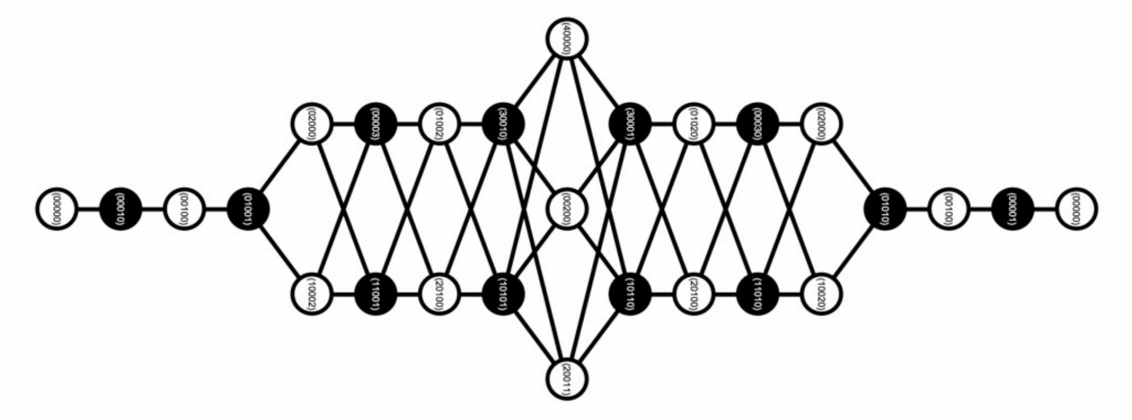
Adinkras in Super-Yang Mills Theory

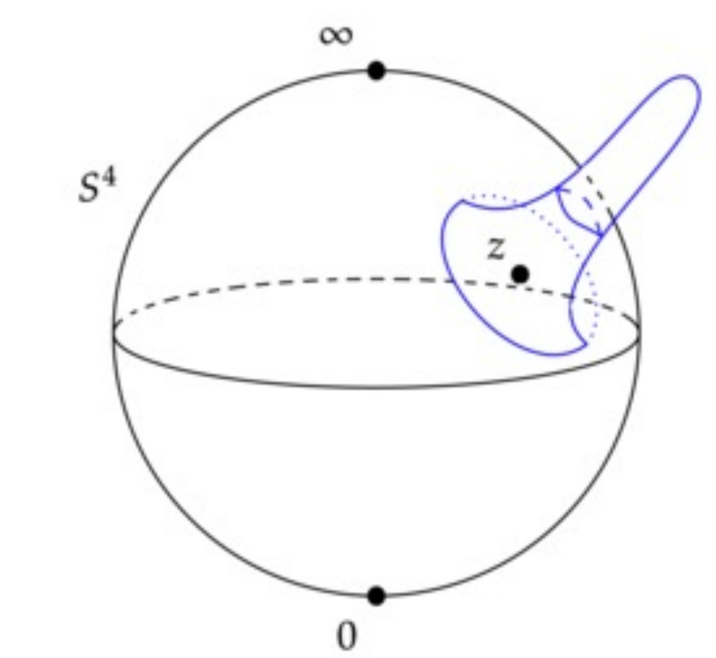


Presenter: Devin Bristow

Advisor: S. James Gates, Jr.

National Society of Black Physicists

Grand Unification in The Diaspora



SUPERSYMMETRY

Supersymmetry is an idea from subatomic particle physics that predicts interactions that change bosons into fermions and fermions into bosons. In other words, fermions have supersymmetric boson partners.

Bosons and fermions are two different kinds of particles. One difference is in their spin. Fermions have angular momentum and spin at a rate that is a half-integer. Bosons have angular momentum and spin at a rate that is an integer. Another (perhaps more notable) difference is that fermions obey the Pauli Exclusion Principle, whereas their counterpart bosons do not.

Supersymmetry has solved many problems since its inception (1970s), including computational problems (i.e., the hierarchy problem) of the Standard Model. The Standard Model consists of a list of all the known particles (and their properties) in our universe.

Ultimately, the goal of supersymmetry is to help us gain a better sense of the quantum mechanics of atoms and subatomic particles.

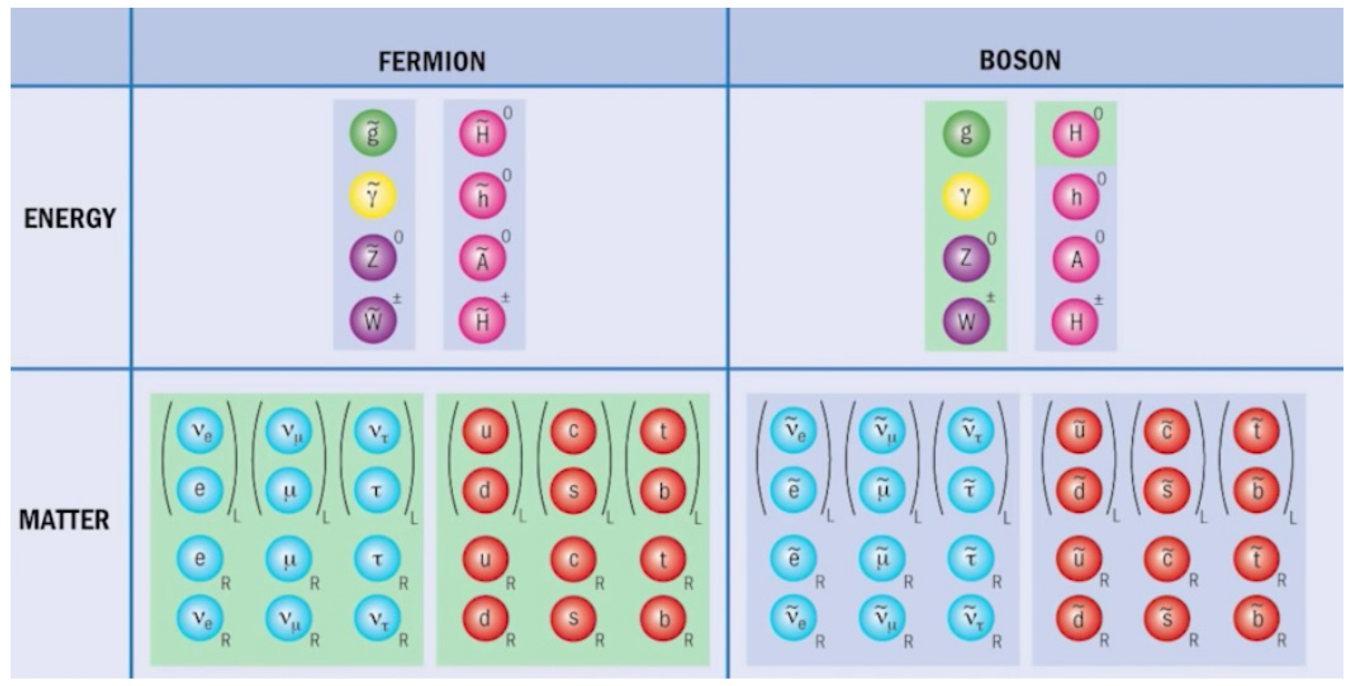


Figure 1: Supersymmetric universe

ADINKRAS

Adinkras are graphical tools used to study representations of supersymmetry algebras. They contain information about the transformation laws of supermultiplets.

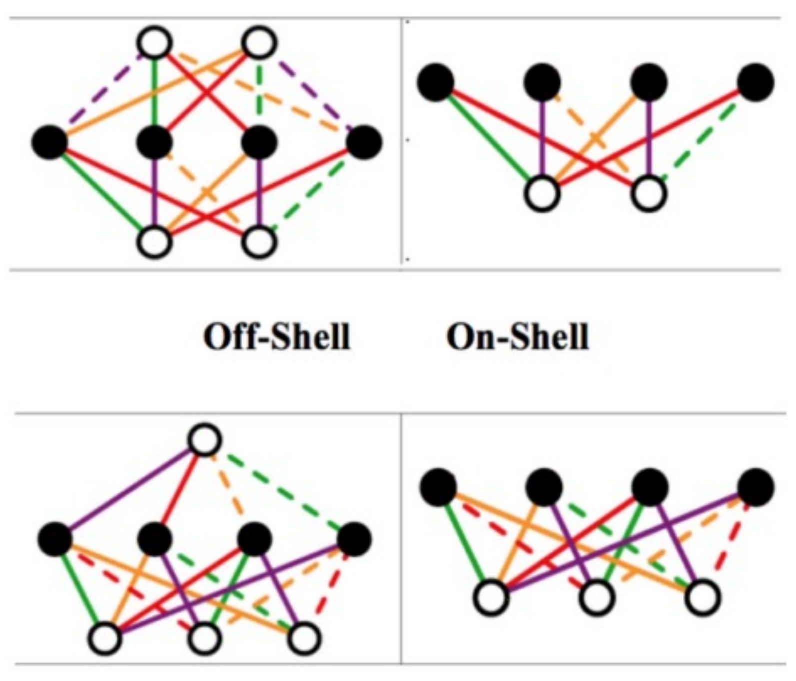


Figure 1: Examples of on-shell and off-shell adinkras [3].

- Black nodes = fermions, white nodes = bosons
- Colored lines = SUSY operator, D, applied to the fermion.
- Solid lines = real transformations, dashed lines = imaginary transformations.
- Colors correspond to spinor index in the transformation

ABSTRACT

This paper seeks to examine several extended SUSY Yang-Mills Theories on the 0-Brane by obtaining the L and R matrices, generate the corresponding adinkra, and studying their correlators. The transformation laws of the on-shell 10D, N=1 Super Yang-Mills Theory are given, and the SUSY algebra is shown to exhibit closure when the equations of motion are satisfied. The closure of the algebra for the 4D N=4 theory was calculated using new computational methods. The resulting adinkra matrices and SUSY algebra structure are investigated for these theories, and from this comparisons are made.

GENERATING THE ADINKRA

In order to generate an adinkra, we must first describe certain transformation laws (following 0-Brane reduction) as a set of vectors, from which these vectors are thought of as matrices. Only then may we obtain the L and R matrices, which we use to generate adinkras. The adinkra that is generated from a set of adinkra matrices in Super Yang-Mills Theory is shown below:

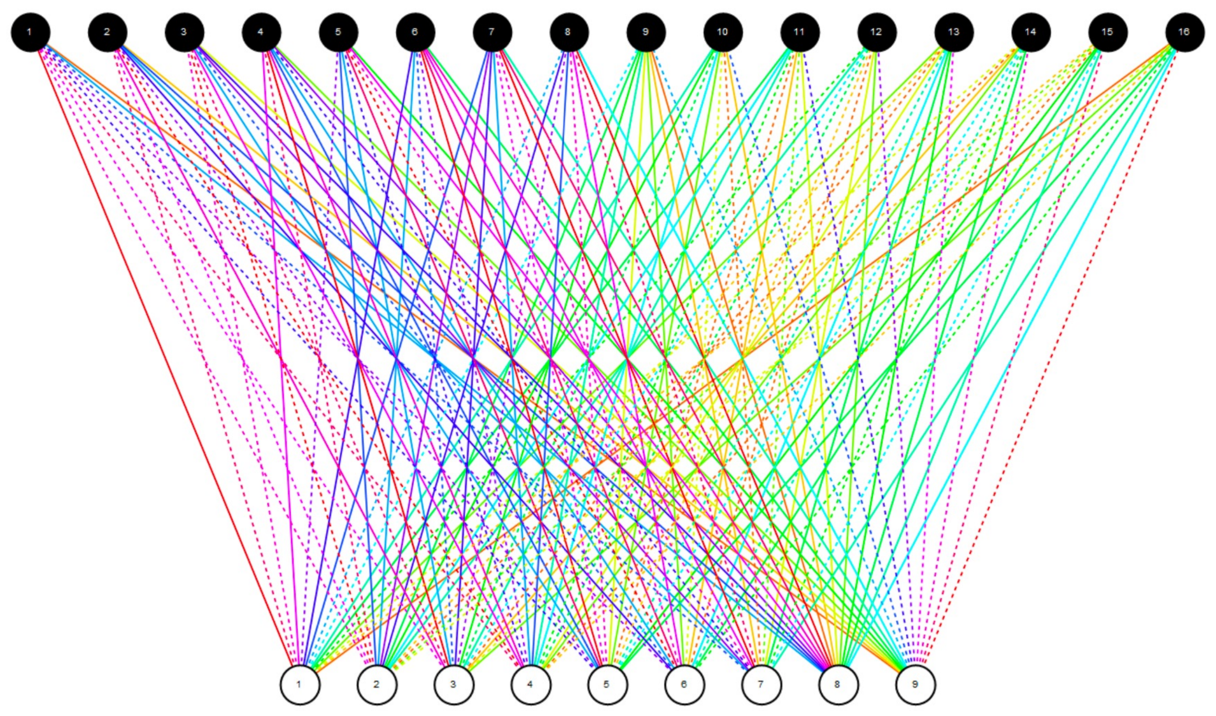


Figure 3: 1D, N=10 Super-Yang Mills Theory Adinkra

SUPER YANG-MILLS THEORY

Yang-Mills theory is a non-Abelian gauge theory, which means particles interact with each other through gauge bosons (or specific boson particles that act as force carriers). We use Yang-Mills theory to describe the behavior of elementary particles using non-Abelian Lie groups. Super Yang-Mills theory is Yang-Mills theory equipped with a supersymmetric structure.

In this research project, the goal was to generate a series of adinkras in several dimensions and study their properties. To do so, we started with Super Yang-Mills Theory in higher dimensions, performed closure on the D-algebra, reduced it to 1D, obtain the L and R matrices, and generate the Adinkra.

Thus far, we have only completed this story in 10 dimensions. In 10 dimensions, we consider 2 fields: the gauge and gravitino. Indeed, that from this, we closed the D-algebra on 2 fields. Because we are in 10 dimensions, the L and R matrices are huge: 16 x 16.

Upon generating an Adinkra, we then examine its correlators. Since an Adinkra is a graph, it comprises of vertices and edges. The correlators on an Adinkra are its edges. By studying its correlators, we can discuss a whole host of other things, like other adinkras that are isomorphic to it, how supersymmetry transforms the particle it acts upon, and so forth.

RESULTS

When we talk about Adinkras, what we're after is the garden algebra. The reason being is because the garden algebra is believed to be highly related to 1D supersymmetric algebra. That is, if the L and R matrices that are used to generate an adinkra satisfy a garden algebra, then a supersymmetry occurs within a given system of particles.

Recall that the garden algebra is as follows:

$$(L_I)_i{}^{\hat{j}}(R_J)_{\hat{j}}{}^k + (L_J)_i{}^{\hat{j}}(R_I)_{\hat{j}}{}^k = 2\delta_i{}^k\delta_{IJ}$$

$$(R_I)_{\hat{i}}{}^j(L_J)_j{}^{\hat{k}} + (R_J)_{\hat{i}}{}^j(L_I)_j{}^{\hat{k}} = 2\delta_{\hat{i}}{}^{\hat{k}}\delta_{IJ}$$

For the adinkra we generated, the first equation is satisfied, that is, when only bosonic nodes are left. The second equation however is not satisfied, that is when only fermionic nodes are left. The number of fermionic fields in the on-shell theory is greater than the number of bosonic fields, and every particle can only have one S-partner. This means that not all fermions will have a bosonic counterpart for each color in the SUSY group.

COLLABORATION

This research project was done in collaboration with Dr. Sylvester James Gates, Jr., Gabriel Yerger, John Caporaletti, Yuqin Wang, Eric Johnson-Greer, Aleksander Cianciara, Shane Weiner, Jude Bedessem, Jeth Arunseangroj, Aaron Beisaw.

We also thank Dr. Kory Stiffler, Yangrui Hu, and Hazel Mak for their suggestions in helping with the project.

Each person's knowledge in various areas like Lagrangian mechanics, Lie algebras, string theory, supersymmetry, graph theory, and geometry brings this project closer to fruition.

REFERENCES

[1] C. Sutton. (1998). Supersymmetry, *Encyclopaedia Britannica. https://www.britannica.com/science/supersymmetry* [2] A. Bilal. (2000). Introduction to Supersymmetry. https://arxiv.org/abs/hep-th/0101055 [3] S. J. Gates, Jr., J. Gonzales, B. MacGregor, J. Parker, R. Polo-Sherk, V. G. J. Rodgers, and L. Wassink. (2009). 4D, N = 1 Supersymmetry Genomics. arXiv: 0902.3830 [hep-th]. [4] S. J. Gates, Jr., Y. Hu, and K. Stiffler. (2019). Adinkra Height Yielding Matrix Numbers: Eigenvalue Equivalence Classes for Minimal Four-Color Adinkras. arXiv: 1904.01738 [hep-th] [5] D. D. Bristow, J. C. Caporaletti, A. J. Cianciara, S. J. Gates, Jr., D. Levine, G. Yerger. (2020). A Note On Exemplary Off-Shell Constructions Of 4D, N = 2 Supersymmetry Representations. arXiv: 2012.14015 [hep-th] [6] S. J. Gates, Jr. and K. Stiffler. (2014). Adinkra 'color' confinement in exemplary off-shell constructions of 4D, N = 2 supersymmetry representations. *Journal of High Energy Physics, 2014*(51). arXiv: 0902.3830 [hep-th]