufficient, as in QED, to ensure perturbative unitarity.

In the above, however, we have cheated, for the amplitude with the full expression for the massive vector boson propagator is

$$\mathcal{L}_{\rm eff} = e^2\, J^\mu_{\rm (h)} \frac{g_{\mu\nu} - p_\mu p_\nu / M_W^2}{E_{\rm cm}^2 - M_W^2} J^\nu_{(\ell)} \tag{22}$$

where p is the four-momentum flowing through the W propagator. While the first (photon-like) term in the propagator does indeed lead to a good high-energy behaviour, the second p-dependent term again provides a factor of $E_{\rm cm}^2$ in the amplitude. Thus, with this term, the bad behaviour, viz. unitarity violation, comes back. For this reason, though the IVB hypothesis was first made in 1949 [7], it began to be taken seriously only after the establishment of the V-A form of the Fermi current [29]. This was because a solution for the problem had been discovered by Schwinger [30] in 1957. His idea was to point out that the weak amplitude can be written

$$\mathcal{L}_{\rm eff} = e^2\, J^\mu_{\rm (h)} \frac{g_{\mu\nu}}{E_{\rm cm}^2 - M_W^2} J^\nu_{(\ell)} - \frac{e^2}{M_W^2} \frac{p_\mu J^\mu_{\rm (h)}\; p_\nu J^\nu_{(\ell)}}{E_{\rm cm}^2 - M_W^2} \tag{23}$$

and this will become exactly like a massive photon if

$$p_\mu J^\mu_{\rm (h)} = p_\nu J^\nu_{(\mu)} = 0 \tag{24}$$

which, in position space, becomes

$$\partial_\mu J^\mu_{\rm (h)} = \partial_\nu J^\nu_{(\ell)} = 0 \tag{25}$$

Now these are conserved currents, and therefore, by the inverse of Nöther's theorem. there must be an underlying symmetry. Taken together with the existence of a new vector boson, it was not difficult to guess that this would surely be a gauge symmetry, with the gauge boson as the IVB. In fact, it would have to be not one gauge boson, but *two*, for the W^- in the figure must have its antiparticle, the W^+.

Schwinger actually went further — he assumed that the photon may be a neutral companion to these two charged gauge bosons and hence what we would then have is a *unified* theory of electromagnetism and weak interactions, i.e. an *electroweak* theory. Such a triplet would arise as a vector in an isospin-like gauge theory[8], just as Yang, Mills and Shaw had shown [31] in 1954. It was an exciting idea, and so Schwinger made his student Glashow work out the phenomenological details of this first idea of electroweak interactions for his 1959 Ph.D. thesis [32]. A similar idea was also proposed by Salam and Ward around the same time [33]. In fact, Salam and Ward stuck to this problem, writing a series of papers, of increasing sophistication, trying to set up a gauge theory of electroweak interactions. This eventually led to Salam being included in the Nobel Prize for developing the Standard Model.

The realisation soon dawned on Glashow that if we take the iso-triplet $(\,W^+ \quad W^0 \quad W^-\,)$ then the group symmetry will compel the W^0 to couple to a pair of neutrinos. In fact, the couplings of the

[7] The IVB hypothesis was originally proposed, almost casually, in Ref. [28].

[8] In group theoretic language: as the adjoint representation in an $SU(2)$ gauge theory

W triplet to electrons and (electron) neutrinos would have the form

$$\begin{aligned}\mathcal{L}_W &= \frac{g_2}{\sqrt{2}}\,\overline{e}(x)\gamma^\mu\nu_e(x)W^-_\mu(x) + \frac{g_2}{\sqrt{2}}\,\overline{\nu}_e(x)\gamma^\mu e(x)W^+_\mu(x)\\ &\quad -\frac{g_2}{2}\,\overline{e}(x)\gamma^\mu e(x)W^0_\mu(x) + \frac{g_2}{2}\,\overline{\nu}_e(x)\gamma^\mu\nu_e(x)W^0_\mu(x)\end{aligned} \tag{26}$$

If the last term is to be absent, we must put $g_2 = 0$, which will remove *all* the interactions giving rise to beta and muon decay. Therefore, one cannot associate the W^0_μ field with the photon as he, Schwinger, as well as Salam and Ward, had done.

Sticking manfully to his task, however, Glashow in 1961 decided to extend his model by adding an extra $U(1)$ symmetry as a direct product to the SU symmetry [34]. Thus, there were now *two* neutral gauge bosons W^0 and B^0.

$$\begin{aligned}\mathcal{L}^0_{W,B} &= \frac{g_2}{2}\,\overline{\nu}_e(x)\gamma^\mu\nu_e(x)W^0_\mu(x) - \frac{g_2}{2}\,\overline{e}(x)\gamma^\mu e(x)W^0_\mu(x)\\ &\quad -\frac{g_1}{2}\,\overline{\nu}_e(x)\gamma^\mu\nu_e(x)B^0_\mu(x) - \frac{g_1}{2}\,\overline{e}(x)\gamma^\mu e(x)B^0_\mu(x)\end{aligned} \tag{27}$$

Though the B_μ corresponds to a $U(1)$ symmetry in Eqn. (27) it cannot be the photon, for it will only decouple from neutrinos if $g_1 = 0$, which is trivial. Glashow then made the heuristic assumption that the W^0 and B^0 are *mixed* states (for some unknown reason), i.e.

$$\begin{aligned}W^0_\mu &\stackrel{\text{def}}{=} Z_\mu(x)\cos\theta - A_\mu(x)\sin\theta\\ B^0_\mu &\stackrel{\text{def}}{=} Z_\mu(x)\sin\theta + A_\mu(x)\cos\theta\end{aligned} \tag{28}$$

leading to

$$\begin{aligned}\mathcal{L}^0_{W,B} &= \frac{1}{2}\,(g_2\sin\theta - g_1\cos\theta)\;\overline{\nu}_e(x)\gamma^\mu\nu_e(x)\;A_\mu(x)\\ &\quad -\frac{1}{2}\,(g_2\sin\theta + g_1\cos\theta)\;\overline{e}(x)\gamma^\mu e(x)\;A_\mu(x)\ldots\end{aligned} \tag{29}$$

By *tuning* the mixing angle θ such that

$$\tan\theta \stackrel{\text{def}}{=} \frac{g_1}{g_2} \tag{30}$$

Glashow was able to ensure that the $A_\mu(x)$'s coupling to a neutrino pair would cancel — and thus the $A_\mu(x)$ could be identified with the photon, with $e = g_2\sin\theta$ to get the right QED coupling of electrons to photons[9]. It was the introduction of an additional $U(1)$ symmetry – and hence an additional IVB – which did the trick, by cancelling the W^0 coupling to neutrinos against the extra $U(1)$ contribution to the same. This produced a phenomenologically viable model of leptons[10], albeit with a fine-tuned parameter θ, for which fine-tuning there was no natural explanation. We may note, however, that if we replace the lepton doublet $\overline{L} = (\overline{\nu}_e \;\; \overline{e})$ by a nucleon doublet

[9] It is often asked why Glashow chose the direct product group $SU(2)\times U(1)$ rather than the simpler $U(2)$. The reason is as follows. In the above, that would mean putting $g_1 = g_2$, i.e. $\theta = \pi/4$. We would then have $e = g_2/\sqrt{2}$ and hence $G_W = 2e^2/M_W^2$, leading to $M_W \approx 23$ GeV. In Glashow's time, this would have seemed absurdly high. Today, on the other hand – with hindsight – we know that this is actually about 3.5 times too small.

[10] It proved ultimately good enough to win Glashow a Nobel Prize (1979).

$\overline{N} = (\overline{p} \quad \overline{n})$ we will obtain a similar-looking interaction

$$\begin{aligned} \mathcal{L}^0_{W,B} &= \frac{1}{2}\left(g_2 \cos\theta - g_1 \sin\theta\right)\ \overline{p}(x)\gamma^\mu p(x)\ A_\mu(x) \\ &\quad -\frac{1}{2}\left(g_2 \sin\theta + g_1 \cos\theta\right)\ \overline{n}(x)\gamma^\mu n(x)\ A_\mu(x) + \ldots \end{aligned} \tag{31}$$

To make the photon-neutron coupling vanish, however, we must have

$$\tan\theta = -\frac{g_1}{g_2} \tag{32}$$

which would then give the right sign for the photon-proton coupling. But we cannot have both the fine-tuning relations (30) and (32) simultaneously, unless $g_1 = 0$, which is trivial. It may have been for this reason that Glashow's 1961 paper restricts itself to electroweak interactions of leptons[11].

Moreover, in Glashow's work, no explanation was attempted for the fact that the photon is massless and the $W^\pm$ are massive (and so, presumably is the W^0, or its mixed version). This, in fact, was the elephant in the room, '*a stumbling block we must overlook*', as Glashow, somewhat apologetically, put it[12]. However, that it did bother him is clear from the fact that he modestly put the words 'partial symmetry' into the title of his paper, that is to say, a symmetry which would not apply to the mass terms for the W-bosons[13], but to the rest of the Lagrangian.

Glashow's *tour de force*, like many another famous work, 'fell stillborn from the press'. By the time Weinberg's classic paper on electroweak interactions appeared in 1967, Glashow had acquired only 4 (non-self) citations. The tension between gauge theory and IVB masses proved too much for most scientists to swallow, and as a result, gauge theory (except QED) was cast into the shade. The 1960s were a period when current algebras and S-matrix theory dominated the world of quantum field theory, and, on the phenomenological side, the composite nature of hadrons was postulated and proved. Only a few intrepid souls were still bothered about electroweak unification.

Today Glashow's 1961 paper has well over 9,000 citations and is regarded as the seminal paper in the development of the Standard Model. What made the difference? It was the solution of the mass problem, the removal of the 'stumbling block' — to which we must now turn.

2 Dark Ages: The Mass Problem

How does one postulate both a symmetry and also introduce terms that break it? This is not difficult to envisage. To take a common example, the Earth may be approximated, for many purposes, as a perfect sphere, i.e. a solid having rotational symmetry (red curves in Figure 2). However, we

[11] The failure of the $SI(2) \times U(1)$ theory for hadrons has its explanation in the simple fact that hadrons are not elementary articles. When the same idea is applied to an $SU(2)$ doublet of quarks, it works perfectly. However, when Glashow wrote his paper in 1961, composite nucleons would have seemed too far-fetched an idea. This came in only with the quark model in 1964.

[12] Italics by the present author.

[13] In justification, Glashow quoted two well-known examples – the scale and chiral invariance of massless theories. These are discussed in the next section.

are aware of irregularities (sketched highly exaggerated as the black curve in Figure 2), which matter only when the Earth's gravitational field has to be considered to a high level of accuracy. In technical language, the Earth's gravitational potential has a dominant monopole term, which represents the spherical symmetry. Then there are small dipole terms, quadrupole terms, etc., i.e. terms which represent the breaking of rotational symmetry.

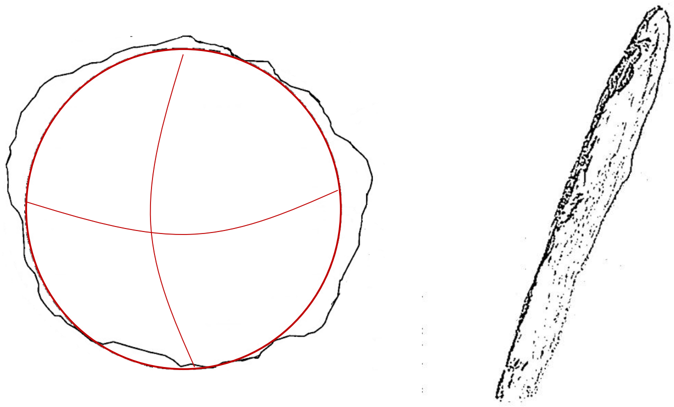


Figure 2: Sketch of the Earth (left) and Oumuamua (right).

The above is an example of *explicit* symmetry-breaking, where the higher multipole terms which do not respect the spherical symmetry are very small. However, this cannot work in the case when the deviation from the symmetry is very large. For example, no one would dream of approximating the cigar-shaped interstellar object Oumuamua (see Figure 2) as a sphere with small corrections. In particle physics, large mass terms, such as are demanded for the $W^\pm$ bosons, would correspond to these large deviations from gauge theory. Where could they be coming from?

A similar question had plagued the theory of strong interactions as well. In the 1950s, the dominant theory was still the 1935 theory of Yukawa [12], with some group theoretic modifications over the years. This theory of pions and nucleons has a nucleon N of mass around 938 MeV and a pion $\vec{\pi}$ of mass around 140 MeV, transforming under $SU(2)$ of isospin as a doublet and a triplet respectively. However, the free theory has a large global symmetry $SU(2)_L \times SU(2)_R$ if it is written in terms of *massless* left- and right-handed nucleons, viz.

$$\mathcal{L}_{\pi N\bar{N}} \stackrel{\text{def}}{=} i\overline{N}_L \, \partial\!\!\!/ N_L + i\overline{N}_R \, \partial\!\!\!/ N_R + \partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} \tag{33}$$

where $\overline{N} \stackrel{\text{def}}{=} (\overline{p} \quad \overline{n})$ is the nucleon doublet and $\vec{\pi} \stackrel{\text{def}}{=} (\pi^+ \quad \pi^0 \quad \pi^-)$ is the pion triplet. Obviously, this is invariant under global transformations

$$N_L \to N_L' = \mathbb{U}_L N_L \qquad\qquad N_R \to N_R' = \mathbb{U}_R N_R \tag{34}$$

where $\mathbb{U}_L$ and $\mathbb{U}_R$ are $SU(2)$ matrices. However, a mass term

$$\mathcal{L}_m = m_N \left(\overline{N}_L N_R + \overline{N}_R N_L \right) \tag{35}$$

immediately breaks this symmetry, unless, indeed $\mathbb{U}_L = \mathbb{U}_R$, i.e. the global $SU(2)_L \times SU(2)_R$ breaks down to the diagonal $SU(2)$ of isospin. At low energies, the nucleon mass $m_N \approx 938$ MeV is a large value, which cannot be attributed to explicit symmetry-breaking.

The solution had, in fact, already been discovered in condensed matter physics, where the theory of superconductivity incorporates a phenomenon now called *spontaneous symmetry-breaking.* It had appeared, in a non-relativistic form, in the 1950 theory of Landau and Ginzburg [35], where the photon effectively acquires a mass inside a superconducting medium below the critical temperature and cannot propagate — making the macroscopic medium at once a perfect conductor and a perfect diamagnet. This phase transition occurs when a dynamical variable of the material, called an *order parameter* by Landau, is close to a ground state, which does not obey the symmetry of the Lagrangian. The idea was given a firm basis in the BCS theory of superconductivity [36], where the hitherto-esoteric order parameter assumes a concrete form as the density of a Bose-Einstein condensate of paired electrons (Cooper pairs), forming quasi-bosons. As one would expect, the BCS ground state breaks the gauge symmetry of electromagnetism, which would otherwise preclude a photon mass.

Could there be a similar phenomenon happening in the pion-nucleon regime? This was the question asked by Nambu and Jona-Lasinio in 1960 [37], and they managed to construct an elaborate theory based closely on the BCS theory, with a nucleonic condensate $\langle N\bar{N}\rangle$ which would break the chiral $SU(2)_L \times SU(2)_R$ down to $SU(2)$ of isospin. Once the symmetry is broken spontaneously, there is no restriction on the mass of the nucleon, for it is related to the energy gap between the ground state and the first excited state of the condensate. Some mechanism was postulated to create the $\langle N\bar{N}\rangle$ bound state — this would be the analogue of the Fröhlich effective interaction which creates the Cooper pairs. Inspired by the success of Nambu and Jona-Lasinio, Gell-Mann and Lévy [38] then created a simpler model where the condensate $\langle N\bar{N}\rangle$ is replaced by a scalar field σ. This came to be known as the (linear) sigma model. In this model, one creates an $SU(2)$ doublet

$$\Phi(x) \stackrel{\text{def}}{=} \mathbb{I}\,\sigma(x) + i\,\vec{\mathbb{T}}\cdot\vec{\pi}(x) \tag{36}$$

and adds to the isospin Hamiltonian a potential energy term of the form

$$V(\Phi) \stackrel{\text{def}}{=} -\frac{\mu^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4!}(\Phi^\dagger\Phi)^4 \tag{37}$$

which is closely analogous to the Ginzburg-Landau Hamiltonian if μ and λ are real and positive. This potential breaks the $SU(2)$ symmetry spontaneously and permits the nucleons to have large masses[14]. At the same time, the pions remain necessarily massless, following a 1962 theorem proved by Goldstone, Salam and Weinberg [39]. As a matter of fact, Goldstone had discovered [40] in 1961 that theories of the sigma model type always contain massless bosons, but a year later, he teamed up with Salam and Weinberg to prove the *Goldstone theorem*, which states [39] that whenever a continuous global (or local) symmetry is spontaneously broken, there will be massless bosons. Today they are called *Goldstone bosons* and are ubiquitous in quantum field theories. The pions in the sigma model are Goldstone bosons, and necessarily massless. However, it is possible to introduce pion mass terms as *explicit* symmetry-breaking terms, which necessarily makes the pion

[14]Some of the details are indicated, in the electroweak context, in the next section.

masses small. Here, it seems, lay the explanation of why the pions are so much lighter than the nucleons.

Nucleons and pions are not, however, the only hadrons. Throughout the 1950s and early 1960s, new hadrons with exotic properties were being discovered [41], and these were then postulated by Gell-Mann [42] and Zweig [43] in 1964 to be composites of a set of three basic quarks. The analysis of deep-inelastic scattering data in 1969 established the existence of these quarks [44], and a gauge interaction between them was established by the early 1970s [45]. The sigma model, therefore, never really achieved the status of a fundamental theory; however, it proved to be the first and paradigm case for a whole class of *effective* field theories. But it was in the theory of electroweak interactions that the ideas introduced in the sigma model secured their biggest triumph.

3 Early Renaissance: The Higgs Mechanism

It may have seemed natural, in the early 1960s, to create a model of electroweak interactions on the analogy of the sigma model. This would certainly have been an excellent means of generating masses for the IVB particles, and would nicely round off Glashow's unified electroweak theory. However, the concept foundered on the rock of the Goldstone theorem. A sigma model analogue would indeed make the vector bosons massive (except for the photon), but in their place we would have three massless scalars, with interactions of the same strength. Such bosons would have surely shown up long before in cosmic rays, or nuclear reactions, for the weak interactions are ubiquitous. There were not even light bosons analogous to the pions, to which one could attach an explicit symmetry-breaking. For this reason, no one interested in particle physics hastened to create a sigma model for electroweak interactions.

Once again, the clue came from condensed matter physics. In 1963, Anderson published a study [46] of superconductivity, in which he suggested that the Goldstone degree of freedom arising from spontaneous symmetry-breakdown in a metallic superconductor actually reappears as a longitudinal mode of the photon[15], rather than as an independent massless boson. The two massless bosons 'cancel', in Anderson's language, leaving a massive (vector) boson. This is, in effect, what is known today as the *Higgs mechanism.* There was really no reason for Anderson not to go on to rewrite his non-relativistic model in a relativistic avatar, since he did mention Yang-Mills theory in passing. However, he drew back from this step, contenting himself with the statement that 'it is not at all clear' that a similar mechanism would work in Sakurai's theory [47] of the strong interactions.

In this 1960 paper, Sakurai had tried to write down a Yang-Mills theory for the $SU(2)_I \times U(1)_Y$ flavour symmetry of hadrons [47]. This had not been taken seriously for the same reason as Glashow's electroweak paper, because the gauge bosons were necessarily massless, leading to a long-range interaction; and even if the gauge symmetry could be spontaneously broken, there would be massless strongly-interacting Goldstone scalars which are known not to exist. Interestingly, the group symmetry is the same as that discovered by Glashow, and the mathematical structure of Sakurai's theory is almost identical to that of Weinberg's electroweak model. Of course, Sakurai's

[15] which corresponds to longitudinal oscillations in the free electron gas, quantised as 'plasmons'.

model did not survive the discovery that flavour symmetries are global in nature — and inexact to boot. But it is ironic that what was dismissed as 'not at all clear' in Anderson's 1963 paper, was precisely what became crystal clear in Weinberg's 1967 paper[16].

It did not take long for the relativistic generalisation to come. Three groups arrived at practically the same formulation independently during 1964. In those pre-arXiv days, neither of them knew of the others' work till it was actually published. The first off the mark were Englert and Brout [49] at Brussels, followed by Higgs [50] at Edinburgh, and a little later, by Guralnik, Hagen and Kibble [51] at London. Thus, the Anderson-Brout-Englert-Guralnik-Hagen-Higgs-Kibble mechanism – or Higgs mechanism for short if we do not wish to mention all the 'magnificent seven' – came into being. Its application to the Abelian gauge theory of Section 1 is outlined below in terms of three 'miracles'.

We start with the scalar gauge theory of Eqn. (6) keeping the scalar massless and add to it two self-interaction terms similar to those of the sigma model, i.e. Eqn. (37), thereby getting

$$\begin{aligned} S &\stackrel{\text{def}}{=} \int d^4x \left[(D^\mu\varphi)^*(x)\, D_\mu\varphi(x) - \frac{1}{4}\, F^{\mu\nu}(x)\, F_{\mu\nu}(x) - V(\varphi) \right] \\ V(\varphi) &= -\frac{\mu^2}{2}\, \varphi^*(x)\varphi(x) + \frac{\lambda}{4!}\, \{\varphi^*(x)\varphi(x)\}^2 \end{aligned} \tag{38}$$

where, as in the sigma model, μ and λ are real and positive constants. It should be noted that the μ term has the wrong sign for a mass term[17] and should be regarded as an interaction term.

If we plot the potential energy term $V(\varphi)$ as a function of φ, we will obtain the so-called 'Mexican hat' potential shown in Figure 3. Since the potential depends only on $|\varphi|^2$, it will have rotational symmetry in the complex φ-plane. Close to the origin, i.e. at small values of $|\varphi|^2$, the quadratic term will dominate, producing approximately an inverted paraboloid of revolution, but at higher values of $|\varphi|$ the quartic term will begin to dominate, causing a turnaround and making the potential positive. These contrary terms will therefore create a minimum for some value $|\varphi| = v$, which is easily calculated as

$$\frac{v}{\sqrt{2}} = \sqrt{\frac{6\mu^2}{\lambda}} \tag{39}$$

However, there is no restriction on the *phase* of φ, which indicates that in the complex φ-plane there is a ring of possible minima of the potential energy with radius $|\varphi| = v$. In the quantum theory, these will correspond to degenerate vacua $\langle\varphi\rangle = v/\sqrt{2}$. Of these, only one can be the physical vacuum, and this will be a random choice denoted in Figure 3 by a red dot. The unstable point $\varphi = 0$ is denoted by a white dot. We now notice that the phase invariance — or $U(1)$ gauge invariance — is spontaneously broken by the choice of a single vacuum state along the ring.

[16]Weinberg later wrote, in a characteristically insightful article [48]: "In fact, Anderson was right: the reason for the exception noted by Higgs *et al.* is that it is not possible to quantize a theory with a local symmetry in a way that preserves both manifest Lorentz invariance and the usual rules of quantum mechanics, including the requirement that probabilities be positive. In fact, there are two ways to quantize theories with local symmetries: one way that preserves positive probabilities but loses manifest Lorentz invariance, and another that preserves manifest Lorentz invariance but seems to lose positive probabilities, so in fact these theories actually do respect both Lorentz invariance and positive probabilities; they just don't respect our theorem." Here, by 'our' theorem, he meant the Goldstone theorem [39].

[17]If we try to identify it as a mass term, then φ would be a *tachyon* and the theory would therefore be unphysical.

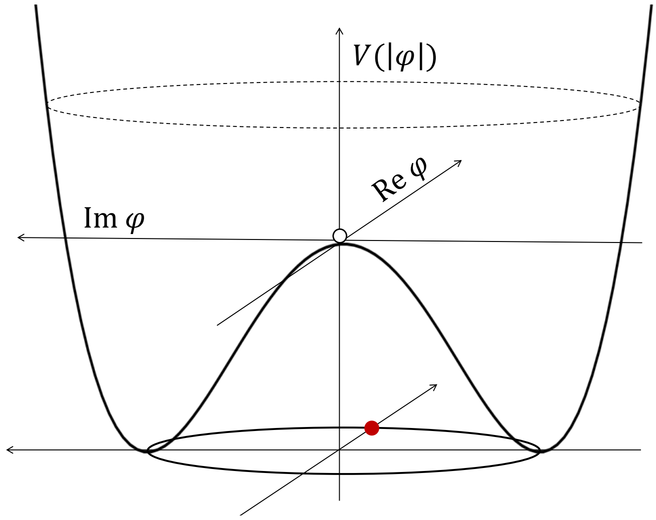


Figure 3: Spontaneous symmetry-breaking potential in a $U(1)$ gauge theory..

Recognising that at energies (temperatures) far above $V(v/\sqrt{2})$ the $U(1)$ symmetry will be there, but it will only be at low energies (temperatures) close to $V(v/\sqrt{2})$, that the symmetry is broken, we realise that this is really a *phase transition* from a symmetric phase to a broken-symmetric phase. As the system in the symmetric phase cools down, the false vacuum at $\varphi = 0$ will become manifestly unstable, and thus any quantum fluctuation will cause the system to 'roll down' to ons of the degenerate true vacua.

To write down a quantisable field theory, we must now expand φ around the vacuum state. Before we do so, however, it is convenient to choose the axes in the complex φ-plane such that the vacuum point lies along the real axis — this can be done without loss of generality and at the same time enables us to eliminate an extraneous phase parameter, which would otherwise have to be carried through the calculation. We now write

$$\varphi \stackrel{\text{def}}{=} \frac{v}{\sqrt{2}} + \omega(x) \tag{40}$$

where $\omega(x)$ is the scalar degree of freedom which is amenable for a quantum field theory. In terms of this, the action becomes

$$S = \int d^4x \left[(\partial^\mu + ieA^\mu)(\frac{v}{\sqrt{2}} + \omega^*)\ (\partial_\mu - ieA_\mu)(\frac{v}{\sqrt{2}} + \omega) - \frac{1}{4}\, F^{\mu\nu}\, F_{\mu\nu} - V(\omega) \right] \tag{41}$$

with a potential energy term

$$V(\omega) = \mu^2\, (\mathrm{Re}\ \omega)^2 + \mu^2\ \omega^*\omega\ \mathrm{Re}\ \omega + \frac{\lambda}{4!}\, (\omega^*\omega)^2 \tag{42}$$

where we need to substitute $\mu^2 = \lambda v^2/12$ and a constant term has been dropped. The kinetic term in Eqn. (41), when expanded, contains a term

$$\mathcal{L}_m = +\frac{1}{2}\, e^2 v^2\, A^\mu A_\mu \tag{43}$$

The above is a mass term for the photon, and therefore, we identify $M_\gamma = ev$. This is the *first miracle* of the sigma model, or more generally of the spontaneously-broken theory, that the choice of one out of multiple possible vacua generates a gauge boson mass. If we consider the quadratic term in Eqn. (42), we will note that it is a mass term with the correct sign, i.e., the real scalar field Re ω has a mass $\sqrt{2}\mu$. This is the *second miracle* of Higgs [50], that the tachyonic scalar theory, once the proper vacuum is chosen, becomes a real scalar theory with a real mass. This scalar is known as the *Higgs boson*[18].

The action in Eqn. (41) still has a problem, since it contains the complex scalar field $\omega = \text{Re}\,\omega + i\text{Re}\,\omega$. but only the real field Re ω has a finite mass term. The other real field Im ω remains massless and indeed, it is the Goldstone boson of the theory. This is where the *third miracle* comes in, i.e. the one devised by Anderson and the seven authors of 1964. The trick is go back and parametrise the complex scalar field φ as

$$\varphi(x) \stackrel{\text{def}}{=} \eta(x)\; e^{i\theta(x)} \tag{44}$$

At this stage, the gauge symmetry is not broken, and hence, with hindsight, we can perform a $U(1)$ gauge transformation as in Eqn. (2) to obtain

$$\varphi(x) \to \varphi'(x) = \eta(x)\; e^{ie[\theta(x)+\alpha(x)]} \tag{45}$$

Since we are free to choose the gauge function $\alpha(x)$, we simply choose it such that

$$\alpha(x) \stackrel{\text{def}}{=} -\theta(x) \tag{46}$$

in which case,

$$\varphi(x) \to \varphi'(x) = \eta(x) \tag{47}$$

The physical system remains the same at this stage, because of the gauge invariance. The action in this so-called *unitary* gauge now becomes

$$\begin{aligned} S &= \int d^4x \left[(D^\mu \eta)^*(x)\; D_\mu \eta(x) - \frac{1}{4}\; F^{\mu\nu}(x)\, F_{\mu\nu}(x) - V(\eta) \right] \\ V(\eta) &= -\frac{\mu^2}{2}\; \eta^2(x) + \frac{\lambda}{4!}\eta^4(x) \end{aligned} \tag{48}$$

where only the real scalar field $\eta(x)$ appears. This is still tachyonic, but we now note that the minimum of the potential lies at η given by $\pm v = \pm\sqrt{12\mu^2/\lambda}$. Choosing the positive value, and making the shift $\eta(x) = v/\sqrt{2} + h(x)$, now produces both the desirable features of the sigma model, viz., a mass term for the photon $A_\mu(x)$ and a mass term of the proper sign for the Higgs boson $h(x)$. Obviously, the degree of freedom denoted by the field θ has vanished from the action in Eqn. (48).

[18] This name was first used, with justification, by B.W. Lee [52], since it was Higgs who had pointed out the presence of 'incomplete multiplets' i.e. the real part without the imaginary part of the scalar field.

It is still there, however — as Anderson had realised — disguised as the longitudinal polarisation mode of the photon.

For the Higgs mechanism to work, then, we require a tachyonic scalar field theory with a Mexican hat potential, where we make two successive reparametrisations of the complex scalar field — the first is a gauge choice in the unbroken theory, and then, when the symmetry breaks, a constant shift in the remaining real scalar theory. This produces the three desirable features ('miracles') mentioned above. viz.,

- The photon acquires a mass.
- The real scalar mode acquires a mass.
- The massless scalar mode disappears from the theory.

Even today, more than a quarter of a century later, the deep insight into Nature provided by spontaneous symmetry-breaking and the Higgs mechanism is quite awe-inspiring. It was soon to provide the key to developing a theory of the electroweak interactions. We must recognise, however, that the choice of parameters in the potential is quite arbitrary, and is in no way demanded by the gauge symmetry. The spontaneously-broken gauge theory, therefore, is a mixed marriage of a pure gauge theory and a specific potential choice, which is driven purely by phenomenological demands.

4 High Renaissance: the Glashow-Salam-Weinberg Model

Given the breakthrough of 1964, it took surprisingly long for someone to come up with a generalisation of the Abelian $U(1)$ Higgs model to the non-Abelian case of an $SU(2) \times U(1)$ local gauge theory *à la* Glashow or Sakurai. However, it was elegantly done in a 1967 paper by T.W.B. Kibble, one, indeed, of the 'magnificent seven' who had proposed the Abelian Higgs mechanism [53]. The essence of this work is very close to the sigma model[19]. Instead of one complex scalar $\varphi(x)$, the theory has four complex scalars $\varphi_0(x)$ and $\vec{\varphi}(x) = \{\varphi_i(x) | i = 1, 2, 3\}$ forming a doublet of $SU(2)$

$$\Phi(x) \stackrel{\text{def}}{=} \mathbb{I}\, \varphi_0(x) + i\, \vec{\mathbb{T}} \cdot \vec{\varphi}(x) \tag{49}$$

and the potential energy terms $V(\Phi)$ will be identical to those in Eqn. (37). We start, as before, with the action

$$S \stackrel{\text{def}}{=} \int d^4x \left[(\mathbb{D}^\mu \Phi)^\dagger\, \mathbb{D}_\mu \Phi - \frac{1}{8}\, \mathrm{Tr}\, [\mathbb{F}^{\mu\nu}\, \mathbb{F}_{\mu\nu}] - \frac{1}{4}\, B^{\mu\nu} B_{\mu\nu} - V(\Phi) \right] \tag{50}$$

where $\mathbb{W}_\mu \stackrel{\text{def}}{=} \vec{\mathbb{T}} \cdot \vec{W}_\mu = \frac{1}{2} \vec{\sigma} \cdot \vec{W}_\mu$, and

$$\begin{aligned} \mathbb{D}_\mu &\stackrel{\text{def}}{=} \mathbb{I}\, \partial_\mu - ig\, \mathbb{W}_\mu - \frac{ig'}{2}\, \mathbb{I}\, B_\mu \\ \mathbb{F}_{\mu\nu} &\stackrel{\text{def}}{=} \partial_\mu \mathbb{W}_\nu - \partial_\nu \mathbb{W}_\mu - ig\, [\mathbb{W}_\mu, \mathbb{W}_\nu] \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{51}$$

[19] Actually, Kibble's paper first discusses a general non-Abelian gauge group before taking up the special case of the Glashow-Sakurai group $SU(2) \times U(1)$.

Following the Abelian case, we can make a polar parametrisation, writing

$$\Phi(x) \stackrel{\text{def}}{=} \begin{pmatrix} 0 \\ \eta(x) \end{pmatrix} e^{i\,\vec{\mathbb{T}}\cdot\vec{G(x)}} \tag{52}$$

and then apply on it a $SU(2)$ local gauge transformation to get rid of the exponentiated (Goldstone) fields, leaving

$$\Phi(x) = \begin{pmatrix} 0 \\ \eta(x) \end{pmatrix} \tag{53}$$

The potential term now becomes identical to that in Eqn. (48) and a shift $\eta(x) = v/\sqrt{2} + H(x)$ will produce gauge boson mass terms of the form

$$S_m = \int d^4x \begin{pmatrix} 0 & v/\sqrt{2} \end{pmatrix} \left(g\,\mathbb{W}^\mu + \frac{g'}{2}\mathbb{I}\,B^\mu\right)\left(g\,\mathbb{W}_\mu - \frac{g'}{2}\mathbb{I}\,B_\mu\right)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \tag{54}$$

$$\begin{aligned} &= \int d^4x \left[\frac{g^2v^2}{8}(W_1^\mu - iW_2^\mu)(W_{1\mu} + iW_{2\mu}) \right. \\ &\qquad \left. + \frac{v^2}{8}\begin{pmatrix} W_3^\mu & B^\mu \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_{3\mu} \\ B_\mu \end{pmatrix}\right] \end{aligned} \tag{55}$$

Kibble's paper stopped at this point, simply stating that the gauge bosons would have masses[20]. However, the thread was brilliantly taken up by Weinberg [54] later the same year. It was Weinberg who pointed out that Eqn. (55) means that the $\vec{W}_\mu$ and B_μ fields are unphysical because of the presence of mixing terms. However, this can be dealt with quite easily. From the first two W's we can define conjugate complex fields

$$W_\mu^\pm \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}}(W_{1\mu} \mp iW_{2\mu}) \tag{56}$$

and for the $W_{3\mu}$ and the B_μ, the mass matrix is diagonalised by the orthonormal physical states

$$Z_\mu \stackrel{\text{def}}{=} \frac{gW_{3\mu} + g'B_\mu}{\sqrt{g^2 + g'^2}} \qquad\qquad A_\mu \stackrel{\text{def}}{=} \frac{g'W_{3\mu} - gB_\mu}{\sqrt{g^2 + g'^2}} \tag{57}$$

corresponding to eigenvalues $g^2 + g'^2$ and 0 respectively. Eqn. (55) then reduces to

$$S = \int d^4x \left[\frac{g^2v^2}{4} W^{+\mu}W_\mu^- + \frac{v^2}{8}(g^2 + g'^2)Z^\mu Z_\mu\right] \tag{58}$$

We thus have charged vector bosons $W_\mu^\pm$ with mass $M_W = \frac{1}{2}gv$, a neutral vector boson Z_μ with mass $M_Z = \frac{1}{2}v\sqrt{g^2 + g'^2}$ and a massless vector boson A_μ. This certainly fits the pattern of massive weak bosons and a photon – if the A_μ can be identified with the photon. For this, it must couple to fermion exactly like the photon. It was here that Weinberg pointed out that we can recast

[20] In fact, Kibble stated that *all* the gauge bosons have mass.

Eqn. (57) in the exact form of Eqn. (28) in Glashow's model, provided we set

$$\tan\theta \stackrel{\text{def}}{=} \frac{g'}{g} \tag{59}$$

Now this, if we write $g = g_2$ and $g' = g_1$, is precisely what Glashow had obtained — see Eqn. (30) — by fine tuning the mixing so that the photon coupling to neutrinos vanishes. We thus have a beautiful understanding of where this particular relation comes from – it arises from the pattern of spontaneous symmetry-breaking and is not a fine tuning at all! Naturally, this relation also means that we can identify the A_μ field as Glashow's photon. It is for this insight that the mixing angle θ in Eqn. (30) has, ever since, been known as the *Weinberg angle* and denoted θ_W. In terms of this we have the useful relation

$$M_Z = \frac{M_W}{\cos\theta_W} \tag{60}$$

Profound as it may be, this $SU(2) \times U(1)$ electroweak model is predicated on the existence of a bunch of scalar fields, of which — before 2012 — there was no evidence. What made Weinberg's model a runaway success was its ability to explain fermion masses and couplings. Here again, the spadework had been done by Salam and Ward [55], as early as 1964. They did not, however, incorporate any solution for the gauge boson masses, contenting themselves with the comment that the weak bosons must be 'outrageously' heavy.

Weak interaction theory, in fact, has a serious difficulty, not just with gauge boson masses, but also with fermion masses. The reason lies in their parity-violating nature. This had been proposed as early as 1956 by Yang and Lee [56], and was soon proved in beautiful experiments by Wu *et al* [57] and by Goldhaber *et al* [58]. This was followed by the discovery [29] that the weak interaction currents are of the V-A form, which, in the IVB theory would mean that the couplings of the W boson are of the form

$$\begin{aligned} S_W &\sim \int d^4x\; \overline{\psi}(x)\gamma^\mu\,(1-\gamma_5)\,\psi(x)\;W_\mu(x) + \text{H.c.} \\ &\sim \int d^4x\; \overline{\psi_L}(x)\gamma^\mu\psi_L(x)\;W_\mu(x) + \text{H.c.} \end{aligned} \tag{61}$$

where we define

$$\psi_L(x) \stackrel{\text{def}}{=} \frac{1}{2}\,(1-\gamma_5)\;\psi(x) \qquad\qquad \psi_R(x) \stackrel{\text{def}}{=} \frac{1}{2}\,(1+\gamma_5)\;\psi(x) \tag{62}$$

However, the W boson is charged and it couples to a $SU(2)$ doublet of scalars. Accordingly, we must create an $SU(2)$ doublet of fermions. For the moment, we just choose the electron and its neutrino, *à la* Weinberg, and create a doublet with the left-handed components

$$L_L(x) \stackrel{\text{def}}{=} \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \qquad\qquad \overline{L_L}(x) = \begin{pmatrix} \overline{\nu_{eL}}(x) & \overline{e_L}(x) \end{pmatrix} \tag{63}$$

with a gauge-kinetic term

$$S_L = \int d^4x\; i\,\overline{L_L}(x)\,\gamma^\mu\,\mathbb{D}_\mu^{(L)}\,L_L(x) \tag{64}$$

where

$$\mathbb{D}_\mu^{(L)} \stackrel{\text{def}}{=} \mathbb{I}\,\partial_\mu + ig\,\mathbb{W}_\mu - \frac{ig'}{2} Y_L\,\mathbb{I}\,B_\mu \tag{65}$$

to be added to the rest of the electroweak action. Since the right-handed components do not interact with the W boson (but the e_R most certainly interacts with the photon), we make them $SU(2)$ *singlets*, with gauge-kinetic terms

$$S_R = \int d^4x\; i\,\overline{e_R}(x)\,\gamma^\mu\, D_\mu^{(e)} e_R(x) \tag{66}$$

where

$$D_\mu^{(e)} \stackrel{\text{def}}{=} \partial_\mu - \frac{ig'}{2} Y_e\, B_\mu \tag{67}$$

also to be added to the rest of the electroweak action. In view of the experiments of Goldhaber *et al* [58], no right-handed neutrino field is included in Eqn. (67).

It may be noted that the gauge-covariant derivatives of Eqns. (65) and (67) differ from those in Eqns. (51) and (4) respectively because of the presence of the *weak hypercharges* Y_L and Y_e. To find these quantum numbers of the chiral fermions, we need to expand the interaction terms in Eqns. (64) and (66), thereby obtaining

$$S_{\text{int}}^{\text{cc}} = \int d^4x\; \frac{g}{2\sqrt{2}}\; \left[\overline{\nu}_e(x)\gamma^\mu\,(1-\gamma_5)\,e(x)W_\mu^+(x) + \text{H.c.}\right] \tag{68}$$

for the *charged current interactions* i.e. terms with currents[21] that couple to the charged $W^\pm$. We can now write the Fermi interaction of Eqn. (14) (with a V-A modification) as a second order effective interaction of this theory in the low-energy limit to get the identification

$$\frac{G_F}{\sqrt{2}} \stackrel{\text{def}}{=} \frac{g^2}{8M_W^2} \tag{69}$$

where G_F is the Fermi coupling constant. The presence of γ^μ as well as $\gamma^\mu\gamma_5$ interactions ensures that both Fermi-type and Gamow-Teller type interactions are taken care of.

The corresponding *electromagnetic interactions* are now obtained by substituting Eqn. (28) in Eqns. (64) and (66) to get

$$\begin{aligned} S_{\text{int}}^{\text{em}} \;=\; \int d^4x\; \Big[& -\frac{1}{2} g\sin\theta_W\,(1-Y_L)\;\overline{\nu_{eL}}\gamma^\mu\nu_{eL}A_\mu \\ & + \frac{1}{2} g\sin\theta_W\,(1+Y_L)\;\overline{e_L}\gamma^\mu e_L A_\mu + \frac{1}{2} g\sin\theta_W Y_e\;\overline{e_R}\gamma^\mu e_R A_\mu \Big] \end{aligned} \tag{70}$$

where we also substitute $g' = g\tan\theta_W$ from Eqn. (59). This will yield the QED interactions if we put

$$Y_L = 1 \qquad\qquad Y_e = 2 \qquad\qquad g\sin\theta_W = e \tag{71}$$

Once the quantum numbers Y_L and Y_e are known, we can determine the *neutral current interactions*,

[21] These currents will themselves transform like charged objects under the $U(1)$ of QED.

i.e. those involving the vector boson Z_μ, as

$$S_{\rm int}^{\rm nc} = \int d^4x \, \left[\frac{g}{4\cos\theta_W} \overline{\nu_e}\gamma^\mu (1-\gamma_5)\,\nu_e Z_\mu - \frac{g}{4\cos\theta_W} \overline{e}\gamma^\mu \left(1 - 4\sin^2\theta_W - \gamma_5\right) e Z_\mu \right] \tag{72}$$

All of this falls into place very nicely, and today it has all been verified experimentally in a myriad ways.

5 Late Renaissance: quarks, mixing and a third generation

Despite the elegance of the Weinberg-Salam construction, in 1967, there still remained three serious issues with this version of the electroweak model of leptons.

- The first of these was the same problem which had plagued Glashow's 1961 model, viz., we cannot write this interaction for a nucleon doublet, for there would then be a non-vanishing neutron-photon coupling. In fact, the neutron does have a magnetic moment, and hence it does couple to the magnetic field components in the photon, but the above form of interaction would couple it to the electric field.

- The second problem was that the separation of chiral fermions into left-handed doublets and right handed singlets does not permit the fermions to have a mass. For example, an electron mass term
$$\mathcal{L}_m^{(e)} = m_e\,\overline{e}\,e = m_e\,(\overline{e_L}\,e_R + \overline{e_R}\,e_L) \tag{73}$$
will not be gauge invariant, since (i) e_L has weak isospin $T_3 = -1/2$ and e_R has $T_3 = 0$, and (ii) e_L has $Y = Y_L = -1$ and e_R has $Y - Y_e = -2$. The only way to retain the gauge invariance would be to set $m_e = 0$, whereas the electron is known to have a finite mass[22].

- A more subtle issue attaches itself to the axial vector coupling of the Z. The axial vector current should be conserved in the limit of unbroken gauge symmetry, but sny axial current is well known to have a chiral anomaly. This arises from the B_μ part of the interaction and is, therefore, proportional to the hypercharge Y. To have a renormalisable quantum field theory, we must arrange for the total anomaly to cancel [59, 60], i.e. we demand $\sum_i Y_i = 0$ for the particles coupling to the Z boson. Clearly, if the particles in the theory are ν_{eL}, e_L and e_R, then $\sum_i Y = Y_L + Y_L + Y_e = -4$. This non-cancellation of the anomaly was the third problem.

The first and third problem were solved only with the introduction of quarks, but the second problem – that of how chiral fermions acquire masses – was solved by Weinberg [54]. The idea is that the symmetry-breaking mechanism would generate the fermion masses. This is because the

[22] One might argue that the electron mass is small and may be treated as an explicit breaking of the gauge symmetry. However, with hindsight, we know that the same argument applies to the top quark, and its huge mass çan by no means be regarded as an explicit symmetry-breaking.

symmetries of the leptonic theory permit the existence of Yukawa couplings

$$\begin{aligned} S_{\rm Yuk} &\stackrel{\rm def}{=} \int d^4x \left[y_e \overline{L_L}(x)\, \Phi(x)\, e_R(x) + \text{H.c.} \right] \\ &= \int d^4x \left[y_e \overline{\nu_{eL}}(x) \varphi^+(x) e_R(x) + y_e \overline{e_L}(x) \varphi^0(x) e_R(x) + \text{H.c.} \right] \end{aligned} \tag{74}$$

After symmetry-breaking, we replace $\varphi^0 \to \frac{v}{\sqrt{2}} + \eta^0(x)$, and obtain

$$\begin{aligned} S_{\rm Yuk} &= \int d^4x \Big[y_e \overline{\nu_{eL}}(x) \varphi^+(x) e_R(x) + y_e \overline{e_L}(x) \eta^0(x) e_R(x) \\ &\qquad\qquad + \frac{y_e v}{\sqrt{2}} \overline{e_L}(x)\, e_R(x) + \text{H.c.} \Big] \end{aligned} \tag{75}$$

The last of these terms is clearly a mass term. Writing it separately, we obtain

$$S_{\rm m}^{(e)} = \int d^4x\ \frac{y_e v}{\sqrt{2}} \left[\overline{e_L}(x)\, e_R(x) + \overline{e_R}(x)\, e_L(x) \right] = \int d^4x\ m_e \bar{e} e \tag{76}$$

where

$$m_e \stackrel{\rm def}{=} \frac{y_e v}{\sqrt{2}} \tag{77}$$

The electron mass is thus a product of the vacuum expectation value v with the Yukawa coupling[23] y_e, and obviously will vanish as $v \to 0$. This may be regarded as a *fourth miracle* of electroweak theory. There will, however, be no neutrino mass, since there is no ν_R, as pointed out by Goldhaber *et al* [58] and hence, there will be no neutrino Yukawa coupling.

The 1969 discovery of quarks [61] provided a new impetus to attempts to include strongly-interacting states in the electroweak model [62]. For the u and d quarks, it was a simple matter. They can be combined into a $SU(2)$ doublet for the left-handed components

$$Q_L(x) \stackrel{\rm def}{=} \begin{pmatrix} u_L(x) \\ d_L(x) \end{pmatrix} \qquad\qquad \overline{Q_L}(x) = \begin{pmatrix} \overline{u_L}(x) & \overline{d_L}(x) \end{pmatrix} \tag{78}$$

while the right-handed components u_R and d_R are singlets under $SU(2)$. We now write the gauge kinetic term for these quarks as

$$S_q = \int d^4x\ \left[i\, \overline{Q_L}(x)\, \gamma^\mu\, \mathbb{D}_\mu^{(Q)}\, Q_L(x) + i\, \overline{u_R}(x)\, \gamma^\mu\, D_\mu^{(u)} u_R(x) + i\, \overline{u_R}(x)\, \gamma^\mu\, D_\mu^{(u)} u_R(x) \right] \tag{79}$$

where, as in the leptonic case, we have covariant derivatives

$$\mathbb{D}_\mu^{(Q)} \stackrel{\rm def}{=} \mathbb{I}\, \partial_\mu + ig\, \mathbb{W}_\mu - \frac{ig'}{2} Y_Q\, \mathbb{I}\, B_\mu \qquad D_\mu^{(u)} \stackrel{\rm def}{=} \partial_\mu - \frac{ig'}{2} Y_u\, B_\mu \qquad D_\mu^{(d)} \stackrel{\rm def}{=} \partial_\mu - \frac{ig'}{2} Y_d\, B_\mu \tag{80}$$

Expanding this, we get charged current interactions

$$S_{\rm int}^{\rm cc} = \int d^4x\ \frac{g}{2\sqrt{2}} \left[\overline{u}(x) \gamma^\mu \left(1 - \gamma_5\right) d(x)\, W_\mu^+(x) + \text{H.c.} \right] \tag{81}$$

[23] Conversely, we can write the Yukawa coupling $y_e = \sqrt{2} m_e / v$.

and electromagnetic interactions

$$\begin{aligned} S_{\rm int}^{\rm em} &= \int d^4x \left[\frac{1}{2}e\left(1+Y_Q\right)\ \overline{u_L}\gamma^\mu u_L A_\mu + \frac{1}{2}eY_u\ \overline{u_R}\gamma^\mu u_R A_\mu \right. \\ & \qquad \left. -\frac{1}{2}e\left(1-Y_Q\right)\ \overline{d_L}\gamma^\mu d_L A_\mu + \frac{1}{2}eY_d\ \overline{d_R}\gamma^\mu d_R A_\mu\right] \end{aligned} \tag{82}$$

using Eqns. (59) and (83) to replace g' and g by e. It is now easy to obtain the proper quark charges by setting

$$Y_Q = \frac{1}{3} \qquad\qquad Y_u = \frac{4}{3} \qquad\qquad Y_d = -\frac{2}{3} \tag{83}$$

leading to the usual electromagnetic interaction

$$S_{\rm int}^{\rm em} = \int d^4x \left[\frac{2e}{3}\,\overline{u}(x)\gamma^\mu u(x)A_\mu(x) - \frac{e}{3}\,\overline{d}(x)\gamma^\mu d(x)A_\mu(x)\right] \tag{84}$$

Both these quarks have nonzero charge, but when combined into $n = udd$, of course, the net charge is zero. This solves the first problem, and also explains why the neutron has a magnetic (dipole) moment.

Inclusion of these quarks also solves the second problem, but with a twist. If we naively sum the hypercharges of the neutrino-electron and up-down pairs (constituting a *generation*), we will get

$$2Y_L + Y_e + 2Y_Q + Y_u + Y_d = -2 - 2 + \frac{2}{3} + \frac{4}{3} - \frac{2}{3} = -\frac{8}{3} \tag{85}$$

indicating that the chiral anomaly does not cancel. However, if we take into account that every quark comes in 3 colours, then the addition must be

$$2Y_L + Y_e + 3\times\left(2Y_Q + Y_u + Y_d\right) = -2 - 2 + 3\left(\frac{2}{3} + \frac{4}{3} - \frac{2}{3}\right) = 0 \tag{86}$$

We now have almost everything fitting in the Glashow-Salam-Weinberg model, except for the strange quark s [24]. That it cannot be a singlet of $SU(2)$ is clear from the fact that we have weak decays like $K^+(u\bar{s}) \to \mu^+\nu_\mu$ which must involve a $\bar{s}dW$ vertex. This puzzle led to the creation of different models [63], of which Lee [59] pointed out that the simplest, and, as it later turned out, correct one was that due to Glashow, Iliopoulos and Maiani (or GIM for short) [64]. This was simply to postulate the existence of a new quark c — called the *charm quark* — which would be the $SU(2)$ partner of the d quark, just as the u is the $SU(2)$ partner of the d quark. Thus, the GIM model envisages *two* fermion generations, viz.,

$$\begin{aligned} L_L^{(1)} &= \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \ell_R^{(1)} = e_R \quad Q_L^{(1)} = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad u_R^{(1)} = u_R \quad d_R^{(1)} = d_R \\ L_L^{(2)} &= \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \ell_R^{(2)} = \mu_R \quad Q_L^{(2)} = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad u_R^{(2)} = c_R \quad d_R^{(2)} = s_R \end{aligned} \tag{87}$$

[24] It is an interesting social phenomenon that the nomenclature of hadronic $SU(2)$ doublets has proceeded from the classical (proton, neutron), to the colloquial (up, down), to the fanciful (charm, strange) and finally the downright irreverent (top, bottom). It is, perhaps, for the best that there are no more.

with the chiral anomaly cancelling separately over each generation. This was brilliantly confirmed by the 1974 discovery [65] of the c-quark — or rather of a $c\bar{c}$ bound state, called the J/ψ.

The existence of two generations also provided an explanation for the mystery of flavour-mixing, which had been postulated by Cabibbo [66] as early as 1963. This was elaborated by GIM, using the Higgs doublet, as follows. The existence of two generations permits the existence of mixed Yukawa couplings in the quark sector of the form

$$\mathcal{S}_{\rm Yuk}^{(q)} = \int d^4x \, \sum_{a=1}^{2}\sum_{b=1}^{2} \left[y_{ab}^{(u)} \, \overline{Q_L^{(a)}}(x) \, \widetilde{\Phi}(x) \, u_R^{(b)}(x) + y_{ab}^{(d)} \, \overline{Q_L^{(a)}}(x) \, \Phi(x) \, d_R^{(b)}(x) + \text{H.c.} \right] \tag{88}$$

using the notation of Eqn. (87), where, in order to keep the $U(1)_Y$ invariance, the first term uses the charge conjugate doublet

$$\widetilde{\Phi}(x) \stackrel{\rm def}{=} i\sigma_2 \Phi^*(x) = \begin{pmatrix} \varphi^0 \\ -\varphi^- \end{pmatrix} \tag{89}$$

Expanding and shifting $\varphi^0 = v/\sqrt{2} + \eta^0(x)$, we obtain mass terms of the form

$$S_m = \int d^4x \, \sum_{a=1}^{2}\sum_{b=1}^{2} \left[\frac{y_{ab}^{(u)} v}{\sqrt{2}} \overline{u_L} u_R + \frac{y_{ab}^{(d)} v}{\sqrt{2}} \overline{d_L} d_R + \text{H.c.} \right] \tag{90}$$

which can be written in matrix form as

$$S_m = \int d^4x \, \left[\overline{U_L} \, \mathbb{M}^{(u)} \, U_R + \overline{D_L} \, \mathbb{M}^{(d)} \, D_R + \text{H.c.} \right] \tag{91}$$

where $\overline{U_{L,R}} = \begin{pmatrix} \overline{u_{L,R}} & \overline{c_{L,R}} \end{pmatrix}$ and $\overline{D_{L,R}} = \begin{pmatrix} \overline{d_{L,R}} & \overline{s_{L,R}} \end{pmatrix}$, while the mass matrices are

$$\mathbb{M}_{ab}^{(u)} \stackrel{\rm def}{=} \frac{y_{ab}^{(u)} v}{\sqrt{2}} \qquad\qquad \mathbb{M}_{ab}^{(d)} \stackrel{\rm def}{=} \frac{y_{ab}^{(d)} v}{\sqrt{2}} \tag{92}$$

Each of these mass matrices $\mathbb{M}^{(u)}$ and $\mathbb{M}^{(d)}$ can be diagonalised by bi-unitary transformations [67]

$$U_L = \mathbb{V}_L^{(u)} U_L^0 \qquad\qquad U_R = \mathbb{V}_R^{(u)} U_R^0 \qquad\qquad \mathbb{V}_L^{(u)} \mathbb{M}^{(u)} \mathbb{V}_R^{u\dagger} = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix} \tag{93}$$

and

$$D_L = \mathbb{V}_L^{(d)} D_L^0 \qquad\qquad D_R = \mathbb{V}_R^{(d)} D_R^0 \qquad\qquad \mathbb{V}_L^{(d)} \mathbb{M}^{(d)} \mathbb{V}_R^{d\dagger} = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix} \tag{94}$$

from which

$$S_m = \int d^4x \, \left[m_u \, \overline{u_L^0} u_R^0 + m_c \, \overline{c_L^0} c_R^0 + m_d \, \overline{d_L^0} d_R^0 + m_s \, \overline{s_L^0} s_R^0 + \text{H.c.} \right] \tag{95}$$

where the superscript 0 denotes the physical quark state (often referred to as the *mass basis.*)

When we come to the charged current interactions of Eqn. (81), however, we can rewrite them as

$$\begin{aligned}
S_{\rm int}^{\rm cc} &= \int d^4x\ \left[\frac{g}{2\sqrt{2}}\,\overline{U_L}\gamma^\mu(1-\gamma_5)D_L W_\mu^+ + {\rm H.c.}\right] \\
&= \int d^4x\ \left[\frac{g}{2\sqrt{2}}\,\overline{U_L^0}\mathbb{V}_L^{(u)\dagger}\gamma^\mu(1-\gamma_5)\mathbb{V}_L^{(d)}D_L^0 W_\mu^+ + {\rm H.c.}\right] \\
&= \int d^4x\ \left[\frac{g}{2\sqrt{2}}\,\overline{U_L^0}\gamma^\mu(1-\gamma_5)\,\mathbb{K}_2 D_L^0 W_\mu^+ + {\rm H.c.}\right]
\end{aligned} \tag{96}$$

where $\mathbb{K}_2 \stackrel{\rm def}{=} \mathbb{V}_L^{(u)\dagger}\mathbb{V}_L^{(d)}$ is known as the *mixing matrix.* The subscript 2 indicates that it is a 2×2 matrix, and, of course, since all the $\mathbb{V}$'s are unitary, $\mathbb{K}$ is also unitary. Such matrix can be parametrised in terms of one angle parameter and three phase parameters; however, it can be shown [67] that these three phase parameters can be absorbed in the phases of the four quark fields. This makes $\mathbb{K}_2$ real, i.e. it has the form

$$\mathbb{K}_2 \stackrel{\rm def}{=} \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \tag{97}$$

where θ_C is the Cabibbo angle — postulated by Cabibbo in 1963. The matrix $\mathbb{K}_2$ may, therefore, be referred to as the *Cabibbo matrix*[25].

The electroweak model developed above was almost complete, but one important feature was left unexplained. When parity $\mathcal{P}$ was shown not to be conserved [56, 57], Landau had speculated [68] that what is conserved is the combination[26] of parity $\mathcal{P}$ and charge conjugation $\mathcal{C}$, i.e., the discrete quantum number $\mathcal{CP}$. This is precisely what happens in the GIM model of the electroweak interactions, even if the Yukawa couplings $y_{ab}^{(u,d)}$ are taken to be complex, since the phases of the mixing matrix $\mathbb{K}_2$ will not appear. $\mathcal{CP}$, however, is not conserved, as was proved in 1964 by the iconic Cronin-Fitch experiment [69]. In fact, unless $\mathcal{CP}$ is violated, there would be no baryon asymmetry in the Universe, as pointed out by Sakharov [70], and hence the Universe of matter which we live in and carry out experiments and observations could not exist. It was in a perhaps desperate attempt[27] to save the electroweak model that Kobayashi and Maskawa, in 1973, suggested [71] that there may be a *third* generation of fermions, viz.

$$L_L^{(3)} = \begin{pmatrix} \nu_L^{(3)} \\ \ell_L^{(3)} \end{pmatrix} \qquad \ell_R^{(3)} \qquad\qquad Q_L^{(3)} = \begin{pmatrix} u_L^{(3)} \\ d_L^{(3)} \end{pmatrix} \qquad u_R^{(3)} \qquad d_R^{(3)} \tag{98}$$

such that the Yukawa couplings and mass matrices would become 3×3 matrices, and we would have to replace the 2×2 mixing matrix $\mathbb{K}_2$ by a 3×3 version $\mathbb{K}_3$, which is now called the Cabibbo-Kobayashi-Maskawa matrix, or *CKM matrix* for short. Now, a 3×3 unitary matrix has 3 angles and 6 phases, of which 5 phases can be absorbed into the phases of the six quark fields. One phase, however, remains, and this, according to Kobayashi and Maskawa [71], is the source of $\mathcal{CP}$

[25] Strictly speaking, in 1963 Cabibbo [66] himself wrote down only the mixing between the d and the s quarks, or rather, the corresponding charged currents with a $\bar{u}$. The full matrix, which requires the c quark, was written down only in 1970 by Glashow, Iliopoulos and Maiani [64].

[26] This is really to say that while elementary particles may have a parity-violating excess, the number of antiparticles in the Universe will have an exactly complementary deficit, or vice versa, keeping the total number unchanged.

[27] Reminiscent of Pauli's 'desperate attempt' to save energy conservation by postulating the neutrino [16].

violation.

Like Pauli's bold prediction of the neutrino and GIM's bold prediction of the charm quark, the equally bold speculation of Kobayashi and Maskawa was proved absolutely correct, when the fermions of the third generation began to be discovered one by one. First came the tau lepton $\ell^{(3)} = \tau^-$ in 1975 [72], closely followed by the bottom quark $d^{(3)} = b$ in 1977 [73]. There followed a 17-year hiatus till the 1994 discovery of the top quark $u^{(3)} = t$ [74], and another 6 years wait till the existence of the tau neutrino $\nu^{(3)} = \nu_\tau$ was confirmed [75] in 2000.

By the turn of the century, it was clear that the electroweak model — dubbed the 'Standard Model', or SM for short, by Pais and Treiman [76] as early as 1975 — provides a logical and coherent structure for the weak and electromagnetic interactions, and also intimately ties in with QCD through the importance of three colours for anomaly cancellation. All that remained was the cornerstone of the whole theory, viz., the Higgs boson, which had proved elusive to experimental searches for several decades We may quickly recall that it is by undergoing a phase transition in the early Universe, as it was cooling down, that the Higgs field gave mass to all the other particles in the SM (except photons and gluons with which it does not interact). And it is the large masses of W, Z bosons that makes the weak interaction weak, since they are responsible for the smallness of the Fermi coupling constant. Therefore, the Higgs boson lies at the very heart of weak interactions theory. However, the electroweak action does not tell us anything about the mass of the Higgs boson, which is a free parameter in the Glashow-Salam-Weinberg model. Fortunately, there are elegant arguments why the mass of the Higs boson cannot be very high. These ars outlined below.

It one considers the elastic scattering of W bosons — a permitted process in the GSW model — then it is easy to see that the longitudinal mode, i.e.

$$W_L + W_L \longrightarrow W_L + W_L \tag{99}$$

will have bad high energy behaviour because in that limit $\varepsilon_L^\mu(p) \simeq p^\mu/M_W$. In fact, in the limit when the W and Z masses can be neglected (but not the Higgs boson mass), the amplitude for this process assumes the form

$$\mathcal{A}_L(s,t) \approx -\sqrt{2}M_H^2 \left(\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right) \tag{100}$$

If we allow M_H to grow without limit, we find that as $M_H \to \infty$ the amplitude reduces to

$$\mathcal{A}_L(s,\theta^*) \approx \frac{G_F}{\sqrt{2}}\, s\,(1 + \cos\theta^*) \tag{101}$$

where θ^* is the scattering angle. Obviously, as s increases, this will grow without limit, violating unitarity. It follows that we cannot let M_H grow without limit.

This argument can be sharpened [19] by invoking partial-wave unitarity of the above amplitude, which can be expanded as

$$\mathcal{A}_L(\theta^*) = \sum_{\ell=0}^{\infty} (2\ell + 1) A_\ell P_\ell(\cos\theta^*) \tag{102}$$

with the requirement $|A_j|^2 \leq |\text{Im}A_j|$, which can be rewritten as $| A_j |\leq \frac{1}{2}$. Applying this to the $j = 0$ partial wave (which happens to give the best bound among all the values of j)

$$A_0 = \frac{1}{16\pi s} \int_{-s}^{\infty} dt \ \mathcal{A}_L(s,t) \tag{103}$$

one obtains an upper bound

$$M_H^2 \leq \frac{4\sqrt{2}\pi}{3G_F} \approx (700 \text{ GeV})^2 \tag{104}$$

Thus it was clear that the Higgs boson could not be very heavy, for in 1977, thoughts of a 20 TeV LHC and a 40 TeV SSC were already in the air. In fact, these machines (of which the SSC never materialised) were designed as machines where the Higgs boson would certainly be discovered. However, successive experiments came and went, but there was no sign[28] of the elusive particle. This explains the jubilation when it was finally discovered at CERN [78] in 2012, and of course, the prompt award of a much-belated Nobel Prize to François Englert and Peter Higgs in the following year[29]. The long road to this discovery makes, of itself, a fascinating story, and it has been covered in several books published after 2012 [79].

With the discovery of the Higgs boson, the long development of the Standard Model was complete. Till date, we do not have any better model for explaining the experimental measurements made in all terrestrial experiments.

Part 2 : After the Higgs, What?

During the US Senate hearings on the SSC project, one Senator is reported to have commented "You have quite a few quarks already. What will you do with another one?" While this story may be apocryphal, a similar question can be asked in the context of extended Higgs sectors and, more generally, of all physics beyond the SM. We need to briefly justify this before moving on to more Higgs-specific discussions.

6 Impressionism: Beyond the Standard Model

The initial impetus to go beyond the SM (bSM) came from the fact that one requires three colours — the basis of QCD — to get an anomaly-free theory of electroweak interactions. The fact that QCD is also a gauge theory led to speculations that a further level of unification, viz. strong and electroweak, may occur if we embed both in a common gauge theory with a larger gauge group. The earliest ideas in this direction came from Pati and Salam [80], who, in 1974, created a gauge theory based on the group $SO(4)$, where lepton number is a fourth colour. A more comprehensive theory,

[28] It is well-known that when Leon Lederman wrote a popular book with Dick Teresi on the hard-to-find Higgs boson, he wanted to name his book *The Goddamn Particle* to express the frustration of the high energy physics community. It is equally famous that his scandalised publisher changed the title to *The God Particle* [77], thereby creating an endless source of confusion in the minds of the public when the elusive particle finally showed up.

[29] Robert Brout, Englert's co-author, had died in 2011.

based on the group $SU(5)$ was written down by Georgi and Glashow soon after [81]. This set off a vast industry of *Grand Unified Models* or GUTs, which generically provided strong-electroweak unification in addition to explaining one or more of the arbitrary features of the SM.

A common feature of GUTs, however, is the prediction of proton decay through the process

$$p \longrightarrow \pi^0 + e^+ \tag{105}$$

which would be mediated by a (massive) gauge boson X of the higher (broken) symmetry. The decay width will be suppressed by the mass of this heavy gauge boson X, and the lifetime can be estimated as [82]

$$\tau_p \sim \frac{M_X^4}{m_p^5} \tag{106}$$

Though originally conceived by Sakharov [70], this prediction of proton decay in the context of GUTs was first made by Pati and Salam [80] in 1974. Since then, there have been intensive searches for proton decay[30], but none have been successful in finding any evidence whatsoever. In the 1970s, the lower bound on proton decay[31] stood at around 10^{29} years, which means that M_X should be at least 10^{15} GeV. This was known as the GUT scale. However, over the years, the lower bound on the proton lifetime has crept up and up to around 1.67×10^{34} years today [85], requiring M_X to be greater than about 3×10^{16} GeV. This is still about 400 times smaller then the Planck scale $M_P = (\hbar c/G_N)^{1/2}$, where G_N is the gravitational constant, and therefore, it should be just about possible to construct a GUT without having to seriously take into account the threshold effects of strong gravity.

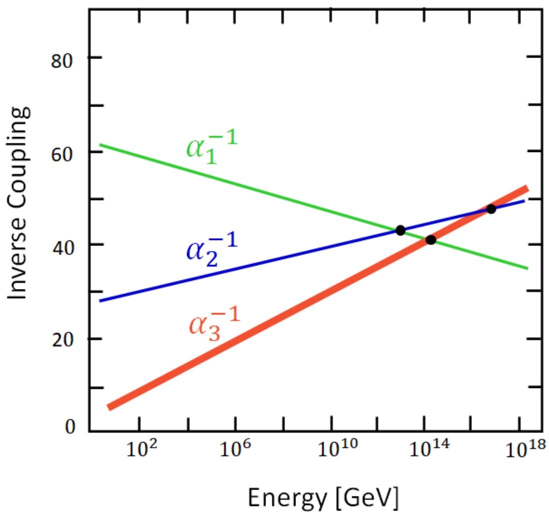


Figure 4: Running coupling constants in the SM.

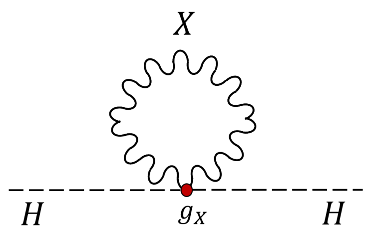
A more serious problem with GUTs arose after 1991 when the LEP-1 collider at CERN produced its precision measurements of all the three gauge couplings α_1, α_2 and α_3 corresponding to $U(1)$,

[30] Including pioneering searches at the Kolar Gold Fields in India [83].

[31] Since the age of the Universe is estimated to be only about 1.38×10^{10} years [84], there is no imminent danger of the Universe disappearing into a puff of pions and leptons.

$SU(2)$ and $SU(3)$ respectively, at the mass scale M_Z. One could now determine the beta functions of these couplings, and thereby predict their running till the Planck scale. This exercise was done by several groups [86] with a startling result. Instead of evolving logarithmically so that all three met at a common scale where the expected GUT unification would occur, the renormalisation group evolution of these couplings seemed to lead to pairwise crossing at three different scales (the black dots in Figure 4) — just as if there is no GUT unification. This was to prove a blow from which GUTs never really recovered. Of course, it was almost immediately shown that the single-scale unification could be restored by embedding the SM in a supersymmetric model, and later it was established that one could also achieve it with a bigger GUT group, with symmetry-breaking in multiple stages, where the beta functions would change at each threshold [87]. But the pristine beauty of $SU(5)$ grand unification was gone, and thenceforth, slowly but surely, GUT theories moved from the mainstream to the fringes of high energy research.

Before fading into the background, however, GUT theories brought to the fore a serious issue with the Higgs boson mass. The Higgs doublet, in order to transform under the $SU(2) \times U(1)$ gauge group, must also transform under the larger GUT gauge group $G \supset SU(2) \times U(1)$. The kinetic term for Higgs bosons will, then, induce seagull terms of the form



$$S_{\rm sg} = \int d^4x \; g_X X^\mu(x) X_\mu(x) \Phi^\dagger(x) \Phi(x0 \tag{107}$$

leading to the so-called 'sunset' diagrams as shown in the figure, with a heavy X boson contributing to the Higgs boson self-energy at the one-loop level. This contribution may be calculated, using the dimensional regularisation scheme in $4 - \varepsilon$ dimensions, as

$$\delta M_H^2(M_X) = \frac{g_X}{32\pi^2} M_X^2 \left(\frac{2}{\varepsilon} - \gamma + \ln \pi - 1 + \ln M_X^2 \right) \tag{108}$$

where γ is the Euler-Mascheroni constant [88]. Even apart from the divergence as $\varepsilon \to 0$, this contribution is intolerably large, given that $M_X \sim 10^{16}$ GeV and M_H is 125.4 GeV. This is known as the *hierarchy problem* and it can appear whenever there are two widely-separated energy scales in a quantum field theory [89]. In this case, the renormalisation technique cannot save the situation. One can, of course, introduce a counterterm in the action to cancel this divergence in the $\overline{\rm MS}$ scheme. However, if we recall that the M_X is a physical mass and will surely run with energy, this means that the divergence will reappear at a different energy. One would then have to introduce a running counterterm, which is hard to justify. This is known as the *fine-tuning problem.*

It is important to point out at this stage that the hierarchy and fine-tuning problems arise only when there is a GUT or other energy scale M_X lying between the electroweak scale M_H and the Planck scale M_P. If there is no such scale, and the SM holds all the way up to the Planck scale M_P, there will be no hierarchy problem, since δM_H^2 will depend only on M_P — which does not run and can therefore be removed once and for all in an $\overline{\mathrm{MS}}$ cancellation. We therefore need to look for a stronger reason to have an intermediate scale M_X – other than a naive belief that there is a GUT somewhere in the background. This may be sought in a consideration of the stability of the electroweak vacuum against quantum corrections.

As is the case with all the coupling constants in the SM, the scalar quartic coupling λ in Eqn. (48) will also run. At the electroweak scale, we can solve the two mass relations

$$M_W = \frac{1}{2} g v \qquad\qquad M_H = \sqrt{2\lambda} v \tag{109}$$

to obtain

$$\lambda = \frac{\pi\alpha}{2\sin^2\theta_W} \left(\frac{M_H}{M_W}\right)^2 \simeq 0.129 \tag{110}$$

plugging in the measured values. This value of λ is already quite small, and, as it happens, with increasing energy it decreases steadily, its evolution being driven mostly by the large top quark Yukawa coupling. At a scale somewhere between 10^9 GeV and 10^{20} GeV, in fact, it will become zero, and thenceforth run to negative values. The state of the art in this analysis is reproduced un Figure 5 from Ref. [90], where the computation has been carried to the three-loop level in the SM. The grey shaded region shows the uncertainty in the extrapolated value of λ as it runs with increasing energy (μ in the figure). The two main sources of this error are the values of the strong coupling constants α_3 and the uncertainty in the Yukawa coupling of the top quark, as currently measured.

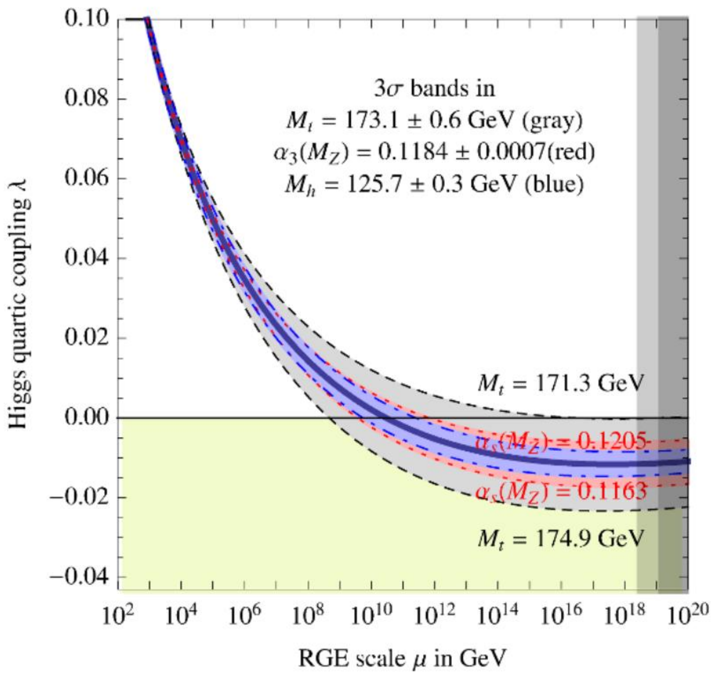


Figure 5: Running of the Higgs quartic coupling in the SM. Reproduced from Ref. [90]). The parameters have values as measured in 2013. Values of λ is the light green-shaded region will correspond to an unstable vacuum.

A glance at Figure 5 will make it obvious, that if λ is non-positive, it will be an absolute disaster for the SM, for then the potential will become unbounded from below, and it will be possible for any particle to go down the potential, emitting an infinite amount of energy, i.e. there will be a catastrophic explosion of the Universe. Of course, a catastrophe is not precluded just because it is a catastrophe. The argument here is that just after the Big Bang, the newly-created Universe consisted of particles at energies at the Planck scale, or just below it. If the electroweak vacuum had been unstable at these energies, there would have been plenty of radiated energy to keep the Universe in that hot condition, or make it even hotter. This clearly did not happen, since the Universe has cooled down to as low as 2.73 K. Ergo, the electroweak vacuum has stayed stable throughout, and the λ coupling has never gone negative. This can only happen if, at some higher scale, there is new physics beyond the SM, which changes the running of the λ coupling and keeps it from falling to zero.

Once can therefore make a strong claim that if λ becomes negative at a high energy scale below the Planck scale, we are bound to have new physics at that scale, and this will immediately reinstate the hierarchy problem for the Higgs boson mass. But can we be sure that λ does, indeed become negative at a value below the Planck scale? Figure 5 shows that because of the uncertainty in parameters[32] we cannot yet be sure about this, since the range of λ values has one edge that lies just around the limiting value, i.e. there is still a possibility that the vacuum will remain stable all the way to the Planck scale without invoking new physics.

There have been attempts to save the electroweak vacuum by postulating a more complicated structure, equivalent to putting in a (non-renormalisable) $\epsilon\varphi^6(x)$ term in the potential. In these models, there is another, deeper vacuum, which is stable, and the current vacuum is a 'false vacuum', i.e., a metastable state [91]. The tunnelling time for this has been calculated as 10^{10} years [90], which is reassuring, since it makes the probability that it will happen in the next 10,000 years around 10^{-129}. However, it has been pointed out [92] that if the Universe underwent an inflationary stage, as is commonly believed, then the tunnelling time would have been much shorter — in fact short enough for the transition to the true vacuum to have occurred. However,it was then pointed out [93] that this is true only in a flat space calculation; if there is Higgs-curvature mixing with a mixing parameter ξ, then it can be tuned to the so-called 'conformal point' $\xi = -1/6$ to keep the electroweak vacuum stable throughout the inflation period. This was reinforced by a calculation of reheating effects [94] which established that the fine tuning $\xi = -1/6$ is the only way to save the electroweak vacuum. There the case rests.

However, long before the value of λ was measured at any scale, the influence of GUTs had inspired several bSM solutions for the hierarchy problem. These are of two classes, as follows.

[32]There are several uncertainties in the value of the top quark mass which goes into the RGE evolution of λ. The principal problem is that there is no measurable pole mass for the top quark, since quark states are not asymptotically free. The experimentally measured mass at the LHC is an invariant mass of final states where the interpretation of hadronisation effects is model dependent. The best inference of the top quark 'pole' mass comes from the measurement of the $t\bar{t}$ production cross section, but that is dependent on PDF uncertainties as well as the renormalisation scheme. Most of these uncertainties do not affect common processes, but even small changes affect the RGE scale, which has the top quark-dependent parameters in the argument of the exponential function.

1. Models where the cutoff scale is brought down to just above the electroweak scale.

   - *Technicolor*: It was Wilson in 1971 who first pointed out [89] that gauge boson and fermion self-energy corrections are protected from having quadratic divergences by the gauge symmetry and the chiral symmetry, respectively. For a scalar, however, there is no such symmetry, and hence, quadratic divergences, such as Eqn. (108), cannot be avoided. Wilson went on to point out that this was not a problem for scalars like the pions, since they were composites, and the scale set by the binding energy would be a natural cutoff for self-energy corrections. For pions, this is the QCD scale, which, at around 280 MeV, is not so much larger than the pion mass.

     By the later half of the decade, as the SM went from triumph to triumph, it was clear that the hierarchy problem in the Higgs boson mass needed to be addressed. The first solution to this was proposed by Weinberg [95] in 1976 and backed up by Susskind [96] in 1978. This was more-or-less a repetition of Wilson's argument for pions. It was assumed that the Higgs boson is actually a composite of a new kind of confined fermions called *techni-fermions*, analogous to qiarks, which were bound into a scalar by a $SU(N)$ gauge interaction called *technicolor*. The cutoff scale could then be around a few TeV, the analogy of the QCD scale, and there would be no hierarchy problem.

     An obvious experimental check of the technicolour model would be the presence of a heavy techni-rho ρ_T^0, being the spin-1 bound state partnering the spin-0 Higgs boson, in the same way as the spin-1 ρ^0 partners the π^0. This could be searched for through decays $\rho_T^0 \to W_L^+ W_L^-$, where the $W_L^\pm$ are longitudinally-polarised W bosons. Needless to say, no such signals have been found at the LHC or other experiments. An even stronger argument against technicolor ideas came in 1990, when Peskin and Takeuchi [97] showed that the prediction of the so-called oblique parameter S in technicolor models was much too large to be compatible with the value obtained from experimental measurements at the CERN LEP-1 collider. Since then, technicolor models have had few takers, except for a few persistent souls who have crafted modifications of the minimal technicolor model [98] to evade all experimental constraints. However, these models have been acquiring an increasingly baroque nature, just as modern GUT models have[33].

   - *Extra Dimensions*: Extra dimensions had been introduced as a vehicle for unification of gravity with electromagnetism in the early twentieth century [99], but largely abandoned when it became clear that such ideas could not be verified experimentally. In the 1970s, as gauge theories stated to become popular, extra dimensions became a mainstay of string theories [100], which visualised extra compact dimensions with a size around M_P^{-1} and were not unduly worried about experimental verification. In 1998, a sensation was created when the trio of Arkani-Hamed, Dimopoulos and Dvali, or ADD for short, proposed a model [101] where there would be extra dimensions of the size of a millimetre, with the SM fields (which don't seem to sense any extra dimensions) confined to a wafer-thin slice of four-dimensional space inside the higher-dimensional bulk[34]. This

[33]The partly-pejorative term 'epicycles' is often applied to these efforts, which is not without justification. However, a sobering thought is that epicycles are just a two-dimensional version of the Fourier series, discovered many centuries after Ptolemy.

[34]The thickness of this slice must be smaller that the smallest length scale to which the SM has curently been

scenario was, in fact, nicely fulfilled by a topological defect, called a D_3-brane, predicted in Type-II string theories [102], and this led to all such subspaces being dubbed *branes*, whether the model was embedded in a string theory or not. The ADD construction would immediately bring the effective Planck scale down to around a TeV, obviously removing the hierarchy problem. The model predicted novel spin-2 interactions from towers of Kaluza-Klein excitations of the graviton, which would lead to observable consequences in high energy experiments.

Soon, however, it became apparent that the ADD model suffers from its own hierarchy problem — not in the mass of the Higgs boson, but in the size of the extra dimensions. Here too, quantum corrections were the problem, for they would tend to shrink these dimensions down to the smallest length scale in the theory, i.e. the Planck scale — thereby reinstating the hierarchy problem *in toto*. To solve this problem, an ingenious construction was effected by Randall and Sundrum in 1999 [103]. The Randall- Sundrum model, or RS model for short, envisages two branes, separated by a small distance not much larger than the Planck length, which communicate through gravitational interactions. By fine-tuning cosmological constants on the two branes and in the intervening bulk (which has to have the topology of a circle folded about one of its diameters [104], i.e. a $\mathbb{S}^1/\mathbb{Z}_2$ orbifold) RS obtained a solution for the gravitational field which falls off exponentially from one brane to the other. It is thus possible for gravity to have electroweak strength on one brane (dubbed the *Planck brane*) and its usual ultra-weak Newtonian strength on the other (dubbed the *TeV brane*). The electroweak scale is the only scale in this model and all large numbers are generated from this by exponentials with small arguments. This theory predicts graviton Kaluza-Klein modes with masses and coupling constants of electroweak strength, which can be easily looked for as resonances in high energy collisions [105]. This model also has an issue with the stability of the inter- brane distance, which was brilliantly solved by Goldberger and Wise [106] through the introduction of a bulk scalar, which appears on the TeV brane as a Higgs-like particle, called the *radion*.

It is worth mentioning, in this context, a third model with an $\mathbb{S}^1/\mathbb{Z}_2$ orbifolded extra dimension — not because it solves the hierarchy problem (it does not!), but because it is relevant for Higgs boson physics. This model places all the SM fields in the bulk, with, presumably some topological defects at the two ends of the extra dimension. In this model, called the Universal Extra Dimensions, or UED, scheme [107], all the SM fields have Kaluza-Klein excitations and the only new parameter in the theory is the radius of the bulk.

All three extra-dimensional models are of *decoupling* nature, i.e. the observable effects can be made arbitrarily small by tuning some parameter – generally the size of the extra dimension. Indeed, this has been forced upon us by the non-observation of any trace of extra dimensions. The negative aspect of this is that the effective Planck mass keeps rising, and thereby we have a *little* hierarchy problem, i.e. a ratio of order 10^4 between the Higgs boson mass and the minimum (bulk) Planck scale. This is a slightly uncomfortable

probed, i.e. about 10^{-17} cm

situation, but by no means as intolerable as having a cutoff at the GUT scale of 10^{16} GeV.

2. Models where the quadratic divergences in the Higgs self-energy cancel.

   - *Supersymmetry*: In Wilson's pioneering work [89], it had been pointed out that there are symmetries protecting the fermion and gauge boson masses from quadratic divergences, but none protecting the scalar Higgs boson mass. If one could find such a symmetry, clearly there would be no quadratic divergence in M_H^2 and hence no hierarchy problem. As it happens, by the late 1970s, such a symmetry lay ready at hand. This was *supersymmetry*. As is well known, quantum mechanics assigns different roles to particles of spin-$\frac{1}{2}$ which are fermions and particles of integer spin which are bosons — this being known as the *spin-statistics theorem*. Supersymmetry, however, transcends this by allowing fermionic degrees of freedom to mix with bosonic degrees of freedom and vice versa. This revolutionary idea was first used in string theory, where the basic string excitations are all bosonic, and therefore one needs supersymmetry to generate fermionic degrees of freedom [108]. It was discovered by Wess and Zumino in 1974 [109] that in fact the effect of this extra symmetry is to protect the Higgs boson mass from developing quadratic divergences, thereby removing the hierarchy problem. The Wess-Zumino model is the simplest supersymmetic model, being the supersymmetric version of a $\lambda\varphi^4$ theory. In 1976, Pierre Fayet was the first to develop a supersymmetric SM [110], but a phenomenologically viable model came only with the work of Dimopoulos and Georgi in 1981 [111]. In all of these models, the Higgs boson mass remains free of quadratic divergences.

     In the minimal supersymmetric SM, called the MSSM, and all of its different variations, every boson in the SM has a fermionic partner and every fermion in the SM has a bosonic partners. These additional fields are called *superpartners*. In the limit of exact supersymmetry, the SM particles and their respective superpartners have the same masses and coupling strengths. However, since no such superpartners have been observed, it must be assumed that they are too heavy to have been produced in experiments till date, i.e. supersymmetry is explicitly broken by large masses of the superpartners[35] Different patterns of this symmetry-breaking create a landscape of supersymmetric models, almost all of which have been diligently searched for at high energy experiments, including the LHC. Unfortunately, nothing has been seen as of date, as a result of which the lower bounds on superpartner masses are getting steadily pushed up. Once again, there is nothing wrong in principle with such heavy superpartners; however their failure to show up has led to a certain degree of despondency in the high energy community.

     One important feature of the MSSM and its variants is that there necessarily has to be a second Higgs doublet. This has inspired a whole new field of investigation, which is described in the next section.

   - *Little Higgs Models*: An ingenious construction, whose proximate inspiration was no doubt extra-dimensional models, was proposed in 2008. However, this idea really goes

[35]and by some other operators besides.

back to the times when the pion was thought to be an elementary particle and the pion-nucleon Lagrangian had a spontaneously-broken global symmetry, of which the pion was a Goldstone boson — or rather (since the pion gets a small mass through explicit symmetry-breaking), a pseudo-Goldstone boson. This was an elegant argument to keep the pion light, and the idea of little Higgs models was to explain the lightness of the Higgs boson using similar ideas. There are different versions of the model, but the essential features are captured by a version where the scalar sector has a local $[SU(2) \times U(1)]^2$ at high energies. This breaks spontaneously around 8–10 TeV to the SM group $SU(2)_L \times U(1)_Y$, and the Higgs boson is the Goldstone boson of this theory. This symmetry again breaks spontaneously to $U(1)_{\rm em}$ at the electroweak scale, and it is at that scale that the Higgs boson acquires a mass, becoming a pseudo-Goldstone boson. However, quantum corrections to this cannot be too large, since masslessness is guaranteed at the not-too-high scale 8–10 TeV. In practice, the way this happens is that the one-loop corrections from the pair of gauge bosons corresponding to the two $SU(2) \times U(1)$ symmetries, and these will have opposite signs due to group theoretic factors which are induced by the unbroken symmetry. Therefore the Higgs boson mass is generated at two loops and is perturbatively small. In the so-called 'littlest Higgs model', the $[SU(2) \times U(1)]^2$ symmetry is obtained from the breaking chain $SU(5) \to SO(5) \to SU(3)_c \times [SU(2) \times U(1)]^2$.

An immediate problem in such a model is that these are extra contributions to the S and T precision oblique parameters at LEP, and these have a non-decoupling component which makes them incompatible with the data, which are very close to the Standard Model predictions. If there are *two* sets of gauge bosons, however, we can assign a $\mathbb{Z}_2$ symmetry called T-parity, such that the SM particles have $T = 1$ and the new particles have $T = -1$. This would ensure a cancellation between the corresponding contributions to the oblique parameters, and indeed to any loop corrections, with any residual effects being due to the mass differences. An immediate consequence is that the lightest particle with $T = -1$ will not decay and can be a candidate for dark matter, just as the LSP is.

A great deal of the research with bSM Higgs bosons has been in the context of one or other of these models. In fact, there was a time when supersymmetry was widely regarded as the 'standard' bSM theory, but that alas! is no longer the case.

We now come to the greatest failing of the SM, viz., its complete inability to explain most of the composition of the Universe. These mysterious components have been dubbed *dark matter* and *dark energy*, and the SM does not have any candidate for either.

Dark matter was postulated by Zwicky in 1937, when he could not reconcile the proper motion of galaxies in the Virgo super cluster with the mass inferred from observation of the luminous matter [112]. Essentially the galaxies were moving at speeds higher than the escape velocity corresponding to this mass, and yet the system remains in a bound state, presumably under Newtonian gravity. The solution, according to Zwicky, was that the supercluster contains a great deal of matter which is not luminous, i.e. it does not radiate any energy in the electromagnetic spectrum, and is, therefore,'dark'. Nearly four decades later, Rubin [113] measured the transverse (rotational) velocity of stars in the neighbouring Andromeda galaxy and a few others. The expectation from Newtonian mechanics was that if the mass M of the galaxy is concentrated in the central region, the transverse velocity of stars in the outer arms would fall off as $v(r) = \sqrt{G_N M/r}$. Rubin found, however, that the measured velocities remain essentially constant as r increases. She correctly interpreted thus to mean that the galaxy is immersed in a ball of invisible — dark — matter, such that the mass inside a sphere of radius r is $M(r) \propto r$. This immediately means that the dark matter distribution inside a galaxy is $\rho(r) \propto r^{-2}$, with a cusp at the centre, compatible with the existence of a black hole there. Since the 1970s, it has been shown that practically every galaxy, and cluster of galaxies shows similar behaviour. The existence of dark matter is also proved by a consideration of gravitational lensing phenomena. Moreover, there is an increasing body of evidence that the dark matter is particulate in nature and does not consist of compact objects like mini black holes [114].

The problem with dark matter is that there is no particle in the SM which can be described as dark matter. Since dark matter is believed to be non-baryonic [115] and neutral, the only possible candidates are neutrinos and the Higgs boson. The latter is ruled out because it is unstable, having a very short lifetime of around 1.6×10^{-22} s. Neutrinos, on the other hand, may be stable, but they are too light to satisfy the requirement that the dark matter has cooled to the observed temperature (2.73 K) of the Universe. In fact, the ideal dark matter candidate is a weakly-interacting massive particle (WIMP) which is stable, or at least has a lifetime greater than the age of the Universe [114]. This is simply not available in the SM.

The mystery that is dark matter pales in significance compared to what is called 'dark energy', which essentially means a positive cosmological constant. The cosmological constant Λ was originally introduced by Einstein in 1917 to get a static Universe [116]. The best way to understand it [117] is to write down the Robertson-Walker metric for a spherically-symmetric Universe

$$ds^2 \stackrel{\text{def}}{=} dt^2 - R^2(t)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{111}$$

and require the radial factor $R(t)$ to satisfy the Friedmann equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{112}$$

where p and ρ stand for pressure and density respectively (assumed uniform over cosmic scales) and Λ is the cosmological constant. Assuming p and ρ are positive, as would be the case for both matter and energy, then $\Lambda = 0$ would lead to $\ddot{R} < 0$, i.e. a Universe which is expanding but at a decelerated rate. The truth is exactly the opposite – it has been shown by a study of very distant supernovae that the Universe is, in fact expanding at a small *accelerated* rate [118]. This makes it imperative to make the right side of the Friedmann equation positive, and the obvious way to do this is to postulate a value $\Lambda > 0$ which will lead to a small positive residue on the right side.

Since the cosmological constant acts as a uniform matter density in the Universe, it is commonly referred to as *dark energy.* Studies of the cosmic microwave background indicate that the Universe is composed of 68% of this mysterious material, about 27% of dark matter, and the remaining 5% of baryonic, i.e. SM matter. It would be easiest to identify the cosmological constant or dark energy density with the vacuum energy associated with the electroweak transition – unfortunately, that leads to an overestimate by a factor of the order of 10^{60}, leading to the so-called *cosmological constant problem.* Many solutions of this have been proposed, from modification of Newtonian dynamics (MOND) [120] to the use of effective field theories to cancel the vacuum energy. There has also been a suggestion of a new kind of particle called *quintessence*, which on cosmic length scales, has a repulsive effect [119], thereby changing the sign of the pressure term and making a nonzero cosmological constant unnecessary. These ideas are only peripheral to this article, becoming relevant only when the Higgs sector acts as a portal to these exotic particles through some kind of feeble coupling.

The signal failure of the first three runs of the LHC to find any evidence for physics beyond the Standard Model has brought to the fore an alternative paradigm. This is the development of bottom-up theories, i.e. those which retain the gauge and global symmetries of the SM and make minimal (or just beyond minimal) extensions to either the field content, or the specific interactions. Among the first type are extensions of the gauge boson sector (often associated with extra gauge symmetries of the simplest kind), extensions of the Higgs sector, extensions of the fermion sector, or addition of exotica such as leptoquarks and dileptons – almost all leading to new interactions with the SM Higgs boson. There are also suggestions for modification of the SM interaction vertices by introducing anomalous terms, which can mostly be generated in effective field theories where there are new heavy fields whose interactions at a high scale can be integrated out leaving effective operators which then modify the SM vertices. In a combination of these, models of dark matter often assume the existence of dark particles, fermions or bosons, which interact with the SM fields through mediator fields, and this can also modify the Higgs boson interactions.

The USP of these bottom-up approaches is claimed to be their model-independent nature. However, while these models generally do not attempt to solve the basic problems with the SM itself, they often make assumptions about new fields and couplings which amount to a degree of model-building. A comprehensive description of these ideas would require a different article altogether. Hence, in the present work, we shall concentrate only on that part of these extensions which is relevant specifically to the Higgs sector.

7 Expressionism: Extended Higgs Sectors

If we do not consider models which are derivable from some symmetry at a high energy scale, then there is just one guideline to the kind of extra scalars which can be added to the Standard Model. This comes from the so-called ρ parameter (also called T parameter), given by

$$\rho \stackrel{\text{def}}{=} \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \tag{113}$$

In the Standard Model, $\rho = 1$, and indeed its experimental value is [121]

$$\rho_{\text{exp}} = 1.00038 \pm 0.00020 \tag{114}$$

In a model with several extra Higgs multiplets $\Phi_1, \Phi_2, \ldots$ with different weak isospin and weak hypercharge assignments (T_i, Y_i) where $i = 1, 2, \ldots$, the rho parameter (at tree-level) can be shown to have the form

$$\rho = \frac{2v^2 + \sum_i \left\{4T_i(T_i+1) - Y_i^2\right\} v_i^2}{2v^2 + \sum_i 2Y_i^2 v_i^2} \tag{115}$$

where v_i is the vev of the neutral component in the i-th multiplet. It is trivial to check that this remains unity if we add any number of scalar doublets ($T = \frac{1}{2}, Y = \pm 1$) or singlets ($T = 0, Y = 0$), irrespective of the vacuum expectation values v_i. For other multiplets, the v_i must be fine-tuned suitably. It is not surprising, therefore, that the most popular extensions of the Higgs sector are singlets and doublets. Some of the specific ideas are now discussed in more detail.

7.1 *Extra Real Singlet Scalar*

The simplest extension of the Higgs sector is to add on a real scalar which is a gauge singlet of $SU(2)_L \times U(1)$. An early exploration of this [122] in the wake of the top quark discovery tried to satisfy the Veltman condition of cancellation of quadratic divergences in the Higgs boson self-energy, but this led to the prediction of a Higgs boson mass above 300 GeV, which is now known to be unphysical. However, more recent analyses [123] take a less ambitious approach, confining themselves to studying the parameter space and discovery limits of such models. A singlet scalar does not, by definition, have gauge interactions and hence its decays are solely governed by the dynamics of the (extended) Higgs sector. This makes it, in many models, practically stable, making it an attractive candidate for dark matter [124].

Denoting the real singlet by σ, the scalar potential of the theory in such models becomes

$$V(\Phi,\sigma) = -\mu^2\Phi^\dagger\Phi + \lambda\left(\Phi^\dagger\Phi\right)^2 + \lambda_1\sigma + \frac{1}{2}\lambda_2\sigma^2 + \lambda_3\sigma^3 + \lambda_4\sigma^4 + \lambda_1'\Phi^\dagger\Phi\sigma + \lambda_2'\Phi^\dagger\Phi\sigma^2 \tag{116}$$

where $\Phi^\dagger \stackrel{\text{def}}{=} \begin{pmatrix} \varphi^- & \varphi^{0*} + \frac{v}{\sqrt{2}} \end{pmatrix}$. Taking advantage of the singlet nature of the scalar σ, it is always possible to make a redefinition such that its vacuum expectation value vanishes, i.e. $\langle\sigma\rangle = 0$. Minimising the potential now leads to a mixing between the erstwhile Higgs boson and the singlet scalar, as follows

$$\begin{aligned} h &= \mathrm{Re}\,\varphi^0\ \cos\alpha + \sigma\ \sin\alpha \\ H &= -\mathrm{Re}\,\varphi^0\ \sin\alpha + \sigma\ \cos\alpha \end{aligned} \tag{117}$$

where $M_h = 125.4$ GeV is the mass of the elementary scalar discovered at the LHC and $v = 246.2$ GeV as in the SM. The free parameters of this theory can be taken to be [123]

$$M_H \qquad \sin\alpha \qquad \lambda_3 \qquad \lambda_4 \qquad \lambda_2'$$

in terms of which

$$\begin{aligned} \lambda &= \frac{1}{2v^2}\left(M_h^2\cos^2\alpha + M_H^2\sin^2\alpha\right) \\ \lambda_2 &= M_h^2\sin^2\alpha + M_H^2\cos^2\alpha - \lambda_2' v^2 \\ \lambda_1' &= \frac{1}{2v}\left(M_h^2 - M_H^2\right)\sin 2\alpha \end{aligned} \tag{118}$$

The mixing results will modify all the couplings of the h and this will lead to testable consequences at experimental facilities. For example, if $M_h < 2M_H$, it is easy to see that the Higgs production cross-section will scale as $\cos^2\alpha$. Taking the experimental data from the LHC experiments, it is shown in Ref. [123] that

$$|\sin\alpha| < 0.2 \tag{119}$$

In case $M_h > 2M_H$, the decay channel $h \to HH$ will open up and the bound will then be dependent on the other parameters as well. There will also be a unitarity bound from the consideration of $h + h \to h + h$, which appears as

$$M_H^2 \leq \frac{16\pi v^2}{3}\csc^2\alpha - M_h^2\cot^2\alpha \tag{120}$$

Taken in conjunction with Eqn. (119) this leads to an upper bound $M_H \lesssim 5$ TeV, which gets relaxed as $\sin\alpha \to 0$. Only about two-fifths of this range on M_H would be accessible at the LHC.

In a variant of this idea, an extra $\mathbb{Z}_2$ symmetry $\sigma \to -\sigma$ is included in the theory. In this case, $\lambda_1 = \lambda_3 = \lambda_1'$, but a glance at the third line of Eqn. (118) will show that this is too simplistic. In fact, in such models, we must set $\langle\sigma\rangle = v' \neq 0$ in order to get nontrivial results.

In a more exotic variant of this idea, a model with a $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ is considered, where there is an extra Z' boson, which is massless even when the electroweak symmetry is broken. Since there is no evidence for such a massless gauge boson, it must acquire mass somehow. One way to do this is to break the extra $U(1)_{Y'}$ symmetry spontaneously by introducing some extra Higgs multiplet which develops a vacuum expectation value of its own. Another possibility, which has been investigated recently [125], is to add a Stückelberg-type real scalar σ (which transforms under the adjoint representation of the gauge group, and is therefore not a singlet *per se*) preserving the gauge symmetry, but giving a mass to the Z'. This model is highly predictive, since the $U(1)_{Y'}$ symmetry forbids many operators which would otherwise have been present.

7.2 *Two Extra Real Singlets*

If one can add on one real scalar singlet, why not two? This Two Real Singlet Model (TRSM) [126] is equivalent to adding a single complex scalar doublet (CxSM) [127]. The main motivation of such models is to obtain a viable dark matter candidate in one of the physical scalars which result when these extra scalars mix among themselves and with the SM scalar. The basic formalism is not too different from a single real scalar. If we have two extra real scalars σ and σ', the most general scalar potential becomes very messy. It is more usual, therefore, to assume an additional $\mathbb{Z}_2 \times \mathbb{Z}_2'$ symmetry under transformations of the form

$$\begin{aligned} \mathbb{Z}_2 &: \quad \sigma \to -\sigma \qquad\qquad \sigma' \to \sigma' \\ \mathbb{Z}_2' &: \quad \sigma \to \sigma \qquad\qquad \sigma' \to -\sigma' \end{aligned} \tag{121}$$

where the SM fields are left invariant by both the transformations. The scalar potential now becomes

$$V(\Phi,\sigma,\sigma') = -\mu^2\Phi^\dagger\Phi + \lambda\left(\Phi^\dagger\Phi\right)^2 + \frac{1}{2}\lambda_2\sigma^2 + \lambda_4\sigma^4 + \frac{1}{2}\lambda_2'\sigma'^2 + \lambda_4'\sigma'^4 + \eta_1\Phi^\dagger\Phi\sigma^2 + \eta_2\Phi^\dagger\Phi\sigma'^2 + \eta_3\sigma^2\sigma'^2 \tag{122}$$

where

$$\Phi = \begin{pmatrix} \varphi^+ \\ \frac{\varphi_0+v}{\sqrt{2}} \end{pmatrix} \qquad \sigma = \frac{s+w}{\sqrt{2}} \qquad \sigma' = \frac{s'+w'}{\sqrt{2}} \tag{123}$$

where the vacuum expectation values $\langle\sigma\rangle = w$ and $\langle\sigma'\rangle = w'$ result in the spontaneous breaking of the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ symmetry.

It is convenient to take all the quartic couplings, viz., $\lambda, \lambda_4, \lambda_4', \eta_1, \eta_2, \eta_3$ as independent parameters. The remaining parameters are then given by the minimisation conditions as

$$\begin{aligned}
\mu^2 &= \lambda v^2 + \frac{1}{2}\eta_1 w^2 + \frac{1}{2}\eta_2 w'^2 \\
\lambda_2 &= \frac{1}{2}\lambda_4 w^2 + \frac{1}{4}\eta_1 v^2 + \frac{1}{4}\eta_3 w'^2 \\
\lambda_2' &= \frac{1}{2}\lambda_4' w'^2 + \frac{1}{4}\eta_2 v^2 + \frac{1}{4}\eta_3 w^2
\end{aligned} \tag{124}$$

where we can set $v = 246.2$ GeV and choose w, w' appropriately as two more free parameters of the theory. With these conditions, the potential can be expanded to obtain mass terms resulting in a mixing between the neutral scalar φ_0 of the SM and the two new scalars, of the form

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \mathbb{O} \begin{pmatrix} \varphi_0 \\ \sigma \\ \sigma' \end{pmatrix} \tag{125}$$

where the h_i masses M_i satisfy $M_1 \leq M_2 \leq M_3$ and the orthogonal mixing matrix $\mathbb{O}$ satisfies the equations

$$\begin{aligned}
\sum_{i=1}^{3} M_i^2 O_{i1}^2 &= 2\lambda v^2 \quad \sum_{i=1}^{3} M_i^2 O_{i2}^2 = 2\lambda_4 w^2 \quad \sum_{i=1}^{3} M_i^2 O_{i3}^2 = 2\lambda_4' w'^2 \\
\sum_{i=1}^{3} M_i^2 O_{i1} O_{i2} &= 2\eta_1 v w \quad \sum_{i=1}^{3} M_i^2 O_{i1} O_{i3} = 2\eta_2 v w' \quad \sum_{i=1}^{3} M_i^2 O_{i2} O_{i3} = 2\eta_3 w w'
\end{aligned} \tag{126}$$

It is simple to now parametrise the mixing matrix $\mathbb{O}$ in terms of 3 mixing angles $\theta_1, \theta_2, \theta_3$, or rather their sines $s_i = \sin\theta_i$ $(i = 12, 3)$ and take these as well as as the 3 physical masses $M_1, M_2, M3$ to replace the 6 free parameters in the potential using Eqns. (127). We can also set one of the $M_i = 125.4$ GeV and $v = 246.2$ GeV. This leaves 7 free parameters — 3 mixing angkes, 2 scalar masses and 2 vev's of the singlets. The potential and all the couplings of the model are then determined in terms of these, and phenomenological studies can be made.

There are mild unitarity bounds on this model, but the phenomenological bounds from electroweak precision tests and from high-energy collider searches can be fairly complicated, depending on which of the different channels of $h_i \to h_j + h_k$, where ijk is some combination of 123 is kinematically permitted. Detailed studies may be found in Ref. [126]. What emerges is that the current data permit a range of parameter space where one of the physical scalars h_i is coupled very lightly with matter. It can be produced by co-annihilation of two of its sister scalars $h_j + h_k \to h_i$ $(j, k \neq i)$, but cannot decay back because of kinematic constraints, i.e. $M_i < M_j + M_k$. This gives it the stability to become a candidate for dark matter.

7.3 *Two Higgs Doublet Models*

While one can, in principle, go on adding more singlets scalars to the SM, there is not much motivation for going beyond two singlets. Instead, the bulk of efforts in this field has been in adding an extra doublet of scalars, leading to the so-called two-Higgs doublet models (2HDM).

The basic formalism resembles the SM or its singlet extensions. If there are two Higgs doublets Φ_1 and Φ_2 with identical gauge quantum numbers, the most general potential has the form

$$\begin{aligned} V(\Phi_1, \Phi_2) &= -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 - \mu_3^2 \left(\Phi_1^\dagger \Phi_2 + \text{H.c.} \right) \\ &\quad + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ &\quad + \lambda_4 \left\{ (\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right\} + \lambda_5 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \end{aligned} \tag{127}$$

As usual, we write

$$\Phi_1 \stackrel{\text{def}}{=} \begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(\eta_1^0 + i g_1^0 + v_1) \end{pmatrix} \qquad \Phi_2 \stackrel{\text{def}}{=} \begin{pmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(\eta_2^0 + i g_2^0 + v_2) \end{pmatrix} \tag{128}$$

The minimisation conditions for this potential are

$$\mu_1^2 = 2\lambda_1 v_1, \qquad \mu_2^2 = 2\lambda_2 v_2, \qquad \mu_3 = 0, \qquad \lambda_5 = -2\left(\lambda_3 + \lambda_4\right) \tag{129}$$

Substituting these in the potential and expanding, we obtain mass terms of the form

$$\begin{aligned} \mathcal{L}_{\text{m}} &= -\frac{1}{2}\left(3\lambda_3 + 8\lambda_4\right) \begin{pmatrix} g_1^+ & g_2^+ \end{pmatrix} \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} g_1^- \\ g_2^- \end{pmatrix} \\ &\quad - \begin{pmatrix} h_1^+ & h_2^+ \end{pmatrix} \begin{pmatrix} -\lambda_1 v_1^2 & \lambda_3 v_1 v_2 \\ \lambda_3 v_1 v_2 & (\lambda_1 - 2\lambda_2) v_2^2 \end{pmatrix} \begin{pmatrix} h_1^- \\ h_2^- \end{pmatrix} \\ &\quad + \lambda_3 \begin{pmatrix} \varphi_1^+ & \varphi_2^+ \end{pmatrix} \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \varphi_1^- \\ \varphi_2^- \end{pmatrix} \end{aligned} \tag{130}$$

Diagonalising these mass matrices, we obtain the following mixed states as the neutral mass eigenstates of the theory

$$\begin{aligned} h^0 &= \eta_1^0 \cos\alpha + \eta_2^0 \sin\alpha & G^0 &= g_1^0 \cos\beta + g_2^0 \sin\beta \\ H^0 &= -\eta_1^0 \sin\alpha + \eta_2^0 \cos\alpha & A^0 &= -g_1^0 \sin\beta + g_2^0 \cos\beta \end{aligned} \tag{131}$$

as well as the charged mass eigenstates

$$\begin{aligned} G^+ &= \varphi_1^+ \cos\beta + \varphi_2^+ \sin\beta \\ H^+ &= -\varphi_1^+ \sin\beta + \varphi_2^+ \cos\beta \end{aligned} \tag{132}$$

of these, the $G^0, G^\pm$ are the massless Goldstone bosons of the theory, which can be removed by a redefinition of the fields (which we call choosing the unitary gauge) and the remaining 5 are physical fields, viz. the charged Higgs bosons $H^\pm$ with mass M_+, the $\mathcal{CP}$-odd neutral Higgs boson A^0 with mass M_A and the two $\mathcal{CP}$-even neutral Higgs bosons h^0 and H^0 with masses M_h and M_H respectively, where it is usual to identify $M_h = 125.4$ GeV. In the above equations, we have

$$v_1^2 + v_2^2 = v^2 = (246.2 \text{ GeV})^2 \qquad\qquad \tan\beta = \frac{v_2}{v_1} \tag{133}$$

while the physical masses M_+ and M_A are given by

$$M_+^2 = \lambda_3 v^2 \qquad\qquad M_A^2 = -\frac{1}{2}\left(3\lambda_3 + 8\lambda_4\right) v^2 \tag{134}$$

and M_h and M_H can be obtained from the roots of the secular equation

$$x^2 + \left\{\lambda_1(v_1^2 - v_2^2) + 2\lambda_2 v_2^2\right\} x - \left(\lambda_1^2 - 2\lambda_1\lambda_2 + \lambda_3^3\right) v_1^2 v_2^2 = 0 \tag{135}$$

while the mixing angle α satisfies

$$\tan 2\alpha = \frac{\lambda_3 \sin 2\beta}{2\lambda_2 \cos^2\beta - \lambda_1 \cos 2\beta} \tag{136}$$

The free parameters of the theory are λ_1, λ_2, λ_3, λ_4 and $\tan\beta$. It is common to trade the $\lambda_{1,2,3,4}$ for the physical masses, and take the free parameter set as $M_\pm$, M_A, M_h, M_H and $\tan\beta$. Since M_h is a known value, we are left with a 4-parameter theory.

The above description is incomplete because we have assumed that the scalar potential is $\mathcal{CP}$-conserving, for which there is no a priori justification. If we include $\mathcal{CP}$ violating terms, we will have one more coupling λ_6 and a phase ξ. Interested readers are referred to Ref. [129] for more details.

However, a new complication arises because a doublet of scalars can have Yukawa couplings with sequential fermions and this could affect the flavour structure of the model. To see this, we go back to Eqns. (79) and (80), from the expansion of which we can obtain the *neutral current* (nc) interactions

$$
\begin{aligned}
S_{\text{int}}^{\text{nc}} &= \int d^4x \; \frac{g}{2\cos\theta_W} \left[g_L^u \overline{U_L} \gamma^\mu U_L + g_R^u \overline{U_R} \gamma^\mu U_R \right. \\
&\qquad\qquad \left. + g_L^d \overline{D_L} \gamma^\mu D_L + g_R^d \overline{D_R} \gamma^\mu D_R \right] Z_\mu
\end{aligned} \tag{137}
$$

where $(1 - g_L^u)/2 = -g_R^u/2 = 1 + g_R^d = g_L^d = 2\sin^2\theta_W/3$ and as in Eqn. (91) we have

$$
U_{L,R} = \begin{pmatrix} u_{L,R} \\ c_{L,R} \\ t_{L,R} \end{pmatrix} \qquad\qquad D_{L,R} = \begin{pmatrix} d_{L,R} \\ s_{L,R} \\ b_{L,R} \end{pmatrix} \tag{138}
$$

In the SM we can replace the unphysical fields $U_{L,R}$ and $D_{l,R}$ with the physical fields $U^0_{L,R}$ and $D^0_{L,R}$ defined in Eqn. (94) to get

$$
\begin{aligned}
S_{\text{int}}^{\text{nc}} &= \int d^4x \; \frac{g}{2\cos\theta_W} \left[g_L^u \overline{U_L^0} \mathbb{V}_L^{(u)\dagger} \gamma^\mu \mathbb{V}_L^{(u)} U_L^0 + g_R^u \overline{U_R^0} \mathbb{V}_R^{(u)\dagger} \gamma^\mu \mathbb{V}_R^{(u)} U_R^0 + (u \leftrightarrow d) \right] Z_\mu \\
&= \int d^4x \; \frac{g}{2\cos\theta_W} \left[g_L^u \overline{U_L^0} \gamma^\mu \left(\mathbb{V}_L^{(u)\dagger} \mathbb{V}_L^{(u)} \right) U_L^0 + g_R^u \overline{U_R^0} \gamma^\mu \left(\mathbb{V}_R^{(u)\dagger} \mathbb{V}_R^{(u)} \right) U_R^0 + (u \leftrightarrow d) \right] Z_\mu \\
&= \int d^4x \; \frac{g}{2\cos\theta_W} \left[g_L^u \overline{U_L^0} \gamma^\mu U_L^0 + g_R^u \overline{U_R^0} \gamma^\mu U_R^0 + (u \leftrightarrow d) \right] Z_\mu
\end{aligned} \tag{139}
$$

since $\mathbb{V}_L^{(u)\dagger} \mathbb{V}_L^{(u)} = \mathbb{V}_R^{(u)\dagger} \mathbb{V}_L^{(u)} = \mathbb{I}$. Expanding this, we obtain the explicit interactions

$$
\begin{aligned}
S_{\text{int}}^{\text{nc}} &= \int d^4x \; \frac{g}{2\cos\theta_W} \left[g_L^u \overline{u_L^0} \gamma^\mu u_L^0 + g_R^u \overline{u_R^0} \gamma^\mu u_R^0 \right. \\
&\qquad\qquad + g_L^u \overline{c_L^0} \gamma^\mu c_L^0 + g_R^u \overline{c_R^0} \gamma^\mu c_R^0 \\
&\qquad\qquad \left. + g_L^u \overline{t_L^0} \gamma^\mu t_L^0 + g_R^u \overline{t_R^0} \gamma^\mu c_t^0 + (u \leftrightarrow d) \right] Z_\mu
\end{aligned} \tag{140}
$$

Unlike the charged current interactions, the neutral currents connect only quarks and antiquarks of the same flavour, i.e. flavour-changing neutral currents (FCNC) are absent in the tree-level action of the gauge sector. This is a strong prediction of the SM which is backed up by a massive amount of experimental data.

When we consider the Yukawa couplings of the Higgs boson, we note that when the quarks are in the unphysical (gauge) basis, it has the form of Eqn. (88), which can be rewritten as

$$
S_{\text{Yuk}}^{(H)} = \int d^4x \; \left[\overline{U_L} \, \mathbb{Y}^{(u)} \, U_R + \overline{D_L} \, \mathbb{Y}^{(d)} \, D_R + \text{H.c.} \right] H^0 \tag{141}
$$

where the Yukawa couplings are given in Eqn. (92). Writing this in terms of the physical basis, we get

$$
\begin{aligned}
S_{\text{Yuk}}^{(H)} &= \int d^4x \; \frac{\sqrt{2}}{v} \left[\overline{U_L} \, \mathbb{M}^{(u)} \, U_R + \overline{D_L} \, \mathbb{M}^{(d)} \, D_R + \text{H.c.} \right] H^0 \qquad (142) \\
&= \int d^4x \; \frac{\sqrt{2}}{v} \left[\overline{U_L^0} \mathbb{V}_L^{(u)\dagger} \, \mathbb{M}^{(u)} \, \mathbb{V}_L^{(u)} U_R^0 + \overline{D_L^0} \mathbb{V}_R^{(d)\dagger} \, \mathbb{M}^{(d)} \, \mathbb{V}_R^{(d)} D_R^0 + \text{H.c.} \right] H^0
\end{aligned}
$$

However, we have, following Eqn. (94),

$$\mathbb{V}_L^{(u)\dagger}\,\mathbb{M}^{(u)}\,\mathbb{V}_L^{(u)} = \left(\begin{array}{ccc} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{array}\right) \qquad \mathbb{V}_L^{(d)\dagger}\,\mathbb{M}^{(d)}\,\mathbb{V}_L^{(d)} = \left(\begin{array}{ccc} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{array}\right) \tag{143}$$

using which Eqn. (143) reduces to the simple form

$$\begin{aligned} S_{\rm Yuk}^{(H)} &= \int d^4x\,\frac{\sqrt{2}}{v}\Big[m_u\,\overline{u_L^0}u_R^0 + m_c\,\overline{c_L^0}c_R^0 + m_t\,\overline{t_L^0}t_R^0 \\ &\qquad +m_d\,\overline{d_L^0}d_R^0 + m_s\,\overline{s_L^0}s_R^0 + m_b\,\overline{b_L^0}b_R^0 + \text{H.c.}\Big]\,H^0 \end{aligned} \tag{144}$$

which shows that there are no tree-level FCNCs in the scalar sector either.

Stated in words, since the matrix of Yukawa coupkings in either sector is proportional to the mass matrix, the basis in which the mass matrices are diagonalised will also have diagonal Yukawa couplimngs, and hence no FCNCs. This result — that the SM has no tree-level FCNCs in either gauge or Yukawa sectors — was discovered as early as 1977 and is generally known as the *Glashow-Weinberg theorem* [128].

Let us now consider a model with two similar Higgs doublets

$$\Phi_1 \stackrel{\rm def}{=} \left(\begin{array}{c} \varphi_1^+ \\ \varphi_1^0 + \frac{v_1}{\sqrt{2}} \end{array}\right) \qquad\qquad \Phi_2 \stackrel{\rm def}{=} \left(\begin{array}{c} \varphi_2^+ \\ \varphi_2^0 + \frac{v_2}{\sqrt{2}} \end{array}\right) \tag{145}$$

where the vacuum expectation values v_1 and v_2 are, in general, different. In this case, the Yukawa part of the SM will have to be changed from that in Eqn. (88) to

$$\begin{aligned} \mathcal{S}_{\rm Yuk}^{(q)} &= \int d^4x\,\sum_{a=1}^{2}\sum_{b=1}^{2}\Big[y_{ab}^{(u,1)}\,\overline{Q_L^{(a)}}(x)\,\widetilde{\Phi_1}(x)\,u_R^{(b)}(x) + y_{ab}^{(d,1)}\,\overline{Q_L^{(a)}}(x)\,\Phi_1(x)\,d_R^{(b)}(x) \\ &\qquad +y_{ab}^{(u,2)}\,\overline{Q_L^{(a)}}(x)\,\widetilde{\Phi_2}(x)\,u_R^{(b)}(x) + y_{ab}^{(d,2)}\,\overline{Q_L^{(a)}}(x)\,\Phi_2(x)\,d_R^{(b)}(x) + \text{H.c.}\Big] \end{aligned} \tag{146}$$

Expanding this leads to the mass terms

$$\begin{aligned} S_m &= \int d^4x\,\sum_{a=1}^{2}\sum_{b=1}^{2}\left[\left(\frac{y_{ab}^{(u,1)}v_1}{\sqrt{2}} + \frac{y_{ab}^{(u,2)}v_2}{\sqrt{2}}\right)\overline{u_L}u_R \right. \\ &\qquad \left. + \left(\frac{y_{ab}^{(d,1)}v_1}{\sqrt{2}} + \frac{y_{ab}^{(d,2)}v_2}{\sqrt{2}}\right)\overline{d_L}d_R + \text{H.c.}\right] \end{aligned} \tag{147}$$

which can be written in the matrix form of Eqn. (91) where the mass matrices are now

$$\mathbb{M}_{ab}^{(u)} \stackrel{\rm def}{=} \frac{y_{ab}^{(u,1)}v_1}{\sqrt{2}} + \frac{y_{ab}^{(u,2)}v_2}{\sqrt{2}} \qquad \mathbb{M}_{ab}^{(d)} \stackrel{\rm def}{=} \frac{y_{ab}^{(d,1)}v_2}{\sqrt{2}} + \frac{y_{ab}^{(d,2)}v_2}{\sqrt{2}} \tag{148}$$

We can still diagonalise these mass matrices by bi-unitary transformations à la Eqn. (94) and this will lead to a charge current interaction with a CKM matrix and no FCNC in the gauge neutral current. However, the mass matrices are no longer proportional to any of the Yukawa couplings individually, but to a linear combination of them. Therefore if $\mathbb{V}_L^{(u)\dagger}\,\mathbb{M}^{(u)}\,\mathbb{V}_L^{(u)}$ is diagonal, there is no guarantee that $\mathbb{V}_L^{(u)\dagger}\,\mathbb{Y}_1^{(u)}\,\mathbb{V}_L^{(u)}$ or $\mathbb{V}_L^{(u)\dagger}\,\mathbb{Y}_2^{(u)}\,\mathbb{V}_L^{(u)}$ will individually be diagonal, unless indeed $\mathbb{Y}_1^{(u)}$ and $\mathbb{Y}_2^{(u)}$ are proportional to each other. Similar considerations apply to the case of $\mathbb{M}^{(d)}$ and its components $\mathbb{Y}_1^{(d)}$ and $\mathbb{Y}_2^{(d)}$. As a result, the Yukawa couplings of both φ_1^0 and φ_2^0 will, in general, have off-diagonal elements, leading to FCNC in the tree-level action, i.e. a violation of the Glashow-Weinberg Theorem.

What are the different ways to have a 2HDM without violating the Glashow-Weiberg theorem? The simplest way, as mentioned above, is to make the $\mathbb{Y}_1^{(u,d)}$ and $\mathbb{Y}_2^{(u,d)}$ proportional to each other. However, it has been shown by Pich and Tuzon [130] that this proportionality breaks down under radiative corrections. On the other hand, we have an ingenious work due to Botella, Cornet-Gomez and Nebot [131] where the $\mathbb{Y}_1^{(u,d)}$ and $\mathbb{Y}_2^{(u,d)}$ are constructed in such a way that they are not proportional and yet *simultaneously* diagonalisable. However, though the mathematical sleight-of-hand involved in this construction is to be admired, the constructed mass matrices have many textures (i.e. zero elements) and it would require a rash of ad hoc assumptions to justify them theoretically, apart from the difficulty of maintaining the textures as zero at all energy scales.

If we discount the above ideas, then we must end up with a model in which the mass matrices have only one Yukawa matrix term, for then it will become similar to the SM case with simultaneous diagonalisation of the mass and Yukawa matrices. There are 4 ways to achieve this and these are enlisted below.

- Type I models

  Here the Φ_1 is *fermiophobic* and does not couple to the quarks or leptons; all the Yukawa couplings are those of the Φ_2, and hence the $\mathbb{Y}_2^{(u,d)}$ get diagonalised with the masses just as in the SM. The Yukawa sector would be of the form

$$\begin{aligned}\mathcal{S}_{\rm Yuk}^{(q)} &= \int d^4x\, \sum_{a=1}^{2}\sum_{b=1}^{2}\Big[y_{ab}^{(u)}\,\overline{Q_L^{(a)}}(x)\,\widetilde{\Phi_2}(x)\,u_R^{(b)}(x)\\ &\qquad\qquad +y_{ab}^{(d)}\,\overline{Q_L^{(a)}}(x)\,\Phi_2(x)\,d_R^{(b)}(x)+\text{H.c.}\Big]\end{aligned}\tag{149}$$

  To make the Φ_1 fermiophobic, all that we need to do is to assume a $\mathbb{Z}_2$ symmetry $\Phi_1\to-\Phi_1$ which leaves all the other fields invariant. All terms with an odd number of Φ_1's will then be forbidden, but the Φ_1 will still couple to the gauge bosons and the other scalar doublet Φ_2 of course, which would lead to phenomenological signals.

  Type I models have been studied phenomenologically but not in the same extensive way as the Type II models (see below). However, an interesting variant, the so-called *inert doublet* model, where the Φ_1 does not acquire a vacuum expectation value, has an excellent dark matter candidate in the heavier $\mathcal{CP}$-even scalar H^0, which has been extensively studied in the literature [132].

- Type II models

Here the two scalars Φ_1 and Φ_2 carry opposite hypercharges $Y = -1$ and $Y = +1$ respectively. Gauge invariance then demands that the Φ_1 couple only to u-type quarks and the Φ_2 couple only to d-type quarks, i.e the Yukawa interaction would have the form

$$\mathcal{S}_{\text{Yuk}}^{(q)} = \int d^4x \sum_{a=1}^{2}\sum_{b=1}^{2}\Big[y_{ab}^{(u)}\,\overline{Q_L^{(a)}}(x)\,\Phi_1(x)\,u_R^{(b)}(x) + y_{ab}^{(d)}\,\overline{Q_L^{(a)}}(x)\,\Phi_2(x)\,d_R^{(b)}(x) + \text{H.c.}\Big] \tag{150}$$

where it may be noted that the doublets Φ_1 and Φ_2 will have the explicit forms

$$\Phi_1 \stackrel{\text{def}}{=} \begin{pmatrix} \frac{1}{\sqrt{2}}(\eta_1^0 + ig_1^0 + v_1) \\ \varphi_1^- \end{pmatrix} \qquad \Phi_2 \stackrel{\text{def}}{=} \begin{pmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(\eta_2^0 + ig_2^0 + v_2) \end{pmatrix} \tag{151}$$

and it is not necessary to introduce the charge-conjugated doublets $\widetilde{\Phi}_{1,2}(x)$ at all. Such a condition arises naturally in the MSSM (see below), and hence this models is said to be *MMSM-like*.

It is possible to obtain a model of this form by imposing *two* $\mathbb{Z}_2$ symmetries,, viz.

$$\begin{array}{ll} \Phi_1 \to -\Phi_1 & \qquad \Phi_2 \to -\Phi_2 \\ u_R \to -u_R & \qquad d_R \to -d_R \end{array} \tag{152}$$

where each keeps all the other fields invariant. These will then prevent any operators which have Φ_1 with d_R and Φ_2 with u_R.

- Type X models

Till now, we have not brought the leptons into the discussion, since flavour-violation in the lepton sector (if at all) is a very small effect depending on neutrino masses. However, the charged leptons (e, μ, τ) do get masses from their Yukawa couplings. If the leptons are brought into play, we get the so-called Type X (or Type III) or *leptophilic* model, where the Φ_2 couples only to quarks (as in Type 1 models), and the Φ_1, instead of decoupling from fermions altogether, couples to the leptons. The Yukawa sector of the model now looks like

$$\mathcal{S}_{\text{Yuk}}^{(q)} = \int d^4x \sum_{a=1}^{2}\sum_{b=1}^{2}\Big[y_{ab}^{(u)}\,\overline{Q_L^{(a)}}(x)\,\widetilde{\Phi_2}(x)\,u_R^{(b)}(x) \tag{153}$$
$$+ y_{ab}^{(d)}\,\overline{Q_L^{(a)}}(x)\,\Phi_2(x)\,d_R^{(b)}(x) + y_{ab}^{(\ell)}\,\overline{L_L^{(a)}}(x)\,\Phi_1(x)\,e_R^{(b)}(x) + \text{H.c.}\Big]$$

and it is possible to achieve this by the single $\mathbb{Z}_2$ symmetry

$$\begin{array}{lcl} \Phi_1 & \to & -\Phi_1 \\ e_R & \to & -e_R \end{array} \tag{154}$$

Clearly in this case, each of the Yukawa matrices will be proportional to a separate mass matrix, and these will all get diagonalised in tandem.

- Type Y models

  Of course, the roles of the Φ_1 and the Φ_2 can always be flipped; this makes little or no difference in the Type 1 and Type 2 cases. However, the leptonic $\mathbb{Z}_2$ induces different behaviour, and hence we have a fourth model, which is called Type Y (or Type IV) or *flipped leptophilic*, with the discrete symmetry

$$\begin{aligned} \Phi_2 &\to -\Phi_2 \\ e_R &\to -e_R \end{aligned} \tag{155}$$

  and the Yukawa interactions have the form

$$\begin{aligned} \mathcal{S}^{(q)}_{\text{Yuk}} &= \int d^4x \sum_{a=1}^{2}\sum_{b=1}^{2}\Big[y^{(u)}_{ab}\, \overline{Q^{(a)}_L}(x)\, \widetilde{\Phi_1}(x)\, u^{(b)}_R(x) \\ &\quad + y^{(d)}_{ab}\, \overline{Q^{(a)}_L}(x)\, \Phi_1(x)\, d^{(b)}_R(x) + y^{(\ell)}_{ab}\, \overline{L^{(a)}_L}(x)\, \Phi_2(x)\, e^{(b)}_R(x) + \text{H.c.}\Big] \end{aligned} \tag{156}$$

As we have just seen, the fact that the two doublets transform differently under these discrete symmetries profoundly affects the Yukawa sector, but it hardly changes the scalar potential in Eqn. (127), except to require $\mu_3 = 0$, which is anyway set by the minimisation condition. Therefore, the Goldstone bosons and physical scalar states will remain the same in all these versions of the 2HDM. The actual phenomenology will, of course, be different, and there exists an extensive literature on the subject [133].

7.4 *MSSM Higgs Sector*

Supersymmetry, the much sought-after symmetry which mixes boson and fermion states, has been a mainstay of theories and experimental searches beyond the Standard Model, and it still commands the largest proportion of searches at the LHC. Though the literature on this is vast, a short summary of the phenomenological motivations of the *minimal* supersymmetric Standard Model (MSSM) and its so-called 'constrained' version (cMSSM), together with the theoretical and experimental constraints on it may be found in Ref. [134].

One of the basic features of the MSSM – and indeed of any supersymmetric model – is the fact that it always calls for two Higgs doublets. There are two paths to understanding this. One, which may be called the high road, notes that if we have a single Higgs doublet Φ, then we require its charge conjugate $\widetilde{\Phi}$ (which includes a complex conjugation) to give mass to both u and d types of quarks. In a supersymmetric model, these would have to be embedded in superfields $\hat{\Phi}$ and $\hat{\Phi}^\dagger$ and then incorporated into the superpotential to get the interaction Lagrangian. However, it is known that the superpotential will not give a Lagrangian invariant under supersymmetry unless it is *holomorphic* in the superfields, i.e. once cannot use the $\hat{\Phi}^\dagger$. Its place must be taken by a different superfield, with opposite hypercharge, and thus there will be two scalar doublets, just as in the Type II models described above. A more pedestrian argument, which may be called the low road, notes that every Higgs boson will have a fermionic partner, called the *higgsino*. Since the

Higgs boson couples to Z bosons, so will the higgsino, and being a fermion, we will get an anomaly proportional to the hypercharge Y. Since all the other anomalies cancel out (see Eqn. (86)), this will leave a residual anomaly. There is no help for it, then, other than to introduce another Higgs doublet, with opposite hypercharge $-Y$, so that these higgsino-induced anomalies mutually cancel out.

Moreover, the requirement that the action be invariant under supersymmetric transformations imposes stringent restrictions on the scalar potential of the theory. In particular, the quartic couplings get related to the gauge couplimgs and are no longer free parameters. The potential now has the form [129]

$$\begin{aligned} V(\Phi_1,\Phi_2) &= -\mu_1^2\Phi_1^\dagger\Phi_1 - \mu_2^2\Phi_2^\dagger\Phi_2 - \mu_3^2\left(\Phi_1\times\Phi_2 + \text{H.c.}\right) \\ &\quad +\frac{1}{8}(g^2+g'^2)\left(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right)^2 + \frac{1}{2}g^2\left(\Phi_1^\dagger\Phi_2\right)^2 \end{aligned} \tag{157}$$

with

$$\Phi_1 = \begin{pmatrix} \Phi_1^1 = \frac{1}{\sqrt{2}}(\eta_1^0 + ig_1^0 + v_1) \\ \Phi_1^2 = \varphi_1^- \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} \Phi_2^1 = \varphi_2^+ \\ \Phi_2^2 = \frac{1}{\sqrt{2}}(\eta_2^0 + ig_2^0 + v_2) \end{pmatrix} \tag{158}$$

and $\Phi_1\times\Phi_2 = \Phi_1^1\Phi_2^2 - \Phi_1^2\Phi_2^1$, which is a $SU(2)$-invariant construction. There are only three free parameters in this potential, viz., μ_1, μ_2 and μ_3. Imposition of the minimisation conditions leads to

$$\mu_3^2 = -\mu_1^2\frac{v_1}{v_2} = -\mu_2^2\frac{v_2}{v_1} \tag{159}$$

There is thus, just one free parameter in the potential apart from the ratio $\tan\beta = v_2/v_1$, i.e., just 2 free parameters in the MSSM Higgs sector. This makes it highly predictive and leads to a very important constraint on the mass of the lighter $\mathcal{CP}$-even scalar.

Expanding the potential and obtaining the mass matrices leads to the same mixing pattern as in the general 2HDM, i.e. Eqns. (131) and (132) where, it is now common to choose M_A and $\tan\beta$ as the free parameters. The remaining parameters are now given by [129]

$$\begin{aligned} M_+^2 &= M_A^2 + M_W^2 \\ M_h^2 &= \frac{1}{2}\left[M_A^2 + M_Z^2 - \sqrt{(M_A^2+M_Z^2)^2 - 4M_Z^2M_Z^2\cos^2 2\beta}\right] \\ M_H^2 &= \frac{1}{2}\left[M_A^2 + M_Z^2 + \sqrt{(M_A^2+M_Z^2)^2 - 4M_Z^2M_Z^2\cos^2 2\beta}\right] \\ \tan 2\alpha &= \tan 2\beta\,\frac{M_A^2+M_Z^2}{M_A^2-M_Z^2} \end{aligned} \tag{160}$$

Allowing M_A and $\tan\beta$ to range freely, this leads to some interesting inequalities, viz.,

$$M_+ \geq M_W \qquad M_h \leq M_Z \qquad M_H \geq M_Z \qquad M_A > M_h \tag{161}$$

In particular, the constraint $M_h < M_Z$ would immediately rule out the MSSM (since, after all, $M_h \simeq 125.4$ GeV), had it not been for the fact that radiative corrections drive M_h to higher values. However, there is a limit beyond which perturbative processes cannot drive it and hence, we have an upper limit around 135 GeV [135], and this can be extended by maximum 8-10 GeV even when further extensions of the MSSM are taken into account [136]. If, indeed, the Higgs boson had been found with a mass above this limit, e.g., at 150 GeV, then the MSSM and its more restrictive variants would have been definitively ruled out. As it happens, however, the mass of the discovered particle lies squarely within the permitted region. This is not, of course, a proof of the existence of supersymmetry (specifically the MSSM), but just a tantalising hint that it may exist. Not until one discovers one or more of the supersymmetric partners can anything definite be said on this account.

The MSSM story is not yet complete, however, for these Higgs scalars will have fermionic partners called the *higgsinos'*. When the electroweak symmetry breaks sponntaneously, these will mix with the fermionic partners of the gauge bosons – the *gauginos* – to produce physical states. Thus there will be mixing with the 'Wino' states $\widetilde{W}^{\pm}, \widetilde{H}^{\pm}$ to produce a pair of charged fermions $\widetilde{\chi}_1^{\pm}$ and $\widetilde{\chi}_2^{\pm}$ called the *charginos*. There will also be mixing of the 'photino' $\widetilde{\gamma}$ and 'Zino' $\widetilde{Z}$ states with the neutral Higgsinos $\widetilde{h}_1^0$ and $\widetilde{h}_2^0$ to produce four neutral fermions $\widetilde{\chi}_i^0$ $i = 1, 2, 3, 4$ called the *neutralinos*. These physical states are generally labelled in order of increasing mass, and the lightest neutralino $\widetilde{\chi}_1^0$ is generally the lightest supersymmetric particle (LSP) and, in a whole class of supersymmetric models, a prime candidate for dark matter. Since the parameters of the Higgs sector, especially $\tan\beta$, enter into the couplings of these new fermions with SM fields, clearly the phenomenological constraints arising from these new particles will affect the Higgs potential as well. This, one cannot treat the Higgs sector of the MSSM — or any supersymmetric model — independently, even as a first approximation. It is for this reason, that most modern analyses of supersymmetry involve making a global fit to a slew of physical measurables, and constraints on the Higgs sector arise as a by-product.

7.5 *NMSSM Higgs Sector*

Since singlet scalars can be added to the SM, they can also be added to the MSSM, since the ρ parameter remains the same. Thus, adding a complex singlet $\sigma(x) = s(x) + ia(x)$ in a way which preserves supersymmetry leads [137] to extra terms added to the potential in Eqn. (157)

$$\begin{aligned} V_\sigma &= \frac{1}{2}\mu_\sigma^2\sigma^2 + \lambda_1^2\sigma^*\sigma\left(\Phi^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2\right) \\ &\quad + \left(\lambda_1\Phi_1\times\Phi_2 - \nu_1\sigma^2\right)\left(\lambda_1\Phi_2\times\Phi_1 - \nu_1\sigma^{*2}\right) - \left(\lambda_2\sigma\Phi_1\times\Phi_2 + \nu_2\sigma^3 + \text{H.c.}\right] \end{aligned} \tag{162}$$

where we note that making σ real would violate supersymmetry. Once again, the physical states may be found by the usual process of minimisation of the potential, isolation of the mass term and diagonalisation of the mass matrices. These are omitted here in the interests of brevity, but may be found in all details in Ref. [137]. It suffices to say that there will be mixing of triplets of scalars

$$\left(h_1^0, h_2^0, s\right) \to \left(h^0, H^0, H'^0\right) \qquad \left(g_1^0, g_2^0, a\right) \to \left(G^0, A^0, A'^0\right) \tag{163}$$

with 3×3 mass matrices, while the mixing of charged scalars is the same as that in the MSSM. The mass spectrum of these states is as follows.

$$\begin{aligned}
M_+^2 &= M^2 + M_W^2 - \frac{1}{2}\lambda_1 v^2 \\
M_A^2 &\approx g\left(\frac{1}{2}\nu_1 v^2 \sin 2\beta + 3\nu_2 v_s\right) \\
M_{A'}^2 &\approx M^2\left(1 + \frac{v^2}{8\lambda_1 v_s^2}\sin^2 2\beta\right) \\
M_h^2 &\approx v_s\left(4\nu_1 v_s^2 - \nu_2\right) \\
M_H^2 &\approx M_Z^2 \\
M_{H'}^2 &\approx M^2\left(1 + \frac{v^2}{32\lambda_1 v_s^2}\sin^2 4\beta\right)
\end{aligned} \tag{164}$$

where $v_s \stackrel{\text{def}}{=} \langle\sigma\rangle$ and

$$M^2 = 2v_s \csc 2\beta\,(\lambda_1\nu_1 v_s + \lambda_2)\ . \tag{165}$$

Like the MSSM, this model must rely on radiative corrections to get a neutral $\mathcal{CP}$-even scalar up to 125.4 GeV. There are 6 free parameters, viz., μ_s, λ_1, λ_2, ν_1, ν_2 and $\tan\beta$, which can be traded for the 5 masses M_A, $M_{A'}$, M_H, $M_{H'}$ and $\tan\beta$.

The fermionic partner of the σ, i.e. the $\widetilde{\sigma}$, which is known as the *singlino*, will naturally not have any gauge couplings. Its only coupling will be to the Higgs sector through the mixing matrices and this can be arranged to be very feeble. Hence, this singlino is an excellent candidate for dark matter, even for regions of parameter space where the neutralino is ruled out as a dark matter candidate. All other phenomenology of the model will be rather similar to that of the MSSM, and once again, constraints on the parameter space from chargino and neutralino searches will impact the Higgs sector as well.

7.6 *Higgs Portal Models for Dark Matter*

Since the Higgs sector is the least known sector of the Standard Model, it offers a fertile ground for speculation that what has been seen, viz., a single, seemingly elementary, scalar, is merely the tip of the iceberg and that it may offer the first glimpse into a whole new world with new fields and new interactions. More specifically, we can create models of dark matter and dark energy where the dark fields do not couple to any of the sectors of the SM except the Higgs sector. These are known as *Higgs portal* models, for then it is only by studying the Higgs boson that we can get any empirical insights into the dark sector.

Since there is plenty of room for speculation about dark matter (less so for dark energy), an extensive literature has grown up around Higgs portal models. A proper discussion would merit a small review in its own right. Here we merely list a few of the more exciting ideas.

1. The idea of the Higgs sector as a portal to 'hidden' sectors was first proposed by Patt and Wilczek in 2006 [138]. In this article they mooted the idea of a 'hidden' copy of the Standard

Model with the gauge symmetry $SU(3)' \times SU(2)' \times Y(1)'$ and a full complement of SM-like fields.

2. The idea was quickly taken up by Bertolami and Rosenfeld [139], who speculated on the Higgs sector being coupled to a singlet scalar which is a quintessence field (solving the dark energy problem) and showing that the symmetry-breaking also generates a dark matter candidate.

3. In the wake of the 2012 Higgs boson discovery, it was pointed out by Djouadi, Falkowski, Mambrini and Quevillon [140] that the fairly stringent LHC limits on invisible decays of the observed H^0 would lead to severe constraints on Higgs portal models.

4. It was perhaps inevitable that speculation about a scalar dark matter candidate would be followed by speculation about a vector dark matter candidate. Higgs portal models with a dark vector field, with a Z_2 symmetry forbidding it to couple to all SM fields except the Higgs field where it can have seagull-type interactions were introduced by Lebedev, Lee and Mambrini [141] in 2012 and have been followed up by several studies of a similar nature [142].

5. In 2013, Weinberg [143] speculated that an extra complex scalar carrying a conserved $U(1)$ quantum number, which was spontaneously broken through its mixing with the SM Higgs scalar, could be feebly interacting and might 'masquerade' as a cosmic ray neutrino. It was soon realised (2014) that if the singlet can be thought of as dark matter rather than a fake neutrino, then this is also a suitable model for Higgs portal dark matter [144].

6. Taking into account the existence of neutrino masses, a right-chiral neutrino becomes a perfect candidate for Higgs portal dark matter, sicne it has no gauge interactions and couples to the Higgs sector only very feebly through its Yukawa couplings, which are proportional to the tiny neutrino mass [145].

This above list is illustrative but not exhaustive. It may be mentioned at this point that Higgs portal models belong to the genre of *simplified models* where a few extra fields are added to the SM. mostly to explain dark matter, without worrying about the other issues in the SM. Obviously, these theories have modest aims, and hence, they represent necessary but modest progress in the vast area of dark matter research.

7.7 *Higgs Triplets*

If there is an additional Higgs triplet, obviously it would cause deviations in the ρ parameter, which is highly constrained, and this would require fine-tuning of the vacuum expectation value of the triplet state [146]. An example of such a model is that due to Gelmini and Roncadelli [147] which considers an extra triplet with $T = 1$ and $Y = 2$ and a small vacuum expectation value v_t of the neutral component of the triplet. This leads to

$$\rho = \frac{2v^2 + 4v_t^2}{2v^2 + 8v_t^2} \simeq 1 - 2\left(\frac{v_t}{v}\right)^2 \tag{166}$$

and the existing bound in Eqn. (114) would require $v_t < 2.6$ GeV at 3σ. Accepting this fine tuning, one can build a model with a Higgs triplet Φ_t (also parametrisable as a bidoublet Δ_t which are given by

$$\Phi_t = \begin{pmatrix} \varphi_t^{++} \\ \varphi_t^{+} \\ \varphi_t^0 + \frac{v_t}{\sqrt{2}} \end{pmatrix} \qquad\qquad \Delta_t = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_t^{+} & \sqrt{2}\varphi_t^{++} \\ \sqrt{2}\varphi_t^0 + v_t & -\varphi_t^{+} \end{pmatrix} \tag{167}$$

The attractive feature of this model was that the low scale v_t could be used to generate small Majorana neutrino masses through the lepton number-violating operator

$$\mathcal{L}_{\rm Maj} = \sum_{a=1}^{3} \sum_{b=1}^{3} y_{ab}^{\rm Maj} (L_L^a)^c i\sigma_2 \Delta_t L_L^b \tag{168}$$

leading to a neutrino Majorana mass matrix $\mathbb{M}_a b^\nu = \sqrt{2} v_t y_{ab}^{\rm Maj}$. This can be incorporated in a seesaw mechanism yielding correct neutrino masses, without making the neutrino-scalar couplings intolerably small. This is essentially the same, in a specific context, as the so-called Majoron mechanism [148], and therefore it is common to call this the *triplet Majoron* model.

A more ingenious variant of this idea, due to Georgi and Machacek [152] is to introduce *two* extra triplets, one of them Φ_t with $Y = 2$, i.e., similar to the previous case, and one Φ_t' with $Y = 0$. The contributions to the ρ parameter then become

$$\rho = \frac{2v^2 + 4v_t^2 + 8{v'}_t^2}{2v^2 + 8v_t^2} \tag{169}$$

which can easily be made unity by choosing ${v'}_t^2 = \frac{1}{2} v_t^2$. This is a fine tuning, of course, but it can be induced by a global symmetry and obviously allows the value of v_t to range freely, unlike in the previous case. As a result, the Georgi-Machacek model, as it is called, has been the popular choice in which Higgs triplet studies have taken place.

However, the existence of a Higgs triplet permits a tree-level $H^\pm W^\mp Z$ vertex, which would have distinct signatures which so far have not been seen [149]. There has also been no signal for the double-charged scalar φ_t^{++}, leading to a fairly robust lower bond around 200 GeV [150]. Last, but not least, a triplet Majoron model would lead to an invisible decay mode for the Z boson, on which there are stringent constraints, essentially ruling out SM extensions by a single multiplet with $Y > 1$ [151], which includes the Gelmini-Roncadelli model, but is evaded when there are more triplets, as in the Georgi-Machacek model.

Apart from collider signatures, in an interesting twist to the triplet story, it has been recently proposed [153] that the discrepancy between the W mass measurement from CDF and from the LHC collaborations can be explained by introducing triplets à la Georgi-Machacek.

In the above discussion, we have discussed Higgs multiplets with $T = 1, 2$ and 3. The literature on extended Higgs sectors is rich in ideas and it would require a much longer work to do justice to more exotic ideas such as Higgs-radion mixing [154], composite Higgses [155] and S_3-based models [156].

8 Post-Modernism: Higgs Effective Theories

As the first two runs of the LHC have failed to turn up any signals for physics beyond the SM, the euphoria caused by the Higgs discovery a decade ago has died down, to be replaced by a more agnostic attitude. This has led to the rise of *effective theories* as a tool to understand the data from the LHC and a host of other sources, including astrophysical ones. The basic idea is very simple. If there is new physics at a higher scale, it will induce new operators of dimension more than four in the action, in the same way as Fermi's six-dimensional current-current operator was induced by the four-dimensional operators of the IVB theory in the low-energy limit.The Standard Model, then, is the low-energy effective theory of the unknown high scale theory if we add on a full set of these higher-dimensional operators. These effective operators will contain only the SM fields and will respect all the symmetries of the SM.

Just as the Fermi coupling was the free parameter of Fermi's theory, each higher-dimensional operator will have an unknown coupling constant — generically called a *Wilson coefficient* — and these are the 'free' parameters of the theory. Obviously, the same operator, or set of operators, can give rise to multiple observables, and thus it is economical to consider all these processes to obtain the values of the Wilson coefficients experimentally. This is in the same spirit as beta decay and muon decay could be compared to find the same Fermi constant and thereby discover charged current universality. This method is already the standard tool in flavour physics. It must be noted that the Wilson coefficients will have some dimension, just as the Fermi constant has dimension, and this will reflect the scale of the new physics, just as the Fermi constant reflects the scale of the electroweak symmetry-breaking. In fact, if we were to write the Fermi constant as

$$G_F \stackrel{\text{def}}{=} \frac{c_F}{v^2} \tag{170}$$

then $c_F = \frac{1}{\sqrt{2}}$ is the Wilson coefficient of the beta decay operator. It is, therefore usual to write the unknown couplings in terms of a common high scale Λ as $c_5/\Lambda, c_6/\Lambda^2, c_7/\Lambda^3, \ldots$ and use the label 'Wilson coefficients' for the c_i's specifically and the subscript i refers to the dimensionality of the operator.

Once we include higher-dimensional operators with couplings which are not dimensionless, the theory ceases to be renormalisable. This is not a problem, since we expect Λ to be a cutoff scale, above which the new theory becomes valid, and that will, presumably, be renormalisable. This is analogous to the fact that the Fermi theory is not renormalisable, but when we go to higher energies close to the electroweak scale, it is replaced by the Glashow-Salam-Weinberg theory, which is renormalisable. However, just as the Fermi coupling G_F runs with energy, so will the Wilson coefficients in an effective theory of the SM (SMEFT) run with energy. In doing so, they will also mix with other operators. Thus, we require to define clumps of operators which mix with each other, but not with others because of one conservation law or the other. These sets are then said to be *closed* under renormalisation group evolution. Early study in the context of flavour physics were carried out by Leung, Love and Rao [157] and by Bigi, Kopp and Zerwas (1986) [158], but a complete enumeration of all the SMEFT operators was done — a real *tour de force* — by Buchmüller and Wyler in 1986 [159]. Alas! there are 2499 operators in the list when all flavours are taken into account, each with an unknown Wilson coefficient [160]. All is not lost however, for it has been pointed out (2010) by a group from Warsaw University [161] that of the dimension-6 operators only a set of 59 will affect electroweak measurables, as opposed to the total number 80 in the Buchmüller-Wyler enumeration. This set of 59 operators has come to be known as the *Warsaw basis* and is a popular choice for SMEFT studies. However, there are many more basis choices, and it quite a task to translate between the different notations and conventions of these different choices [162].

Obviously many of the SMEFT operators will involve the Higgs doublet $\Phi(x)$. Isolating this set leads to a so-called Higgs EFT, or HEFT. In the following the HEFT outlined by Contino *et al* [163] is briefly described. In this framework, the HEFT action is divided into three parts, having the form

$$S_{\text{HEFT}} = S^{(1)}_{\text{HEFT}} + S^{(2)}_{\text{HEFT}} + S^{(3)}_{\text{HEFT}} \tag{171}$$

where each of them is written as an operator sum

$$S^{(X)}_{\text{HEFT}} \stackrel{\text{def}}{=} \int d^4x \; \frac{1}{\Lambda^2} \sum_{i=1}^{n_X} c_i^{(X)} \; \mathcal{O}_i^{(X)} \tag{172}$$

where $n_X = 12, 8, 8$ for $X = 1, 2, 3$ respectively, Λ is the scale of the new physics and the $c_i^{(X)}$ are the Wilson coefficients of the dimension-6 operators $\mathcal{O}_i^{(X)}$. The list of operators in $S^{(1)}_{\text{HEFT}}$, which matches with the strongly-interacting light Higgs (SILH) scenario of Ref. [164], is given below.

$$\begin{array}{lll} \mathcal{O}_1^{(1)} & = & \mathbb{D}^\mu(\Phi^\dagger\Phi)\mathbb{D}_\mu(\Phi^\dagger\Phi) \qquad\qquad \mathcal{O}_2^{(1)} = (\Phi^\dagger\overleftrightarrow{\mathbb{D}}^\mu\Phi)(\Phi^\dagger\overleftrightarrow{\mathbb{D}}_\mu\Phi) \\ \mathcal{O}_3^{(1)} & = & -(\Phi^\dagger\Phi)^3 \qquad\qquad \mathcal{O}_4^{(1)} = (\Phi^\dagger\Phi)\overline{Q}_L\widetilde{\Phi}u_R + \text{H.c.} \\ \mathcal{O}_5^{(1)} & = & (\Phi^\dagger\Phi)\overline{Q}_L\Phi d_R + \text{H.c.} \qquad\qquad \mathcal{O}_6^{(1)} = (\Phi^\dagger\Phi)\overline{L}_L\widetilde{\Phi}e_R + \text{H.c.} \\ \mathcal{O}_7^{(1)} & = & \Phi^\dagger(\mathbb{D}^\nu\mathbb{W}_{\mu\nu})\overleftrightarrow{\mathbb{D}}^\mu\Phi \qquad\qquad \mathcal{O}_8^{(1)} = (\Phi^\dagger\overleftrightarrow{\mathbb{D}}^\mu\Phi)\partial^\nu B_{\mu\nu} \\ \mathcal{O}_9^{(1)} & = & (\mathbb{D}^\mu\Phi)^\dagger\mathbb{W}_{\mu\nu}(\mathbb{D}^\nu\Phi)) \qquad\qquad \mathcal{O}_{10}^{(1)} = (\mathbb{D}^\mu\Phi)^\dagger(\mathbb{D}^\nu\Phi))B_{\mu\nu} \\ \mathcal{O}_{11}^{(1)} & = & (\Phi^\dagger\Phi)B_{\mu\nu}B^{\mu\nu} \qquad\qquad \mathcal{O}_{12}^{(1)} = (\Phi^\dagger\Phi)W^a_{\mu\nu}W^{a\mu\nu} \end{array} \tag{173}$$

where the covariant derivatives are defined in Eqn. (51). Similarly the operators for $S^{(2)}_{\text{HEFT}}$ are

$$\begin{array}{lll} \mathcal{O}_1^{(2)} & = & (\overline{Q_L}\gamma^\mu Q_L)(\Phi^\dagger\overleftrightarrow{\mathbb{D}}_\mu\Phi) \qquad\qquad \mathcal{O}_2^{(2)} = (\overline{Q_L}\gamma^\mu\vec{\mathbb{T}}Q_L)\cdot(\Phi^\dagger\vec{\mathbb{T}}\overleftrightarrow{\mathbb{D}}_\mu\Phi) \\ \mathcal{O}_3^{(2)} & = & (\overline{u_R}\gamma^\mu u_R)(\Phi^\dagger\overleftrightarrow{\mathbb{D}}_\mu\Phi) \qquad\qquad \mathcal{O}_4^{(2)} = (\overline{d_R}\gamma^\mu d_R)(\Phi^\dagger\overleftrightarrow{\mathbb{D}}_\mu\Phi) \\ \mathcal{O}_5^{(2)} & = & (\overline{u_R}\gamma^\mu d_R)(\Phi^\dagger\overleftrightarrow{\mathbb{D}}_\mu\Phi) + \text{H.c.} \qquad\qquad \mathcal{O}_6^{(2)} = (\overline{L_L}\gamma^\mu L_L)(\Phi^\dagger\overleftrightarrow{\mathbb{D}}_\mu\Phi) \\ \mathcal{O}_7^{(2)} & = & (\overline{L_L}\gamma^\mu\vec{\mathbb{T}}L_L)\cdot(\Phi^\dagger\vec{\mathbb{T}}\overleftrightarrow{\mathbb{D}}_\mu\Phi) \qquad\qquad \mathcal{O}_8^{(2)} = (\overline{e_R}\gamma^\mu e_R)(\Phi^\dagger\overleftrightarrow{\mathbb{D}}_\mu\Phi) \end{array} \tag{174}$$

and the operators for $S^{(3)}_{\text{HEFT}}$ are

$$\begin{array}{lll} \mathcal{O}_1^{(3)} & = & \overline{Q_L}\widetilde{\Phi}\sigma^{\mu\nu}u_R B_{\mu\nu} + \text{H.c.} \qquad\qquad \mathcal{O}_2^{(3)} = \overline{Q_L}\mathbb{W}_{\mu\nu}\widetilde{\Phi}\sigma^{\mu\nu}u_R + \text{H.c.} \\ \mathcal{O}_3^{(3)} & = & \overline{Q_L}\widetilde{\Phi}\sigma^{\mu\nu}\mathbb{G}_{\mu\nu}d_R + \text{H.c.} \qquad\qquad \mathcal{O}_4^{(3)} = \overline{Q_L}\Phi\sigma^{\mu\nu}d_R B_{\mu\nu} + \text{H.c.} \\ \mathcal{O}_5^{(3)} & = & \overline{Q_L}\mathbb{W}_{\mu\nu}\Phi\sigma^{\mu\nu}d_R + \text{H.c.} \qquad\qquad \mathcal{O}_6^{(3)} = \overline{Q_L}\sigma^{\mu\nu}\mathbb{G}_{\mu\nu}d_R + \text{H.c.} \\ \mathcal{O}_7^{(3)} & = & \overline{L_L}\Phi\sigma^{\mu\nu}e_R B_{\mu\nu} + \text{H.c.} \qquad\qquad \mathcal{O}_8^{(3)} = \overline{L_L}\mathbb{W}_{\mu\nu}\Phi\sigma^{\mu\nu}e_R + \text{H.c.} \end{array} \tag{175}$$

In the above formulae, we must take note of the iso-vector currents generated by the $\mathbb{T}$ as well as the fact that when the 3×3 matrix $\mathbb{G}_{\mu\nu}$ is introduced between quark fields, it is implicit that each quark field is a colour triplet. Moreover, these 28 operators have been written for a single flavour. If we assume that there is no flavour-violation from the new physics, we could simply write down three copies of these operators for the three fermion generations. Alternatively, if we allow there to be flavour-changing effects in the EFT, then we can write mixed flavour operators, e.g.,

$$\left[\mathcal{O}_4^{(1)}\right]_{ij} \stackrel{\text{def}}{=} (\Phi^\dagger \Phi)\overline{Q}_{Li}\widetilde{\Phi}u_{Rj} + \text{H.c.} \tag{176}$$

where $i, j = 1, 2, 3$ for the different generations. Thus, there will be a 3×3 matrix of Wilson coefficients $[c_4^{(1)}]_{ij}$, which will complement the Yukawa couplings in generating flavour violation. In any case, even if we do not make this assumption, the above operators are written in terms of the gauge bosons and fermions in the (unbroken) gauge basis and some mixing will be induced when the electroweak symmetry is spontaneously broken.

Calculations in the HEFT are long and messy, but fortunately we live in the age of computer algebra, and hence several software packages have been created to carry out these calculations. A summary of the available tools may be found in Ref. [165]. It would lead us too far afield in this article to discuss the details of EFT calculations and the different constraints obtainable on the Wilson coefficients. However, Higgs EFTs have become the framework of choice for the LHC Collaborations to present their (so far negative) results. A recent summary of these findings may be found in Ref. [166].

9 Concluding Remarks

The Higgs boson represents both an end and a beginning. It most definitely marks the end of the long road to building up a viable framework which explains and correctly predicts the value of different measurables in the area of elementary particle interactions at high energies, a framework which we call the Standard Model. However (and this has been explained at some length), that that is precisely what it is — a model and not a theory. There are too many *ad hoc* inclusions and experimentally-fitted parameters in the SM for it to really qualify as a theory. Nevertheless, that the Standard Model has been supremely successful in explaining a few thousand experimental results cannot be denied. Essentially, the SM is like a quack doctor whose motley treatments are suspect, but who has enormous success in curing people. After all (to flog a cliché), nothing succeeds like success.

For the last four decades, scientists have been beating at the stone wall of the SM, hoping to find a way to new physics, which is desirable for the many reasons explained in the text. In this wall, perhaps the weakest point is the Higgs sector, about which very little is really known, and hence provides a possible way to glimpse the new physics which lies at a higher scale. In this sense, the Higgs boson is also a beginning. Only the future will tell if this beginning is an auspicious one.

Before concluding this long story, it is worth noting the fact that the Higgs discovery required the linkage of elementary particle physics with several other branches of science. The idea of gauge theory itself came from general relativity and that of spontaneous symmetry-breaking came from condensed matter physics. The issues of dark matter and dark energy arose from astrophysical measurements. Among the tools of particle physicists, computers have always played a major role in the number crunching, but over the past couple of decades, they have begun to take over the burden of analytical calculations as well. This has permitted theorists to take up issues which would earlier have been totally intractable. Likewise, the enormous amounts of data which come pouring out of the LHC and similar machines, requires the handling of large data structures, and inevitably, the use of machine learning and artificial intelligence to interpret these masses of data. And yet, unless and until we find something which establishes the existence of physics beyond the SM in a terrestrial environment, much of all this effort and technical sophistication will be, to quote Lord Rutherford, 'mere stamp-collecting'. It may well be that a single small idea of a revolutionary nature is needed to break the impasse. One can only hope that this will not be a long time in coming.

The author acknowledges support of the Department of Atomic Energy, Government of India, under Project Identification No. RTI 4002.

References

[1] E. Nöther, *Nachr. d. König. Gesellsch. d. Wiss. zu Göttingen, Math-phys. Klasse* **918**, 235 (1918); for a modern re-appraisal, see N. Byers, arXiv: physics/9807044 [physics.hist-ph] (1998).

[2] A. Einstein, *Sitz. Preuss. Akad. Wiss. Berlin (Math. Phys. 1915*, 844 (1915).

[3] D. Hilbert, *Nachr. d. König. Gesellsch. d. Wiss. zu Göttingen, Math-phys. Klasse* **3**, 395 (1915).

[4] H. Weyl, *Sitz. Preuss. Akad. Wiss. Berlin (Math. Phys. 1918*, 465 (1918).

[5] W. Heisenberg, *Z. Phys.* **33**, 879 (1925); M. Born and P. Jordan, *ibid.*, **34**, 858 (1925); M. Born, W. Heisenberg and P. Jordan, *ibid.* **35**, 557 (1926).

[6] V. Fock, *Z. Phys.* **39**, 226; F. London, *ibid.* **42**, 375 (1927).

[7] See, for example, the insightful introduction to gauge field theories by D. Tong, available at https://www.damtp.cam.ac.uk/user/tong/gaugetheory.html

[8] P.A.M. Dirac, *Proc. Roy. Soc. Lond.* **A117**, 610 (1928).

[9] W. Pauli and V.F. Weisskopf, *Helv. Phys. Acta* **7**, 709 (1934).

[10] J. Chadwick, *Nature*, **129**, 312.

[11] W. Heisenberg, *Z. Phys.* **77**, 1 (1932).

[12] H. Yukawa, *Proc. Phys. Math. Soc. Jap.* **17**, 48 (1935).

[13] E. Rutherford, *Phil. Mag.* **47**, 109 (1899).

[14] H. Becquerel, *Nature*, **63**, 396 (1901).

[15] V. Ambarzumian and D. Iwanenko, *Compt. Rend. Acad Sci. Paris*, **190**, 582 (1930).

[16] W. Pauli, letter to L. Meitner at the Tübingen Conference on Radioactivity (1930), reproduced verbatim in *Phys. Today* **31N9**, 27 (1978).

[17] E. Fermi, *Nuovo Cim.* **11**, 1 (1934) and *Z. Phys.* **88**, 161 (1934).

[18] A. Pais, *Inward Bound*, (Oxford U. Press 1986) pp. 488.

[19] B.W. Lee, C. Quigg and H.B. Thacker, *Phys. Rev.* **D16** 1519 (1977).

[20] Fermi is quoted as stating this by T.D. Lee, *Int. J. Mod. Phys.* **A16**, 3633 (2001).

[21] P.A.M. Dirac (*Proc. Roy. Soc. Lond.*, **A114**, 243 (1927)).

[22] F.N.D. Kurie, J.R. Richardson and H.C. Paxton, *Phys. Rev* **49**, 368 (1936).

[23] D.C. Camp and L.M. Langer, *Phys. Rev.* **129**, 1782 (1963).

[24] KATRIN Collaboration, *Nature Phys.* **18**, 160 (2022) (2022).

[25] G. Gamow and E. Teller, *Phys. Rev.* **49**, 895 (1936) and *ibid.* **51**, 289 (1937).

[26] O. Klein, *Nature* **161**, 897 (1948); G. Puppi, *Nuov. Cim.* **6**, 194 (1949); J. Tiomno and J.A. Wheeler, *Rev. Mod. Phys.* **21**, 153 (1949).

[27] See, for example, F. Halzen and A.D. Martin, it Quarks and Leptons (John Wiley & Sons, 1984), pp. 125.

[28] T.D. Lee, M. Rosenbluth and C.N. Yang, *Phys. Rev.* **75**, 905 (1949).

[29] R.P. Feynman and M. Gell-Mann, *Phys. Rev.* **109**, 193 (1958); E.C.G. Sudarshan and R. Marshak, *ibid.* **109**, 1860 (1958).

[30] J. Schwinger, *Ann. Phys.* **2**, 407 (1957).

[31] C.N. Yang and R. Mills, *Phys. Rev.* **96**, 191 (1954); R. Shaw, Ph.D. thesis, (U. of Cambridge, 1956), pp. 34.

[32] S.L. Glashow, Ph.D. thesis (Harvard U., 1959).

[33] A. Salam and J.C. Ward, *Nuovo Cim.* **11**, 568 (1959).

[34] S.L. Glashow, *Nucl. Phys.* **22**, 579 (1961).

[35] V.L. Ginzburg and L.D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950).

[36] J. Bardeen, L.N. Cooper and J.R. Schrieffer, *Phys. Rev* **106**, 162 (1957).

[37] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961) and *ibid.* **124**, 246 (1961).

[38] M. Gell-Mann and M. Levy, *Nuovo Cim.* **16**, 705 (1960).

[39] J. Goldstone, A. Salam and S. Weinberg, *Phys. Rev.* **127**, 965 (1962).

[40] J. Goldstone, *Nuovo Cim.* **19**, 154 (1961).

[41] For a very readable account, see F. Close, *The Particle Explosion* (Oxford University Press, 1994). The same author has written a textbook on this topic, entitled *An Introduction to Quarks and Partons* (Academic Presss, 1979).

[42] M. Gell-Mann, M., *Phys. Lett.* **8**, 214 (1964).

[43] G. Zweig, CERN Report No.8182/TH.401 (1964).

[44] R.P. Feynman, in Proceedings of *High Energy Collisions: Third International Conference at Stony Brook, N.Y.*, (Gordon & Breach. 1969), p. 237; J. Bjorken and E. Paschos, *Phys. Rev.*, **185**, 1975 (1968); M. Breidenbach *et al*, *Phys. Rev. Lett.* **23**, 935 (1969).

[45] O.W. Greenberg, *Phys. Rev. Lett.* **13**, 598 (1964); M.-Y. Han and Y. Nambu, *Phys. Rev.* **139**, B1006 (1965); H. Fritzsch, M. Gell-Mann and H. Leutwyler, *Phys. Lett.* **47B**, 365 (1973).

[46] P.W. Anderson, *Phys. Rev.* **130**, 439 (1963).

[47] J.J. Sakurai, *Annals Phys.* **11**, 1 (1960).

[48] S. Weinberg, *CERN Courier*, **48**, 17 (2008).

[49] F. Englert and R. Brout, *Phys. Rev. Lett.* **13**, 321 (1964).

[50] P.W. Higgs, *Phys. Lett.* **12**, 132 (1964) and *Phys. Rev. Lett.* **13**, 508 (1964).

[51] G. Guralnik, C.R. Hagen and T.W.B. Kibble, *Phys. Rev. Lett.* **13**, 585 (1964).

[52] For the history of how the Higgs boson came to be named after only one of its progenitors, see I. Sample, *Nature*, **466**, 689 (2010).

[53] T.W.B. Kibble, *Phys. Rev.* **155**, 1554 (1967).

[54] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967).

[55] A. Salam and J.C. Ward, *Phys. Lett.* **13**, 168 (1964).

[56] T.D. Lee and C.-N. Yang, *Phys. Rev.* **104**, 254 (1956).

[57] C.S. Wu *et al*, *Phys. Rev.* **105**, 1413 (1957).

[58] M. Goldhaber, L. Grodzins and A.W. Sunyar, *Phys. Rev.* **109**, 1015 (1958).

[59] M. Veltman, quoted in B.W. Lee, Fermilab preprint FERMILAB-CONF-72-034-T (1971).

[60] D.J. Gross and R. Jackiw, *Phys. Rev.* **D6**, 477 (1972); C. Bouchiat, J. Iliopoulos and P. Meyer, *Phys. Lett.* **B38**, 519 (1972).

[61] M. Breidenbach *et al*, *Phys. Rev. Lett.* **23**, 935 (1969),

[62] J. Schechter and Y. Ueda, *Phys. Rev.* **D2**, 736.

[63] M.Y. Han and Y. Nambu, *Phys. Rev.* **139B**, 1006 (1965); N. Cabibbo, L. Maiani and G. Preparata, *Phys. Rev. Lett.* **B25**, 132 (1970); B. Stech, *Nucl. Phys.* **B64**, 194 (1973); J. Prentki and B. Zumino, *Nucl. Phys.* **B47**, 99 (1972); B.W. Lee, *Phys. Rev.* **D6**, 1188 (1972).

[64] S.L. Glashow, J. Iliopoulos and L. Maiani, *Phys. Rev.* **D2**, 1285 (1970).

.

[65] J.E. Augustin *et al*, *Phys. Rev. Lett.* **33**, 1406 (1974); J.J. Aubert *et al*, *Phys. Rev. Lett.* **33**, 1404 (1974).

[66] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963).

[67] See, for example, T.-P. Chenga and L. F. Li, *Gauge Theory of Elementary Particle Physics* World Scientific, 1984), pp. 357-359.

[68] L. Landau, *Nucl. Phys.* **3**, 127 (1957).

[69] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, *Phys. Rev. Lett.* **13**, 138 (1964).

[70] A.D. Sakharov, *JETP Lett.* **5**, 24 (1967).

[71] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys. 49*, 652 (1973).

[72] M.L. Perl *et al*, *Phys. Rev. Lett.* **35**, 1489 (1975).

[73] S.W. Herb *et al*, *Phys. Rev. Lett.* **39**, 252 (1977).

[74] F. Abe *al* (CDF Collaboration), *Phys. Rev. Lett.* **74**, 2626 (1995);
S. Abachi *et al.* (DØ Collaboration), *Phys. Rev. Lett.* **74**, 2632 (1995).

[75] K. Kodama *et al*, *Phys. Lett.*, **504**, 218 (2001).

[76] A. Pais and S.B. Treiman, *Phys. Rev. Lett.* **35**, 1556 (1975).

[77] L. Lederman and D. Teresi, *The God Particle: if the Universe is the Answer, what is the Question?* (Dell Publishing, 1993).

[78] ATLAS Collaboration, G. Aad *et al*, *Phys. Lett.* **B716**, 1 (2012); CMS Collaboration, S. Chatrchyan *et al*, *Phys. Lett.* **B716**, 30 (2012).

[79] See, for example,
L.R.F. Castillo, *The Search and Discovery of the Higgs Boson* (Morgan & Claypool Publishers, 2015); A. Nisati and V. Sharma (Ed.), *Discovery of the Higgs Boson* (World Scientific, 2017); R. Wolf, *The Higgs Boson Discovery at the Large Hadron Collider* (Springer, 2015.

[80] J.C. Pati and A. Salam, *Phys. Rev.* **D10**, 275 (1974).

[81] H. Georgi and S.L. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974).

[82] M.S. Chanowitz, J. Ellis and M.K. Gaillard, *Nucl. Phys.* **B128**, 506 (1977).

[83] M.R. Krishnaswamy *et al.*, *AIP Conference Proceedings* **96**, 168 (1983).

[84] N. Aghanim *et al.* [Planck Collaboration), *A&A* **641**, A6 (2020).

[85] F. Di Lodovico (Hyper-Kamiokande Collaboration), *J. Phys.: Conf. Ser.* **888**, 012020 (2017).

[86] U. Amaldi, W. de Boer and H. Furstenau, *Phys. Lett.* **B260**, 447 (1991); P. Langacker and M.-X. Luo, *Phys. Rev.* **D44**, 817-822 (1991); A. Zichichi, *NATO Sci. Ser.* **C427**, 301 (1994).

[87] The best introduction to the subject is probably still the classic text by G. Ross, *Grand Unified Theories* Ed. 2 (Avalon Publishing, 2003). For a compact and updated review, see D. Croon, *Front. in Phys.* **7**, 76 (2019).

[88] G. 't Hooft and M.J.G. Veltman, *Nucl. Phys.* **B153**, 365 (1979).

[89] K.G. Wilson, *Phys. Rev.* **D3**, 1818 (1971); E. Gildener, *ibid.* **14**, 1667 (1976); For a review of recent developments, see A. Echhorn *et al*, *JHEP*, **64**, 022 (2015).

[90] D. Buttazzo *et al*, *JHEP* **12**, 089 (2013).

[91] C. Callan and S. Coleman, *Phys. Rev.* **D16**, 1762 (1977).

[92] A. Kobakhidze and A. Spencer-Smith, *Phys. Lett.* **B722**, 130 (2013).

[93] J.R. Espinosa, *et al*, *JHEP* **09**, 174 (2015).

[94] C. Gross, O. Lebedev and M. Zatta, *Phys. Lett.* **B753**, 178 (2016).

[95] S. Weinberg, *Phys. Rev.* **D13**, 974 (1976).

[96] L. Susskind, *Phys. Rev.* **D20**, 2619 (1979).

[97] M.E. Peskin and T. Takeuchi, *Phys. Rev. Lett.* **65**, 964 (1990).

[98] C.D. Carone, *Phys. Rev.* **D86**, 055011 (2012); A. Belyaev *et al*, *Phys. Rev.* **D99**, 095006 (2019) and *ibid.* **D101**, 086013 (2020).

[99] G. Nordström, *Phys. Z.* **15**, 504 (1914);
Th. Kaluza (1921), reprinted in *Int. J. Mod. Phys.* **D27**, 1870001 (2018); O. Klein, *Nature* **118**, 516 (1926) and *Z. Phys.* **37**, 895 (1926).

[100] P. Ramond, *Phys. Rev.* **D3**, 2415 (1971); A. Neveu and J. Schwartz, *Nucl. Phys.* **B31**, 86 (1971); T. Yoneya, *Prog. Theor. Phys.* **51**, 1907 (1974); J. Scherk and J. Schwarz, *Nucl. Phys.* **B81**, 118 (1974).

[101] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B429**, 263 (1998) and *Phys. Rev.* **D59**, 086004 (1999).

[102] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B436**, 257 (1998).

[103] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).

[104] P. Hořava, *Phys. Lett.* **B231**, 251 (1989).

[105] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Phys. Rev. Lett.* **84**, 2080 (2000).

[106] W.D. Goldberger and M.B. Wise, *Phys. Rev. Lett.* **83**, 4922 (1999) and *Phys. Lett.* **B475**, 275 (2000).

[107] T. Appelquist, H.-C. Cheng and B. Dobrescu, *Phys. Rev.* **D64**, 035002 (2001).

[108] P. Ramond, *Phys. Rev.* **D3**, 2415 (1971); A. Neveu and J.H. Schwarz, *Phys. Lett.* **B34**, 517 (1971); C.B. Thorn, *Phys. Rev.* **D4**, 1112 (1971).

[109] J. Wess and B. Zumino, *Nucl. Phys.* **B70**, 1 (1974) and *ibid.* **B70**, 39 (1974).

[110] P. Fayet, *Phys. Lett.* **B64**, 159 (1976).

[111] S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193**, 150 (1981).

[112] F. Zwicky, *Helv. Phys. Act.* **6**, 110 (1933).

[113] V.C. Rubin and W. Kent Fird Jr., *Astrophys. J.* **159**, 379 (1970).

[114] C.J. Copi, D.M. Schramm and M.S. Turner, *Science* **267**, 192 (1995).

[115] M. Markevitch, *et al.*, *Astrophys. J.* **606**, 819 (2004).

[116] A. Einstein, *Sitz. Preuss. Akad. Wiss. Berlin (Math. Phys.)*, **1917**, 142 (1917).

[117] S. Weinberg, *Gravitation and Cosmology* (John Wiley & Sons,1972).

[118] See, for example, S. Perlmutter and B.P. Schmidt, *Lect. Notes Phys.* **598**, 195 (2003).

[119] R.R. Caldwell and P.J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998).

[120] R. Brada and M. Milgrom, *Astrophys. J. Lett.* **512**, L17 (1999).

[121] R.L. Workman *et al.* (Particle Data Group) (Chapter 10), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).

[122] A. Kundu and S. Raychaudhuri, *Phys. Rev.* **D53**, 4042 (1996).

[123] S. Dawson, P.P. Giardino and S. Homiller, *Phys. Rev.* **D103**, 075016 (2021).

[124] T. Binoth and J.J. van der Bij, *Z. Phys.* **C75**, 17 (1997).

[125] B. Kors and P. Nath, *Phys. Lett.* **B586**, 366 (2004) and *JHEP* **07**, 069 (2005); R. Vinze and S. Raychaudhuri, arXiv: 2107.08840 [hep-ph] (2021).

[126] T. Robens, T. Stefaniak and J. Wittbrodt, *Eur. Phys. J.* **C80**, 151 (2020).

[127] R. Coimbra, M.O.P. Sampaio and R. Santos, *Eur. Phys. J.* **C73**, 2428 (2013); R. Costa, M. Mühlleitner, M.O.P. Sampaio and R. Santos, *JHEP* **06**, 034 (2016).

[128] S.L. Glashow and S. Weinberg, *Phys. Rev.* **D15**, 1958 (1977); E. Paschos, *Phys. Rev.* **D15**, 1966 (1977).

[129] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs Hunter's Guide* (CRC Press, 1990), Ch.4.

[130] A. Pich and P. Tuzon, *Phys. Rev.* **D80**, 091702 (2009).

[131] F.J. Botella, F. Cornet-Gomez and M. Nebot, *Phys. Rev.* **D98**, 035046 (2018).

[132] L.L. Honorez, E. Nezri, J.F. Oliver and M.H.G. Tytgat *JCAP* **02**, 028 (2007); L.L. Honorez and C.E. Vaguna, *JHEP* **09**, 046 (2010); For a more recent analysis, see, for example, G. Belangér, A. Mjallal and A. Pukhov, *Phys. Rev.* **D105**, 035018 (2022).

[133] See for example, A. Djouadi, *Phys. Rept.* **459**, 1 (2008) and references therein; For a more recent update, see F. Kling, S. Su and W. Su, *JHEP* **06**, 163 (2020).

[134] D. Ghosh, M. Guchait, S. Raychaudhuri and D. Sengupta, *Phys. Rev.* **D86**, 055007 (2012).

[135] Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys. Lett.* **85**, 1 (1991) and *Phys. Lett.* **B262**, 54 (1991); J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B257**, 83 (1991) and *Phys. Lett.* **B262**, 477 (1991); R. Barbieri, M. Frigeni and M. Caravaglios, *Phys. Lett.* **B258** 167 (1991); R. Barbieri and F. Frigeni, *Phys. Lett.* **B258**, 395 (1991); H.E. Haber and R. Hempfling, *Phys. Rev. Lett.* **66**, 1815 (1991).

[136] See, for example, P.N. Pandita, *Phys. Lett.* **B318**, 338 (1993) and *Z. Phys.* **C59**, 575 (1993); B. Ananthanarayan and P.N. Pandita, *Int. J. Mod. Phys.* **A12**, 2321 (1997).

[137] U. Ellwanger and C. Hugonie, Rutherford Lab preprint RAL-TR-2000-027, arXiv:hep-ph/0006222; D. Miller, contribution to *SUSY02*, p. 694 (2002); For a recent update, see A. Adhikary *et al*, arXiv: 2207.00600 [hep-ph].

[138] B. Patt and F. Wilczek, MIT preprint MIT-CTP-3745 (2006), arXiv:hep-ph/0605188 [hep-ph].

[139] O. Bertolami and R. Rosenfeld, *Int. J. Mod. Phys.* **A23**, 4817 (2008).

[140] A. Djouadi, A. Falkowski, Y. Mambrini, J. Quevillon, *Eur. Phys. J.* **C73**, 2455 (2013).

[141] O. Lebedev, H.M. Lee and Y. Mambrini, *Phys. Lett.* **B707**, 570 (2012).

[142] M. Duch, B. Grzadkowski and M. McGarrie, *JHEP* **09**, 162 (2015); C.-H. Chen and T. Nomura, *Phys. Rev.* **D93**, 074019 (2016); Y. Cui, and F. D'Eramo, *Phys. Rev.* **D96**, 095006 (2017); J. Heisig, M. Kramer, E. Madge and A. Mück, *JHEP* **03**, 183 (2020); G. Arcadi, A. Djouadi and M. Kado, *Phys. Lett.* **B805**, 135427 (2020);

[143] S. Weinberg, *Phys. Rev. Lett.* **110**, 241301 (2013).

[144] L.A. Anchordoqui *et al*, *Phys. Rev.* **D89**, 083513 (2014).

[145] N. Haba, H. Ishida and R. Takahasi, *PTEP* **2015**, 053B01 (2015).

[146] U. Amaldi, *et al*, *Phys. Rev.* **D36**, 1385 (1987).

[147] G.B. Gelmini and M. Roncadelli, *Phys. Lett.* **B99**, 411 (1981).

[148] Y. Chikashige, R.N. Mohapatra and R.D. Peccei, *Phys. Lett.* **B98**, 265 (1981).

[149] See, for example, E. Asakawa and S. Kanemura, *Phys. Lett.* **B626**, 111 (2005); For a more recent update, see J.-Y. Cen *et al*, *JHEP* **01**, 148 (2019).

[150] ATLAS Collaboration, M. Aaboud *et al*, *Eur. Phys. J.* **C79**, 58 (2019); CMS Collaboration, A.M. Sirunyan *et al*, *Eur. Phys. J.* **C81**, 723 (2021).

[151] M.C. Gonzalez-Garcia and Y. Nir, *Phys. Lett.* **B232**, 83 (1989).

[152] H. Georgi and M. Machacek, *Nucl. Phys.* **B262**, 463 (1985); M.B. Chanowitz and M. Golden, *Phys. Lett.* **B165**, 105 (1985).

[153] X.-K. Du, Z. Li, F. Wang and Y. K. Zhang, arXiv:2204.05760 [hep-ph]; P. Mondal, *Phys. Lett.* **B833**, 137357 (2022); T.-K. Chen, C.-W. Chiang and K. Yagyu, *Phys. Rev.* **D106**, 055035 (2022).

[154] For a detailed study, see A. Chakraborty, U. Maitra, S. Raychaudhuri and T. Samui, *Nucl. Phys.* **B922**, 41 (2017).

[155] For a typical analysis, see L. Da Rold and A.N. Rossia, *JHEP* **12**, 023 (2019).

[156] See, for example, G. Bhattacharyya *et al*, *Phys. Rev.* **D106**, 055032 (2022).

[157] C.N. Leung, S.T. Love and S. Rao, *Z. Phys.* **C31**, 433 (1986).

[158] I.I.Y. Bigi, G. Kopp and P.M.Zerwas, *Phys. Lett.* **B166**, 238 (1986).

[159] W. Buchmüller and D. Wyler, *Nucl. Phys.* **B268**, 621 (1986).

[160] R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, *JHEP* **04**, 159 (2014).

[161] B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek, *JHEP* **10**, 085 (2010). See also, J.A. Aguilar-Saavedra, *Nucl. Phys.* **B843**, 638 (2011), Erratum: *ibid.* **B851**, 443 (2011).

[162] J. Aebischer, *et al*, *Comput. Phys. Commun.* **232**, 71 (2018).

[163] R. Contino *et al*, *JHEP* **07**, 035 (2013).

[164] G.F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, *JHEP* **0706**, 045 (2007).

[165] J. Aebischer, *et al*, Proc. of *SMEFT-Tools 2019*, arXiv: 1910.11003 [hep-ph].

[166] N. Belyaev, for ATLAS and CMS Collaborations, in Proc. of *LHCP 2020*, p. 137,, doi: 10.22323/1.382.0137 (2021).