

# Product Groups, Discrete Symmetries, and Grand Unification

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## Abstract

We study grand unified theories based on an  $SU(5) \times SU(5)$  gauge group in which the GUT scale,  $M_{\text{GUT}}$ , is the VEV of an exact or approximate modulus, and in which fast proton decay is avoided through a combination of a large triplet mass and small triplet couplings. These features are achieved by discrete symmetries. In many of our models,  $M_{\text{GUT}}$  is generated naturally by the balance of higher dimension terms that lift the GUT modulus potential, and soft supersymmetry breaking masses. The theories often lead to interesting patterns of quark and lepton masses. We also discuss some distinctions between grand unified theories and string unification.

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## I. INTRODUCTION

Lacking direct evidence, the unification of couplings is one of the few experimental hints both of supersymmetry and of unification of the gauge interactions of the standard model. The separation of the unification scale from the Planck scale has long suggested that the methods of effective field theory might be appropriate in understanding unification. Probably the most troubling puzzle of such grand unified field theories (GUTs) is why Higgs doublets should be light, while their colored partners are massive.

String theory offers an interesting perspective on these issues. In string theories, one often has unification of couplings, even though there is typically no scale at which one can speak of a four dimensional unifying gauge group. The unification scale, instead, is to be identified with the radius of some internal space, or with the string scale. In most cases, this scale is the threshold, not for a finite number of states, but for an infinite number of states. One of the appealing features of string models, stressed even in textbooks [1], is that they can readily produce light doublets without light triplets.

String theory also sheds light on some of the traditional questions of model building. In particular, it was long suspected that one should not expect continuous global symmetries in a quantum theory of gravity, and indeed there are no such symmetries in string theory [2]. String theory does frequently yield rather intricate patterns of discrete symmetries.

This discussion suggests that we make a distinction between *Grand Unification*, i.e. theories where, for some range of energy scales, there are a finite number of fields and the gauge groups of the standard model are unified in a larger group, and *String Unification*, where the couplings are unified, but there is no such range of energies. There are a number of reasons to pursue Grand Unification, even if one imagines that the underlying theory is string theory. Perhaps the most important of these is coupling unification itself. Coupling unification is a robust property of the weakly coupled heterotic string. It is also true of theories in which some larger gauge symmetry is broken by Wilson lines (e.g. strongly coupled heterotic string,  $G_2$  compactifications, etc.). But it does not hold more generally, and it is thus not clear in what sense it is a *prediction* of string theory. Certainly one could imagine that the explanation of coupling unification, even in string theory, is Grand Unification. Second, even though the appearance of massless Higgs doublets is an impressive feature of string theory, the absence of proton decay suggests that discrete symmetries may play an important role in the structure of the theory and, as Witten has recently stressed [3], such discrete symmetries might provide an alternative explanation of the presence of massless doublets.

So even from the perspective of string theory, it is interesting to explore conventional grand unification, with particular emphasis on the role of discrete symmetries. From this perspective, however, there is a second puzzle: the origin of  $M_{\text{GUT}}$ . In field theory models of unification, this scale often appears as an explicit input parameter. One might expect that in a string theory construction, the Higgs fields required to break the grand unified group would be massless in some approximation. In order that they obtain large expectation values, they would need flat or nearly flat potentials. This need not necessarily be the case – after all, a parameter of only  $10^{-2}$  is required in order to obtain this scale from the Planck scale. Still, the existence of a flat or nearly flat direction gives, as we will see, an appealing picture, allowing for a natural mechanism of generating the GUT scale in the effective field theory. This possibility will be the focus of much of this paper.

We consider models in which  $M_{\text{GUT}}$  is given by some combination of the Planck scale

and the supersymmetry breaking scale [4]. The GUT group,  $G_{\text{GUT}}$ , in our models, is broken by the expectation values of some fields, which correspond to an exact or approximate flat direction in the limit of unbroken supersymmetry. The location of the minimum, and the value of  $M_{\text{GUT}}$ , are ultimately determined by supersymmetry breaking effects.

It is easy to generate such an approximately flat direction by the use of discrete symmetries. With symmetries such as a  $Z_{10}$ , the potential along this direction arises from non-renormalizable terms suppressed by a high power of the Planck scale, such that, when balanced against a supersymmetry-breaking soft mass for the GUT breaking field, the correct value of  $M_{\text{GUT}}$  is obtained.

Alternatively,  $M_{\text{GUT}}$  may be an exact modulus of the theory. This can easily be achieved with a continuous global symmetry [5], but such global symmetries are unlikely to arise in a theory of gravity. It is possible to obtain exact flat directions with discrete symmetries, but typically one finds additional light fields which spoil the predictions of unification. We explain these difficulties, and construct some examples of exact flat directions which are almost, but not entirely, satisfactory.

The discrete symmetries of our models also play a role in addressing the first puzzle mentioned above, namely, the presence of colored partners of the Higgs doublets. In a conventional grand unified model, because of the GUT symmetry, the Yukawa couplings of these partners are related by the gauge symmetry to standard-model Yukawa couplings of quarks and leptons, giving rise to fast proton decay. In the models we study, following ideas of [3, 6, 7], a discrete symmetry distinguishes the triplets from the doublets. This symmetry not only allows a triplet mass while forbidding a doublet mass, but, as stressed in [3], may also suppress the triplet couplings to matter fields relative to the usual Yukawa couplings. Thus, proton decay is avoided by splitting not just the *masses* but also the *couplings* of the doublets and triplets.

It is important to note that, as explained recently in [8], gauge coupling unification in minimal  $SU(5)$  implies an allowed *window* for the triplet masses, which is somewhat below  $M_{\text{GUT}}$ . Proton decay, on the other hand, implies a lower bound, which lies above this window. The idea of Yukawa splitting between triplets and doublets may avoid this problem in more general models. (For another mechanism, see ref. [9].)

As articulated by Witten, a symmetry that distinguishes doublets from triplets requires a semi-simple group, and indeed our models have  $SU(5) \times SU(5)$  as  $G_{\text{GUT}}$ . Still, the standard model gauge group lies in the diagonal  $SU(5)$ , so that the main achievements of grand unification are maintained: hypercharge is quantized, and all standard model couplings originate from a single coupling, that of the diagonal  $SU(5)$ . No large couplings are therefore required in order to ensure unification. In addition, the models require relatively few fields to achieve the initial stage of symmetry breaking, so that they remain weakly coupled up to the Planck scale.

We review the mechanism of doublet-triplet splitting by discrete symmetries of [3, 6, 7] in Section II and describe the basic  $SU(5) \times SU(5)$  theory and flat direction we consider. We explain how  $M_{\text{GUT}}$  may arise naturally along an approximate flat direction in Section III. Some explicit models that realize these two ideas are exhibited in Section IV. We argue that these are the simplest models which satisfy all of our requirements: along this direction, all these fields get  $M_{\text{GUT}}$  masses, and a discrete symmetry, which forbids a doublet mass, remains unbroken. The models contain two pairs of bifundamentals, three adjoints and a gauge singlet. As we show in Appendix A, within such models a continuous symmetry is required to obtain an exact flat direction.

In Section V, we show that an exact flat direction, which is nearly satisfactory, can be obtained by considering a different pattern of expectation values. However, in these models, not all fields with standard model quantum numbers gain mass. In fact, a full  $SU(5)$  adjoint representation remains light, so that  $SU(3)$  becomes strongly coupled near  $M_{\text{GUT}}$ .

In the theories we consider, the standard-model Higgs doublets may originate from a  $5$  and a  $\bar{5}$  either of the same  $SU(5)$  factor, or of different  $SU(5)$  factors. To cancel  $SU(5)$  anomalies, one can then arrange the standard model matter fields in different representations of  $SU(5) \times SU(5)$ , or add an extra  $5 + \bar{5}$  pair. We consider these different possibilities in Section VI. It should be emphasized that the discrete symmetry must be broken at some stage in order to allow a  $\mu$  term for the standard model Higgs doublets, and we discuss some possibilities for accomplishing this. In particular, we show that a Higgs doublet mass may be generated automatically once the discrete symmetry is broken, without having to add any new Higgs couplings.

Since, in this framework, the standard model matter and Higgs fields may originate from representations of the different  $SU(5)$ 's, some of their Yukawa couplings arise from non-renormalizable terms. This situation introduces a small parameter of order  $M_{\text{GUT}}/M_{\text{Pl}} \sim 10^{-2}$  into the Yukawa matrices. As we show in Section VII (with further details given in Appendix C), the resulting quark and lepton mass matrices are viable. Furthermore, some of the mass hierarchies are automatically explained by the gauge quantum numbers. When the standard Yukawa couplings of the Higgs doublets arise from non-renormalizable terms, the Yukawa couplings of their colored partners can be highly suppressed. This *Yukawa-splitting* is also explained in Section VII.

The framework described above, of an effective theory in which  $M_{\text{GUT}}$  corresponds to an exact or approximate modulus, as we have argued, seems a plausible outcome of string theory. String theory typically leaves many light, undetermined moduli, and possesses discrete symmetries. We discuss some distinctions between field theory unification and string unification in Section VIII.

## II. SPLITTING TRIPLETS FROM DOUBLETS

In this section and the next, we describe the key elements of our models. As explained in the introduction, we wish to distinguish between Higgs doublets and triplets by a discrete symmetry. (Here and in the following, we use ‘‘Higgs’’ to denote the standard model doublets and their triplet partners, as opposed to GUT breaking fields.) As discussed most clearly by Witten [3], if one wants to explain the masslessness of Higgs doublets through discrete symmetries, there should be an unbroken discrete symmetry (at least at a high energy scale) which acts differently on doublets and triplets, and is not a subgroup of hypercharge. This is not possible with a single  $SU(5)$ ,  $SO(10)$  or  $E_6$ , but it is not difficult to achieve with the group  $SU(5) \times SU(5)$ , and other direct product groups, provided one has fields transforming as bifundamentals. In this paper, we study the simplest such case, with an  $SU(5) \times SU(5)$  gauge group. Here, one wants a symmetry which is a linear combination of a discrete symmetry acting on the bifundamentals, and a gauge transformation acting in one of the  $SU(5)$ 's. The latter transformation can be taken to lie in the first  $SU(5)$ , without loss of

generality (by combining with a hypercharge transformation),

$$g_1 = \begin{pmatrix} \alpha^{-1} & & & & \\ & \alpha^{-1} & & & \\ & & \alpha^{-1} & & \\ & & & \alpha^{\frac{N+3}{2}} & \\ & & & & \alpha^{\frac{N+3}{2}} \end{pmatrix}. \quad (1)$$

Here  $\alpha$  is an  $N$ 'th root of unity, and for simplicity we have taken  $N$  odd (for  $N$  even it is a simple matter to modify our formulas).

Suppose that one has two pairs of bifundamentals,  $\Phi_i, \bar{\Phi}_i$ ,  $i = 1, 2$ , and that the superpotential respects the  $Z_N$  symmetry

$$\Phi_1 \rightarrow \alpha \Phi_1, \quad \bar{\Phi}_1 \rightarrow \alpha^{-1} \bar{\Phi}_1, \quad \Phi_2 \rightarrow \alpha^{-\frac{N+3}{2}} \Phi_2, \quad \bar{\Phi}_2 \rightarrow \alpha^{\frac{N+3}{2}} \bar{\Phi}_2. \quad (2)$$

Then the vacuum expectation values (VEVs)

$$\langle \Phi_1 \rangle = \langle \bar{\Phi}_1 \rangle = \begin{pmatrix} v_1 & & & & \\ & v_1 & & & \\ & & v_1 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \langle \bar{\Phi}_2 \rangle = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & v_2 & \\ & & & & v_2 \end{pmatrix}, \quad (3)$$

preserve a  $Z'_N$  symmetry which is a combination of the original  $Z_N$  symmetry of eq. (2) and the hypercharge transformation of eq. (1). Because the symmetry is unbroken, this structure of expectation values is natural; it is automatically an extremum of the potential. We will discuss another possible set of expectation values, with only  $\langle \Phi_1 \rangle$  and  $\langle \Phi_2 \rangle$  nonzero, in section V.

In fact, Barr constructed a model with precisely these features, for the case  $Z_N = Z_2$  [7]. The superpotential in his model is:

$$\begin{aligned} W = & M_1(\Phi_1 \bar{\Phi}_1 + \Phi_2 \bar{\Phi}_2) \\ & + \frac{1}{M_2} [(\bar{\Phi}_1 \bar{\Phi}_1 \Phi_1 \bar{\Phi}_1) + (\Phi_2 \bar{\Phi}_2 \Phi_2 \bar{\Phi}_2) + (\Phi_1 \bar{\Phi}_1 \Phi_2 \bar{\Phi}_2) + (\Phi_1 \bar{\Phi}_2 \Phi_2 \bar{\Phi}_1) \\ & + (\bar{\Phi}_1 \Phi_1)(\bar{\Phi}_1 \Phi_1) + (\bar{\Phi}_2 \Phi_2)(\bar{\Phi}_2 \Phi_2) + (\bar{\Phi}_1 \Phi_1)(\bar{\Phi}_2 \Phi_2) + (\bar{\Phi}_1 \Phi_2)(\bar{\Phi}_2 \Phi_1) \\ & + (\Phi_1 \bar{\Phi}_2 \Phi_1 \bar{\Phi}_2) + (\Phi_2 \bar{\Phi}_1 \Phi_2 \bar{\Phi}_1) + (\bar{\Phi}_1 \bar{\Phi}_2)(\bar{\Phi}_1 \bar{\Phi}_2) + (\bar{\Phi}_2 \bar{\Phi}_1)(\bar{\Phi}_2 \bar{\Phi}_1)]. \end{aligned} \quad (4)$$

(For simplicity, we have not distinguished the different couplings.) It is easy to see that for such a superpotential, there is a solution of the form (3). Moreover, one can also see that all fields gain mass. Note, in particular, that the only continuous symmetry of the superpotential is  $SU(5) \times SU(5)$ . As a result, if we turn on the gauge coupling, the only massless fields we expect to find are those associated with the Goldstone bosons for the breaking of this symmetry and their superpartners, which are Higgsed.

This superpotential is not of the form we seek. The VEVs (3) are not a flat direction of the superpotential. Instead, the potential has a minimum with VEVs of order  $M_{\text{GUT}}$  if  $M_1$  and  $M_2$  are of order  $M_{\text{GUT}}$ .  $M_{\text{GUT}}$  appears as an explicit input parameter; both as mass terms ( $M_1$ ), and as the scale suppressing higher-dimension terms ( $M_2$ ). This superpotential is puzzling if the  $\Phi$ 's are the only  $M_{\text{GUT}}$  fields. Generating these couplings in a theory with

only fields of  $\mathcal{O}(M_{\text{GUT}})$ -masses and marginal or relevant couplings requires the addition of, at least, a  $(24, 24)$  [7] with mass of order  $M_{\text{GUT}}$  and couplings to the  $\Phi$  fields.

However, for the purposes of splitting doublets and triplets, Barr’s model is the simplest example of the mechanism we will use. It is now clear how triplets can gain mass while doublets remain massless. Assigning  $Z_N$  charges to the Higgses so that they cannot gain mass means that they can have couplings to  $\Phi_1$  or  $\bar{\Phi}_1$ , but not to  $\Phi_2$  or  $\bar{\Phi}_2$ . From the point of view of the low energy theory, it is easy to choose  $Z_N$  charges for the Higgses, such that the unbroken  $Z'_N$  will forbid a doublet mass.

If we want the unbroken  $Z'_N$  to distinguish between doublet and triplet *mass terms*, the standard model Higgses must arise from fields charged under different  $SU(5)$  factors.<sup>1</sup> The standard model Higgses therefore come from, for example,

$$h(5, 1) , \quad \bar{h}'(1, \bar{5}) . \quad (5)$$

(Here and in the following, unprimed (primed) fields transform under the first (second)  $SU(5)$ .) This possibility was discussed in [3]. We refine this statement and consider additional possibilities in Section VI.

The fact that the unbroken  $Z'_N$  symmetry in the low energy theory below  $M_{\text{GUT}}$  distinguishes Higgs doublets and triplets may be used to relax the bound on the triplet mass from proton decay. Suppose that triplets are lighter than  $M_{\text{GUT}}$ . Since triplets and doublets have different charges under the unbroken  $Z'_N$  of the low-energy theory, their couplings to standard-model matter fields are typically not the same. From the point of view of the full theory, if, as mentioned above, the Higgs doublets come from a  $5$  and  $\bar{5}$  of different  $SU(5)$ ’s, then some standard-model Yukawa couplings originate from higher-dimension terms, involving the bifundamentals. Thus, the triplets couplings to matter fields may be suppressed, so that their contribution to proton decay is small. Of course, the triplet mass has to be sufficiently high in order to preserve the unification of couplings. But in minimal  $SU(5)$  models, this requirement actually gives both an *upper* and a lower bound on the triplet mass. While in our models the triplet masses arise from renormalizable couplings to  $\Phi_1$  and  $\bar{\Phi}_1$ , it may be possible to construct models in which these masses are lower, as low as permitted by gauge coupling unification.

### III. THE ORIGIN OF $M_{\text{GUT}}$

We are interested in models in which  $M_{\text{GUT}}$  corresponds to an exact, or nearly exact flat direction. The main difficulty in constructing such models is that, while no potential should be generated along this direction (or at least no sizable potential), all fields charged under the standard model gauge group should acquire  $\mathcal{O}(M_{\text{GUT}})$  masses. As we will see, however, it is possible to construct  $SU(5) \times SU(5)$  models that are fairly simple yet possess these two features.

We note that various constructions of this type have been put forward in the past. For example, a model based on  $SO(10)$ , which also solves the doublet-triplet problem (using the so-called “Dimopoulos-Wilczek mechanism”) was presented in [5]<sup>2</sup>. In this model, the

<sup>1</sup> For  $h(5, 1)$  and  $\bar{h}'(\bar{5}, 1)$ , the doublet and triplet mass terms  $\bar{h}_3 h_3$  and  $\bar{h}_2 h_2$  have the same  $Z'_N$  charges.

<sup>2</sup> Other examples have been given in [10, 11, 12]. However, in most cases, these models were only studied at the level of renormalizable terms.

existence of flat directions, as well as the masslessness of the Higgs doublets, are insured by global continuous  $R$ -symmetries, which restrict the Lagrangian to renormalizable terms. The pattern of expectation values which give massless doublets does not respect any particular symmetry; it is an accident of the restriction of the model to renormalizable terms. Allowing only discrete symmetries, the flat direction is no longer flat, and the desired pattern of expectation values requires fine tuning. It should be noted that, even with the use of global symmetries, the construction requires six adjoints, two symmetric tensors and two spinor representations just in order to obtain the first stage of symmetry breaking.

If one does not worry about the spectrum, it is not difficult to obtain models in which the flat direction is an exact modulus using discrete  $R$  symmetries. For example, suppose one has a conventional  $SU(5)$  theory with a single adjoint. Requiring that the adjoint,  $A$ , be neutral under the  $R$  symmetry, while the superpotential transforms with a non-trivial phase, forbids any superpotential for  $A$ . So the potential for  $A$  is flat. However, this simple example also illustrates the main difficulty: if we choose to break to  $SU(3) \times SU(2) \times U(1)$ , there are additional massless fields transforming as  $(8, 1)$  and  $(1, 3)$  under  $SU(3) \times SU(2)$ . This difficulty arises also in the case of approximate flat directions. E.g., if one has an ordinary (non- $R$ )  $Z_{10}$  under which  $A \rightarrow e^{\frac{2\pi i}{10}} A$ , then the leading term in the superpotential involving  $A$  alone is  $A^{10}$ , so the flat direction is very flat, but the octet and triplet are very light.

We will consider, as above, product groups. Most of our models will involve two pairs of bifundamentals, as described in section IV, with the VEVs (3). To avoid the problem of light fields, we will study models with additional fields beyond the minimal set of bifundamentals. The  $Z_N$  global symmetry we discussed does not forbid a superpotential in the would-be flat directions. In fact, all nonzero gauge invariants, such as  $\Phi_1 \bar{\Phi}_1$ ,  $\Phi_2 \bar{\Phi}_2$ , and  $\Phi_1^3 \Phi_2^2$  (contracted with  $\varepsilon$ -tensors) are also  $Z_N$  singlets, and can appear in the superpotential to any power. It is easy to forbid such terms using an additional continuous symmetry in order to obtain an exact flat direction. Discrete  $R$ -symmetries can give exact flat directions involving the  $\Phi$  fields but, as in our simple  $SU(5)$  example, there are unwanted massless fields in these directions. As we will show, with the simplest possible field content, one cannot obtain exact flat directions with all fields massive using only discrete symmetries (continuous global symmetries can do the job).

But the fact that the flat directions are only approximate can be a virtue, yielding a simple mechanism for generating  $M_{\text{GUT}}$  [4]. Suppose that the first contribution to the  $F$  terms comes from the superpotential term

$$W = \frac{1}{M_{\text{Pl}}^{n-3}} X^n, \quad (6)$$

where  $X$  stands collectively for the fields of the theory and  $M_{\text{Pl}}$  is the Planck scale. Suppose further that, once supersymmetry is broken,  $X$  acquires a negative soft mass squared,  $m^2$ . Then the potential for  $X$  is

$$V = -m^2 |X|^2 + \frac{1}{M_{\text{Pl}}^{2n-6}} |X|^{2n-2}, \quad (7)$$

leading to an  $X$ -VEV of the order

$$\langle X \rangle \sim \left( \frac{m}{M_{\text{Pl}}} \right)^{\frac{1}{n-2}} M_{\text{Pl}}. \quad (8)$$

For  $n$  around 10, this is around  $M_{\text{GUT}}$ .

We therefore aim, in our models, for superpotentials which generate  $F$  terms only from nonrenormalizable terms such as  $(\Phi_1 \bar{\Phi}_1)^5$ , or  $(\Phi_1^3 \Phi_2^2)^2$ .

In models in which  $M_{\text{GUT}}$  corresponds to an exactly flat direction, we will assume that  $M_{\text{GUT}}$  is generated by supersymmetry breaking effects. Explicit examples of this type were studied in [13] in the context of gauge mediated supersymmetry breaking, and in [14] in the context of anomaly mediated supersymmetry breaking. The mechanism of [14] leads however to large flavor violation, unless extra structure is invoked. We note that other mechanisms for generating  $M_{\text{GUT}}$  exist in the literature [11, 12]. In these models,  $M_{\text{GUT}}$  is related to the dynamical scale of some gauge group.

In the context of string theory, there is another mechanism which one might imagine could generate  $M_{\text{GUT}}$ . This is the appearance of a Fayet-Iliopoulos term, with a coefficient of order  $\frac{\alpha_{\text{GUT}}}{4\pi}$ , perhaps small enough to explain the scale [15]. In this case,  $M_{\text{GUT}}$  might not be related to supersymmetry breaking. It is still necessary to have directions in field space which are flat or nearly flat with respect to the unified gauge group and the superpotential.

#### IV. MODELS WITH ‘MESONIC’ FLAT DIRECTIONS

We now turn to some examples which demonstrate the various ideas we have discussed. As described in the introduction, our models contain two pairs of bifundamentals  $\Phi_i$  and  $\bar{\Phi}_i$ ,  $i = 1, 2$ . We impose a  $Z_N$  global symmetry, which ensures the masslessness of the Higgs doublets. The  $Z_N$  charges of the bifundamentals are listed in table I. Note that the bifundamentals form vectorlike representations of the  $Z_N$  – the charges of  $\Phi_1$  and  $\bar{\Phi}_1$  sum to zero, and similarly the charges of  $\Phi_2$  and  $\bar{\Phi}_2$ .

We would like to find a vanishing potential along the direction discussed in the introduction [eq. (3)]. Along this direction,  $SU(5) \times SU(5) \times Z_N$  breaks into  $SU(3)_D \times SU(2)_D \times U(1)_D \times Z'_N$ .  $Z'_N$  is a combination of the original discrete symmetry and a discrete “hypercharge” subgroup of  $SU(5)_1$ . Various components of  $\Phi_i$ ,  $\bar{\Phi}_i$  are un eaten. In particular, the following standard-model  $SU(3) \times SU(2)$  representations appear:  $3 \times (8, 1)$ ,  $3 \times (1, 3)$ ,  $2 \times (3, 2)$  and  $2 \times (\bar{3}, 2)$ .

It is easy to see that we cannot give mass to the bifundamentals by couplings among the bifundamentals alone, without spoiling the flat directions. Adding two fields transforming as adjoints under the diagonal  $SU(5)$  forces  $Z_N$  charges that are inconsistent with an exact low-energy  $Z'_N$  and, furthermore, leaves light, incomplete  $SU(5)$  multiplets. At a minimum, we need to include in the model three fields transforming as adjoints under the diagonal  $SU(5)$ . We add three adjoints of  $SU(5)_1$ ,  $A_{i=1,2,3}$ , as well as a gauge singlet  $S$ , with the superpotential

$$W = \lambda_{12} \Phi_1 A_1 \bar{\Phi}_2 + \lambda_{21} \Phi_2 A_2 \bar{\Phi}_1 + \lambda_{11} \Phi_1 A_3 \bar{\Phi}_1 + \lambda_{22} \Phi_2 A_3 \bar{\Phi}_2 + \eta_{12} S A_1 A_2 + \eta_{33} S A_3 A_3 . \quad (9)$$

This superpotential indeed preserves a  $Z_N$  symmetry, for appropriate choices of the singlet and adjoint charges. The field content of the model is summarized in Table I. The fields appearing below the first horizontal line contain the Higgs doublets. Most often, these will come from  $h$  and  $\bar{h}'$ , which we always include. The remaining two fields,  $h'$  and  $\bar{h}$ , may or may not appear. We postpone further discussion of these Higgses until Section VI, and for now concentrate on the GUT breaking fields. In Appendix B we show that the field content in Table I is still small enough that there are no Landau poles below the Planck scale.



Field	$SU(5) \times SU(5) \times Z_N$
$\Phi_1$	$(5, \bar{5}, 1)$
$\bar{\Phi}_1$	$(\bar{5}, 5, -1)$
$\Phi_2$	$(5, \bar{5}, (N-3)/2)$
$\bar{\Phi}_2$	$(\bar{5}, 5, (N+3)/2)$
$A_1$	$(24, 1, (N-5)/2)$
$A_2$	$(24, 1, (N+5)/2)$
$A_3$	$(24, 1, 0)$
$S$	$(1, 1, 0)$
$h$	$(5, 1, 1)$
$\bar{h}'$	$(1, \bar{5}, 0)$
$\bar{h}$	$(\bar{5}, 1, 0)$
$h'$	$(1, 5, -1)$

TABLE I: Basic fields and their charge assignments

Indeed, the potential vanishes for

$$\begin{aligned}
\langle \Phi_1 \rangle &= \langle \bar{\Phi}_1 \rangle = v_1 \times \text{diag}(1, 1, 1, 0, 0), \\
\langle \Phi_2 \rangle &= \langle \bar{\Phi}_2 \rangle = v_2 \times \text{diag}(0, 0, 0, 1, 1), \\
\langle S \rangle &= s, \\
\langle A_i \rangle &= 0,
\end{aligned} \tag{10}$$

and all pseudo-Goldstone bosons get mass. The flatness condition requires<sup>3</sup>

$$\lambda_{11}v_1^2 = \lambda_{22}v_2^2. \tag{11}$$

For superpotential couplings of order one, the two VEVs are not very different.

Clearly, however, various terms which are allowed by the symmetries would spoil the flatness of the potential. For example, any power of  $\Phi_1\bar{\Phi}_1$ ,  $\Phi_1^3\Phi_2^2$ , or  $S$ , generates a potential. In the following, we consider different symmetries that forbid all or some of these terms.

### A. Exact flat directions

If one allows continuous global symmetries, it is easy to obtain models in which the direction (10) is exactly flat. The superpotential (9) preserves a  $U(1)_R$  symmetry under which all adjoints  $A_i$  have charge 1, the singlet  $S$  has charge  $-1$ , and the bifundamentals  $\Phi_i$ ,  $\bar{\Phi}_i$  have charge zero. The superpotential has charge 1. For completeness we list the charges of all fields in Table II.

The  $U(1)_R$  symmetry forbids all the ‘dangerous’ non-renormalizable terms that would otherwise lift the flat direction. It does so by requiring that in each allowed term there is at least one adjoint. All terms with two adjoints or with a single  $A_1$  or  $A_2$  are trivially

<sup>3</sup> To see that one needs to include Lagrange multiplier terms ensuring the tracelessness of the adjoints.

Field	$SU(5) \times SU(5) \times Z_N \times U(1)_R$
$\Phi_1$	$(5, \bar{5}, 1, 0)$
$\bar{\Phi}_1$	$(\bar{5}, 5, N - 1, 0)$
$\Phi_2$	$(5, \bar{5}, (N - 3)/2, 0)$
$\bar{\Phi}_2$	$(\bar{5}, 5, (N + 3)/2, 0)$
$A_1$	$(24, 1, (N - 5)/2, 1)$
$A_2$	$(24, 1, (N + 5)/2, 1)$
$A_3$	$(24, 1, 0, 1)$
$S$	$(1, 1, 0, -1)$

TABLE II: Exact flat direction

harmless. The non-renormalizable terms with a single  $A_3$  modify the flatness condition (11); the precise relation between  $v_1$  and  $v_2$  is then changed, but not the fact that there is an exactly flat direction of the form (3).

If, however, we consider just a discrete  $R$  symmetry, the flat direction is lifted. The problem is easy to understand. To obtain an exact flat direction for the  $\Phi$ 's, we would like them to be neutral under some  $R$  symmetry. But this fixes the  $R$  transformation laws of the  $A_i$ 's and, as a result, of  $S$ . In particular,  $S$  cannot be neutral. We show in Appendix A that there are always terms of the form  $S^n(\bar{\Phi}\Phi)^m$  which transform properly under the symmetries. So in this class of models, it is easy to obtain an exact flat direction for the bifundamentals using only discrete  $R$  symmetries, but it is not possible to obtain an exact flat direction simultaneously for  $S$ . In the next section, we discuss approximate flat directions.

### B. Discrete Symmetry: Approximate flat directions

As we just saw, we can get a model with an approximate flat direction by replacing the continuous  $R$  symmetry above by a discrete  $R$  symmetry,  $Z_M^R$ . (The field content and charge assignments of this model are presented explicitly in Table V.) Then the terms

$$\frac{1}{M_{\text{Pl}}^{M-4}} S^{M-1}, \quad \frac{1}{M_{\text{Pl}}^{M-2}} S^{M-1} \Phi_i \bar{\Phi}_i, \quad i = 1, 2, \quad (12)$$

are allowed and lift the flat direction.

In this case, the  $S$  flat direction is lifted by a superpotential term of dimension  $M - 1$ , while the  $\Phi$  flat direction (here  $\Phi$  stands for all bifundamentals) is lifted by superpotential terms of dimension  $M + 1$ . As discussed in Section III, if the singlet and bifundamentals get negative masses-squared by supersymmetry-breaking, they will be stabilized near  $M_{\text{GUT}}$  for  $M \sim 9, 10$ .

We thus obtain a model in which the GUT breaking VEVs are naturally obtained at the right scale, all fields associated with the GUT breaking obtain masses of order  $M_{\text{GUT}}$ , and the low energy theory has an unbroken global  $Z'_N$  symmetry, which can distinguish between Higgs doublets and triplets.

It is straightforward to generate other models that share these features. For example, consider giving the  $\Phi$ 's nonzero charges under the  $R$  symmetry. We take the  $R$  symmetry

Field	$SU(5) \times SU(5) \times Z_N \times Z_{11}^R$
$\Phi_1$	$(5, \bar{5}, 1, 0)$
$\bar{\Phi}_1$	$(\bar{5}, 5, N-1, 0)$
$\Phi_2$	$(5, \bar{5}, (N-3)/2, 3)$
$\bar{\Phi}_2$	$(\bar{5}, 5, (N+3)/2, 8)$
$A_1$	$(24, 1, (N-5)/2, 4)$
$A_2$	$(24, 1, (N+5)/2, 9)$
$A_3$	$(24, 1, 0, 1)$
$S$	$(1, 1, 0, 10)$

TABLE III: Model B: Approximate flat direction

to be  $Z_{11}^R$  with the charges given in Table III. The symmetries allow, among others, the following terms:

$$\frac{1}{M_{\text{Pl}}^6} S^4 (\bar{\Phi}_1^3 \bar{\Phi}_2^2), \quad \frac{1}{M_{\text{Pl}}^7} (\Phi_1^3 \Phi_2^2)^2. \quad (13)$$

The flat direction is lifted by terms of dimensions 9 and 10 and consequently a reasonable value for  $M_{\text{GUT}}$  is obtained.

## V. MODELS WITH ‘BARYONIC’ FLAT DIRECTIONS

As we saw in the previous section, it is impossible in the models considered so far to obtain an exact mesonic flat direction using discrete symmetries only. This can easily be done, however, if we consider the following direction:

$$\begin{aligned} \langle \Phi_1 \rangle &= v \times \text{diag}(1, 1, 1, 0, 0), \\ \langle \Phi_2 \rangle &= v \times \text{diag}(0, 0, 0, 1, 1), \\ \langle \bar{\Phi}_i \rangle &= 0. \end{aligned} \quad (14)$$

Again, these VEVs break the gauge symmetry to the standard-model gauge group, and preserve the  $Z'_N$  global symmetry irrespectively of the  $\bar{\Phi}_i$  charge assignments [the  $Z_N$ -charges of  $\Phi_1$  and  $\Phi_2$  are still 1 and  $(N-3)/2$ ].

Apart from the possibility of obtaining exact flat directions, these directions are also promising for obtaining potentials which are flat to a very high degree. The only non-vanishing gauge invariant combinations are powers of the baryon operator  $\Phi_1^3 \Phi_2^2$ . If the first such term appearing in the superpotential is the baryon squared, it can naturally generate the scale  $M_{\text{GUT}}$  once supersymmetry is broken.

It is easy to see that discrete symmetries can guarantee an exactly vanishing potential in this case. For example, with no additional fields in the model, we can choose a  $Z_M^R$  symmetry under which the  $\Phi$ 's have charge zero and the  $\bar{\Phi}$ 's have charge one. Then, assuming the superpotential carries  $R$ -charge 2, the only terms allowed contain two  $\bar{\Phi}$ 's, and the potential vanishes.

It is, however, very hard to generate masses for all GUT breaking fields in these models, or even for all fields but one (or more) sets in a complete  $SU(5)$  representation. In particular,

note that the uneaten fields in the  $\Phi$ 's are an  $SU(3)$  octet and an  $SU(2)$  triplet, and these can only get mass from the 'baryon' operators  $\Phi_1^5$  and  $\Phi_1\Phi_2^4$ .

In the following, we present three types of models. Each of these models demonstrates some nice features that can be achieved with 'baryonic' flat directions, but also suffers from some problems. Model A is closest in spirit to the models with 'mesonic' flat directions discussed in the previous section. It has an approximately flat potential but a light 24 of  $SU(5)$ . With such a large representation, the  $SU(3)$  coupling blows up below  $M_{\text{GUT}}$ , unless the fields in the 24 get masses of at least 1 TeV. Model B has no light fields beyond the MSSM fields, but the flat direction is lifted by a dimension-5 superpotential term. Model C has an exact flat direction but, again, has a light 24. Both models B and C have no adjoints, but in addition to the light 24 have incomplete multiplets at a scale of order  $10^{12}$  GeV which might spoil coupling unification.

### A. Model A: Approximate Flat Direction and a Light 24

The field content of our first model appears in Table IV. We take the superpotential to carry  $R$ -charge 1. The symmetry allows the superpotential

Field	$SU(5) \times SU(5) \times Z_N \times Z_9^R$
$\Phi_1$	$(5, \bar{5}, 1, 1)$
$\bar{\Phi}_1$	$(\bar{5}, 5, N-1, 0)$
$\Phi_2$	$(5, \bar{5}, (N-3)/2, 1)$
$\bar{\Phi}_2$	$(\bar{5}, 5, (N+3)/2, 0)$
$A$	$(24, 1, 0, 0)$
$B_1$	$(1, 24, (N+5)/2, 0)$
$B_2$	$(1, 24, (N-5)/2, 0)$
$S_1$	$(1, 1, -5, 0)$
$S_2$	$(1, 1, 5, 0)$

TABLE IV: 'Baryon' model A

$$\begin{aligned}
W = & B_2\Phi_1\bar{\Phi}_2 + B_1\Phi_2\bar{\Phi}_1 + A\Phi_1\bar{\Phi}_1 + A\Phi_2\bar{\Phi}_2 + S_1B_1^2 + S_2B_2^2 \\
& + \frac{1}{M_{\text{Pl}}^7}(\Phi_1^3\Phi_2^2)^2 + \frac{1}{M_{\text{Pl}}^9}(S_1S_2)(\Phi_1^3\Phi_2^2)^2 + \dots
\end{aligned}
\tag{15}$$

Here we are interested in nonzero VEVs for  $S_1, S_2$  as well. We only show terms that either give mass to some fields, or generate a potential. We get an approximate flat direction, allowing for a natural generation of  $M_{\text{GUT}}$ . Note, however, that for general  $N$ , the  $Z_N$  symmetry is spontaneously broken by the singlet VEVs, and one needs to carefully choose the Higgs charges, so that the doublets remain light. In addition, one needs to ensure that higher-dimension terms involving the bifundamentals and the singlets, that destabilize the VEV (14), are sufficiently suppressed. Alternatively, if we insist on having an unbroken global symmetry, we can work with  $N = 2$ , with only  $\Phi_2, \bar{\Phi}_2$ , and  $B_i$  odd under the  $Z_2$ . The unpleasant feature of this model is that it leaves a light 24.

## B. Model B: Approximate Flat Direction with No Light Fields

We can write down a much simpler model with just the  $\Phi$ 's and  $\bar{\Phi}$ 's. We impose a  $Z_2 \times Z_2$  global symmetry. The first  $Z_2$  is our original  $Z_N$  with  $N = 2$ , under which only  $\Phi_2$  and  $\bar{\Phi}_2$  are odd. Under the second  $Z_2$  the two  $\bar{\Phi}$ 's are odd. The allowed superpotential is:

$$W = \frac{1}{M} \left( \Phi_1 \bar{\Phi}_1 \Phi_1 \bar{\Phi}_1 + \Phi_1 \bar{\Phi}_2 \Phi_1 \bar{\Phi}_2 + \Phi_2 \bar{\Phi}_1 \Phi_2 \bar{\Phi}_1 + \Phi_2 \bar{\Phi}_2 \Phi_2 \bar{\Phi}_2 + \Phi_1 \bar{\Phi}_1 \Phi_2 \bar{\Phi}_2 + \Phi_1 \bar{\Phi}_2 \Phi_2 \bar{\Phi}_1 \right) + \frac{1}{M^2} \left( \Phi_1^5 + \Phi_1 \Phi_2^4 + \Phi_1^3 \Phi_2^2 \right) + \dots \quad (16)$$

The dimension-4 terms give mass to all components of the  $\bar{\Phi}$ 's (these therefore get mass two orders of magnitude below  $M_{\text{GUT}}$ ). The first two dimension-5 terms give mass to the  $SU(2)$ -triplet and  $SU(3)$ -octet in the  $\Phi$ 's (these therefore get mass four orders of magnitude below  $M_{\text{GUT}}$ , a potential problem for coupling unification). Finally, the problematic term is the last one; it lifts the flat direction already at dimension-5, and is thus inappropriate for generating  $M_{\text{GUT}}$ . In order that the model is acceptable, we need to identify  $M = M_{\text{GUT}}$  in eq. (16), that is, put  $M_{\text{GUT}}$  by hand.

## C. Model C: Exactly Flat Direction

Consider a model with the bifundamentals and singlets, where the  $\bar{\Phi}_i$  fields carry a  $Z_M^R$ -charge +1 and all other fields ( $\Phi_i$  and  $S_i$ ) are  $Z_M^R$ -neutral. The superpotential has  $Z_M^R$ -charge +2. Thus all terms must include two  $\bar{\Phi}_i$ -fields and consequently the baryonic direction (14) is exactly flat. Consider additional symmetries that allow only the following terms up to dimension-5:

$$W = \frac{1}{M_{\text{Pl}}^2} \left( S_1 \Phi_1 \bar{\Phi}_1 \Phi_1 \bar{\Phi}_2 + S_2 \Phi_2 \bar{\Phi}_1 \Phi_2 \bar{\Phi}_2 + S_3 \Phi_1 \bar{\Phi}_1 \Phi_2 \bar{\Phi}_1 \right). \quad (17)$$

This superpotential leaves a light 24, while all other fields [which include  $(8, 1) + (1, 3)$  plus complete  $SU(5)$ -multiplets] get their masses at a scale of order  $M_{\text{GUT}}^3/M_{\text{Pl}}^2$  which may be problematic for coupling unification.

## VI. STANDARD MODEL HIGGSSES

So far, we have concentrated on the GUT breaking fields. In Section IV, we found models in which  $M_{\text{GUT}}$  corresponds to an exact or approximate flat direction. Along this direction, all fields associated with the GUT breaking are heavy. In addition, there is an unbroken  $Z'_N$  global symmetry. We will now see in more detail how this symmetry splits the doublets and triplets. For concreteness, we focus on the first model described in Section IV B, but it is straightforward to repeat this discussion for different versions of the model – only the charges under the discrete  $R$ -symmetry will change. For convenience, we list in Table V the relevant fields.

Let us next write down the most general renormalizable superpotential that involves the  $h$  fields:

$$W_1 = h \bar{\Phi}_1 \bar{h}' + h' \Phi_1 \bar{h}. \quad (18)$$

Field	$SU(5) \times SU(5) \times Z_N \times Z_M^R$
$\Phi_1$	$(5, \bar{5}, 1, 0)$
$\bar{\Phi}_1$	$(\bar{5}, 5, N - 1, 0)$
$\Phi_2$	$(5, \bar{5}, (N - 3)/2, 0)$
$\bar{\Phi}_2$	$(\bar{5}, 5, (N + 3)/2, 0)$
$A_1$	$(24, 1, (N - 5)/2, 1)$
$A_2$	$(24, 1, (N + 5)/2, 1)$
$A_3$	$(24, 1, 0, 1)$
$S$	$(1, 1, 0, M - 1)$
$h$	$(5, 1, 1, 1)$
$\bar{h}'$	$(1, \bar{5}, 0, 0)$
$\bar{h}$	$(\bar{5}, 1, 0, 0)$
$h'$	$(1, 5, N - 1, 1)$

TABLE V: The fields and symmetries of the model of Section IV B

Since the  $h$  fields do not couple to the  $\Phi_2$  and  $S$  fields, only the triplets acquire masses.

It is useful to look at the charges of the doublets and triplets under the unbroken  $Z'_N$  of the low energy theory. We list these in Table VI. Clearly, no doublet mass term is allowed

Field	$Z'_N$
$h_3$	0
$h_2$	$(N + 5)/2$
$\bar{h}_3$	1
$\bar{h}_2$	$(N - 3)/2$
$h'_3$	-1
$h'_2$	-1
$\bar{h}'_3$	0
$\bar{h}'_2$	0

TABLE VI: Doublet ( $h_2$ ) and triplet ( $h_3$ )  $Z'_N$ -charges.

in the low energy theory below  $M_{\text{GUT}}$ . Therefore, if we start from four Higgses, the theory contains four light doublets.

It is easy to see that this result always holds if we insist on an unbroken  $Z'_N$  and masses of order  $M_{\text{GUT}}$  for all triplets. In order to give mass to all triplets, we need to allow both terms in  $W_1$  of eq. (18). Since  $\Phi_1$  and  $\bar{\Phi}_1$  have opposite  $Z_N$  charges, the  $Z_N$  charges of the Higgses must satisfy

$$Q(h) + Q(\bar{h}') + Q(h') + Q(\bar{h}) = 0 . \quad (19)$$

As a result, the terms

$$h\bar{\Phi}_2\bar{h}' , \quad h'\Phi_2\bar{h} , \quad (20)$$

have the same  $Z_N$  charges: they are either both allowed, or both forbidden. We are therefore left with four light doublets. There are then three options a priori:

- **A:** The theory does not contain  $h'$  and  $\bar{h}$ .  $SU(5)$  anomalies are cancelled by appropriate choices of  $SU(5) \times SU(5)$  representations for the standard model matter fields. The standard model Higgses come from fields charged under different  $SU(5)$ 's. Some standard model Yukawa couplings arise from non-renormalizable terms.  $Z'_N$  can be broken by supersymmetry breaking effects to generate the  $\mu$  term.
- **B:** The theory does contain  $h'$  and  $\bar{h}$ , but these remain massless<sup>4</sup> [3]. Witten speculates that these could be the messengers of supersymmetry breaking. The heavy triplets are from  $h$  and  $\bar{h}'$ .
  - **B1:** The standard model Higgses come from fields charged under different  $SU(5)$ 's, say,  $h$  and  $\bar{h}'$ , so that, again, some standard model Yukawa couplings arise from non-renormalizable terms.
  - **B2:** The standard model Higgses come from fields charged under a single  $SU(5)$ , say,  $h$  and  $\bar{h}$ . Then, standard model fields can all be charged under the same  $SU(5)$ , and all Yukawa couplings are renormalizable.
- **C:** The theory does contain  $h'$  and  $\bar{h}$ . All triplets gain mass through the couplings (18). The  $Z'_N$  is broken at a high scale, so that one doublet pair also gets mass around  $M_{\text{GUT}}$ . It is possible to arrange for an acceptable  $\mu$  term for the remaining two doublets, for example, through the mechanisms proposed in [16] or in [7]. This is most easily done by adding a gauge-singlet,  $S_H$ , charged under the  $Z_N$ , with a GUT scale VEV. Then, in order to have masses of order  $M_{\text{GUT}}$ , the doublets that couple to  $S_H$  must be charged under the same  $SU(5)$ , and the higgs doublets are charged under the second  $SU(5)$ .

In this case, one should also make sure that the pattern of VEVs we are considering, eqn. (3), is not destabilized, that is, that  $\Phi_1, \bar{\Phi}_1$  do not get VEVs in the last two entries. It is easy to see that such VEVs can be very small. For example, if  $S_H$  has charge 1 under the  $Z_N$ , then the operator  $S_H^{(N+5)/2} S^{M-1} \bar{\Phi}_1 \Phi_2 \bar{\Phi}_1 \Phi_1$  is allowed, and can generate the dangerous VEV. However, it is very suppressed. In fact, such operators might even generate a  $\mu$  term of precisely the right size, without having to add any new coupling for the Higgses. The doublet mass will simply arise from the first term in (18).<sup>5</sup>

In the next section, we will study the implications for quark and lepton mass matrices.

## VII. YUKAWA MATRICES AND YUKAWA SPLITTING

We now turn to the Yukawa couplings for quarks and leptons. We will see that some of the possibilities mentioned above are excluded, and that in others, the  $SU(5) \times SU(5)$  gauge symmetry automatically suppresses some Yukawa couplings. Moreover, in many cases, triplet-Higgses couplings to matter are naturally suppressed, demonstrating the mechanism of “Yukawa splitting”.

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<sup>4</sup> It is easy to forbid the relevant mass terms by choosing appropriate charges for  $h'$  and  $\bar{h}$ .

<sup>5</sup> With a  $Z_2$  instead of a  $Z_N$ , one does not even have to break the  $Z_2$  in order to generate a large mass for the extra triplets. This is the mechanism employed in [7].

The chiral part of our model contains the Higgs and matter fields. Taking into account the various possibilities for the Higgs fields discussed in the previous section and the requirement of anomaly cancellation, we have the following options:

**A:** Standard model higgses from  $(5, 1)_H + (1, \bar{5})_H$ , no additional fundamentals, and the standard model fermion generations coming from one of the following sets of representations:

1.  $3 \times (\bar{5}, 1) + 2 \times (10, 1) + (1, 10)$
2.  $2 \times [(\bar{5}, 1) + (1, 10)] + (10, 1) + (1, \bar{5})$
3.  $(\bar{5}, 1) + 2 \times (1, \bar{5}) + 3 \times (1, 10)$

**B1:** Standard model higgses from  $(5, 1)_H + (1, \bar{5})_H$ , additional fundamentals in  $(\bar{5}, 1)_H + (1, 5)_H$ , and the standard model fermion generations coming from one of the following sets of representations:

1.  $3 \times [(\bar{5}, 1) + (10, 1)]$
2.  $2 \times [(\bar{5}, 1) + (10, 1)] + (1, \bar{5}) + (1, 10)$
3.  $(\bar{5}, 1) + (10, 1) + 2 \times [(1, \bar{5}) + (1, 10)]$
4.  $3 \times [(1, \bar{5}) + (1, 10)]$

**B2, C:** Standard model higgses from  $(5, 1)_H + (\bar{5}, 1)_H$ , additional fundamentals in  $(1, \bar{5})_H + (1, 5)_H$ , and the standard model fermion generations coming from one of the following sets of representations:

1.  $3 \times [(\bar{5}, 1) + (10, 1)]$
2.  $2 \times [(\bar{5}, 1) + (10, 1)] + (1, \bar{5}) + (1, 10)$
3.  $(\bar{5}, 1) + (10, 1) + 2 \times [(1, \bar{5}) + (1, 10)]$
4.  $3 \times [(1, \bar{5}) + (1, 10)]$

Note that the only renormalizable Yukawa couplings involve

$$(10, 1)(10, 1)(5, 1)_H, \quad (1, 10)(1, \bar{5})(1, \bar{5})_H, \quad (10, 1)(\bar{5}, 1)(\bar{5}, 1)_H. \quad (21)$$

Non-renormalizable terms will involve one or two bifundamental fields. The resulting Yukawa couplings are suppressed therefore by one or two powers of  $\epsilon = M_{\text{GUT}}/M_{\text{Pl}} \sim 10^{-2}$ . This leads to several interesting consequences.

First, a phenomenologically viable model must have a renormalizable top-Yukawa. This requirement excludes model (3) of class **A** and models (4) of classes **B** and **C**.

Second, there is a single possibility to have all Yukawa couplings coming from renormalizable terms, and that is model (1) of classes **B2** and **C**. Here the gauge symmetry plays no role in the Yukawa hierarchy.

Third, when down-type Yukawas involve matter fields charged under different  $SU(5)$ 's, say, a  $(10, 1)$  and a  $(1, \bar{5})$ , they give *either* a down-quark *or* a charged-lepton mass term. This allows for the exciting possibility that, just by the gauge quantum numbers, bottom-tau unification is maintained, but similar relations for the two light generations are not!



Furthermore, we see that it is possible to explain some features of the Yukawa hierarchy by the  $SU(5) \times SU(5)$  symmetry. Take, for example, model (1) of class **B1**. The mass matrices here are of the form

$$M_u \sim \langle (5, 1)_H \rangle \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad M_d \sim \langle (1, \bar{5})_H \rangle \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}. \quad (22)$$

Thus, the small ratio  $m_b/m_t$  is explained by the gauge symmetry. The hierarchy within each sector requires, however, some additional flavor physics. We show the mass matrices obtained in the remaining cases in Appendix C.

Finally, we can see how doublet-triplet *Yukawa-splitting* works. Take again model (1) of class **B1**. The entries in  $M_d$  of eq. (22) come from the non-renormalizable terms

$$(10, 1)(\bar{5}, 1)(1, \bar{5})_H(\bar{5}, 5)_{\bar{\Phi}_2}. \quad (23)$$

Note that these terms induce Yukawa couplings for the doublet Higgs fields only. Yukawa couplings for the triplet Higgs fields could have been induced by the following terms:

$$(10, 1)(\bar{5}, 1)(1, \bar{5})_H(\bar{5}, 5)_{\bar{\Phi}_1}. \quad (24)$$

However, if the discrete charge assignments are such that the terms in (23) are allowed, those in (24) are, in general, forbidden. This situation can be understood also in terms of the unbroken low-energy  $Z'_N$  symmetry that can allow Yukawa couplings for the doublets and forbid them for the triplets. Thus, the triplet-Yukawa couplings will only appear with  $Z'_N$ -breaking and can therefore be very strongly suppressed. As explained in section II, this mechanism of doublet-triplet Yukawa splitting is very helpful in solving the problem of proton decay.

## VIII. CONCLUSIONS: DISTINCTIONS BETWEEN GRAND UNIFICATION AND STRING UNIFICATION

Various puzzles of grand unification find attractive solutions in models that combine product gauge groups and discrete symmetries. In this work we focused on a framework that combines an  $SU(5) \times SU(5)$  gauge group with Abelian ( $Z_N$ ) discrete symmetries.

The gauge group breaks directly, at a scale  $M_{\text{GUT}}$ , into the standard model  $SU(3) \times SU(2) \times U(1)$  group which resides in the diagonal  $SU(5)$ . Let us first emphasize again that, in spite of the fact that there is no range of energies where there is a simple unifying gauge group, the attractive features of GUTs are maintained: there is coupling unification and hypercharge is quantized. At the same time, some of the problems associated with GUTs find solutions for which the combination of product groups and discrete symmetries is essential:

(i) The doublet-triplet splitting problem has a symmetry-based solution. A discrete symmetry (that remains unbroken below  $M_{\text{GUT}}$ ) can forbid doublet masses while allowing triplet masses. At the same time, this symmetry can allow Yukawa couplings for the doublet while forbidding them for the triplet. Thus the inconsistency between the constraints from coupling unification and from proton decay that arises in minimal  $SU(5)$  can be avoided in this framework.

(ii) The GUT scale corresponds to an exact or an approximate flat direction. In the latter case, the value of  $M_{\text{GUT}}$  is not an input parameter but is induced by an interplay between the Planck scale and the supersymmetry breaking scale. In addition to its intrinsic appeal, we argued that if the underlying theory is string or M theory, it is almost inevitable that  $M_{\text{GUT}}$  should be a modulus.

The combination of product gauge groups and discrete symmetries might have interesting implications for a third puzzle:

(iii) A non-trivial flavor structure arises since, in general, some of the Yukawa couplings arise from non-renormalizable terms. Such couplings are suppressed therefore by powers of  $M_{\text{GUT}}/M_{\text{Pl}}$ .

Our models demonstrate that it is possible to have fully consistent models of grand unification, with no light, extra incomplete (or even complete)  $SU(5)$  multiplets and with a relatively simple particle content at  $M_{\text{GUT}}$ , and in which the value of  $M_{\text{GUT}}$  is explained in terms of the Planck scale and the supersymmetry breaking scale.

The assumption that  $M_{\text{GUT}}$  is a modulus opens up the possibility of a cosmological moduli problem. In the context of string theory, the difficulty is that moduli typically dominate the energy density at nucleosynthesis. Unless they are very massive, their subsequent decays reheat the universe only to temperatures of order a few keV, and the successful predictions of big bang nucleosynthesis are spoiled [17, 18]. In the present case, however, the couplings of the modulus may be barely strong enough to evade this problem. Explicitly, the coefficient  $c$  in the leading contribution to the decay amplitude,  $\mathcal{L}_{\mathcal{M}} = \frac{c\alpha_{\text{GUT}}}{4\pi M_{\text{GUT}}}\mathcal{M}FF$ , can be  $\mathcal{O}(50)$  due to the large number of fields.

In the introduction, we stressed that while string theory does not admit continuous global symmetries, it often yields discrete symmetries. These symmetries are usually gauge symmetries, and are subject to various consistency conditions. In weakly coupled string models as well as a range of strongly coupled string theories [3, 19, 20, 21], it is possible to cancel all anomalies in discrete symmetries by postulating a discrete transformation law for some moduli fields. A priori, however, it is not clear that this provides a constraint on field theory models, since one might postulate the existence of several such moduli with Planck scale couplings. It is true that in most instances which have been studied, a single field can cancel all anomalies, but it seems unlikely that this holds in general [21]. We have not explored possible anomaly constraints in our work, but this is certainly an issue worthy of further study.

Whether or not of stringy origin, unification in the framework of product groups opens up new possibilities regarding the flavor structure of quarks and leptons and the question of proton decay. We have presented some aspects of the quark Yukawa hierarchy and of the doublet-triplet Yukawa splitting in Section VII. We are currently pursuing further investigations of these intriguing possibilities.

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## APPENDIX A: EXACT FLAT DIRECTIONS

In this appendix we show that it is impossible to forbid all higher-dimension operators using a  $Z_N \times Z_M^R$  discrete symmetry in a model with bifundamentals, adjoints and singlets.

We denote the  $Z_M^R$ -charge of each field by the corresponding small letter. For example, the charge of  $\Phi_i$  is  $\phi_i$ . The superpotential has charge  $w$ . All the equations involving the  $Z_M^R$ -charges are mod  $M$ , and all charges are integer.

In order to allow the terms  $SA_3^2$  and  $\Phi_1 A_3 \bar{\Phi}_1$  of eq. (9), the charges should satisfy

$$\phi_1 + \bar{\phi}_1 + a_3 = w, \quad s + 2a_3 = w. \quad (\text{A1})$$

We will now show that we can always find  $n, m$  so that

$$S^n (\Phi_1 \bar{\Phi}_1)^m \quad (\text{A2})$$

is allowed. For this term to appear we need:

$$n(w - 2a_3) + m(w - a_3) = w \implies (n + m - 1)w = (2n + m)a_3. \quad (\text{A3})$$

This condition is satisfied for

$$n + m - 1 = qM, \quad 2n + m = q'M, \quad (\text{A4})$$

where  $q, q'$  are some integers. Now consider the charges under the  $Z_N$  symmetry. We can repeat the above, with  $w \rightarrow 0$  and  $M \rightarrow N$ . Therefore we get one additional equation for  $n, m$ :

$$2n + m = q''N. \quad (\text{A5})$$

The three equations (A4)-(A5) can be solved with  $q' = q'' = kN$ , with integer  $k$ , giving:

$$n = kNM - qM - 1, \quad m = (2q - kN)M + 2. \quad (\text{A6})$$

Since the charges are only defined mod  $(MN)$  we can choose:

$$n = MN - qM - 1, \quad m = 2qM + 2. \quad (\text{A7})$$

A simple possibility is  $q = 0$ : the term

$$S^{NM-1} (\Phi_1 \bar{\Phi}_1)^2 \quad (\text{A8})$$

is always allowed.

Note that if we have effectively just the  $Z_M^R$  symmetry, (as in the models of section IV, where both  $S$  and  $\Phi_i \bar{\Phi}_i$  are neutral under the  $Z_N$ ), then

$$S^{M-1} (\Phi_1 \bar{\Phi}_1)^2 \quad (\text{A9})$$

is allowed (corresponding to  $N = 1$  in (A8)).

## APPENDIX B: LANDAU POLES

Since our models require a large number of representations at  $M_{\text{GUT}}$ , there is a potential problem of hitting a Landau pole below the Planck scale. We investigate this issue here. The RGE reads

$$\frac{1}{\alpha(M_1)} = \frac{1}{\alpha(M_2)} + \frac{b}{2\pi} \ln \frac{M_2}{M_1}. \quad (\text{B1})$$

We will hit a Landau pole,  $\alpha \rightarrow \infty$ , at a scale  $M_{\text{LP}}$  given by

$$\frac{M_{\text{LP}}}{M} = \exp\left(\frac{2\pi}{b\alpha(M)}\right). \quad (\text{B2})$$

Using  $\alpha(M_{\text{GUT}}) \sim 0.04$  and requiring that the Landau pole is not reached below the Planck scale, that is  $M_{\text{LP}}/M_{\text{GUT}} \gtrsim 10^2$ , we find  $b \lesssim 34$ .

For a supersymmetric  $SU(N)$  group, we have

$$b = \sum_a C(\phi_a) - 3C(\text{adj}),$$

$$C(\text{fun}) = 1/2, \quad C(\text{adj}) = N, \quad C(\text{as}) = N/2 - 1, \quad (\text{B3})$$

where fun, adj and as stand for, respectively, the fundamental, adjoint and antisymmetric representations. Defining for an  $SU(5) \times SU(5)'$  theory

$$\begin{aligned} n_h &= \#(5, 1) + \#(\bar{5}, 1), \\ n_\Phi &= \#(5, \bar{5}) + \#(\bar{5}, 5), \\ n_A &= \#(24, 1), \\ n_T &= \#(10, 1) + \#(\bar{10}, 1), \end{aligned} \quad (\text{B4})$$

we obtain for  $b(SU(5))$ :

$$b = \frac{1}{2}n_h + \frac{5}{2}n_\Phi + 5n_A + \frac{3}{2}n_T - 15. \quad (\text{B5})$$

In particular, the maximal  $b$  arises if we all the MSSM fields ( $n_h = 5$ ,  $n_T = 3$ ) and all  $M_{\text{gut}}$  fields ( $n_\Phi = 4$ ,  $n_A = 3$ ) transform under the same  $SU(5)$ :

$$n_h = 5, \quad n_\Phi = 4, \quad n_A = 3, \quad n_T = 3 \quad \implies \quad b = 17. \quad (\text{B6})$$

Putting that in eq. (B2), with  $\alpha_{\text{GUT}} \simeq 1/25$ , we find

$$\frac{M_{\text{LP}}}{M_{\text{GUT}}} \simeq \exp\left(\frac{50\pi}{17}\right) \simeq 10^4. \quad (\text{B7})$$

Thus our models are safe against Landau poles.

## APPENDIX C: QUARK AND LEPTON MASSES

In this Appendix, we give expressions for the Yukawa matrices that arise in the various models we consider. The representations for the quark (or lepton) fields are given explicitly. The Higgs fields are denoted by  $h(5, 1)$ ,  $\bar{h}(\bar{5}, 1)$  and  $\bar{h}'(1, \bar{5})$ . Bifundamental fields are denoted by  $\Phi(5, \bar{5})$  and  $\bar{\Phi}(\bar{5}, 5)$ .

For the up sector, the following combinations are relevant for inducing the Yukawa couplings:

$$\begin{aligned}
& (10, 1) (10, 1) h, \\
& (1, 10) (1, 10) h \bar{\Phi}, \\
& (10, 1) (1, 10) h \Phi \Phi.
\end{aligned} \tag{C1}$$

For the down and charged lepton sectors, the following combinations are relevant for inducing the Yukawa couplings from  $\bar{h}'$ :

$$\begin{aligned}
& (1, 10) (1, \bar{5}) \bar{h}', \\
& (10, 1) (\bar{5}, 1) \bar{h}' \bar{\Phi}, \\
& (1, 10) (\bar{5}, 1) \bar{h}' \Phi [d/\ell], \\
& (10, 1) (1, \bar{5}) \bar{h}' \bar{\Phi} \bar{\Phi} [d/\ell],
\end{aligned} \tag{C2}$$

and from  $\bar{h}$ :

$$\begin{aligned}
& (10, 1) (\bar{5}, 1) \bar{h}, \\
& (1, 10) (1, \bar{5}) \bar{h} \Phi, \\
& (10, 1) (1, \bar{5}) \bar{h} \bar{\Phi} [d/\ell], \\
& (1, 10) (\bar{5}, 1) \bar{h} \Phi \Phi [d/\ell].
\end{aligned} \tag{C3}$$

Entries marked with  $[d, \ell]$  mean that these terms can give masses to either the down sector or the charged lepton sector but not to both. In the full high energy theory this depends on whether the bifundamental representations are  $\Phi_1$  or  $\Phi_2$  (or  $\bar{\Phi}_1$  or  $\bar{\Phi}_2$ ). From the point of view of the low energy theory, below  $M_{\text{GUT}}$ , this depends on the different  $Z'_N$  charges carried by  $\bar{d}_L d_R$  and  $\bar{\ell}_R \ell_L$ . ( $Z'_N$ -breaking will eventually lift the zero masses.)

Defining  $\epsilon = M_{\text{GUT}}/M_{\text{Pl}}$ , we get the following parametric suppression due to the  $SU(5) \times SU(5)$  gauge symmetry (entries in parenthesis vanish in either  $M_d$  or  $M_\ell$ ):

Model **A**(1), with (1, 10) as first generation:

$$M_u \sim \langle h \rangle \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \\ \epsilon^2 & 1 & 1 \end{pmatrix}, \quad M_d \sim \langle \bar{h}' \rangle \begin{pmatrix} (\epsilon) & (\epsilon) & (\epsilon) \\ \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}. \tag{C4}$$

Model **A**(2), with (10, 1) as third generation and ( $\bar{5}, 1$ ) as first generation:

$$M_u \sim \langle h \rangle \begin{pmatrix} \epsilon & \epsilon & \epsilon^2 \\ \epsilon & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}, \quad M_d \sim \langle \bar{h}' \rangle \begin{pmatrix} 1 & (\epsilon) & (\epsilon) \\ 1 & (\epsilon) & (\epsilon) \\ (\epsilon^2) & \epsilon & \epsilon \end{pmatrix}. \tag{C5}$$

Model **B**1(1):

$$M_u \sim \langle h \rangle \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad M_d \sim \langle \bar{h}' \rangle \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}. \tag{C6}$$

Model **B1(2)**, with  $(1, 10) + (1, \bar{5})$  as first generation:

$$M_u \sim \langle h \rangle \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \\ \epsilon^2 & 1 & 1 \end{pmatrix}, \quad M_d \sim \langle \bar{h}' \rangle \begin{pmatrix} 1 & (\epsilon) & (\epsilon) \\ (\epsilon^2) & \epsilon & \epsilon \\ (\epsilon^2) & \epsilon & \epsilon \end{pmatrix}. \quad (\text{C7})$$

Model **B1(3)**, with  $(10, 1) + (\bar{5}, 1)$  as third generation:

$$M_u \sim \langle h \rangle \begin{pmatrix} \epsilon & \epsilon & \epsilon^2 \\ \epsilon & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}, \quad M_d \sim \langle \bar{h}' \rangle \begin{pmatrix} 1 & 1 & (\epsilon) \\ 1 & 1 & (\epsilon) \\ (\epsilon^2) & (\epsilon^2) & \epsilon \end{pmatrix}. \quad (\text{C8})$$

Model **B2/C(1)**:

$$M_u \sim \langle h \rangle \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad M_d \sim \langle \bar{h} \rangle \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (\text{C9})$$

Model **B2/C(2)**, with  $(1, 10) + (1, \bar{5})$  as first generation:

$$M_u \sim \langle h \rangle \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \\ \epsilon^2 & 1 & 1 \end{pmatrix}, \quad M_d \sim \langle \bar{h} \rangle \begin{pmatrix} \epsilon & (\epsilon^2) & (\epsilon^2) \\ (\epsilon) & 1 & 1 \\ (\epsilon) & 1 & 1 \end{pmatrix}. \quad (\text{C10})$$

Model **B2/C(3)**, with  $(10, 1) + (\bar{5}, 1)$  as third generation:

$$M_u \sim \langle h \rangle \begin{pmatrix} \epsilon & \epsilon & \epsilon^2 \\ \epsilon & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}, \quad M_d \sim \langle \bar{h} \rangle \begin{pmatrix} \epsilon & \epsilon & (\epsilon^2) \\ \epsilon & \epsilon & (\epsilon^2) \\ (\epsilon) & (\epsilon) & 1 \end{pmatrix}. \quad (\text{C11})$$

Note that, in principle, the  $Z_N$  symmetry may constrain these matrices further. We checked that none of the above models is excluded by these constraints.

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