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A codex on linearized Nordström supergravity in eleven and ten dimensional superspaces

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Abstract: As the full off-shell theories of supergravity in the important dimensions of eleven and ten dimensions are currently unknown, we introduce a superfield formalism as a foundation and experimental laboratory to explore the possibility that the scalar versions of the higher dimensional supergravitation theory can be constructed.

Keywords: Supergravity Models, Superspaces, Supersymmetric Effective Theories

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1 Introduction

In 1907, Einstein described his "happiest thought" [1] which marked the commencement of the race to create the Theory of General Relativity. Unrealized, he was already decidedly at a disadvantage. As early 1900, the astronomer Karl Schwarzschild (1873–1916) [2] had written about Riemann's geometrical concepts to describe curved space — but not curved space-time. The latter would not emerge until Hermann Minkowski introduced the concept of "four-geometry" into physics [3, 4].

By 1914, there were a number of competitors. At a minimum, these included Max Abraham (1875–1922), Gustav Mie (1868–1957), and Gunnar Nordström (1881–1923). Even the accomplished mathematician David Hilbert (1862–1943) became involved but at the conclusion made the comment in his own work, "The differential equations of gravitation that result are, as it seems to me, in agreement with the magnificent theory of general relativity established by Einstein [5]."

Along the pathway to the end of the race, the idea of scalar gravitational theories was explored. It is of note that Nordström created two such sets of equations [6, 7] and even Einstein looked at this idea before discarding it. In the scalar approach, the usual metric of the space-time manifold is replaced by a single scalar field. A way to do this is to begin with the Minkowski metric and simply multiply it by a scalar field. This implies that, geometrically, scalar theories of gravitation are all members of the same conformal class as the usual flat Minkowski metric. Mathematically, scalar gravitation theories are perfectly consistent models they simply do not describe the physical laws observed in our universe.

In the nineteen eighties, superspace geometrical descriptions of supergravity in eleven and ten dimensions were presented in the physics literature. To the best of our knowledge a list of these inaugural publications looks as:

(a). 11D, $\mathcal{N} = 1$ supergravity [8, 9],

(b). 10D, $\mathcal{N} = 2$A supergravity, [10],

(c). 10D, $\mathcal{N} = 2$B supergravity [11, 12], and

(d). 10D, $\mathcal{N} = 1$ supergravity [13].

Of course these theories had been obtained in other descriptions even earlier. Interested parties are directed to these works for such references.

If we think of the references in [8–13] as the eleven and ten dimensional analogs of Einstein's "happiest thought," then by analogy all developments since these works are analogs of the race to find the Theory of General Relativity. This also reveals a glaring disappointment. Since the "bell was rung" in this new race, all the competition is still at the starting line.

How would one know the race has been successfully ended? As indicated by the title of the work [9] ("Eleven-Dimensional Supergravity on the Mass-Shell in Superspace"), these descriptions possess sets of Bianchi identities that are consistent *only* when the equations of motion for the component fields in the theory satisfy their mass-shell conditions. This

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holds true for all of the works in [8–13]. So we may take as the sign of the successful completion of the race, if a set of superspace geometries were explicitly found such that their Bianchi identities do *not* require a mass shell condition. By way of comparison, for 4D, $\mathcal{N} = 1$ superspace supergravity, the analogs of the "happiest thought" and the conclusion of the race occurred within one year as is seen via the works completed by Wess and Zumino [14, 15].

A new urgency has recently appeared and which drives a need to improve the current situation. Recent progress [17–19] occurred in the derivation of M-Theory corrections to 11D Supergravity. A series of procedures connecting the corrections to a 3D, $\mathcal{N} = 2$ Chern-Simons theory [20–23] (used in a role roughly analogous to world-sheet σ-model β-function calculations for string corrections) has been successfully demonstrated.

Though the works in [17–19] have presented a method of deriving these corrections beyond the supergravity limit, these *solely* treat purely bosonic M-Theory corrections, with no equivalent results describing fermionic corrections. One traditional way of accomplishing this is to embed the purely bosonic results into a superspace formulation. This impels us to a renewed interest in 11D supergravity in superspace. The goal we are pursuing is to find a Salam-Strathdee superspace [24], as modified by Wess & Zumino [14, 15], such that superspace Bianchi identities in the Nordström limit allow for the appearance of the M-Theory corrections.

In the earliest days of formulating the superspace understanding of supergravity, an important discovery was made in a work by Grimm, Wess and Zumino [16] where it was shown the dynamical equations of motion of the component fields contained within the supergravity supermultiplet could be derived with *no reference at all to a superspace action*. It is important to *note* all the superspace formulations in [8–13] exhibit this feature. In fact, for the case of [12], there exists no universally accepted action principle to describe the theory... even at the component level. Thus, using superspace formulations, the dynamics of the component fields can be obtained without a complete knowledge of the structure of the superfields that contain them. This allows for a "clean separation" between issues of representations from those of dynamics of the component fields so being described. Our investigation will follow this minimalist approach.

As noted by Misner and Watt [25], though scalar gravitational theories are not realistic, they have value as computational tools in numerical relativity. This raises a very intriguing question. The quote, "History doesn't repeat itself, but it often rhymes",[1] has been stated about many situations. Since scalar gravitation models have value as computational tools for General Relativity, might extending them to eleven and ten dimensional supergeometries offer new ways to replicate Einstein's path from the "happiest thought" to the higher level of understanding as indicated by his lectures at Göttingen?

It is the purpose of this work to lay a new foundation for such an exploration. We take as a guiding principle the procedure used to provide the first example of a four dimensional supergravity supermultiplet including auxiliary fields and where the closure of the local simple supersymmetry algebra was not predicated on the use of field equations. This was

[1]Though often attributed to Mark Twain, there is little evidence of this being accurate.

accomplished by Breitenlohner [26] who took his starting point as an off-shell supermultiplet, the so-called "non-abelian vector supermultiplet." The final form of Breitenlohner's initial presentation realized a reducible representation of supersymmetry. Finally, this first work also did *not* consider the issue surrounding the construction of an action for the supermultiplet.

His approach is equivalent to starting with the component fields of the Wess-Zumino gauge 4D, $\mathcal{N} = 1$ vector supermultiplet $(v_{\underline{a}},\, \lambda_b, \mathrm{d})$ together with their familiar SUSY transformation laws,

$$\begin{aligned} \mathrm{D}_a\, v_{\underline{b}} &= (\gamma_{\underline{b}})_a{}^c\, \lambda_c\,, \\ \mathrm{D}_a \lambda_b &= -\, i\, \frac{1}{4}([\,\gamma^{\underline{c}}\,,\, \gamma^{\underline{d}}\,])_{ab}\, (\,\partial_{\underline{c}}\, v_{\underline{d}} - \partial_{\underline{d}}\, v_{\underline{c}}\,) + (\gamma^5)_{a\,b}\, \mathrm{d}\,, \\ \mathrm{D}_a\, \mathrm{d} &= i\, (\gamma^5 \gamma^{\underline{c}})_a{}^b\, \partial_{\underline{c}} \lambda_b\,, \end{aligned} \tag{1.1}$$

followed by choosing the gauge group as the spacetime translations, SUSY generators, and the spin angular momentum generators as well as allowing additional internal symmetries. For the spacetime translations, this requires a series of replacements of the fields according to:

$$v_{\underline{b}} \to h_{\underline{b}\,\underline{c}}\,, \quad \lambda_b \to \psi_{\underline{c}\,b}\,, \quad \mathrm{d} \to A_{\underline{c}}\,, \tag{1.2}$$

(in the notation in [26] $A_{\underline{a}} = B^5{}_{\underline{a}}$) while for the SUSY generators, the replacements occur according to:

$$v_{\underline{b}} \to \chi_{\underline{b}\,c}\,, \quad \lambda_b \to \phi_{b\,c}\,, \quad \mathrm{d} \to \chi_c{}^5\,, \tag{1.3}$$

and finally for the spin angular momentum generator, a replacement of

$$v_{\underline{b}} \to \omega_{\underline{b}\,\underline{c}\,\underline{d}}\,, \quad \lambda_b \to \chi_{b\,\underline{c}\,\underline{d}}\,, \quad \mathrm{d} \to D_{\underline{c}\,\underline{d}}\,, \tag{1.4}$$

was used. However, to be more exact, Breitenlohner also allowed for more symmetries like chirality to be included. Because the vector supermultiplet was off-shell (up to WZ gauge transformations) the resulting supergravity theory was off-shell and included a redundant set of auxiliary component fields, i.e. this is not an irreducible description of supergravity. But as seen from (1.2) the supergravity fields were all present and together with the remaining component fields a complete superspace geometry can be constructed.

With the Breitenlohner approach as a guiding principle to the study of a class of curved supermanifolds containing eleven and ten dimensions, it is thus to be expected the extensions will manifest the same structure of being off-shell but reducible, and not address the issue of the construction of actions. No off-shell gauge vector supermultiplet is known beyond six dimensions, thus one is forced to deviate from completely following the Breitenlohner approach. Since a scalar superfield in any dimension greater than three is guaranteed to be off-shell, but reducible, the expectation is to be led to the study of Nordström supergravity theories in eleven and ten dimensions in this approach.

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In our approach to Nordström SG, the analog of the Wess-Zumino gauge 4D, $\mathcal{N}=1$ vector supermultiplet is played by a scalar superfield in any of the 11D or 10D superspaces to be studied. This scalar superfield guarantees off-shell supersymmetry. However, like the approach taken by Breitenlohner, the resulting theory is expected to be reducible. Also like this earlier approach, the question of an action principle is not addressed.

We organize this current paper in the manner described below.

Section 2 provides a self-consistent introduction to the field-theory and gauge-theory based formulation of gravitation described solely by a metric in D dimensions. We use a frame field/spin-connection formulation from the beginning point of our discussion. This eases the transition to the case of superspace as for these latter theories it is an impossibility [27] to introduce a metric/Christoffel formulation (i.e. a Riemannian formulation) in the context of a superspace geometry appropriate to supersymmetry. The restriction of the full frame field to retain only the degree of freedom associated with its determinant is presented along with:

(a). the well-known vanishing of the Weyl tensor, and

(b). the residual form of the Einstein-Hilbert action under this restriction.

Section 3 is a transitional one where we review 4D, $\mathcal{N}=1$ supergravity as a paradigm setting arena. We show how the structure of this superspace in this well studied theory suggests pathways that can be pursued for how to carry out construction of scalar supergravitation in all higher dimensions including ten and eleven dimensional theories.

In sections 4 through 7, we deploy the lessons found in the third section to work in making respective proposals for linearized theories of scalar supergravitation in the 11D, $\mathcal{N}=1$, 10D, $\mathcal{N}=2$A, 10D, $\mathcal{N}=2$B, and 10D, $\mathcal{N}=1$ superspaces.

Section 8 is devoted to presentations of all the component level results implied by the previous four sections. Each respective higher dimensional theory is treated as subsection of this section.

Section 9 is a short section in comparison to the two that precede it. In 4D, $\mathcal{N}=1$ supergravity [28, 29], the concept of the "conformal compensator" was introduced some time ago. The first of these references is distinguished from the second in that the compensator is described by a chiral superfield, while in the second the alternative case where the compensator is a complex linear superfield was introduced. It is the latter case which is relevant to our exploration. However, we will demonstrate evidence that chiral superfields exist for the 10D, $\mathcal{N}=2$B superspace. This is unique among superspaces of eleven and ten dimensions. However, we also present evidence that though chiral superfields appear to consistently exist in this context, the linearized Nordström superspace is such that a chiral superfield of this type cannot be used as a compensator.

We follow this work with our conclusions, two appendices, and the bibliography.

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2 Gauge theory perspective on ordinary gravity

The traditional geometrical approach to describing gravity can be regarded as having driven an apparent wedge between general relativity and theory of elementary particles. Instead, a gauge theory and field theory based point of view provides a logical foundation for gravity which permits an alternative to geometry.

For gravitational theories in D dimensions, the gauge group can be taken as the Poincaré group, and the Lie algebra generators are momentum $P_{\underline{m}} = -i\partial_{\underline{m}}$ and spin angular momentum generator $\mathcal{M}_{\underline{ab}}$. These are taken to satisfy the following commutation relations,

$$[\, P_{\underline{m}} \,,\, P_{\underline{n}} \,] = 0 \,, \qquad [\, \mathcal{M}_{\underline{ab}} \,,\, P_{\underline{m}} \,] = 0 \,, \qquad [\, \mathcal{M}_{\underline{ab}} \,,\, \partial_{\underline{c}} \,] = \eta_{\underline{ca}}\partial_{\underline{b}} - \eta_{\underline{cb}}\partial_{\underline{a}} \,, \tag{2.1}$$

$$[\, \mathcal{M}_{\underline{ab}} \,,\, \mathcal{M}_{\underline{cd}} \,] = \eta_{\underline{ca}}\mathcal{M}_{\underline{bd}} - \eta_{\underline{cb}}\mathcal{M}_{\underline{ad}} - \eta_{\underline{da}}\mathcal{M}_{\underline{bc}} + \eta_{\underline{db}}\mathcal{M}_{\underline{ac}} \,, \tag{2.2}$$

and it might appear that the definition $P_{\underline{m}} = -i\partial_{\underline{m}}$ together with the second and third equations among (2.1) are in contradiction. The resolution to this conundrum is to note

$$\partial_{\underline{a}} \equiv \delta_{\underline{a}}{}^{\underline{m}}\, \partial_{\underline{m}} \,, \tag{2.3}$$

and the factor of $\delta_{\underline{a}}{}^{\underline{m}}$ actually corresponds to the vacuum value of the inverse frame field $e_{\underline{a}}{}^{\underline{m}}$ whose first index transforms under the action of $\mathcal{M}_{\underline{ab}}$ and whose second index is inert under the action of the spin angular momentum generator. To distinguish between these two types of quantities, we use the "early" latin letters, $\underline{a}$, $\underline{b}$, etc. to denote indices that transform under the action of $\mathcal{M}_{\underline{ab}}$. Similarly, we use the "late" latin letters, $\underline{m}$, $\underline{n}$, etc. to denote indices that do *not* transform under the action of $\mathcal{M}_{\underline{ab}}$.

The covariant derivative with respect to this gauge group is

$$\nabla_{\underline{a}} \equiv e_{\underline{a}}{}^{\underline{m}}\partial_{\underline{m}} + \frac{1}{2}\omega_{\underline{ac}}{}^{\underline{d}}\mathcal{M}_{\underline{d}}{}^{\underline{c}} \,, \tag{2.4}$$

where $e_{\underline{a}}{}^{\underline{m}}$ is related to the metric through its inverse $e_{\underline{m}}{}^{\underline{a}}$ via $g_{\underline{mn}} = e_{\underline{m}}{}^{\underline{a}}\eta_{\underline{ab}}e_{\underline{n}}{}^{\underline{b}}$. The commutator of $\nabla_{\underline{a}}$ generates field strengths torsions $T_{\underline{abc}}$ and curvatures $R_{\underline{abcd}}$

$$[\, \nabla_{\underline{a}} \,,\; \nabla_{\underline{b}} \,] = T_{\underline{ab}}{}^{\underline{c}}\, \nabla_{\underline{c}} + \frac{1}{2}R_{\underline{abc}}{}^{\underline{d}}\, \mathcal{M}_{\underline{d}}{}^{\underline{c}} \,. \tag{2.5}$$

Scalar gravitation can be defined by restricting the form of the inverse frame field to

$$e_{\underline{a}}{}^{\underline{m}} = \psi\, \delta_{\underline{a}}{}^{\underline{m}} \,, \tag{2.6}$$

where ψ is a finite scalar field. By definition, this defines a class of geometries that is conformally flat in the context of strictly Riemannian spaces. To see this we begin by setting $T_{\underline{abc}} = 0$, which implies

$$\nabla_{\underline{a}} = \psi\, [\, \partial_{\underline{a}} - \, (\partial_{\underline{b}} \ln \psi)\mathcal{M}_{\underline{a}}{}^{\underline{b}} \,] \,. \tag{2.7}$$

and allows the full Riemann curvature tensor to be solely expressed in terms of the ψ field as

$$R_{\underline{ab}}{}^{\underline{cd}} = -\psi(\partial_{[\underline{a}}\partial^{[\underline{c}}\psi)\delta_{\underline{b}]}{}^{\underline{d}]} + (\partial^{\underline{e}}\psi)(\partial_{\underline{e}}\psi)\delta_{[\underline{a}}{}^{\underline{c}}\delta_{\underline{b}]}{}^{\underline{d}} \,, \tag{2.8}$$

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similarly for the Ricci curvature we find

$$R_{\underline{a}}{}^{\underline{c}} = R_{\underline{a}\underline{b}}{}^{\underline{c}\underline{b}} = -(D-2)\psi(\partial_{\underline{a}}\partial^{\underline{c}}\psi) - \psi(\Box\psi)\delta_{\underline{a}}{}^{\underline{c}} + (D-1)(\partial^{\underline{e}}\psi)(\partial_{\underline{e}}\psi)\delta_{\underline{a}}{}^{\underline{c}}\,, \tag{2.9}$$

and finally for the curvature scalar by $R = \delta_{\underline{c}}{}^{\underline{a}} R_{\underline{a}}{}^{\underline{c}}$,

$$R = -2(D-1)\psi(\Box\psi) + D(D-1)(\partial^{\underline{e}}\psi)(\partial_{\underline{e}}\psi)\,. \tag{2.10}$$

The Weyl tensor $\mathcal{C}_{\underline{a}\,\underline{b}}{}^{\underline{c}\,\underline{d}}$ is defined by the equation

$$\mathcal{C}_{\underline{a}\,\underline{b}}{}^{\underline{c}\,\underline{d}} = R_{\underline{a}\,\underline{b}}{}^{\underline{c}\,\underline{d}} - \left[\frac{1}{D-2}\right] R_{[\underline{a}}{}^{[\underline{c}}\delta_{\underline{b}]}{}^{\underline{d}]} + \left[\frac{1}{(D-2)(D-1)}\right]\delta_{[\underline{a}}{}^{\underline{c}}\delta_{\underline{b}]}{}^{\underline{d}} R\,, \tag{2.11}$$

and when the results in (2.8)–(2.10) are used, this is found to vanish.

We define $e \equiv \det(e_{\underline{a}}{}^{\underline{m}}) = \psi^D$ and the Einstein-Hilbert action takes the form

$$\begin{aligned} S_{\mathrm{EH}} &= \frac{3}{\kappa^2}\int d^D x\; e^{-1} R(\psi) = \frac{3}{\kappa^2}\int d^D x \psi^{-D}\big[-2(D-1)\psi(\Box\psi) + D(D-1)(\partial^{\underline{e}}\psi)(\partial_{\underline{e}}\psi)\big] \\ &= \frac{3}{\kappa^2}\int d^D x\big[-2(D-1)\psi^{1-D}(\Box\psi) + D(D-1)\psi^{-D}(\partial^{\underline{e}}\psi)(\partial_{\underline{e}}\psi)\big] \\ &= -\frac{3}{\kappa^2}(D-1)\int d^D x\big\{(D-2)\big[\psi^{-D}(\partial^{\underline{e}}\psi)(\partial_{\underline{e}}\psi)\big] + 2\partial^{\underline{e}}\big[\psi^{1-D}(\partial_{\underline{e}}\psi)\big]\big\}\,. \end{aligned} \tag{2.12}$$

As the full off-shell description of 10D and 11D supergravities are yet unknown, we work with a toy model-scalar supergravity in the higher dimensions, which we expect gives part of the complete solutions. In the subsequent sections, we replace ψ by $1+\Psi$, where Ψ is an infinitesimal superfield, and study the corresponding linearized supergravity.

3 Nordström supergravity in 4D, $\mathcal{N}=1$ supergeometry

As a preparatory step for our eventual goals, it is important that we re-visit four dimensional $\mathcal{N}=1$ linearized supergravity as there are important lessons to be gained from asking questions solely in this domain prior to making the leap to eleven and ten dimensions. The formulation of linearized 4D, $\mathcal{N}=1$ supergravity in term of the usual supergravity pre-potential $H^{\underline{a}}$ was identified long ago [30]. It is perhaps of importance to note that supergravity pre-potentials bare some resemblance to other better known concepts important for the mathematical description of theories describing ordinary gravitation.

One of the most computationally enabling formulations of the dynamics of ordinary gravitation is based on the Arnowitt-Deser-Meisner (ADM) formulation [31] wherein the quadratic form involving the metric is expressed in the form

$$dx^{\underline{m}} g_{\underline{m}\,\underline{n}} dx^{\underline{n}} = \Big(-N^2 + N_{\underline{i}}\delta^{\underline{i}\,\underline{j}}N_{\underline{j}}\Big)dt\otimes dt + N_{\underline{i}}\left(dt\otimes dx^{\underline{i}} + dx^{\underline{i}}\otimes dt\right) + \mathrm{G}_{\underline{i}\,\underline{j}}\, dx^{\underline{i}} dx^{\underline{j}}\,, \tag{3.1}$$

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in terms of the "lapse" function N, "shift" vector $N_{\underline{i}}$, and induced 3-metric $\mathrm{G}_{\underline{i}\,\underline{j}}$. For the equation above to be valid, we can write $dx^{\underline{m}} = \big(c\,dt,\; dx^{\underline{1}},\; dx^{\underline{2}},\; dx^{\underline{3}}\big)$ and

$$g_{\underline{m}\,\underline{n}} = \begin{bmatrix} \frac{1}{c^2}\,[\,-N^2 + N_{\underline{i}}\delta^{\underline{i}\,\underline{j}}N_{\underline{j}}\,] & \frac{1}{c}\,N_{\underline{1}} & \frac{1}{c}\,N_{\underline{2}} & \frac{1}{c}\,N_{\underline{3}} \\ \frac{1}{c}\,N_{\underline{1}} & \mathrm{G}_{\underline{1}\,\underline{1}} & \mathrm{G}_{\underline{1}\,\underline{2}} & \mathrm{G}_{\underline{1}\underline{3}} \\ \frac{1}{c}\,N_{\underline{2}} & \mathrm{G}_{\underline{2}\,\underline{1}} & \mathrm{G}_{\underline{2}\,\underline{2}} & \mathrm{G}_{\underline{2}\,\underline{3}} \\ \frac{1}{c}\,N_{\underline{3}} & \mathrm{G}_{\underline{3}\,\underline{1}} & \mathrm{G}_{\underline{3}\,\underline{2}} & \mathrm{G}_{\underline{3}\,\underline{3}} \end{bmatrix} . \tag{3.2}$$

The introduction of frame fields can be accomplished by observing that the quadratic form in (3.1) may also be written as

$$dx^{\underline{m}}\, g_{\underline{m}\,\underline{n}}\, dx^{\underline{n}} \equiv dx^{\underline{m}}\, e_{\underline{m}}{}^{\underline{a}}\, \eta_{\underline{a}\underline{b}}\, e_{\underline{n}}{}^{\underline{b}}\, dx^{\underline{n}} \quad , \tag{3.3}$$

by "factorizing" the metric into the product of two frame fields $e_{\underline{m}}{}^{\underline{a}}(x)$ and $e_{\underline{n}}{}^{\underline{b}}(x)$ multiplied by the constant Minkowski metric, $\eta_{\underline{a}\underline{b}}$, of flat spacetime. Thus, there exist relations between the ADM variables and the frame fields [32, 33].

The point of the above discussion is to note that the inverse frame fields $e_{\underline{a}}{}^{\underline{m}}$ may be regarded as functions of the ADM variables, i.e.

$$e_{\underline{a}}{}^{\underline{m}} = e_{\underline{a}}{}^{\underline{m}}\big(N,\; N_{\underline{i}},\; \mathrm{G}_{\underline{i}\,\underline{j}}\big)\,, \tag{3.4}$$

and that for numerical relativity calculations, the latter are far more useful than the frame fields $e_{\underline{m}}{}^{\underline{a}}$, or even the metric $g_{\underline{m}\,\underline{n}}$ itself. As we will see later, it is the form of 4D, $\mathcal{N} = 1$ supergravity often called the "Breitenlohner auxiliary field set" that is relevant to our work. For this formulation, it was first shown in the work of [29] the super-frame superfields $E_{\underline{A}}{}^{\underline{M}}$ are expressed in terms of a more fundamental set of superfields, i.e. the prepotentials $H^{\underline{m}}$ and Ψ (with the "conformal compensator" explicitly dependent upon a complex linear superfield). In an "echo" of the utility of the ADM variables, the prepotentials are far more useful when component calculations, or quantum calculations are undertaken, with the latter able to utilize the technology of super Feynman graphs.

As in the discussion of section 7.5 in [34], we write (with X being the superfield linearization of Ψ)

$$\begin{aligned} \mathrm{E}_{\alpha} &= \mathrm{D}_{\alpha} + \overline{X}\mathrm{D}_{\alpha} + i\frac{1}{2}\,(\mathrm{D}_{\alpha}H^{\underline{b}})\partial_{\underline{b}}\,, \\ \mathrm{E}_{\dot{\alpha}} &= \overline{\mathrm{D}}_{\dot{\alpha}} + X\overline{\mathrm{D}}_{\dot{\alpha}} - i\frac{1}{2}\,(\overline{\mathrm{D}}_{\dot{\alpha}}H^{\underline{b}})\partial_{\underline{b}}\,, \end{aligned} \tag{3.5}$$

$$\begin{aligned} \mathrm{E}_{\underline{a}} = \partial_{\underline{a}} &+ i\left[\frac{1}{2}\overline{\mathrm{D}}^2\mathrm{D}_{(\alpha}H^{\gamma)}{}_{\dot{\alpha}} - (\overline{\mathrm{D}}_{\dot{\alpha}}\overline{X})\delta_{\alpha}{}^{\gamma}\right]\mathrm{D}_{\gamma} + i\left[-\frac{1}{2}\mathrm{D}^2\overline{\mathrm{D}}_{(\dot{\alpha}}H_{\alpha}{}^{\dot{\gamma})} - (\mathrm{D}_{\alpha}X)\delta_{\dot{\alpha}}{}^{\dot{\gamma}}\right]\overline{\mathrm{D}}_{\dot{\gamma}} \\ &+ \left[-\frac{1}{2}([\mathrm{D}_{\alpha}\,,\;\overline{\mathrm{D}}_{\dot{\alpha}}]H^{\underline{b}}) + (X+\overline{X})\delta_{\underline{a}}{}^{\underline{b}}\right]\partial_{\underline{b}}\,. \end{aligned} \tag{3.6}$$

for the linearized superframe superfields. Similar to the ADM formulation of ordinary gravity, the superframe is expressed in terms of two independent superfields, $H^{\underline{a}}$, and X.

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The remaining structures needed to specify the supergravity supercovariant derivatives are the spin-connections which here take the forms

$$
\begin{aligned}
\Phi_{\alpha\beta\gamma} &= -C_{\alpha(\beta}\mathrm{D}_{\gamma)}\overline{X}\,, \\
\Phi_{\alpha\dot{\beta}\dot{\gamma}} &= \frac{1}{2}\mathrm{D}^2\overline{\mathrm{D}}_{(\dot{\beta}|}H_{\alpha|\dot{\gamma})}\,, \\
\Phi_{\dot{\alpha}\beta\gamma} &= -\frac{1}{2}\overline{\mathrm{D}}^2\mathrm{D}_{(\beta}H_{\gamma)\dot{\alpha}}\,, \\
\Phi_{\underline{a}\beta\gamma} &= i\frac{1}{2}\mathrm{D}_\alpha\overline{\mathrm{D}}^2\mathrm{D}_{(\beta}H_{\gamma)\dot{\alpha}} + iC_{\alpha(\beta|}\overline{\mathrm{D}}_{\dot{\alpha}}\mathrm{D}_{|\gamma)}\overline{X}\,.
\end{aligned}
\tag{3.7}
$$

The superfield X introduced above is a general scalar superfield. This implies that the linearized formulation described above *is* reducible because X is reducible. There are two widely familiar choices that lead to irreducibility. One choice is implemented by picking X to depend on $H^{\underline{a}}$, and a chiral superfield ϕ (i.e. $\overline{\mathrm{D}}_{\dot{\alpha}}\phi = 0$). This is the path that leads to the minimal off-shell formulation of 4D, $\mathcal{N} = 1$ supergravity. For this choice, the commutator algebra of the superspace supergravity covariant derivatives takes the forms

$$
\begin{aligned}
[\nabla_\alpha, \nabla_\beta\} &= -2\overline{R}\,\mathcal{M}_{\alpha\beta}\,, \qquad [\nabla_\alpha, \overline{\nabla}_{\dot{\alpha}}\} = i\nabla_{\underline{a}}\,, \\
[\nabla_\alpha, \nabla_{\underline{b}}\} &= -i\,C_{\alpha\beta}\Big[\overline{R}\,\overline{\nabla}_{\dot{\beta}} - G^\gamma{}_{\dot{\beta}}\nabla_\gamma\Big] - i\,(\overline{\nabla}_{\dot{\beta}}\overline{R})\mathcal{M}_{\alpha\beta} \\
&\quad + i\,C_{\alpha\beta}\Big[\overline{W}_{\dot{\beta}\dot{\gamma}}{}^{\dot{\delta}}\overline{\mathcal{M}}_{\dot{\delta}}{}^{\dot{\gamma}} - (\nabla^\delta G_{\gamma\dot{\beta}})\mathcal{M}_\delta{}^\gamma\Big]\,, \\
[\nabla_{\underline{a}}, \nabla_{\underline{b}}\} &= \bigg\{ \Big[C_{\dot{\alpha}\dot{\beta}}W_{\alpha\beta}{}^\gamma + \frac{1}{2}C_{\alpha\beta}(\overline{\nabla}_{(\dot{\alpha}}G^\gamma{}_{\dot{\beta})}) - \frac{1}{2}C_{\dot{\alpha}\dot{\beta}}(\nabla_{(\alpha}R)\delta_{\beta)}{}^\gamma\Big]\nabla_\gamma \\
&\quad + i\frac{1}{2}C_{\alpha\beta}G^\gamma{}_{(\dot{\alpha}|}\nabla_{\gamma|\dot{\beta})} \\
&\quad - \bigg[\, C_{\dot{\alpha}\dot{\beta}}\Big(\nabla_\alpha W_{\beta\delta\gamma} + \frac{1}{2}C_{\delta(\alpha}C_{\beta)\gamma}\,(\overline{\nabla}^2\overline{R} + 2R\overline{R})\Big) \\
&\quad - \frac{1}{2}C_{\alpha\beta}\,(\overline{\nabla}_{(\dot{\alpha}|}\nabla_\gamma G_{\delta|\dot{\beta})})\bigg]\mathcal{M}^{\gamma\delta}\bigg\} + \text{h.c.}\,.
\end{aligned}
\tag{3.8}
$$

The other widely known choice "the Breitenlohner auxiliary field set" is implemented by picking X to depend on $H^{\underline{a}}$, and a complex linear superfield Σ (i.e. $\overline{\mathrm{D}}^2\Sigma = 0$). This is the path that leads to the non-minimal off-shell formulation of 4D, $\mathcal{N} = 1$ supergravity. For this choice, the commutator algebra of the superspace supergravity covariant derivatives takes the forms

$$
\begin{aligned}
[\nabla_\alpha\,,\ \nabla_\beta\} &= \frac{1}{2}T_{(\alpha}\nabla_{\beta)} - 2\,\overline{R}\mathcal{M}_{\alpha\beta}\,, \\
[\nabla_\alpha\,,\ \overline{\nabla}_{\dot{\beta}}\} &= i\,\nabla_{\alpha\dot{\beta}}\,, \\
[\nabla_\alpha\,,\ \nabla_{\underline{b}}\} &= \frac{1}{2}\,T_\beta\nabla_{\alpha\dot{\beta}} - iC_{\alpha\beta}\left[\overline{R} + \frac{1}{4}(\nabla^\gamma T_\gamma)\right]\overline{\nabla}_{\dot{\beta}} \\
&\quad + i\left[C_{\alpha\beta}\,G^\gamma{}_{\dot{\beta}} - \frac{1}{2}C_{\alpha\beta}\left(\left(\nabla^\gamma + \frac{1}{2}T^\gamma\right)\overline{T}_{\dot{\beta}}\right) + \frac{1}{2}(\overline{\nabla}_{\dot{\beta}}T_\beta)\delta_\alpha{}^\gamma\right]\nabla_\gamma \\
&\quad - i\left[C_{\alpha\beta}\,(\nabla^\gamma G_{\delta\dot{\beta}})\mathcal{M}_\gamma{}^\delta + \big((\overline{\nabla}_{\dot{\beta}} - \overline{T}_{\dot{\beta}})\overline{R}\big)\mathcal{M}_{\alpha\beta}\right] \\
&\quad + iC_{\alpha\beta}\left[\overline{W}_{\dot{\beta}\dot{\gamma}}{}^{\dot{\delta}}\,\overline{\mathcal{M}}_{\dot{\delta}}{}^{\dot{\gamma}} + \frac{1}{6}\left(\nabla^\delta\left(\nabla_\delta + \frac{1}{2}T_\delta\right)\overline{T}_{\dot{\gamma}}\right)\overline{\mathcal{M}}_{\dot{\beta}}{}^{\dot{\gamma}} + \frac{1}{3}\,\overline{R}\,\overline{T}_{\dot{\gamma}}\,\overline{\mathcal{M}}_{\dot{\beta}}{}^{\dot{\gamma}}\right].
\end{aligned}
\tag{3.9}
$$

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The final commutator $[\nabla_{\underline{a}}\,,\ \nabla_{\underline{b}}\}$, derived from equation (3.9), is explicitly found to be

$$
\begin{aligned}
[\nabla_{\underline{a}}, \nabla_{\underline{b}}\} = \Big\{ & i\tfrac{1}{2}(\nabla_\beta \overline{T}_{\dot\beta})\nabla_{\underline{a}} - i\tfrac{1}{2}(\overline{\nabla}_{\dot\alpha} T_\beta)\nabla_{\alpha\dot\beta} - iC_{\dot\alpha\dot\beta}\Big[G_\beta{}^{\dot\gamma} + \tfrac{1}{2}\Big(\Big(\overline{\nabla}^{\dot\gamma} + \tfrac{1}{2}\overline{T}^{\dot\gamma}\Big)T_\beta\Big)\Big]\nabla_{\alpha\dot\gamma} \\
& + \tfrac{1}{2}\Big[(\overline{\nabla}_{\dot\alpha}\overline{\nabla}_{\dot\beta} T_\beta) - \tfrac{1}{2}\overline{T}_{\dot\beta}(\overline{\nabla}_{\dot\alpha} T_\beta)\Big]\nabla_\alpha \\
& - C_{\dot\alpha\dot\beta}\Big[(\nabla_\alpha R) + \tfrac{1}{4}(\nabla_\alpha \overline{\nabla}^{\dot\gamma}\overline{T}_{\dot\gamma}) + \tfrac{1}{3}RT_\alpha - \tfrac{1}{12}(\overline{\nabla}^{\dot\gamma}(\overline{\nabla}_{\dot\gamma} - \overline{T}_{\dot\gamma})T_\alpha)\Big]\nabla_\beta \\
& + C_{\dot\alpha\dot\beta}W_{\beta\alpha}{}^{\gamma}\nabla_\gamma + C_{\alpha\beta}\Big[(\overline{\nabla}_{\dot\alpha}G^\gamma{}_{\dot\beta}) - \tfrac{1}{2}\overline{T}_{\dot\beta}G^\gamma{}_{\dot\alpha}\Big]\nabla_\gamma + \tfrac{1}{2}C_{\alpha\beta}\Big[\tfrac{1}{2}(\nabla^\gamma \overline{T}_{\dot\alpha})\overline{T}_{\dot\beta} \\
& \qquad - \Big(\overline{\nabla}_{\dot\alpha}\Big(\nabla^\gamma + \tfrac{1}{2}T^\gamma\Big)\overline{T}_{\dot\beta}\Big) + \tfrac{1}{2}\Big(\Big(\overline{\nabla}_{\dot\alpha} + \tfrac{1}{2}\overline{T}_{\dot\alpha}\Big)\overline{T}_{\dot\beta}\Big)T^\gamma\Big]\nabla_\gamma \\
& + \Big[-(\overline{\nabla}_{\dot\alpha}\overline{\nabla}_{\dot\beta}\overline{R}) + \tfrac{1}{2}(\overline{\nabla}_{\dot\alpha}\overline{R})\overline{T}_{\dot\beta} + \overline{R}\Big(\overline{\nabla}_{\dot\alpha} + \tfrac{1}{2}\overline{T}_{\dot\alpha}\Big)\overline{T}_{\dot\beta}\Big]\mathcal{M}_{\alpha\beta} \\
& \qquad + 2C_{\dot\alpha\dot\beta}\overline{R}\Big[R + \tfrac{1}{4}(\overline{\nabla}^{\dot\gamma}\overline{T}_{\dot\gamma})\Big]\mathcal{M}_{\alpha\beta} \\
& - \tfrac{1}{6}C_{\dot\alpha\dot\beta}\Big[RT_\beta T_\gamma + \tfrac{1}{2}T_\beta(\overline{\nabla}^{\dot\delta}\overline{\nabla}_{\dot\delta}T_\gamma) + \tfrac{1}{4}T_\beta T_\gamma(\overline{\nabla}^{\dot\delta}\overline{T}_{\dot\delta})\Big]\mathcal{M}_\alpha{}^\gamma \\
& + \tfrac{1}{6}C_{\dot\alpha\dot\beta}\Big[2(\nabla_\alpha R)T_\gamma + 2R(\nabla_\alpha T_\gamma) + \Big(\nabla_\alpha\overline{\nabla}^{\dot\delta}\Big(\overline{\nabla}_{\dot\delta} + \tfrac{1}{2}\overline{T}_{\dot\delta}\Big)T_\gamma\Big) \\
& \qquad + \tfrac{1}{2}(\overline{\nabla}^{\dot\delta}\overline{T}_{\dot\delta})(\nabla_\alpha T_\gamma)\Big]\mathcal{M}_\beta{}^\gamma \\
& - C_{\alpha\beta}\Big[(\overline{\nabla}_{\dot\alpha}\nabla^\gamma G_{\delta\dot\beta}) - \tfrac{1}{2}\overline{T}_{\dot\beta}(\nabla^\gamma G_{\delta\dot\alpha})\Big]\mathcal{M}_\gamma{}^\delta + C_{\dot\alpha\dot\beta}\Big[(\nabla_\alpha W_{\beta\delta}{}^\gamma) - \tfrac{1}{2}T_\beta W_{\alpha\delta}{}^\gamma\Big]\mathcal{M}_\gamma{}^\delta\Big\} \\
& + \text{h.c.}\,.
\end{aligned}
\tag{3.10}
$$

Under either choice, one can use the definitions of the superframe superfields in (3.5)–(3.7) together with the set of equations of *either* (3.8) or (3.9) and (3.10) to find the dependence of $W_{\alpha\beta\gamma}$, $G_{\underline{a}}$, and $\overline{R}$ (for the minimal theory) on $H^{\underline{a}}$, and ϕ, or the dependence of $W_{\alpha\beta\gamma}$, $G_{\underline{a}}$, $\overline{R}$, and T_α on $H^{\underline{a}}$, and Σ (for the non-minimal theory). These are the standard and well discussed theories of off-shell 4D, $\mathcal{N} = 1$ supergravity, i.e. the consistency of the Bianchi identities associated with (3.8) or (3.9) and (3.10) for the algebra of the superspace supergravity covariant derivatives do *not* require on-shell conditions to be imposed on the component fields contained within the superfields.

The process of imposing the Einstein Field Equations in the non-supersymmetrical case in the absence of matter amounts to the condition

$$
R_{\underline{a}\,\underline{b}} - \frac{1}{2}\eta_{\underline{a}\underline{b}}\,R = 0\,, \tag{3.11}
$$

excluding the cosmological constant.[2] The equivalent condition in the case of superspace supergravity arises by setting $G_{\underline{a}}$, and $\overline{R}$ (for the minimal theory) to zero or by setting $G_{\underline{a}}$, and T_α (for the non-minimal theory) to zero. The condition $T_\alpha = 0$ also forces $\overline{R} = 0$ in the non-minimal theory. Under these conditions, the algebra of superspace supergravity

[2]The reason the cosmological constant can be ignored in our considerations is because unlike in four dimensions, there exists no evidence currently available in the literature that it is possible to construct spaces of constant curvature in ten or eleven dimensions with their respective superspaces.

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covariant derivatives takes the universal form

$$\begin{aligned} &[\nabla_\alpha, \nabla_\beta\} = 0\,, \qquad [\nabla_\alpha, \overline{\nabla}_{\dot\alpha}\} = i\nabla_{\underline{a}}\,, \\ &[\nabla_\alpha, \nabla_{\underline{b}}\} = i\, C_{\alpha\beta}[\, \overline{W}_{\dot\beta\dot\gamma}{}^{\dot\delta}\overline{\mathcal{M}}_{\dot\delta}{}^{\dot\gamma}\,]\,, \\ &[\nabla_{\underline{a}}\,,\ \nabla_{\underline{b}}\} = C_{\dot\alpha\dot\beta}\, W_{\beta\alpha}{}^{\gamma}\,\nabla_\gamma + C_{\dot\alpha\dot\beta}\,(\nabla_\alpha W_{\beta\gamma}{}^{\delta})\,\mathcal{M}_\delta{}^\gamma + \text{h.c.}\,. \end{aligned} \tag{3.12}$$

At this point, we can take a largely unexplored path as it is possible to consider the limit of these equations wherein $H^{\underline{a}} = 0$. This is the route to the 4D, $\mathcal{N} = 1$ superspace version of scalar supergravitation theory à la Nordström in the eleven and ten dimensions that are the targets of our study.

The curious reader may wonder from where does the condition $H^{\underline{a}} = 0$ arise? On page 335, of [34] there appears the following text.

> *Nonsupersymmetric deSitter covariant derivatives can be obtained from gravitational covariant derivatives by eliminating all field components except the (density) compensating field (i.e., the determinant of the metric or vierbein). This follows from the fact that in deSitter space the Weyl tensor vanishes, which says that there is no conformal (spin 2) part to the metric: it is conformally flat.*

From the discussion given in section 2, we saw that Nordström geometries in all dimensions are necessarily such as to describe Weyl tensors that vanish and are hence conformally flat.

Also in the work of [34] it is explained that the conformal part of the metric arises solely from $H^{\underline{a}}$. Since the Nordström limit is a conformally flat bosonic space, it must corresponds to setting $H^{\underline{a}} = 0$. Thus, to our knowledge the passage above from "Superspace" marked the first indication of this. Of course, other authors such as in the work of [35] later reaffirmed this point about the structure of superspace of supergravity.

In the limit of our interest, we find

$$\begin{aligned} &\mathrm{E}_\alpha = \mathrm{D}_\alpha + \overline{X}\mathrm{D}_\alpha\,, \quad \mathrm{E}_{\dot\alpha} = \overline{\mathrm{D}}_{\dot\alpha} + X\overline{\mathrm{D}}_{\dot\alpha}\,, \\ &\mathrm{E}_{\underline{a}} = \partial_{\underline{a}} - i\Big[(\overline{\mathrm{D}}_{\dot\alpha}\overline{X})\delta_\alpha{}^\gamma\Big]\mathrm{D}_\gamma - i\Big[(\mathrm{D}_\alpha X)\delta_{\dot\alpha}{}^{\dot\gamma}\Big]\overline{\mathrm{D}}_{\dot\gamma} + \Big[(X + \overline{X})\delta_{\underline{a}}{}^{\underline{b}}\Big]\partial_{\underline{b}}\,, \\ &\Phi_{\alpha\beta\gamma} = -C_{\alpha(\beta}\mathrm{D}_{\gamma)}\overline{X}\,, \quad \Phi_{\alpha\dot\beta\dot\gamma} = 0\,, \quad \Phi_{\dot\alpha\beta\gamma} = 0\,, \quad \Phi_{\underline{a}\beta\gamma} = iC_{\alpha(\beta|}\overline{\mathrm{D}}_{\dot\alpha}\mathrm{D}_{|\gamma)}\overline{X}\,. \end{aligned} \tag{3.13}$$

In response to this restriction, the forms of the algebras in (3.8), (3.9) and (3.10) also change. In particular, the superfield $W_{\alpha\beta\gamma}$ (and consequently $W_{\alpha\beta\gamma\delta}$) is identically zero. The latter condition is consistent with the component level description of scalar gravitation in the previous section as the Weyl tensor of (2.11) is the leading component field that occurs in $W_{\alpha\beta\gamma\delta}$ and occurs at first order in the θ-expansion of $W_{\alpha\beta\gamma}$. The third result in (3.13) also contains two useful bits of information:

(a). The final term of the equation informs us that the leading term in the θ-expansion of $X + \overline{X}$ corresponds to the linearization of ψ seen in equation (2.6).

(b). The second term of the equation informs us that the leading term in the θ-expansion of $(\overline{\mathrm{D}}_{\dot\alpha}\overline{X})\delta_\alpha{}^\gamma$ corresponds to the spin-1/2 remnant of the gravitino!

Another point to discuss is the dependence of the field strength superfields $G_{\underline{a}}$, and $\overline{R}$ (for the minimal theory) and $G_{\underline{a}}$, $\overline{R}$, and T_α (for the non-minimal theory) on the superfield

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X. Direct calculation shows that the reality of $G_{\underline{a}}$ in both cases implies that it only depends on the difference $i(X - \overline{X})$. The superfield T_α is found to depend on the first spinor derivative (i.e. D_α) of X. Finally, the superfield $\overline{R}$ is found to depend on the second spinorial derivative of an expression linear in X and $\overline{X}$.

We have argued previously [36], the minimal supergravity theory does not extend from four dimensions to eleven dimensions since there is no concept of chirality in the eleven dimensional superspace. This implies that only features seen in the non-minimal theory should be expected to occur in the subsequent sections of this work. As we shall see, this is indeed the case. The commutator algebra for the superspace supergravity covariant derivatives responds to the condition $H^{\underline{a}} = 0$, by the elimination of all terms proportional to $W_{\alpha\beta\gamma}$ and $W_{\alpha\beta\gamma\delta}$. Thus, we find 4D, $\mathcal{N} = 1$ Nordström supergravity *that descends* from ten or eleven dimensions and only contains the generators associated with 4D, $\mathcal{N} = 1$ simple supegravity is described by

$$
\begin{aligned}
[\nabla_\alpha\,,\ \nabla_\beta\} =& \tfrac{1}{2}T_{(\alpha}\nabla_{\beta)} - 2\overline{R}\mathcal{M}_{\alpha\beta}\,,\\
[\nabla_\alpha\,,\ \overline{\nabla}_{\dot{\beta}}\} =& i\nabla_{\alpha\dot{\beta}}\,,\\
[\nabla_\alpha\,,\ \nabla_{\underline{b}}\} =& \tfrac{1}{2}T_\beta\nabla_{\alpha\dot{\beta}} - iC_{\alpha\beta}\left[\overline{R} + \tfrac{1}{4}(\nabla^\gamma T_\gamma)\right]\overline{\nabla}_{\dot{\beta}}\\
&+ i\left[C_{\alpha\beta}G^\gamma{}_{\dot{\beta}} - \tfrac{1}{2}C_{\alpha\beta}\left(\left(\nabla^\gamma + \tfrac{1}{2}T^\gamma\right)\overline{T}_{\dot{\beta}}\right) + \tfrac{1}{2}(\overline{\nabla}_{\dot{\beta}}T_\beta)\delta_\alpha{}^\gamma\right]\nabla_\gamma\\
&- i\Big[C_{\alpha\beta}(\nabla^\gamma G_{\delta\dot{\beta}})\mathcal{M}_\gamma{}^\delta + ((\overline{\nabla}_{\dot{\beta}} - \overline{T}_{\dot{\beta}})\overline{R})\mathcal{M}_{\alpha\beta}\Big]\\
&+ iC_{\alpha\beta}\left[\tfrac{1}{6}\left(\nabla^\delta\left(\nabla_\delta + \tfrac{1}{2}T_\delta\right)\overline{T}_{\dot{\gamma}}\right)\overline{\mathcal{M}}_{\dot{\beta}}{}^{\dot{\gamma}} + \tfrac{1}{3}\overline{R}\,\overline{T}_{\dot{\gamma}}\overline{\mathcal{M}}_{\dot{\beta}}{}^{\dot{\gamma}}\right],\\
[\nabla_{\underline{a}}\,,\ \nabla_{\underline{b}}\} =& \Big\{ i\tfrac{1}{2}(\nabla_\beta\overline{T}_{\dot{\beta}})\nabla_{\underline{a}} - i\tfrac{1}{2}(\overline{\nabla}_{\dot{\alpha}}T_\beta)\nabla_{\alpha\dot{\beta}} - iC_{\dot{\alpha}\dot{\beta}}\left[G_\beta{}^{\dot{\gamma}} + \tfrac{1}{2}\left(\left(\overline{\nabla}^{\dot{\gamma}} + \tfrac{1}{2}\overline{T}^{\dot{\gamma}}\right)T_\beta\right)\right]\nabla_{\alpha\dot{\gamma}}\\
&+ \tfrac{1}{2}\left[(\overline{\nabla}_{\dot{\alpha}}\overline{\nabla}_{\dot{\beta}}T_\beta) - \tfrac{1}{2}\overline{T}_{\dot{\beta}}(\overline{\nabla}_{\dot{\alpha}}T_\beta)\right]\nabla_\alpha\\
&- C_{\dot{\alpha}\dot{\beta}}\left[(\nabla_\alpha R) + \tfrac{1}{4}(\nabla_\alpha\overline{\nabla}^{\dot{\gamma}}\overline{T}_{\dot{\gamma}}) + \tfrac{1}{3}RT_\alpha - \tfrac{1}{12}(\overline{\nabla}^{\dot{\gamma}}(\overline{\nabla}_{\dot{\gamma}} - \overline{T}_{\dot{\gamma}})T_\alpha)\right]\nabla_\beta\\
&+ C_{\alpha\beta}\left[(\overline{\nabla}_{\dot{\alpha}}G^\gamma{}_{\dot{\beta}}) - \tfrac{1}{2}\overline{T}_{\dot{\beta}}G^\gamma{}_{\dot{\alpha}}\right]\nabla_\gamma + \tfrac{1}{2}C_{\alpha\beta}\Big[\tfrac{1}{2}(\nabla^\gamma\overline{T}_{\dot{\alpha}})\overline{T}_{\dot{\beta}}\\
&\qquad - \left(\overline{\nabla}_{\dot{\alpha}}\left(\nabla^\gamma + \tfrac{1}{2}T^\gamma\right)\overline{T}_{\dot{\beta}}\right) + \tfrac{1}{2}\left(\left(\overline{\nabla}_{\dot{\alpha}} + \tfrac{1}{2}\overline{T}_{\dot{\alpha}}\right)\overline{T}_{\dot{\beta}}\right)T^\gamma\Big]\nabla_\gamma\\
&+ \left[-(\overline{\nabla}_{\dot{\alpha}}\overline{\nabla}_{\dot{\beta}}\overline{R}) + \tfrac{1}{2}(\overline{\nabla}_{\dot{\alpha}}\overline{R})\overline{T}_{\dot{\beta}} + \overline{R}(\overline{\nabla}_{\dot{\alpha}} + \tfrac{1}{2}\overline{T}_{\dot{\alpha}})\overline{T}_{\dot{\beta}}\right]\mathcal{M}_{\alpha\beta}\\
&\qquad + 2C_{\dot{\alpha}\dot{\beta}}\overline{R}\left[R + \tfrac{1}{4}(\overline{\nabla}^{\dot{\gamma}}\overline{T}_{\dot{\gamma}})\right]\mathcal{M}_{\alpha\beta}\\
&- \tfrac{1}{6}C_{\dot{\alpha}\dot{\beta}}\left[RT_\beta T_\gamma + \tfrac{1}{2}T_\beta(\overline{\nabla}^{\dot{\delta}}\overline{\nabla}_{\dot{\delta}}T_\gamma) + \tfrac{1}{4}T_\beta T_\gamma(\overline{\nabla}^{\dot{\delta}}\overline{T}_{\dot{\delta}})\right]\mathcal{M}_\alpha{}^\gamma\\
&+ \tfrac{1}{6}C_{\dot{\alpha}\dot{\beta}}\Big[2(\nabla_\alpha R)T_\gamma + 2R(\nabla_\alpha T_\gamma) + \left(\nabla_\alpha\overline{\nabla}^{\dot{\delta}}\left(\overline{\nabla}_{\dot{\delta}} + \tfrac{1}{2}\overline{T}_{\dot{\delta}}\right)T_\gamma\right)\\
&\qquad + \tfrac{1}{2}(\overline{\nabla}^{\dot{\delta}}\overline{T}_{\dot{\delta}})(\nabla_\alpha T_\gamma)\Big]\mathcal{M}_\beta{}^\gamma\\
&- C_{\alpha\beta}\left[(\overline{\nabla}_{\dot{\alpha}}\nabla^\gamma G_{\delta\dot{\beta}}) - \tfrac{1}{2}\overline{T}_{\dot{\beta}}(\nabla^\gamma G_{\delta\dot{\alpha}})\right]\mathcal{M}_\gamma{}^\delta\Big\} + \text{h.c.}\,.
\end{aligned}
\tag{3.14}
$$

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To our knowledge, the results in (3.14) mark the first time that a superspace description using non-minimal supergravity (one irreducible version of the Breitenlohner formulation) of 4D, $\mathcal{N}=1$ Nordström supergravity has appeared in the literature. As we consider higher dimensional theories, we set the cosmological constant to zero and the original non-minimal theory is sufficient for our purposes.

To summarize, the limit of off-shell 4D, $\mathcal{N}=1$ superfield supergravity where we *only* retain the conformal compensator provides a superspace extension of the Nordström supergravitation theory that is discussed in section 3. We will make a working assumption that such an approach is universally applicable to all superspaces. In particular, in the subsequent sections we will apply this assumption to superspaces whose bosonic subspaces possess either eleven or ten dimensions.

4 Linearized Nordström supergravity in 11D, $\mathcal{N}=1$ supergeometry

We begin our discussion by reviewing the work of [8, 9] where it was shown that the entire structure of the torsions, curvatures, and 4-form field strengths could be written in terms of a single superfield denoted by $W_{\underline{abcd}}$. Using the conventions of [36], we can write

$$\begin{aligned}
&T_{\alpha\beta}{}^{\underline{c}} = i(\gamma^{\underline{c}})_{\alpha\beta}\,, \qquad F_{\alpha\beta\underline{cd}} = \frac{1}{2}(\gamma_{\underline{cd}})_{\alpha\beta}\,, \quad F_{\underline{abc}\delta} = 0\,,\\
&F_{\alpha\beta\gamma\delta} = F_{\alpha\beta\gamma\underline{d}} = 0\,, \qquad F_{\underline{cdef}} = W_{\underline{cdef}}\,,\\
&T_{\alpha\beta}{}^{\gamma} = 0\,, \qquad T_{\alpha\underline{b}}{}^{\underline{c}} = 0\,,\\
&T_{\alpha\underline{b}}{}^{\gamma} = i\frac{1}{144}(\gamma_{\underline{b}}{}^{\underline{cdef}} + 8\delta_{\underline{b}}{}^{\underline{c}}\gamma^{\underline{def}})_{\alpha}{}^{\gamma}W_{\underline{cdef}}\,,\\
&R_{\alpha\beta\underline{cd}} = \frac{1}{3}(\gamma^{\underline{ef}})_{\alpha\beta}W_{\underline{cdef}} + \frac{1}{72}(\gamma_{\underline{cd}}{}^{\underline{efgh}})_{\alpha\beta}W_{\underline{efgh}}\,.
\end{aligned} \tag{4.1}$$

In addition to the torsion and curvature supertensors, the formulation above includes the 4-form supertensor, $F_{\underline{ABCD}}$. It should be noted that these equations in (4.1) are the eleven dimensional analog of the equations in (3.12). In other words, the supergeometry in (4.1) is an "on-shell" supergeometry. We must find a supergeometry consistent with the Nordström theory as the analog of (3.14).

We now wish to construct the linearized torsion and curvature supertensors with the property that when all fermions are set to zero, the theory smoothly maps to the linearization of the non-supersymmetrical theory described in section 2.

For this purpose we introduce eleven dimensional supergravity covariant derivatives linear in the infinitesimal conformal compensator Ψ given by

$$\nabla_\alpha = \mathrm{D}_\alpha + \frac{1}{2}\Psi\mathrm{D}_\alpha + l_0(\mathrm{D}_\beta\Psi)(\gamma^{\underline{de}})_\alpha{}^\beta\mathcal{M}_{\underline{de}}\,, \tag{4.2}$$

$$\begin{aligned}
\nabla_{\underline{a}} = \partial_{\underline{a}} + \Psi\partial_{\underline{a}} + il_1(\gamma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha\Psi)\mathrm{D}_\beta + l_2(\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}}\,,\\
+ il_3(\gamma_{\underline{a}}{}^{\underline{de}})^{\alpha\beta}(\mathrm{D}_\alpha\mathrm{D}_\beta\Psi)\mathcal{M}_{\underline{de}}\,,
\end{aligned} \tag{4.3}$$

where the "bare" superderivative operators D_α satisfy

$$\{\mathrm{D}_\alpha, \mathrm{D}_\beta\} = i(\gamma^{\underline{a}})_{\alpha\beta}\partial_{\underline{a}}\,, \tag{4.4}$$

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and the torsion tensors and Riemann curvature tensors can be obtained via

$$
\begin{aligned}
[\nabla_\alpha, \nabla_\beta\} &= T_{\alpha\beta}{}^{\underline{c}}\nabla_{\underline{c}} + T_{\alpha\beta}{}^{\gamma}\nabla_\gamma + \frac{1}{2}R_{\alpha\beta\underline{d}}{}^{\underline{e}}\mathcal{M}_{\underline{e}}{}^{\underline{d}}\,, \\
[\nabla_\alpha, \nabla_{\underline{b}}\} &= T_{\alpha\underline{b}}{}^{\underline{c}}\nabla_{\underline{c}} + T_{\alpha\underline{b}}{}^{\gamma}\nabla_\gamma + \frac{1}{2}R_{\alpha\underline{b}\underline{d}}{}^{\underline{e}}\mathcal{M}_{\underline{e}}{}^{\underline{d}}\,, \qquad (4.5)\\
[\nabla_{\underline{a}}, \nabla_{\underline{b}}\} &= T_{\underline{a}\underline{b}}{}^{\underline{c}}\nabla_{\underline{c}} + T_{\underline{a}\underline{b}}{}^{\gamma}\nabla_\gamma + \frac{1}{2}R_{\underline{a}\underline{b}\underline{d}}{}^{\underline{e}}\mathcal{M}_{\underline{e}}{}^{\underline{d}}\,.
\end{aligned}
$$

The commutation relations of the operators with the 11D Lorentz generators satisfy

$$
[\mathcal{M}_{\underline{a}\underline{b}}, \mathrm{D}_\alpha\} = \frac{1}{2}(\gamma_{\underline{a}\underline{b}})_\alpha{}^\beta \mathrm{D}_\beta\,, \qquad (4.6)
$$

$$
\big[\mathcal{M}_{\underline{a}\underline{b}}, [\mathcal{M}_{\underline{c}\underline{d}}, \mathrm{D}_\alpha\}\big\} + \big[\mathcal{M}_{\underline{c}\underline{d}}, [\mathrm{D}_\alpha, \mathcal{M}_{\underline{a}\underline{b}}\}\big\} + \big[\mathrm{D}_\alpha, [\mathcal{M}_{\underline{a}\underline{b}}, \mathcal{M}_{\underline{c}\underline{d}}\}\big\} = 0\,, \qquad (4.7)
$$

in addition to the relations seen in (2.1) and (2.2).

By imposing the constraints

$$
T_{\underline{a}\underline{b}}{}^{\underline{c}} = 0\,, \quad T_{\alpha\beta}{}^{\underline{c}} = i(\gamma^{\underline{c}})_{\alpha\beta}\,, \qquad (4.8)
$$

we obtain the following parameterization results

$$
l_0 = \frac{1}{10}\,, \quad l_1 = \frac{1}{4}\,, \quad l_2 = -1\,, \quad l_3 = 0\,. \qquad (4.9)
$$

In turn these lead to a set of results that express the torsion and curvature tensors solely in terms of Ψ and its derivatives. We give these in the following.

For the components of the torsion we find the results seen in (4.10)–(4.15).

$$
\begin{aligned}
T_{\alpha\beta}{}^{\underline{c}} &= i(\gamma^{\underline{c}})_{\alpha\beta}\,, && (4.10)\\
T_{\alpha\beta}{}^{\gamma} &= \frac{3}{40}(\gamma^{[2]})_{\alpha\beta}(\gamma_{[2]})^{\gamma\delta}(\mathrm{D}_\delta\Psi)\,, && (4.11)\\
T_{\alpha\underline{b}}{}^{\underline{c}} &= \frac{3}{4}\delta_{\underline{b}}{}^{\underline{c}}(\mathrm{D}_\alpha\Psi) + \frac{9}{20}(\gamma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\beta(\mathrm{D}_\beta\Psi)\,, && (4.12)\\
T_{\alpha\underline{b}}{}^{\gamma} &= i\frac{1}{128}\bigg[-(\gamma_{\underline{b}})_\alpha{}^\gamma C^{\delta\epsilon} + \frac{1}{2}(\gamma^{[2]})_\alpha{}^\gamma(\gamma_{\underline{b}[2]})^{\delta\epsilon} - \frac{1}{3!}(\gamma_{\underline{b}[3]})_\alpha{}^\gamma(\gamma^{[3]})^{\delta\epsilon} + \frac{1}{3!}(\gamma^{[3]})_\alpha{}^\gamma(\gamma_{\underline{b}[3]})^{\delta\epsilon} \\
&\qquad - \frac{1}{4!}(\gamma_{\underline{b}[4]})_\alpha{}^\gamma(\gamma^{[4]})^{\delta\epsilon}\bigg](\mathrm{D}_\delta\mathrm{D}_\epsilon\Psi) + \frac{1}{8}\delta_\alpha{}^\gamma(\partial_{\underline{b}}\Psi) + \frac{3}{8}(\gamma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma(\partial_{\underline{c}}\Psi)\,, && (4.13)\\
T_{\underline{a}\underline{b}}{}^{\underline{c}} &= 0\,, && (4.14)\\
T_{\underline{a}\underline{b}}{}^{\gamma} &= -i\frac{1}{4}(\gamma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\mathrm{D}_\delta\Psi)\,. && (4.15)
\end{aligned}
$$

For the components of the curvature we find the results seen in (4.16)–(4.18).

$$
\begin{aligned}
R_{\alpha\beta}{}^{\underline{d}\underline{e}} &= \frac{1}{80}\bigg[(\gamma^{\underline{d}\underline{e}})_{\alpha\beta}C^{\gamma\delta} + (\gamma_{[1]})_{\alpha\beta}(\gamma^{[1]\underline{d}\underline{e}})^{\gamma\delta} - \frac{1}{2}(\gamma_{[2]})_{\alpha\beta}(\gamma^{[2]\underline{d}\underline{e}})^{\gamma\delta} - \frac{1}{3!}(\gamma^{\underline{d}\underline{e}[3]})_{\alpha\beta}(\gamma_{[3]})^{\gamma\delta} \\
&\qquad + \frac{1}{5!4!}\epsilon^{\underline{d}\underline{e}[5][4]}(\gamma_{[5]})_{\alpha\beta}(\gamma_{[4]})^{\gamma\delta}\bigg](\mathrm{D}_\gamma\mathrm{D}_\delta\Psi)\,, && (4.16)\\
R_{\alpha\underline{b}}{}^{\underline{d}\underline{e}} &= -(\partial^{[\underline{d}}\mathrm{D}_\alpha\Psi)\delta_{\underline{b}}{}^{\underline{e}]} + \frac{1}{5}(\gamma^{\underline{d}\underline{e}})_\alpha{}^\delta(\partial_{\underline{b}}\mathrm{D}_\delta\Psi)\,, && (4.17)\\
R_{\underline{a}\underline{b}}{}^{\underline{d}\underline{e}} &= -(\partial_{[\underline{a}}\partial^{[\underline{d}}\Psi)\delta_{\underline{b}]}{}^{\underline{e}]}\,. && (4.18)
\end{aligned}
$$

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In reaching (4.10)–(4.18), we used the Fierz identities (A.24)–(A.27) listed in appendix A.

It is the last equation that ensures that we have reached our goal. Namely, the choice of constraints in (4.9) has led to a linearized super Riemann curvature tensor expressed solely in terms of an infinitesimal superfield Ψ that has the exact form of the first term in the non-supersymmetrical Riemann curvature tensor given in (2.8). Recall that the supersymmetrical theory here is linearized, so to make a proper comparison to the bosonic theory, that should also be linearized. When this is done, there is a matching of the terms.

We should note the work in [36] also constructs a fully non-linear 11D supergeometry in terms of a finite scalar compensator. However, its linearization is different from the one obtained here. In the next three sections, we will obtain new and never before presented results of this nature for the 10D, $\mathcal{N}=2$A, 10D, $\mathcal{N}=2$B, and 10D, $\mathcal{N}=1$ supergeometries that possess the purely bosonic linearized results as in the linearization of (2.8). The Fierz identities used for simplifying the torsions and curvatures in ten dimensions are listed in appendix B.

5 Linearized Nordström supergravity in 10D, $\mathcal{N}=1$ supergeometry

We begin this discussion by pointing out the on-shell description of 10D, $\mathcal{N}=1$ superspace supergravity. A set of torsion and curvature supertensors can be written in the form

$$
\begin{aligned}
T_{\alpha\beta}{}^{\underline{c}} &= i\,(\sigma^{\underline{c}})_{\alpha\beta}\,, \quad T_{\alpha\beta}{}^{\gamma} = -\frac{1}{2}\sqrt{\frac{1}{2}}\left[\delta_{(\alpha}{}^{\gamma}\,\delta_{\beta)}{}^{\epsilon} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\epsilon}\right]\chi_{\epsilon}\,, \quad T_{\alpha\underline{b}}{}^{\underline{c}} = 0\,,\\
T_{\alpha\underline{b}}{}^{\gamma} &= -\frac{1}{24}(\sigma_{\underline{b}}\,\sigma^{\underline{cde}})_{\alpha}{}^{\gamma}\left[e^{\Phi}\,H_{\underline{cde}} - i\,\frac{1}{8}\,(\,\chi\,\sigma_{\underline{cde}}\,\chi)\right]\\
&\quad -\frac{1}{48}(\sigma^{\underline{cde}}\,\sigma_{\underline{b}})_{\alpha}{}^{\gamma}\left[e^{\Phi}\,H_{\underline{cde}} - i\,\frac{1}{16}\,(\,\chi\,\sigma_{\underline{cde}}\,\chi)\right]\,,\\
R_{\alpha\beta\underline{de}} &= -i\,\frac{1}{4}\,(\sigma^{\underline{c}})_{\alpha\beta}\left[3e^{-\Phi}\,H_{\underline{cde}} - i\,\frac{5}{16}\,(\,\chi\,\sigma_{\underline{cde}}\,\chi)\right]\\
&\quad -i\,\frac{1}{24}\,(\sigma^{\underline{abc}}{}_{\underline{de}})_{\alpha\beta}\left[e^{-\Phi}\,H_{\underline{abc}} - i\,\frac{3}{16}\,(\,\chi\,\sigma_{\underline{abc}}\,\chi)\right]\,,\\
R_{\alpha\underline{cde}} &= -i\,\frac{1}{2}\Big[\,(\sigma_{\underline{c}})_{\alpha\gamma}\,T_{\underline{de}}{}^{\gamma} - (\sigma_{\underline{d}})_{\alpha\gamma}\,T_{\underline{ec}}{}^{\gamma} - (\sigma_{\underline{e}})_{\alpha\gamma}\,T_{\underline{cd}}{}^{\gamma}\Big]\,,
\end{aligned}
\tag{5.1}
$$

as was noted in the work of [37, 38]. In these expression $H_{\underline{abc}}$ refers to the supercovariantized field strength of a two-form $B_{\underline{ab}}$. The results in (5.1) are the 10D, $\mathcal{N}=1$ analogs of the results in (3.12) for the 4D, $\mathcal{N}=1$ superspace geometry. That is the component fields embedded in this supergeometry must obey a set of mass-shell conditions. To release these conditions, one must find the 10D, $\mathcal{N}=1$ analogs of the equations in (3.9) and (3.10). However, as our goal once more is to find a supergeometry consistent with the Nordström theory, we seek the analogs of (3.14).

The covariant derivatives linear in the conformal compensator Ψ are given by

$$
\nabla_{\alpha} = \mathrm{D}_{\alpha} + l_0\Psi\mathrm{D}_{\alpha} + l_1(\sigma^{\underline{ab}})_{\alpha}{}^{\beta}(\mathrm{D}_{\beta}\Psi)\mathcal{M}_{\underline{ab}}\,, \tag{5.2}
$$

$$
\nabla_{\underline{a}} = \partial_{\underline{a}} + l_2\Psi\partial_{\underline{a}} + il_3(\sigma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_{\alpha}\Psi)\mathrm{D}_{\beta} + l_4(\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}} + il_5(\sigma_{\underline{a}}{}^{\underline{de}})^{\gamma\delta}(\mathrm{D}_{\gamma}\mathrm{D}_{\delta}\Psi)\mathcal{M}_{\underline{de}}\,, \tag{5.3}
$$

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and similar to the case of the eleven dimensional theory, here we have

$$\{\mathrm{D}_\alpha, \mathrm{D}_\beta\} = i(\sigma^{\underline{a}})_{\alpha\beta}\partial_{\underline{a}}\,. \tag{5.4}$$

The commutation relations of Poincare generators in 10D

$$[\mathcal{M}_{\underline{ab}}, \mathrm{D}_\alpha\} = \frac{1}{2}(\sigma_{\underline{ab}})_\alpha{}^\beta \mathrm{D}_\beta\,, \tag{5.5}$$

is similar to the eleven dimensional case. Also the equation in (4.7) is valid in all ten dimensional theories. There will be some slight modifications for the dotted and barred spinor indices in type IIA and IIB supergravity, respectively.

By adoption of the constraints

$$T_{\underline{ab}}{}^{\underline{c}} = 0\,, \quad T_{\alpha\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\,, \tag{5.6}$$

we obtain the following parameterization results

$$l_0 = \frac{1}{2}\,, \quad l_1 = \frac{1}{10}\,, \quad l_2 = 1\,, \quad l_3 = -\frac{2}{5}\,, \quad l_4 = -1\,, \quad l_5 = 0\,. \tag{5.7}$$

As the consequence of this choice of parameters, we find the torsion supertensors given in (5.8)–(5.13).

$$T_{\alpha\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\,, \tag{5.8}$$

$$T_{\alpha\beta}{}^{\gamma} = 0\,, \tag{5.9}$$

$$T_{\alpha\underline{b}}{}^{\underline{c}} = \frac{3}{5}\left[\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\delta + (\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\delta\right](\mathrm{D}_\delta\Psi)\,, \tag{5.10}$$

$$\begin{aligned} T_{\alpha\underline{b}}{}^{\gamma} = {}& i\frac{1}{80}\left[-(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} + \frac{1}{3}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\mathrm{D}_\beta\mathrm{D}_\delta\Psi) \\ & - \frac{3}{10}\delta_\alpha{}^\gamma(\partial_{\underline{b}}\Psi) + \frac{3}{10}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma(\partial_{\underline{c}}\Psi)\,, \end{aligned} \tag{5.11}$$

$$T_{\underline{ab}}{}^{\underline{c}} = 0\,, \tag{5.12}$$

$$T_{\underline{ab}}{}^{\gamma} = i\frac{2}{5}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\mathrm{D}_\delta\Psi)\,. \tag{5.13}$$

For the components of the curvatures, we find the results seen in (5.14)–(5.16).

$$R_{\alpha\beta}{}^{\underline{de}} = -i\frac{6}{5}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}\Psi) - \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}}{}^{\underline{de}})^{\gamma\delta}\right](\mathrm{D}_\gamma\mathrm{D}_\delta\Psi)\,, \tag{5.14}$$

$$R_{\alpha\underline{b}}{}^{\underline{de}} = -(\mathrm{D}_\alpha\partial^{[\underline{d}}\Psi)\delta_{\underline{b}}{}^{\underline{e}]} + \frac{1}{5}(\sigma^{\underline{de}})_\alpha{}^\gamma(\partial_{\underline{b}}\mathrm{D}_\gamma\Psi)\,, \tag{5.15}$$

$$R_{\underline{ab}}{}^{\underline{de}} = -(\partial_{[\underline{a}}\partial^{[\underline{d}}\Psi)\delta_{\underline{b}]}{}^{\underline{e}]}\,. \tag{5.16}$$

It has long been suggested [39] that a superfield with the structure of $G_{\underline{abc}}$ should appear in the off-shell structure of 10D, $\mathcal{N} = 1$ supergeometry and that it was related by a superdifferential operator to an underlying unconstrained prepotential $V_{\underline{abc}}$ analogous to $H^{\underline{m}}$ that appears in 4D, $\mathcal{N} = 1$ supergravity. However, there are reasons to believe [37, 38] that $V_{\underline{abc}}$ must be related to an even more fundamental spinorial prepotential $\Psi_{\underline{ab}}{}^\alpha$. In the equations of (5.11) and (5.14) the superfield $G_{\underline{abc}} \equiv (\sigma_{\underline{abc}})^{\gamma\delta}(\mathrm{D}_\gamma\mathrm{D}_\delta\Psi)$ has precisely the structure suggested in the work by Howe, Nicolai, and Van Proeyen.

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6 Linearized Nordström supergravity in 10D, $\mathcal{N} = 2$A supergeometry

We repeat the discussions as seen in the previous two sections with a beginning of the on-shell description of 10D, $\mathcal{N} = 2$A superspace supergravity. A set of torsion and curvature supertensors can be written in the form

$$
\begin{aligned}
&T_{\alpha \dot{\beta}}{}^{\underline{c}} = T_{\dot{\alpha} \dot{\beta}}{}^{\gamma} = T_{\alpha \dot{\beta}}{}^{\gamma} = T_{\dot{\alpha} \beta}{}^{\dot{\gamma}} = T_{\alpha\beta}{}^{\dot{\gamma}} = 0 \,, \\
&T_{\alpha \underline{b}}{}^{\underline{c}} = T_{\dot{\alpha} \underline{b}}{}^{\underline{c}} = T_{\underline{ab}}{}^{\underline{c}} \ = 0 \,, \\
&T_{\alpha\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta} \,, \quad T_{\dot{\alpha} \dot{\beta}}{}^{\underline{c}} \ = \ i(\sigma^{\underline{c}})_{\dot{\alpha} \dot{\beta}} \,, \\
&T_{\alpha\beta}{}^{\gamma} = \Big[\delta_{(\alpha}{}^{\gamma} \delta_{\beta)}{}^{\delta} + (\sigma^{\underline{a}})_{\alpha\beta} (\sigma_{\underline{a}})^{\gamma\delta} \Big] \chi_{\delta} \,, \\
&T_{\dot{\alpha} \dot{\beta}}{}^{\dot{\gamma}} = \Big[\delta_{(\dot{\alpha}}{}^{\dot{\gamma}} \delta_{\dot{\beta})}{}^{\dot{\delta}} + (\sigma^{\underline{a}})_{\dot{\alpha} \dot{\beta}} (\sigma_{\underline{a}})^{\dot{\gamma} \dot{\delta}} \Big] \chi_{\dot{\delta}} \,, \\
&T_{\alpha \underline{b}}{}^{\gamma} = -\frac{1}{8} (\sigma^{\underline{de}})_{\alpha}{}^{\gamma} H_{\underline{bde}} \,, \quad C_{\alpha \dot{\alpha}} C^{\gamma \dot{\gamma}} T_{\dot{\gamma} \underline{b}}{}^{\dot{\alpha}} = -\frac{1}{8} (\sigma^{\underline{de}})_{\alpha}{}^{\gamma} G_{\underline{bde}} \,, \\
&C_{\gamma \dot{\gamma}} T_{\alpha \underline{b}}{}^{\dot{\gamma}} = \frac{1}{16} (\sigma_{\underline{b}})_{\alpha\delta} \left[(\sigma^{[2]})_{\gamma}{}^{\delta} K_{[2]} - \frac{1}{12} (\sigma^{[4]})_{\gamma}{}^{\delta} D_{[4]} \right] \,, \\
&C^{\alpha \dot{\alpha}} T_{\dot{\alpha} \underline{b}}{}^{\gamma} = -\frac{1}{16} (\sigma_{\underline{b}})^{\alpha\delta} \left[(\sigma^{[2]})_{\delta}{}^{\gamma} K_{[2]} - \frac{1}{12} (\sigma^{[4]})_{\delta}{}^{\gamma} D_{[4]} \right] \,,
\end{aligned}
\tag{6.1}
$$

as was noted in the work of [40]. In these expressions $H_{\underline{abc}}$ refers to the supercovariantized field strengths of a two-form $B_{\underline{ab}}$ gauge field, and

$$
\begin{aligned}
K_{\underline{ab}} &= e^{-\Phi} F_{\underline{ab}} - \chi_{\alpha} (\sigma_{\underline{ab}})_{\beta}{}^{\alpha} \chi^{\beta} \,, \\
D_{[4]} &= 2 e^{-\Phi} \tilde{F}_{[4]} + \chi_{\alpha} (\sigma_{[4]})_{\beta}{}^{\alpha} \chi^{\beta} \,.
\end{aligned}
\tag{6.2}
$$

with $F_{\underline{ab}}$ and $\tilde{F}_{[4]}$ denoting supercovariantized field strengths for a gauge 1-form and a gauge 3-form respectively. The results in (6.1) are the 10D, $\mathcal{N} = 2$A analogs of the results in (3.12) for the 4D, $\mathcal{N} = 1$ superspace geometry. Those are the component fields embedded in this supergeometry must obey a set of mass-shell conditions. To release these conditions, one must find the 10D, $\mathcal{N} = 2$A analogs of the equations in (3.9) and (3.10). Again the goal must be to find a supergeometry consistent with the Nordström theory, we seek the analogs of (3.14). In analogy with the 3-form gauge field sector of 11D, $\mathcal{N} = 1$ supergravity, the gauge fields components are:

$$
\begin{aligned}
&F_{\alpha \dot{\beta}} = C_{\alpha \dot{\beta}} e^{\Phi} \,, && F_{\alpha\beta} = F_{\dot{\alpha} \dot{\beta}} = 0 \,, \\
&F_{\underline{c}\alpha} = i e^{\Phi} (\sigma_{\underline{c}})_{\alpha\beta} \chi^{\beta} \,, && F_{\underline{c} \dot{\alpha}} = i C_{\alpha \dot{\alpha}} e^{\Phi} (\sigma_{\underline{c}})^{\alpha\beta} \chi_{\beta} \,, \\
&G_{\alpha\beta\gamma} = G_{\underline{a}\beta \dot{\gamma}} = G_{\underline{ab}\gamma} = G_{\underline{ab} \dot{\gamma}} = 0 \,, && \\
&G_{\underline{c}\alpha\beta} = i (\sigma_{\underline{c}})_{\alpha\beta} \,, && G_{\underline{c} \dot{\alpha} \dot{\beta}} = -i (\sigma_{\underline{c}})_{\dot{\alpha} \dot{\beta}} \,, \\
&\tilde{F}_{\alpha\beta\gamma\underline{d}} = \tilde{F}_{\alpha\beta\underline{cd}} = \tilde{F}_{\dot{\alpha} \dot{\beta} \underline{cd}} = 0 \,, && \\
&\tilde{F}_{\alpha \dot{\beta} \underline{cd}} = e^{\Phi} (\sigma_{\underline{cd}})_{\alpha}{}^{\beta} C_{\beta \dot{\beta}} \,, && \\
&\tilde{F}_{\alpha \underline{bcd}} = -i e^{\Phi} (\sigma_{\underline{bcd}})_{\alpha\beta} \chi^{\beta} \,, && \tilde{F}_{\dot{\alpha} \underline{bcd}} = i C_{\alpha \dot{\alpha}} e^{\Phi} (\sigma_{\underline{bcd}})^{\alpha\beta} \chi_{\beta} \,.
\end{aligned}
\tag{6.3}
$$

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Φ denotes a dilaton superfield, and χ_α is its partner dilatino. All of the equations in (6.1)–(6.3) describe the on-shell 10D, $\mathcal{N}=2$A theory, i.e. these are the analogs of (3.12).

The covariant derivatives linear in the conformal compensator Ψ are given by

$$\nabla_\alpha = \mathrm{D}_\alpha + \frac{1}{2}\Psi \mathrm{D}_\alpha + l_0(\sigma^{\underline{ab}})_\alpha{}^\beta(\mathrm{D}_\beta\Psi)\mathcal{M}_{\underline{ab}} \tag{6.4}$$

$$\nabla_{\dot\alpha} = \mathrm{D}_{\dot\alpha} + \frac{1}{2}\Psi \mathrm{D}_{\dot\alpha} + l_0(\sigma^{\underline{ab}})_{\dot\alpha}{}^{\dot\beta}(\mathrm{D}_{\dot\beta}\Psi)\mathcal{M}_{\underline{ab}} \tag{6.5}$$

$$\begin{aligned}\nabla_{\underline{a}} = {}& \partial_{\underline{a}} + l_1\Psi\partial_{\underline{a}} + il_2(\sigma_{\underline{a}})^{\delta\gamma}(\mathrm{D}_\delta\Psi)\mathrm{D}_\gamma + il_3(\sigma_{\underline{a}})^{\dot\delta\dot\gamma}(\mathrm{D}_{\dot\delta}\Psi)\mathrm{D}_{\dot\gamma} + l_4(\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}} \\ & + il_5(\sigma_{\underline{a}}{}^{\underline{cd}})^{\gamma\delta}(\mathrm{D}_\gamma\mathrm{D}_\delta\Psi)\mathcal{M}_{\underline{cd}} + il_6(\sigma_{\underline{a}}{}^{\underline{cd}})^{\dot\gamma\dot\delta}(\mathrm{D}_{\dot\gamma}\mathrm{D}_{\dot\delta}\Psi)\mathcal{M}_{\underline{cd}}\end{aligned} \tag{6.6}$$

where the Type IIA supersymmetry algebra

$$\{\mathrm{D}_\alpha,\mathrm{D}_\beta\} = i(\sigma^{\underline{a}})_{\alpha\beta}\partial_{\underline{a}}\,,\quad \{\mathrm{D}_{\dot\alpha},\mathrm{D}_{\dot\beta}\} = i(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}\partial_{\underline{a}}\,,\quad \{\mathrm{D}_\alpha,\mathrm{D}_{\dot\beta}\} = 0 \tag{6.7}$$

is satisfied by the bare derivative operators.

By adopting the constraints

$$T_{\underline{ab}}{}^{\underline{c}} = 0\,,\qquad T_{\alpha\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\,,\qquad T_{\dot\alpha\dot\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\dot\alpha\dot\beta}\,, \tag{6.8}$$

we obtain the following paramaterization values

$$l_0 = \frac{1}{10}\,,\quad l_1 = 1\,,\quad l_2 = l_3 = -\frac{1}{5}\,,\quad l_4 = -1\,,\quad l_5 = l_6 = 0\,. \tag{6.9}$$

As the consequence of this choice of parameters, we find the torsion supertensors given in (6.10)–(6.27).

$$T_{\alpha\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\,, \tag{6.10}$$

$$T_{\alpha\beta}{}^{\gamma} = \frac{1}{5}(\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}(\mathrm{D}_\delta\Psi)\,, \tag{6.11}$$

$$T_{\alpha\beta}{}^{\dot\gamma} = -\frac{1}{5}(\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\dot\gamma\dot\delta}(\mathrm{D}_{\dot\delta}\Psi)\,, \tag{6.12}$$

$$T_{\dot\alpha\dot\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\dot\alpha\dot\beta}\,, \tag{6.13}$$

$$T_{\dot\alpha\dot\beta}{}^{\gamma} = -\frac{1}{5}(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}(\sigma_{\underline{a}})^{\gamma\delta}(\mathrm{D}_\delta\Psi)\,, \tag{6.14}$$

$$T_{\dot\alpha\dot\beta}{}^{\dot\gamma} = \frac{1}{5}(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}(\sigma_{\underline{a}})^{\dot\gamma\dot\delta}(\mathrm{D}_{\dot\delta}\Psi)\,, \tag{6.15}$$

$$T_{\alpha\dot\beta}{}^{\underline{c}} = 0\,, \tag{6.16}$$

$$T_{\alpha\dot\beta}{}^{\gamma} = \frac{1}{2}\left[\delta_\alpha{}^\gamma(\mathrm{D}_{\dot\beta}\Psi) + \frac{1}{10}(\sigma^{\underline{ab}})_\alpha{}^\gamma(\sigma_{\underline{ab}})_{\dot\beta}{}^{\dot\delta}(\mathrm{D}_{\dot\delta}\Psi)\right]\,, \tag{6.17}$$

$$T_{\alpha\dot\beta}{}^{\dot\gamma} = \frac{1}{2}\left[\delta_{\dot\beta}{}^{\dot\gamma}(\mathrm{D}_\alpha\Psi) + \frac{1}{10}(\sigma^{\underline{ab}})_{\dot\beta}{}^{\dot\gamma}(\sigma_{\underline{ab}})_\alpha{}^\delta(\mathrm{D}_\delta\Psi)\right]\,, \tag{6.18}$$

$$T_{\alpha\underline{b}}{}^{\underline{c}} = \frac{4}{5}\delta_{\underline{b}}{}^{\underline{c}}(\mathrm{D}_\alpha\Psi) + \frac{2}{5}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\delta(\mathrm{D}_\delta\Psi)\,, \tag{6.19}$$

$$\begin{aligned}T_{\alpha\underline{b}}{}^{\gamma} = {}& i\frac{1}{80}\left[-\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} + \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\mathrm{D}_\beta\mathrm{D}_\delta\Psi) \\ & - \frac{2}{5}\delta_\alpha{}^\gamma(\partial_{\underline{b}}\Psi) + \frac{2}{5}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma(\partial_{\underline{c}}\Psi)\,,\end{aligned} \tag{6.20}$$

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$$T_{\alpha\underline{b}}{}^{\dot{\gamma}} = -i\frac{1}{80}\Big[(\sigma_{\underline{b}})_{\alpha}{}^{\dot{\gamma}}C^{\beta\dot{\delta}} - (\sigma^{\underline{c}})_{\alpha}{}^{\dot{\gamma}}(\sigma_{\underline{bc}})^{\beta\dot{\delta}} - \frac{1}{3!}(\sigma^{[3]})_{\alpha}{}^{\dot{\gamma}}(\sigma_{\underline{b}[3]})^{\beta\dot{\delta}} + \frac{1}{2}(\sigma_{\underline{b}[2]})_{\alpha}{}^{\dot{\gamma}}(\sigma^{[2]})^{\beta\dot{\delta}} + \frac{1}{4!}(\sigma_{\underline{b}[4]})_{\alpha}{}^{\dot{\gamma}}(\sigma^{[4]})^{\beta\dot{\delta}}\Big](\mathrm{D}_{\beta}\mathrm{D}_{\dot{\delta}}\Psi)\,, \tag{6.21}$$

$$T_{\dot{\alpha}\underline{b}}{}^{\underline{c}} = \frac{4}{5}\delta_{\underline{b}}{}^{\underline{c}}(\mathrm{D}_{\dot{\alpha}}\Psi) + \frac{2}{5}(\sigma_{\underline{b}}{}^{\underline{c}})_{\dot{\alpha}}{}^{\dot{\delta}}(\mathrm{D}_{\dot{\delta}}\Psi)\,, \tag{6.22}$$

$$T_{\dot{\alpha}\underline{b}}{}^{\gamma} = -i\frac{1}{80}\Big[(\sigma_{\underline{b}})_{\dot{\alpha}}{}^{\gamma}C^{\delta\dot{\beta}} + (\sigma^{\underline{c}})_{\dot{\alpha}}{}^{\gamma}(\sigma_{\underline{bc}})^{\delta\dot{\beta}} - \frac{1}{3!}(\sigma^{[3]})_{\dot{\alpha}}{}^{\gamma}(\sigma_{\underline{b}[3]})^{\delta\dot{\beta}} - \frac{1}{2}(\sigma_{\underline{b}[2]})_{\dot{\alpha}}{}^{\gamma}(\sigma^{[2]})^{\delta\dot{\beta}} + \frac{1}{4!}(\sigma_{\underline{b}[4]})_{\dot{\alpha}}{}^{\gamma}(\sigma^{[4]})^{\delta\dot{\beta}}\Big](\mathrm{D}_{\delta}\mathrm{D}_{\dot{\beta}}\Psi)\,, \tag{6.23}$$

$$T_{\dot{\alpha}\underline{b}}{}^{\dot{\gamma}} = i\frac{1}{80}\left[-\frac{1}{2}(\sigma^{[2]})_{\dot{\alpha}}{}^{\dot{\gamma}}(\sigma_{\underline{b}[2]})^{\dot{\beta}\dot{\delta}} + \frac{1}{3!}(\sigma_{\underline{b}[3]})_{\dot{\alpha}}{}^{\dot{\gamma}}(\sigma^{[3]})^{\dot{\beta}\dot{\delta}}\right](\mathrm{D}_{\dot{\beta}}\mathrm{D}_{\dot{\delta}}\Psi) - \frac{2}{5}\delta_{\dot{\alpha}}{}^{\dot{\gamma}}(\partial_{\underline{b}}\Psi) + \frac{2}{5}(\sigma_{\underline{b}}{}^{\underline{c}})_{\dot{\alpha}}{}^{\dot{\gamma}}(\partial_{\underline{c}}\Psi)\,, \tag{6.24}$$

$$T_{\underline{ab}}{}^{\underline{c}} = 0\,, \tag{6.25}$$

$$T_{\underline{ab}}{}^{\gamma} = i\frac{1}{5}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\mathrm{D}_{\delta}\Psi)\,, \tag{6.26}$$

$$T_{\underline{ab}}{}^{\dot{\gamma}} = i\frac{1}{5}(\sigma_{[\underline{a}})^{\dot{\gamma}\dot{\delta}}(\partial_{\underline{b}]}\mathrm{D}_{\dot{\delta}}\Psi)\,. \tag{6.27}$$

For the components of the curvatures, we find the results seen in (6.28)–(6.33).

$$R_{\alpha\beta}{}^{\underline{de}} = -i\frac{6}{5}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}\Psi) - \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}}{}^{\underline{de}})^{\gamma\delta}\right](\mathrm{D}_{\gamma}\mathrm{D}_{\delta}\Psi)\,, \tag{6.28}$$

$$R_{\dot{\alpha}\dot{\beta}}{}^{\underline{de}} = -i\frac{6}{5}(\sigma^{[\underline{d}})_{\dot{\alpha}\dot{\beta}}(\partial^{\underline{e}]}\Psi) - \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\dot{\alpha}\dot{\beta}}(\sigma_{[3]})^{\dot{\gamma}\dot{\delta}} + (\sigma^{\underline{a}})_{\dot{\alpha}\dot{\beta}}(\sigma_{\underline{a}}{}^{\underline{de}})^{\dot{\gamma}\dot{\delta}}\right](\mathrm{D}_{\dot{\gamma}}\mathrm{D}_{\dot{\delta}}\Psi)\,, \tag{6.29}$$

$$R_{\alpha\dot{\beta}}{}^{\underline{de}} = \frac{1}{40}\Big[-C_{\alpha\dot{\beta}}(\sigma^{\underline{de}})^{\gamma\dot{\delta}} + (\sigma^{\underline{de}})_{\alpha\dot{\beta}}C^{\gamma\dot{\delta}} - \frac{1}{2}(\sigma_{[2]})_{\alpha\dot{\beta}}(\sigma^{\underline{de}[2]})^{\gamma\dot{\delta}} + \frac{1}{2}(\sigma^{\underline{de}[2]})_{\alpha\dot{\beta}}(\sigma_{[2]})^{\gamma\dot{\delta}} + \frac{1}{4!4!}\epsilon^{\underline{de}[4][\bar{4}]}(\sigma_{[4]})_{\alpha\dot{\beta}}(\sigma_{[\bar{4}]})^{\gamma\dot{\delta}}\Big](\mathrm{D}_{\gamma}\mathrm{D}_{\dot{\delta}}\Psi)\,, \tag{6.30}$$

$$R_{\alpha\underline{b}}{}^{\underline{de}} = -(\mathrm{D}_{\alpha}\partial^{[\underline{d}}\Psi)\delta_{\underline{b}}{}^{\underline{e}]} + \frac{1}{5}(\sigma^{\underline{de}})_{\alpha}{}^{\gamma}(\partial_{\underline{b}}\mathrm{D}_{\gamma}\Psi)\,, \tag{6.31}$$

$$R_{\dot{\alpha}\underline{b}}{}^{\underline{de}} = -(\mathrm{D}_{\dot{\alpha}}\partial^{[\underline{d}}\Psi)\delta_{\underline{b}}{}^{\underline{e}]} + \frac{1}{5}(\sigma^{\underline{de}})_{\dot{\alpha}}{}^{\dot{\gamma}}(\partial_{\underline{b}}\mathrm{D}_{\dot{\gamma}}\Psi)\,, \tag{6.32}$$

$$R_{\underline{ab}}{}^{\underline{de}} = -(\partial_{[\underline{a}}\partial^{[\underline{d}}\Psi)\delta_{\underline{b}]}{}^{\underline{e}]}\,. \tag{6.33}$$

7 Linearized Nordström supergravity in 10D, $\mathcal{N} = 2\mathrm{B}$ supergeometry

Now for a final time we replicate the discussions as seen in the previous three sections with a beginning of the on-shell description of 10D, $\mathcal{N} = 2\mathrm{B}$ superspace supergravity here. A set of torsion and curvature supertensors can be written in the form

$$\begin{aligned} &T_{\alpha\bar{\beta}}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\,, \qquad T_{\alpha\beta}{}^{\underline{c}} = T_{\bar{\alpha}\bar{\beta}}{}^{\underline{c}} = 0\,, \qquad T_{\underline{ab}}{}^{\underline{c}} = 0\,, \\ &T_{\alpha\beta}{}^{\gamma} = T_{\bar{\alpha}\beta}{}^{\gamma} = T_{\alpha\bar{\beta}}{}^{\bar{\gamma}} = \left[\delta_{(\alpha}{}^{\gamma}\delta_{\beta)}{}^{\delta} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}\right]\Lambda_{\delta}\,, \\ &T_{\alpha\beta}{}^{\bar{\gamma}} = T_{\bar{\alpha}\beta}{}^{\gamma} = T_{\bar{\alpha}\bar{\beta}}{}^{\bar{\gamma}} = \left[\delta_{(\alpha}{}^{\gamma}\delta_{\beta)}{}^{\delta} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}\right]\overline{\Lambda}_{\delta}\,, \end{aligned} \tag{7.1}$$

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$$
\begin{aligned}
T_{\alpha\underline{b}}{}^{\bar\gamma} &= \frac{1}{24}(\sigma_{\underline{b}})_{\alpha\delta}(\sigma^{[3]})^{\delta\gamma}\Big[e^{-2\Phi}(1+\overline{W})\overline{G}_{[3]} - e^{-2\Phi}(1-\overline{W})G_{[3]} - i(\sigma_{[3]})^{\epsilon\lambda}(\Lambda_\epsilon\Lambda_\lambda - \overline{\Lambda}_\epsilon\overline{\Lambda}_\lambda)\Big] \\
&\quad + \frac{1}{96}(\sigma^{[3]})_{\alpha\delta}(\sigma_{\underline{b}})^{\delta\gamma}(G_{[3]}+\overline{G}_{[3]})\,, \\
T_{\bar\alpha\underline{b}}{}^{\gamma} &= \frac{1}{24}(\sigma_{\underline{b}})_{\alpha\delta}(\sigma^{[3]})^{\delta\gamma}\Big[e^{-2\Phi}(1+W)G_{[3]} - e^{-2\Phi}(1-W)\overline{G}_{[3]} + i(\sigma_{[3]})^{\epsilon\lambda}(\Lambda_\epsilon\Lambda_\lambda - \overline{\Lambda}_\epsilon\overline{\Lambda}_\lambda)\Big] \\
&\quad + \frac{1}{96}(\sigma^{[3]})_{\alpha\delta}(\sigma_{\underline{b}})^{\delta\gamma}(G_{[3]}+\overline{G}_{[3]})\,, \\
T_{\alpha\underline{b}}{}^{\gamma} &= -T_{\bar\alpha\underline{b}}{}^{\bar\gamma} = \frac{1}{4}(\sigma_{\underline{b}})_{\alpha\delta}(\sigma^{\underline{d}})^{\delta\gamma}\left[e^{-2\Phi}\nabla_{\underline{d}}(W-\overline{W}) + i\frac{7}{4}(\sigma_{\underline{d}})^{\epsilon\lambda}\Lambda_\epsilon\overline{\Lambda}_\lambda\right] \\
&\quad + i\frac{1}{48}(\sigma_{[4]})_\alpha{}^\gamma\left[\frac{1}{8}(\sigma_{\underline{b}}{}^{[4]})^{\epsilon\lambda}\Lambda_\epsilon\overline{\Lambda}_\lambda - \frac{5}{3}e^{-2\Phi}\tilde{F}_{\underline{b}}{}^{[4]}\right]\,,
\end{aligned} \tag{7.2}
$$

$$
\begin{aligned}
R_{\alpha\beta\underline{cd}} &= i\frac{1}{12}(\sigma_{\underline{cd}}{}^{[3]})_{\alpha\beta}\Big\{e^{-2\Phi}(1+\overline{W})\overline{G}_{[3]} - e^{-2\Phi}(1-\overline{W})G_{[3]} - i(\sigma_{[3]})^{\epsilon\lambda}\big[\Lambda_\epsilon\Lambda_\lambda - \overline{\Lambda}_\epsilon\overline{\Lambda}_\lambda\big] \\
&\qquad - \frac{1}{4}(G_{[3]}+\overline{G}_{[3]})\Big\} \\
&\quad - i\frac{1}{2}(\sigma^{\underline{e}})_{\alpha\beta}\Big\{e^{-2\Phi}(1+\overline{W})\overline{G}_{\underline{cde}} - e^{-2\Phi}(1-\overline{W})G_{\underline{cde}} - i(\sigma_{\underline{cde}})^{\epsilon\lambda}\big[\Lambda_\epsilon\Lambda_\lambda - \overline{\Lambda}_\epsilon\overline{\Lambda}_\lambda\big] \\
&\qquad + \frac{1}{4}(G_{\underline{cde}}+\overline{G}_{\underline{cde}})\Big\}\,,
\end{aligned} \tag{7.3}
$$

$$
\begin{aligned}
R_{\bar\alpha\bar\beta\underline{cd}} &= i\frac{1}{12}(\sigma_{\underline{cd}}{}^{[3]})_{\alpha\beta}\Big\{e^{-2\Phi}(1+W)G_{[3]} - e^{-2\Phi}(1-W)\overline{G}_{[3]} + i(\sigma_{[3]})^{\epsilon\lambda}\big[\Lambda_\epsilon\Lambda_\lambda - \overline{\Lambda}_\epsilon\overline{\Lambda}_\lambda\big] \\
&\qquad - \frac{1}{4}(G_{[3]}+\overline{G}_{[3]})\Big\} \\
&\quad - i\frac{1}{2}(\sigma^{\underline{e}})_{\alpha\beta}\Big\{e^{-2\Phi}(1+W)G_{\underline{cde}} - e^{-2\Phi}(1-W)\overline{G}_{\underline{cde}} + i(\sigma_{\underline{cde}})^{\epsilon\lambda}\big[\Lambda_\epsilon\Lambda_\lambda - \overline{\Lambda}_\epsilon\overline{\Lambda}_\lambda\big] \\
&\qquad + \frac{1}{4}(G_{\underline{cde}}+\overline{G}_{\underline{cde}})\Big\}\,.
\end{aligned} \tag{7.4}
$$

as was noted in the portion of the work in [40] devoted to type IIB supergravity. We will end our discussion here. As the astute reader can note the expressions are of increasing complication. But the central message of the expressions in (7.1)–(7.4) is that the on-shell description of the 10D, $\mathcal{N}=2$B theory exists in perfect analogy with the on-shell description of 4D, $\mathcal{N}=1$ superspace given by the equations in (3.12).

Now for the covariant derivative operators linear in the conformal compensator Ψ and necessary for a Nordström theory may be given by

$$
\nabla_\alpha = \mathrm{D}_\alpha + \frac{1}{2}\Psi\mathrm{D}_\alpha + l_0(\sigma^{\underline{ab}})_\alpha{}^\beta(\mathrm{D}_\beta\Psi)\mathcal{M}_{\underline{ab}} \tag{7.5}
$$

$$
\nabla_{\bar\alpha} = \overline{\mathrm{D}}_\alpha + \frac{1}{2}\overline{\Psi}\overline{\mathrm{D}}_\alpha + \bar{l}_0(\sigma^{\underline{ab}})_\alpha{}^\beta(\overline{\mathrm{D}}_\beta\overline{\Psi})\mathcal{M}_{\underline{ab}} \tag{7.6}
$$

$$
\begin{aligned}
\nabla_{\underline{a}} &= \partial_{\underline{a}} + l_1\Psi\partial_{\underline{a}} + l_2\overline{\Psi}\partial_{\underline{a}} + il_3(\sigma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha\overline{\Psi})\overline{\mathrm{D}}_\beta + il_4(\sigma_{\underline{a}})^{\alpha\beta}(\overline{\mathrm{D}}_\alpha\Psi)\mathrm{D}_\beta \\
&\quad + il_5(\sigma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha\Psi)\overline{\mathrm{D}}_\beta + il_6(\sigma_{\underline{a}})^{\alpha\beta}(\overline{\mathrm{D}}_\alpha\overline{\Psi})\mathrm{D}_\beta \\
&\quad + il_7(\sigma_{\underline{a}}{}^{\underline{de}})^{\alpha\beta}(\mathrm{D}_\alpha\overline{\mathrm{D}}_\beta\overline{\Psi})\mathcal{M}_{\underline{de}} + il_8(\sigma_{\underline{a}}{}^{\underline{de}})^{\alpha\beta}(\overline{\mathrm{D}}_\alpha\mathrm{D}_\beta\Psi)\mathcal{M}_{\underline{de}} \\
&\quad + l_9(\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}} + l_{10}(\partial_{\underline{c}}\overline{\Psi})\mathcal{M}_{\underline{a}}{}^{\underline{c}}
\end{aligned} \tag{7.7}
$$

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with the Type IIB supersymmetry algebra

$$\{\mathrm{D}_\alpha, \mathrm{D}_\beta\} = 0\,, \qquad \{\overline{\mathrm{D}}_\alpha, \overline{\mathrm{D}}_\beta\} = 0\,, \qquad \{\mathrm{D}_\alpha, \overline{\mathrm{D}}_\beta\} = i(\sigma^{\underline{a}})_{\alpha\beta}\partial_{\underline{a}}\,. \tag{7.8}$$

By adopting the constraints

$$T_{\underline{ab}}{}^{\underline{c}} = 0\,, \qquad T_{\alpha\bar{\beta}}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\,, \tag{7.9}$$

we have the following parameterization results

$$\begin{aligned} &l_1 = l_2 = \frac{1}{2}\,, \quad l_3 = l_4 = -\frac{1}{32}\,, \quad l_5 = l_6 = -\frac{27}{160}\,, \\ &l_7 = l_8 = 0\,, \quad l_9 = l_{10} = -\frac{1}{2}\,. \end{aligned} \tag{7.10}$$

As the consequence of this choice of parameters, we find the torsion supertensors given in (7.11)–(7.28).

$$T_{\alpha\beta}{}^{\underline{c}} = 0\,, \tag{7.11}$$

$$T_{\alpha\beta}{}^{\gamma} = \frac{2}{5}(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\gamma\delta}(\mathrm{D}_\delta\Psi)\,, \tag{7.12}$$

$$T_{\alpha\beta}{}^{\bar{\gamma}} = 0\,, \tag{7.13}$$

$$T_{\bar{\alpha}\bar{\beta}}{}^{\underline{c}} = 0\,, \tag{7.14}$$

$$T_{\bar{\alpha}\bar{\beta}}{}^{\gamma} = 0\,, \tag{7.15}$$

$$T_{\bar{\alpha}\bar{\beta}}{}^{\bar{\gamma}} = \frac{2}{5}(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\gamma\delta}(\overline{\mathrm{D}}_\delta\overline{\Psi})\,, \tag{7.16}$$

$$T_{\alpha\bar{\beta}}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\,, \tag{7.17}$$

$$\begin{aligned} T_{\alpha\bar{\beta}}{}^{\gamma} = &-\frac{1}{320}\left[(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} + \frac{1}{24}(\sigma^{[5]})_{\alpha\beta}(\sigma_{[5]})^{\gamma\delta}\right](\overline{\mathrm{D}}_\delta\overline{\Psi}) \\ &+ \frac{1}{192}\left[-(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} + \frac{1}{40}(\sigma^{[5]})_{\alpha\beta}(\sigma_{[5]})^{\gamma\delta}\right](\overline{\mathrm{D}}_\delta\Psi)\,, \end{aligned} \tag{7.18}$$

$$\begin{aligned} T_{\alpha\bar{\beta}}{}^{\bar{\gamma}} = &-\frac{1}{320}\left[(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} + \frac{1}{24}(\sigma^{[5]})_{\alpha\beta}(\sigma_{[5]})^{\gamma\delta}\right](\mathrm{D}_\delta\Psi) \\ &+ \frac{1}{192}\left[-(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} + \frac{1}{40}(\sigma^{[5]})_{\alpha\beta}(\sigma_{[5]})^{\gamma\delta}\right](\mathrm{D}_\delta\overline{\Psi})\,, \end{aligned} \tag{7.19}$$

$$T_{\alpha\underline{b}}{}^{\underline{c}} = \left[\frac{53}{160}\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\gamma + \frac{59}{160}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma\right](\mathrm{D}_\gamma\Psi) + \left[\frac{15}{32}\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\gamma + \frac{1}{32}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma\right](\mathrm{D}_\gamma\overline{\Psi})\,, \tag{7.20}$$

$$\begin{aligned} T_{\alpha\underline{b}}{}^{\gamma} = &-\frac{31}{64}\delta_\alpha{}^\gamma(\partial_{\underline{b}}\Psi) + \frac{27}{320}\delta_\alpha{}^\gamma(\partial_{\underline{b}}\overline{\Psi}) + \frac{15}{64}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma(\partial_{\underline{c}}\Psi) + \frac{53}{320}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma(\partial_{\underline{c}}\overline{\Psi}) \\ &- i\frac{1}{512}\left[\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} - \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\mathrm{D}_\beta\overline{\mathrm{D}}_\delta\Psi) \\ &- i\frac{27}{2560}\left[\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} - \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\mathrm{D}_\beta\overline{\mathrm{D}}_\delta\overline{\Psi})\,, \end{aligned} \tag{7.21}$$

$$\begin{aligned} T_{\alpha\underline{b}}{}^{\bar{\gamma}} = &-i\frac{1}{512}\left[\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} - \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\mathrm{D}_\beta\mathrm{D}_\delta\overline{\Psi}) \\ &- i\frac{27}{2560}\left[\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} - \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\mathrm{D}_\beta\mathrm{D}_\delta\Psi)\,, \end{aligned} \tag{7.22}$$

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$$T_{\bar{\alpha}\underline{b}}{}^{\underline{c}} = \left[\frac{15}{32}\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\gamma + \frac{1}{32}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma\right](\overline{\mathrm{D}}_\gamma\Psi) + \left[\frac{53}{160}\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\gamma + \frac{59}{160}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma\right](\overline{\mathrm{D}}_\gamma\overline{\Psi}) \,, \tag{7.23}$$

$$\begin{aligned} T_{\bar{\alpha}\underline{b}}{}^{\gamma} = &-i\frac{1}{512}\left[\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} - \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\overline{\mathrm{D}}_\beta\overline{\mathrm{D}}_\delta\Psi) \\ &- i\frac{27}{2560}\left[\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} - \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\overline{\mathrm{D}}_\beta\overline{\mathrm{D}}_\delta\overline{\Psi}) \,, \end{aligned} \tag{7.24}$$

$$\begin{aligned} T_{\bar{\alpha}\underline{b}}{}^{\bar{\gamma}} = &-\frac{31}{64}\delta_\alpha{}^\gamma(\partial_{\underline{b}}\overline{\Psi}) + \frac{27}{320}\delta_\alpha{}^\gamma(\partial_{\underline{b}}\Psi) + \frac{15}{64}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma(\partial_{\underline{c}}\overline{\Psi}) + \frac{53}{320}(\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma(\partial_{\underline{c}}\Psi) \\ &- i\frac{1}{512}\left[\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} - \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\overline{\mathrm{D}}_\beta\mathrm{D}_\delta\overline{\Psi}) \\ &- i\frac{27}{2560}\left[\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma(\sigma_{\underline{b}[2]})^{\beta\delta} - \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma(\sigma^{[3]})^{\beta\delta}\right](\overline{\mathrm{D}}_\beta\mathrm{D}_\delta\Psi) \,, \end{aligned} \tag{7.25}$$

$$T_{\underline{ab}}{}^{\underline{c}} = 0 \,, \tag{7.26}$$

$$T_{\underline{ab}}{}^{\gamma} = i\frac{1}{32}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\overline{\mathrm{D}}_\delta\Psi) + i\frac{27}{160}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\overline{\mathrm{D}}_\delta\overline{\Psi}) \,, \tag{7.27}$$

$$T_{\underline{ab}}{}^{\bar{\gamma}} = i\frac{1}{32}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\mathrm{D}_\delta\overline{\Psi}) + i\frac{27}{160}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\mathrm{D}_\delta\Psi) \,. \tag{7.28}$$

For the components of the curvatures, we find the results seen in (7.29)–(7.34).

$$R_{\alpha\beta}{}^{\underline{de}} = \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} - (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}}{}^{\underline{de}})^{\gamma\delta}\right](\mathrm{D}_\gamma\mathrm{D}_\delta\Psi) \,, \tag{7.29}$$

$$R_{\bar{\alpha}\bar{\beta}}{}^{\underline{de}} = \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} - (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}}{}^{\underline{de}})^{\gamma\delta}\right](\overline{\mathrm{D}}_\gamma\overline{\mathrm{D}}_\delta\overline{\Psi}) \,, \tag{7.30}$$

$$\begin{aligned} R_{\alpha\bar{\beta}}{}^{\underline{de}} = &-i\frac{3}{5}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}(\Psi+\overline{\Psi})) - i\frac{1}{10}(\sigma^{\underline{def}})_{\alpha\beta}(\partial_{\underline{f}}(\Psi+\overline{\Psi})) \\ &- \frac{1}{80}\left[(\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}}{}^{\underline{de}})^{\gamma\delta} - \frac{1}{2}(\sigma^{[2][\underline{d}})_{\alpha\beta}(\sigma^{\underline{e}]}{}_{[2]})^{\gamma\delta} - \frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta}\right](\overline{\mathrm{D}}_\gamma\mathrm{D}_\delta\Psi) \\ &- \frac{1}{80}\left[(\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}}{}^{\underline{de}})^{\gamma\delta} - \frac{1}{2}(\sigma^{[2][\underline{d}})_{\alpha\beta}(\sigma^{\underline{e}]}{}_{[2]})^{\gamma\delta} - \frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta}\right](\mathrm{D}_\gamma\overline{\mathrm{D}}_\delta\overline{\Psi}) \,, \end{aligned} \tag{7.31}$$

$$R_{\alpha\underline{b}}{}^{\underline{de}} = -\frac{1}{2}(\mathrm{D}_\alpha\partial^{[\underline{d}}(\Psi+\overline{\Psi}))\delta_{\underline{b}}{}^{\underline{e}]} + \frac{1}{5}(\sigma^{\underline{de}})_\alpha{}^\gamma(\partial_{\underline{b}}\mathrm{D}_\gamma\Psi) \,, \tag{7.32}$$

$$R_{\bar{\alpha}\underline{b}}{}^{\underline{de}} = -\frac{1}{2}(\overline{\mathrm{D}}_\alpha\partial^{[\underline{d}}(\Psi+\overline{\Psi}))\delta_{\underline{b}}{}^{\underline{e}]} + \frac{1}{5}(\sigma^{\underline{de}})_\alpha{}^\gamma(\partial_{\underline{b}}\overline{\mathrm{D}}_\gamma\overline{\Psi}) \,, \tag{7.33}$$

$$R_{\underline{ab}}{}^{\underline{de}} = -\frac{1}{2}(\partial_{[\underline{a}}\partial^{[\underline{d}}(\Psi+\overline{\Psi}))\delta_{\underline{b}]}{}^{\underline{e}]} \,. \tag{7.34}$$

8 Higher dimensional component considerations

In the following four subsections, we will appropriately adapt these results to the cases of eleven and ten dimensional formulations appropriate for Nordström supergravity in those contexts. There are four steps:

(a). define the Nordström SG linearized superspace supercovariant derivatives in terms of a scalar prepotential leading to component fields,

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(b). express the geometrical tensors of each respective superspace in terms of the component fields presented in the previous part,

(c). express the "composition rules" of the parameters of general coordinates, local Lorentz, and local SUSY transformations, and

(d). write the component level SUSY transformation laws

that we undertake in each of the four cases of 11D, $\mathcal{N} = 1$, 10D, $\mathcal{N} = 1$, 10D, $\mathcal{N} = 2$A, and 10D, $\mathcal{N} = 2$B theories.

8.1 Adaptation to 11D, $\mathcal{N} = 1$ component/superspace results: step 1

In the case of the 11D N(ordström)-SG covariant derivatives we define

$$\nabla_\alpha = \mathrm{D}_\alpha + \frac{1}{2}\Psi \mathrm{D}_\alpha + \frac{1}{10}(\gamma^{\underline{de}})_\alpha{}^\beta(\mathrm{D}_\beta\Psi)\mathcal{M}_{\underline{de}}\,, \tag{8.1}$$

$$\nabla_{\underline{a}} = \partial_{\underline{a}} + \Psi\partial_{\underline{a}} + i\frac{1}{4}(\gamma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha\Psi)\mathrm{D}_\beta - (\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}}\,, \tag{8.2}$$

and "split" the spatial 11D N-SG covariant derivative into two parts

$$\nabla_{\underline{a}}| = \mathbf{D}_{\underline{a}} + \psi_{\underline{a}}{}^\gamma\nabla_\gamma|\,. \tag{8.3}$$

On taking the $\theta \to 0$ limit the latter terms allows an identification with the gravitino and the leading term in this limit yields a component-level linearized gravitationally covariant derivative operator given by

$$\mathbf{D}_{\underline{a}} = e_{\underline{a}} + \phi_{\underline{a}}{}^\iota\mathcal{M}_\iota = \partial_{\underline{a}} + \Psi\partial_{\underline{a}} + \phi_{\underline{a}}{}^\iota\mathcal{M}_\iota\,. \tag{8.4}$$

By comparison of the l.h.s. to the r.h.s. of (8.4), we see that a linearized frame field $e_{\underline{a}}{}^{\underline{m}} = (1 + \Psi)\delta_{\underline{a}}{}^{\underline{m}}$ emerges to describe a scalar graviton. Finally, comparison of the coefficient of the Lorentz generator $\mathcal{M}_\iota$ as it appears in the latter two forms of (8.4) informs us the spin connection is given by

$$\phi_{\underline{c}}{}^{\underline{de}} = -\frac{1}{2}\delta_{\underline{c}}{}^{[\underline{d}}(\partial^{\underline{e}]}\Psi)\,. \tag{8.5}$$

Comparing the result in (8.2) with the one in (8.3) a component gravitino is identified via

$$\psi_{\underline{a}}{}^\gamma = i\frac{1}{4}(\gamma_{\underline{a}})^{\gamma\delta}(\mathrm{D}_\delta\Psi)\,. \tag{8.6}$$

However, as this expression contains an explicit γ-matrix we see that it really defines the non-conformal *spin*-$\frac{1}{2}$ part of the gravitino to be

$$\psi_\beta \equiv (\gamma^{\underline{a}})_{\beta\gamma}\psi_{\underline{a}}{}^\gamma\,. \tag{8.7}$$

This is to be expected. As a Nordström type theory only contains a scalar graviton, it follows only the "γ-trace" of the gravitino can occur. So then we have

$$\mathrm{D}_\beta\Psi = i\frac{4}{11}(\gamma^{\underline{a}})_{\beta\gamma}\psi_{\underline{a}}{}^\gamma \equiv i\frac{4}{11}\psi_\beta\,, \tag{8.8}$$

in the $\theta \to 0$ limit.

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In order to complete the specification of the geometrical superfields also requires explicit definitions of the bosonic terms to second order in D-derivatives. So we define bosonic fields:

$$K = C^{\gamma\delta}(\mathrm{D}_\gamma \mathrm{D}_\delta \Psi)\,, \quad K_{[3]} = (\gamma_{[3]})^{\gamma\delta}(\mathrm{D}_\gamma \mathrm{D}_\delta \Psi)\,, \quad K_{[4]} = (\gamma_{[4]})^{\gamma\delta}(\mathrm{D}_\gamma \mathrm{D}_\delta \Psi)\,, \tag{8.9}$$

or in other words,

$$\frac{1}{2}\mathrm{D}_{[\gamma}\mathrm{D}_{\delta]}\Psi = \frac{1}{32}\left\{C_{\gamma\delta}K - \frac{1}{3!}(\gamma^{[3]})_{\gamma\delta}K_{[3]} + \frac{1}{4!}(\gamma^{[4]})_{\gamma\delta}K_{[4]}\right\}. \tag{8.10}$$

We emphasize that the component fields (the K's) are defined by the $\theta \to 0$ limit of these equations. The results in (8.9) and (8.10) follow as results from a Fierz identity

$$\delta_{[\gamma}{}^{\alpha}\delta_{\delta]}{}^{\beta} = \frac{1}{16}\left\{C_{\gamma\delta}C^{\alpha\beta} - \frac{1}{3!}(\gamma^{[3]})_{\gamma\delta}(\gamma_{[3]})^{\alpha\beta} + \frac{1}{4!}(\gamma^{[4]})_{\gamma\delta}(\gamma_{[4]})^{\alpha\beta}\right\}, \tag{8.11}$$

valid for 11D spinors.

8.2 Adaptation to 11D, $\mathcal{N} = 1$ component/superspace results: step 2

Torsions:

$$T_{\alpha\beta}{}^{\underline{c}} = i(\gamma^{\underline{c}})_{\alpha\beta}\,, \tag{8.12}$$

$$T_{\alpha\beta}{}^{\gamma} = i\frac{3}{110}(\gamma^{[2]})_{\alpha\beta}(\gamma_{[2]})^{\gamma\delta}\psi_\delta\,, \tag{8.13}$$

$$T_{\alpha\underline{b}}{}^{\underline{c}} = i\frac{3}{11}\left[\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\beta + \frac{3}{5}(\gamma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\beta\right]\psi_\beta\,, \tag{8.14}$$

$$\begin{aligned}T_{\alpha\underline{b}}{}^{\gamma} = i\frac{1}{128}\Big[&-(\gamma_{\underline{b}})_\alpha{}^\gamma K + \frac{1}{2}(\gamma^{[2]})_\alpha{}^\gamma K_{\underline{b}[2]} - \frac{1}{3!}(\gamma_{\underline{b}[3]})_\alpha{}^\gamma K^{[3]} + \frac{1}{3!}(\gamma^{[3]})_\alpha{}^\gamma K_{\underline{b}[3]}\\ &- \frac{1}{4!}(\gamma_{\underline{b}[4]})_\alpha{}^\gamma K^{[4]}\Big] + \frac{1}{8}\left[\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\gamma + 3(\gamma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma\right](\partial_{\underline{c}}\Psi)\,,\end{aligned} \tag{8.15}$$

$$T_{\underline{ab}}{}^{\underline{c}} = 0\,, \tag{8.16}$$

$$T_{\underline{ab}}{}^{\gamma} = \frac{1}{11}(\gamma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\psi_\delta)\,. \tag{8.17}$$

Curvatures:

$$\begin{aligned}R_{\alpha\beta}{}^{\underline{de}} = \frac{1}{80}\Big[&(\gamma^{\underline{de}})_{\alpha\beta}K + (\gamma_{[1]})_{\alpha\beta}K^{[1]\underline{de}} - \frac{1}{3!}(\gamma^{\underline{de}[3]})_{\alpha\beta}K_{[3]} - \frac{1}{2}(\gamma_{[2]})_{\alpha\beta}K^{[2]\underline{de}}\\ &+ \frac{1}{5!4!}\epsilon^{\underline{de}[5][4]}(\gamma_{[5]})_{\alpha\beta}K_{[4]}\Big]\,,\end{aligned} \tag{8.18}$$

$$R_{\alpha\underline{b}}{}^{\underline{de}} = i\frac{4}{11}\left[\delta_{\underline{b}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_\alpha) + \frac{1}{5}(\gamma^{\underline{de}})_\alpha{}^\delta(\partial_{\underline{b}}\psi_\delta)\right], \tag{8.19}$$

$$R_{\underline{ab}}{}^{\underline{de}} = -(\partial_{[\underline{a}}\partial^{[\underline{d}}\Psi)\delta_{\underline{b}]}{}^{\underline{e}]}\,. \tag{8.20}$$

8.3 Adaptation to 11D, $\mathcal{N} = 1$ component/superspace results: step 3

Parameter Composition Rules:

$$\xi^{\underline{m}} = -i\epsilon_1{}^\alpha \epsilon_2{}^\beta (\gamma^{\underline{c}})_{\alpha\beta}\delta_{\underline{c}}{}^{\underline{m}}(1+\Psi)\,, \tag{8.21}$$

$$\begin{aligned}\lambda^{\underline{de}} = -\frac{1}{80}\epsilon_1{}^\alpha \epsilon_2{}^\beta \Big[&(\gamma^{\underline{de}})_{\alpha\beta}K + (\gamma_{[1]})_{\alpha\beta}K^{[1]\underline{de}} - \frac{1}{3!}(\gamma^{\underline{de}[3]})_{\alpha\beta}K_{[3]} - \frac{1}{2}(\gamma_{[2]})_{\alpha\beta}K^{[2]\underline{de}} \\ &+ \frac{1}{5!4!}\epsilon^{\underline{de}[5][4]}(\gamma_{[5]})_{\alpha\beta}K_{[4]}\Big] + i\frac{1}{2}\epsilon_1{}^\alpha \epsilon_2{}^\beta (\gamma^{[\underline{d}}})_{\alpha\beta}(\partial^{\underline{e}]}\Psi)\,,\end{aligned} \tag{8.22}$$

$$\epsilon^\delta = i\frac{1}{11}\epsilon_1^\alpha \epsilon_2^\beta \left[(\gamma^{[1]})_{\alpha\beta}(\gamma_{[1]})^{\delta\epsilon} - \frac{3}{10}(\gamma^{[2]})_{\alpha\beta}(\gamma_{[2]})^{\delta\epsilon}\right]\psi_\epsilon\,. \tag{8.23}$$

8.4 Adaptation to 11D, $\mathcal{N} = 1$ component/superspace results: step 4

SUSY transformation laws:

$$\delta_Q e_{\underline{a}}{}^{\underline{m}} = -i\frac{4}{11}\epsilon^\beta \left[\delta_{\underline{a}}{}^{\underline{d}}\delta_\beta{}^\gamma + \frac{1}{5}(\gamma_{\underline{a}}{}^{\underline{d}})_\beta{}^\gamma\right]\delta_{\underline{d}}{}^{\underline{m}}\psi_\gamma\,, \tag{8.24}$$

$$\begin{aligned}\delta_Q \psi_{\underline{a}}{}^\delta = {}&(1+\Psi)\partial_{\underline{a}}\epsilon^\delta - \epsilon^\delta(\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}} \\ &- i\frac{1}{128}\epsilon^\beta\Big[-(\gamma_{\underline{a}})_\beta{}^\delta K + \frac{1}{2}(\gamma^{[2]})_\beta{}^\delta K_{\underline{a}[2]} - \frac{1}{3!}(\gamma_{\underline{a}[3]})_\beta{}^\delta K^{[3]} + \frac{1}{3!}(\gamma^{[3]})_\beta{}^\delta K_{\underline{a}[3]} \\ &- \frac{1}{4!}(\gamma_{\underline{a}[4]})_\beta{}^\delta K^{[4]}\Big] - \frac{1}{8}\epsilon^\beta\Big[\delta_{\underline{a}}{}^{\underline{c}}\delta_\beta{}^\delta + 3(\gamma_{\underline{a}}{}^{\underline{c}})_\beta{}^\delta\Big](\partial_{\underline{c}}\Psi)\,,\end{aligned} \tag{8.25}$$

$$\delta_Q \phi_{\underline{a}}{}^{\underline{de}} = -i\frac{4}{11}\epsilon^\beta \left[\delta_{\underline{a}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_\beta) + \frac{1}{5}(\gamma^{\underline{de}})_\beta{}^\delta(\partial_{\underline{a}}\psi_\delta)\right]\,. \tag{8.26}$$

In the remaining subsections of the section, the steps described for the case of the 11D, $\mathcal{N} = 1$ theory above will be repeated, essentially line by line, in each of the cases for 10D, $\mathcal{N} = 1$, 10D, $\mathcal{N} = 2$A, and 10D, $\mathcal{N} = 2$B superspaces. This will imply a certain repetitive nature to the respective presentation. There will only be slight variations in explicit details. We are able to minimize this very slightly by noting the result in (8.4) applies universally in all three cases. So we will not explicitly rewrite it nor its resultant implications several more times.

8.5 Adaptation to 10D, $\mathcal{N} = 1$ component/superspace results: step 1

In the case of 10D $\mathcal{N} = 1$ N-SG covariant derivatives we define

$$\nabla_\alpha = \mathrm{D}_\alpha + \frac{1}{2}\Psi\mathrm{D}_\alpha + \frac{1}{10}(\sigma^{\underline{ab}})_\alpha{}^\beta(\mathrm{D}_\beta\Psi)\mathcal{M}_{\underline{ab}}\,, \tag{8.27}$$

$$\nabla_{\underline{a}} = \partial_{\underline{a}} + \Psi\partial_{\underline{a}} - i\frac{2}{5}(\sigma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha\Psi)\mathrm{D}_\beta - (\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}}\,, \tag{8.28}$$

and "split" the spatial 10D $\mathcal{N} = 1$ N-SG covariant derivative into two parts

$$\nabla_{\underline{a}}| = \mathbf{D}_{\underline{a}} + \psi_{\underline{a}}{}^\gamma\nabla_\gamma|\,. \tag{8.29}$$

Comparing the result (8.28) in with the one in (8.29) a component gravitino is identified via

$$\psi_{\underline{a}}{}^\gamma = -i\frac{2}{5}(\sigma_{\underline{a}})^{\gamma\delta}(\mathrm{D}_\delta\Psi)\,. \tag{8.30}$$

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However, as this expression contains an explicit σ-matrix we see that it defines the non-conformal *spin*-$\frac{1}{2}$ part of the gravitino to be

$$\psi_\beta \equiv (\sigma^{\underline{a}})_{\beta\gamma}\psi_{\underline{a}}{}^\gamma \,, \tag{8.31}$$

and it follows only the "σ-trace" of the gravitino can occur. So then we have

$$\mathrm{D}_\beta \Psi = i\frac{1}{4}(\sigma^{\underline{a}})_{\beta\gamma}\psi_{\underline{a}}{}^\gamma \equiv i\frac{1}{4}\psi_\beta \,, \tag{8.32}$$

in the $\theta \to 0$ limit.

The complete specification of the geometrical superfields also requires explicit definitions of the bosonic terms to second order in D-derivatives. We take advantage of the 10D Fierz identity

$$\delta_{[\gamma}{}^\alpha \delta_{\delta]}{}^\beta = \frac{1}{48}\,(\sigma^{[3]})_{\gamma\delta}(\gamma_{[3]})^{\alpha\beta} \,, \tag{8.33}$$

valid for 10D spinors, so we may define a bosonic field:

$$G_{[3]} = (\sigma_{[3]})^{\gamma\delta}(\mathrm{D}_\gamma \mathrm{D}_\delta \Psi) \,, \tag{8.34}$$

or in other words,

$$\frac{1}{2}\mathrm{D}_{[\gamma}\mathrm{D}_{\delta]}\Psi = \frac{1}{16\times 3!}(\sigma^{[3]})_{\gamma\delta}G_{[3]} \,. \tag{8.35}$$

We emphasize that the component field (the G) is defined by the $\theta \to 0$ limit of these equations.

8.6 Adaptation to 10D, $\mathcal{N} = 1$ component/superspace results: step 2

Torsions:

$$T_{\alpha\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta} \,, \tag{8.36}$$

$$T_{\alpha\beta}{}^\gamma = 0 \,, \tag{8.37}$$

$$T_{\alpha\underline{b}}{}^{\underline{c}} = i\frac{3}{20}\left[\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\delta + (\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\delta\right]\psi_\delta \,, \tag{8.38}$$

$$T_{\alpha\underline{b}}{}^\gamma = i\frac{1}{80}\left[-(\sigma^{[2]})_\alpha{}^\gamma G_{\underline{b}[2]} + \frac{1}{3}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma G^{[3]}\right] - \frac{3}{10}\left[\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\gamma - (\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma\right](\partial_{\underline{c}}\Psi) \,, \tag{8.39}$$

$$T_{\underline{ab}}{}^{\underline{c}} = 0 \,, \tag{8.40}$$

$$T_{\underline{ab}}{}^\gamma = -\frac{1}{10}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\psi_\delta) \,. \tag{8.41}$$

Curvatures:

$$R_{\alpha\beta}{}^{\underline{de}} = -i\frac{6}{5}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}\Psi) - \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}G_{[3]} + (\sigma_{[1]})_{\alpha\beta}G^{[1]\underline{de}}\right] , \tag{8.42}$$

$$R_{\alpha\underline{b}}{}^{\underline{de}} = i\frac{1}{4}\left[\delta_{\underline{b}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_\alpha) + \frac{1}{5}(\sigma^{\underline{de}})_\alpha{}^\gamma(\partial_{\underline{b}}\psi_\gamma)\right] , \tag{8.43}$$

$$R_{\underline{ab}}{}^{\underline{de}} = -(\partial_{[\underline{a}}\partial^{[\underline{d}}\Psi)\delta_{\underline{b}]}{}^{\underline{e}]} \,. \tag{8.44}$$

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8.7 Adaptation to 10D, $\mathcal{N} = 1$ component/superspace results: step 3

Parameter Composition Rules:

$$\xi^{\underline{m}} = -i\epsilon_1{}^{\alpha}\epsilon_2{}^{\beta}(\sigma^{\underline{c}})_{\alpha\beta}\delta_{\underline{c}}{}^{\underline{m}}(1+\Psi)\,, \tag{8.45}$$

$$\lambda^{\underline{de}} = \frac{1}{40}\epsilon_1{}^{\alpha}\epsilon_2{}^{\beta}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}G_{[3]} + (\sigma_{[1]})_{\alpha\beta}G^{[1]\underline{de}}\right] + i\frac{17}{10}\epsilon_1{}^{\alpha}\epsilon_2{}^{\beta}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}\Psi)\,, \tag{8.46}$$

$$\epsilon^{\delta} = -i\frac{1}{10}\epsilon_1{}^{\alpha}\epsilon_2{}^{\beta}(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\delta\epsilon}\psi_{\epsilon}\,. \tag{8.47}$$

8.8 Adaptation to 10D, $\mathcal{N} = 1$ component/superspace results: step 4

SUSY transformation laws:

$$\delta_Q e_{\underline{a}}{}^{\underline{m}} = -i\frac{1}{4}\epsilon^{\beta}\left[\delta_{\underline{a}}{}^{\underline{d}}\delta_{\beta}{}^{\gamma} + \frac{1}{5}(\sigma_{\underline{a}}{}^{\underline{d}})_{\beta}{}^{\gamma}\right]\delta_{\underline{d}}{}^{\underline{m}}\psi_{\gamma}\,, \tag{8.48}$$

$$\begin{aligned}\delta_Q \psi_{\underline{a}}{}^{\delta} =& (1+\Psi)\partial_{\underline{a}}\epsilon^{\delta} - \epsilon^{\delta}(\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}}\\ &- i\frac{1}{80}\epsilon^{\beta}\left[-(\sigma^{[2]})_{\beta}{}^{\delta}G_{\underline{a}[2]} + \frac{1}{3}(\sigma_{\underline{a}[3]})_{\beta}{}^{\delta}G^{[3]}\right] + \frac{3}{10}\epsilon^{\beta}\left[\delta_{\underline{a}}{}^{\underline{c}}\delta_{\beta}{}^{\delta} - (\sigma_{\underline{a}}{}^{\underline{c}})_{\beta}{}^{\delta}\right](\partial_{\underline{c}}\Psi)\,,\end{aligned} \tag{8.49}$$

$$\delta_Q \phi_{\underline{a}}{}^{\underline{de}} = -i\frac{1}{4}\epsilon^{\beta}\left[\delta_{\underline{a}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_{\beta}) + \frac{1}{5}(\sigma^{\underline{de}})_{\beta}{}^{\gamma}(\partial_{\underline{a}}\psi_{\gamma})\right]\,. \tag{8.50}$$

8.9 Adaptation to 10D, $\mathcal{N} = 2A$ component/superspace results: step 1

In the case of 10D $\mathcal{N} = 2A$ N-SG covariant derivatives we define

$$\nabla_{\alpha} = \mathrm{D}_{\alpha} + \frac{1}{2}\Psi\mathrm{D}_{\alpha} + \frac{1}{10}(\sigma^{\underline{ab}})_{\alpha}{}^{\beta}(\mathrm{D}_{\beta}\Psi)\mathcal{M}_{\underline{ab}}\,, \tag{8.51}$$

$$\nabla_{\dot{\alpha}} = \mathrm{D}_{\dot{\alpha}} + \frac{1}{2}\Psi\mathrm{D}_{\dot{\alpha}} + \frac{1}{10}(\sigma^{\underline{ab}})_{\dot{\alpha}}{}^{\dot{\beta}}(\mathrm{D}_{\dot{\beta}}\Psi)\mathcal{M}_{\underline{ab}}\,, \tag{8.52}$$

$$\nabla_{\underline{a}} = \partial_{\underline{a}} + \Psi\partial_{\underline{a}} - i\frac{1}{5}(\sigma_{\underline{a}})^{\delta\gamma}(\mathrm{D}_{\delta}\Psi)\mathrm{D}_{\gamma} - i\frac{1}{5}(\sigma_{\underline{a}})^{\dot{\delta}\dot{\gamma}}(\mathrm{D}_{\dot{\delta}}\Psi)\mathrm{D}_{\dot{\gamma}} - (\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}}\,, \tag{8.53}$$

and "split" the spatial 10D $\mathcal{N} = 2A$ N-SG covariant derivative into three parts

$$\nabla_{\underline{a}}| = \mathbf{D}_{\underline{a}} + \psi_{\underline{a}}{}^{\gamma}\nabla_{\gamma}| + \psi_{\underline{a}}{}^{\dot{\gamma}}\nabla_{\dot{\gamma}}|\,. \tag{8.54}$$

On taking the $\theta \to 0$ limit the latter terms allow an identification with the component gravitinos are identified via

$$\psi_{\underline{a}}{}^{\gamma} = -\,i\frac{1}{5}(\sigma_{\underline{a}})^{\gamma\delta}(\mathrm{D}_{\delta}\Psi)\,, \quad \psi_{\underline{a}}{}^{\dot{\gamma}} = -i\frac{1}{5}(\sigma_{\underline{a}})^{\dot{\gamma}\dot{\delta}}(\mathrm{D}_{\dot{\delta}}\Psi)\,. \tag{8.55}$$

However, as this expression contains an explicit σ-matrix we see that it really defines the non-conformal *spin*-$\frac{1}{2}$ part of the gravitino to be

$$\psi_{\beta} \equiv (\sigma^{\underline{a}})_{\beta\gamma}\psi_{\underline{a}}{}^{\gamma}\,, \quad \psi_{\dot{\beta}} \equiv (\sigma^{\underline{a}})_{\dot{\beta}\dot{\gamma}}\psi_{\underline{a}}{}^{\dot{\gamma}}\,. \tag{8.56}$$

It follows only the "σ-trace" of the gravitino can occur. So then we have

$$\mathrm{D}_{\beta}\Psi = i\frac{1}{2}(\sigma^{\underline{a}})_{\beta\gamma}\psi_{\underline{a}}{}^{\gamma} \equiv i\frac{1}{2}\psi_{\beta}\,, \quad \mathrm{D}_{\dot{\beta}}\Psi = i\frac{1}{2}(\sigma^{\underline{a}})_{\dot{\beta}\dot{\gamma}}\psi_{\underline{a}}{}^{\dot{\gamma}} \equiv i\frac{1}{2}\psi_{\dot{\beta}}\,, \tag{8.57}$$

in the $\theta \to 0$ limit.

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In order to complete the specification of the geometrical superfields also requires explicit definitions of the bosonic terms to second order in D-derivatives. So we define bosonic fields:

$$G_{[3]} = (\sigma_{[3]})^{\gamma\delta}(\mathrm{D}_\gamma \mathrm{D}_\delta \Psi)\,, \quad H_{[3]} = (\sigma_{[3]})^{\dot\gamma\dot\delta}(\mathrm{D}_{\dot\gamma} \mathrm{D}_{\dot\delta} \Psi)\,, \tag{8.58}$$

$$N = C^{\gamma\dot\delta}(\mathrm{D}_\gamma \mathrm{D}_{\dot\delta}\Psi)\,, \quad N_{[2]} = (\sigma_{[2]})^{\gamma\dot\delta}(\mathrm{D}_\gamma \mathrm{D}_{\dot\delta}\Psi)\,, \quad N_{[4]} = (\sigma_{[4]})^{\gamma\dot\delta}(\mathrm{D}_\gamma \mathrm{D}_{\dot\delta}\Psi)\,, \tag{8.59}$$

or in other words,

$$\frac{1}{2}\mathrm{D}_{[\gamma}\mathrm{D}_{\delta]}\Psi = \frac{1}{16\times 3!}(\sigma^{[3]})_{\gamma\delta}G_{[3]}\,, \quad \frac{1}{2}\mathrm{D}_{[\dot\gamma}\mathrm{D}_{\dot\delta]}\Psi = \frac{1}{16\times 3!}(\sigma^{[3]})_{\dot\gamma\dot\delta}H_{[3]}\,, \tag{8.60}$$

and

$$\mathrm{D}_\gamma \mathrm{D}_{\dot\delta}\Psi = \frac{1}{16}\left\{ C_{\gamma\dot\delta}N + \frac{1}{2!}(\sigma^{[2]})_{\gamma\dot\delta}N_{[2]} + \frac{1}{4!}(\sigma^{[4]})_{\gamma\dot\delta}N_{[4]} \right\}. \tag{8.61}$$

We emphasize that the component fields (the G's, H's and N's) are defined by the $\theta \to 0$ limit of these equations.

8.10 Adaptation to 10D, $\mathcal{N} = 2$A component/superspace results: step 2

Torsions:

$$T_{\alpha\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\,, \tag{8.62}$$

$$T_{\alpha\beta}{}^{\gamma} = i\frac{1}{10}(\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}\psi_\delta\,, \tag{8.63}$$

$$T_{\alpha\beta}{}^{\dot\gamma} = -i\frac{1}{10}(\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\dot\gamma\dot\delta}\psi_{\dot\delta}\,, \tag{8.64}$$

$$T_{\dot\alpha\dot\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\dot\alpha\dot\beta}\,, \tag{8.65}$$

$$T_{\dot\alpha\dot\beta}{}^{\gamma} = -i\frac{1}{10}(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}(\sigma_{\underline{a}})^{\gamma\delta}\psi_\delta\,, \tag{8.66}$$

$$T_{\dot\alpha\dot\beta}{}^{\dot\gamma} = i\frac{1}{10}(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}(\sigma_{\underline{a}})^{\dot\gamma\dot\delta}\psi_{\dot\delta}\,, \tag{8.67}$$

$$T_{\alpha\dot\beta}{}^{\underline{c}} = 0\,, \tag{8.68}$$

$$T_{\alpha\dot\beta}{}^{\gamma} = i\frac{1}{4}\left[\delta_\alpha{}^\gamma\delta_{\dot\beta}{}^{\dot\delta} + \frac{1}{10}(\sigma^{\underline{ab}})_\alpha{}^\gamma(\sigma_{\underline{ab}})_{\dot\beta}{}^{\dot\delta}\right]\psi_{\dot\delta}\,, \tag{8.69}$$

$$T_{\alpha\dot\beta}{}^{\dot\gamma} = i\frac{1}{4}\left[\delta_{\dot\beta}{}^{\dot\gamma}\delta_\alpha{}^\delta + \frac{1}{10}(\sigma^{\underline{ab}})_{\dot\beta}{}^{\dot\gamma}(\sigma_{\underline{ab}})_\alpha{}^\delta\right]\psi_\delta\,, \tag{8.70}$$

$$T_{\alpha\underline{b}}{}^{\underline{c}} = i\frac{1}{5}\left[2\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\delta + (\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\delta\right]\psi_\delta\,, \tag{8.71}$$

$$T_{\alpha\underline{b}}{}^{\gamma} = i\frac{1}{80}\left[-\frac{1}{2}(\sigma^{[2]})_\alpha{}^\gamma G_{\underline{b}[2]} + \frac{1}{3!}(\sigma_{\underline{b}[3]})_\alpha{}^\gamma G^{[3]}\right] - \frac{2}{5}\left[\delta_{\underline{b}}{}^{\underline{c}}\delta_\alpha{}^\gamma - (\sigma_{\underline{b}}{}^{\underline{c}})_\alpha{}^\gamma\right](\partial_{\underline{c}}\Psi)\,, \tag{8.72}$$

$$\begin{aligned} T_{\alpha\underline{b}}{}^{\dot\gamma} = -i\frac{1}{80}\Big[&(\sigma_{\underline{b}})_\alpha{}^{\dot\gamma}N - (\sigma^{[1]})_\alpha{}^{\dot\gamma}N_{\underline{b}[1]} + \frac{1}{2}(\sigma_{\underline{b}[2]})_\alpha{}^{\dot\gamma}N^{[2]} - \frac{1}{3!}(\sigma^{[3]})_\alpha{}^{\dot\gamma}N_{\underline{b}[3]} \\ &+ \frac{1}{4!}(\sigma_{\underline{b}[4]})_\alpha{}^{\dot\gamma}N^{[4]}\Big]\,, \end{aligned} \tag{8.73}$$

$$T_{\dot\alpha\underline{b}}{}^{\underline{c}} = i\frac{1}{5}\left[2\delta_{\underline{b}}{}^{\underline{c}}\delta_{\dot\alpha}{}^{\dot\delta} + (\sigma_{\underline{b}}{}^{\underline{c}})_{\dot\alpha}{}^{\dot\delta}\right]\psi_{\dot\delta}\,, \tag{8.74}$$

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$$T_{\dot{\alpha}\underline{b}}{}^{\gamma} = -i\frac{1}{80}\Big[(\sigma_{\underline{b}})_{\dot{\alpha}}{}^{\gamma}N + (\sigma^{[1]})_{\dot{\alpha}}{}^{\gamma}N_{\underline{b}[1]} - \frac{1}{2}(\sigma_{\underline{b}[2]})_{\dot{\alpha}}{}^{\gamma}N^{[2]} - \frac{1}{3!}(\sigma^{[3]})_{\dot{\alpha}}{}^{\gamma}N_{\underline{b}[3]} + \frac{1}{4!}(\sigma_{\underline{b}[4]})_{\dot{\alpha}}{}^{\gamma}N^{[4]}\Big]\,, \tag{8.75}$$

$$T_{\dot{\alpha}\underline{b}}{}^{\dot{\gamma}} = i\frac{1}{80}\left[-\frac{1}{2}(\sigma^{[2]})_{\dot{\alpha}}{}^{\dot{\gamma}}H_{\underline{b}[2]} + \frac{1}{3!}(\sigma_{\underline{b}[3]})_{\dot{\alpha}}{}^{\dot{\gamma}}H^{[3]}\right] - \frac{2}{5}\left[\delta_{\underline{b}}{}^{\underline{c}}\delta_{\dot{\alpha}}{}^{\dot{\gamma}} - (\sigma_{\underline{b}}{}^{\underline{c}})_{\dot{\alpha}}{}^{\dot{\gamma}}\right](\partial_{\underline{c}}\Psi)\,, \tag{8.76}$$

$$T_{\underline{ab}}{}^{\underline{c}} = 0\,, \tag{8.77}$$

$$T_{\underline{ab}}{}^{\gamma} = -\frac{1}{10}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\psi_{\delta})\,, \tag{8.78}$$

$$T_{\underline{ab}}{}^{\dot{\gamma}} = -\frac{1}{10}(\sigma_{[\underline{a}})^{\dot{\gamma}\dot{\delta}}(\partial_{\underline{b}]}\psi_{\dot{\delta}})\,. \tag{8.79}$$

Curvatures:

$$R_{\alpha\beta}{}^{\underline{de}} = -i\frac{6}{5}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}\Psi) - \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}G_{[3]} + (\sigma_{[1]})_{\alpha\beta}G^{[1]\underline{de}}\right]\,, \tag{8.80}$$

$$R_{\dot{\alpha}\dot{\beta}}{}^{\underline{de}} = -i\frac{6}{5}(\sigma^{[\underline{d}})_{\dot{\alpha}\dot{\beta}}(\partial^{\underline{e}]}\Psi) - \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\dot{\alpha}\dot{\beta}}H_{[3]} + (\sigma_{[1]})_{\dot{\alpha}\dot{\beta}}H^{[1]\underline{de}}\right]\,, \tag{8.81}$$

$$R_{\alpha\dot{\beta}}{}^{\underline{de}} = \frac{1}{40}\Big[(\sigma^{\underline{de}})_{\alpha\dot{\beta}}N - C_{\alpha\dot{\beta}}N^{\underline{de}} + \frac{1}{2}(\sigma^{\underline{de}[2]})_{\alpha\dot{\beta}}N_{[2]} - \frac{1}{2}(\sigma_{[2]})_{\alpha\dot{\beta}}N^{\underline{de}[2]} + \frac{1}{4!4!}\epsilon^{\underline{de}[4][\bar{4}]}(\sigma_{[4]})_{\alpha\dot{\beta}}N_{[\bar{4}]}\Big]\,, \tag{8.82}$$

$$R_{\alpha\underline{b}}{}^{\underline{de}} = i\frac{1}{2}\left[\delta_{\underline{b}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_{\alpha}) + \frac{1}{5}(\sigma^{\underline{de}})_{\alpha}{}^{\gamma}(\partial_{\underline{b}}\psi_{\gamma})\right]\,, \tag{8.83}$$

$$R_{\dot{\alpha}\underline{b}}{}^{\underline{de}} = i\frac{1}{2}\left[\delta_{\underline{b}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_{\dot{\alpha}}) + \frac{1}{5}(\sigma^{\underline{de}})_{\dot{\alpha}}{}^{\dot{\gamma}}(\partial_{\underline{b}}\psi_{\dot{\gamma}})\right]\,, \tag{8.84}$$

$$R_{\underline{ab}}{}^{\underline{de}} = -(\partial_{[\underline{a}}\partial^{[\underline{d}}\Psi)\delta_{\underline{b}]}{}^{\underline{e}]}\,. \tag{8.85}$$

8.11 Adaptation to 10D, $\mathcal{N} = 2$A component/superspace results: step 3

Parameter Composition Rules:

$$\xi^{\underline{m}} = -i\big[\;\epsilon_1{}^{\alpha}\epsilon_2{}^{\beta}(\sigma^{\underline{c}})_{\alpha\beta} + \epsilon_1{}^{\dot{\alpha}}\epsilon_2{}^{\dot{\beta}}(\sigma^{\underline{c}})_{\dot{\alpha}\dot{\beta}}\;\big]\;\delta_{\underline{c}}{}^{\underline{m}}(1+\Psi)\,, \tag{8.86}$$

$$\begin{aligned}\lambda^{\underline{de}} = &-\frac{1}{40}(\epsilon_1{}^{\alpha}\epsilon_2{}^{\dot{\beta}} + \epsilon_1{}^{\dot{\beta}}\epsilon_2{}^{\alpha})\Big[(\sigma^{\underline{de}})_{\alpha\dot{\beta}}N - C_{\alpha\dot{\beta}}N^{\underline{de}} + \frac{1}{2}(\sigma^{\underline{de}[2]})_{\alpha\dot{\beta}}N_{[2]} - \frac{1}{2}(\sigma_{[2]})_{\alpha\dot{\beta}}N^{\underline{de}[2]} + \frac{1}{4!4!}\epsilon^{\underline{de}[4][\bar{4}]}(\sigma_{[4]})_{\alpha\dot{\beta}}N_{[\bar{4}]}\Big]\\ &+\epsilon_1{}^{\alpha}\epsilon_2{}^{\beta}\left[i\frac{17}{10}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}\Psi) + \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}G_{[3]} + (\sigma_{[1]})_{\alpha\beta}G^{[1]\underline{de}}\right]\right]\\ &+\epsilon_1{}^{\dot{\alpha}}\epsilon_2{}^{\dot{\beta}}\left[i\frac{17}{10}(\sigma^{[\underline{d}})_{\dot{\alpha}\dot{\beta}}(\partial^{\underline{e}]}\Psi) + \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\dot{\alpha}\dot{\beta}}H_{[3]} + (\sigma_{[1]})_{\dot{\alpha}\dot{\beta}}H^{[1]\underline{de}}\right]\right]\,,\end{aligned} \tag{8.87}$$

$$\begin{aligned}\epsilon^{\delta} = &-i\frac{1}{4}(\epsilon_1{}^{\alpha}\epsilon_2{}^{\dot{\beta}} + \epsilon_1{}^{\dot{\beta}}\epsilon_2{}^{\alpha})\left[\delta_{\alpha}{}^{\delta}\delta_{\dot{\beta}}{}^{\dot{\epsilon}} + \frac{1}{10}(\sigma^{[2]})_{\alpha}{}^{\delta}(\sigma_{[2]})_{\dot{\beta}}{}^{\dot{\epsilon}}\right]\psi_{\dot{\epsilon}}\\ &-i\frac{1}{5}\epsilon_1{}^{\alpha}\epsilon_2{}^{\beta}(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\delta\epsilon}\psi_{\epsilon}\,.\end{aligned} \tag{8.88}$$

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8.12 Adaptation to 10D, $\mathcal{N}$ = 2A component/superspace results: step 4

SUSY transformation laws:

$$\delta_Q e_{\underline{a}}{}^{\underline{m}} = -i\frac{1}{2}\epsilon^\beta\left[\delta_{\underline{a}}{}^{\underline{d}}\delta_\beta{}^\gamma + \frac{1}{5}(\sigma_{\underline{a}}{}^{\underline{d}})_\beta{}^\gamma\right]\delta_{\underline{d}}{}^{\underline{m}}\psi_\gamma - i\frac{1}{2}\epsilon^{\dot\beta}\left[\delta_{\underline{a}}{}^{\underline{d}}\delta_{\dot\beta}{}^{\dot\gamma} + \frac{1}{5}(\sigma_{\underline{a}}{}^{\underline{d}})_{\dot\beta}{}^{\dot\gamma}\right]\delta_{\underline{d}}{}^{\underline{m}}\psi_{\dot\gamma}\,, \tag{8.89}$$

$$\begin{aligned}\delta_Q\psi_{\underline{a}}{}^\delta =& (1+\Psi)\partial_{\underline{a}}\epsilon^\delta - \epsilon^\delta(\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}}\\ &-i\frac{1}{80}\epsilon^\beta\left[-\frac{1}{2}(\sigma^{[2]})_\beta{}^\delta G_{\underline{a}[2]} + \frac{1}{3!}(\sigma_{\underline{a}[3]})_\beta{}^\delta G^{[3]}\right] + \frac{2}{5}\epsilon^\beta\left[\delta_{\underline{a}}{}^{\underline{c}}\delta_\beta{}^\delta - (\sigma_{\underline{a}}{}^{\underline{c}})_\beta{}^\delta\right](\partial_{\underline{c}}\Psi)\\ &+i\frac{1}{80}\epsilon^{\dot\beta}\left[(\sigma_{\underline{a}})_{\dot\beta}{}^\delta N + (\sigma^{[1]})_{\dot\beta}{}^\delta N_{\underline{a}[1]} - \frac{1}{2}(\sigma_{\underline{a}[2]})_{\dot\beta}{}^\delta N^{[2]} - \frac{1}{3!}(\sigma^{[3]})_{\dot\beta}{}^\delta N_{\underline{a}[3]}\right.\\ &\qquad\left. + \frac{1}{4!}(\sigma_{\underline{a}[4]})_{\dot\beta}{}^\delta N^{[4]}\right],\end{aligned} \tag{8.90}$$

$$\delta_Q\phi_{\underline{a}}{}^{\underline{de}} = -i\frac{1}{2}\epsilon^\beta\left[\delta_{\underline{a}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_\beta) + \frac{1}{5}(\sigma^{\underline{de}})_\beta{}^\gamma(\partial_{\underline{a}}\psi_\gamma)\right] - i\frac{1}{2}\epsilon^{\dot\beta}\left[\delta_{\underline{a}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_{\dot\beta}) + \frac{1}{5}(\sigma^{\underline{de}})_{\dot\beta}{}^{\dot\gamma}(\partial_{\underline{a}}\psi_{\dot\gamma})\right]. \tag{8.91}$$

8.13 Adaptation to 10D, $\mathcal{N}$ = 2B component/superspace results: step 1

In the case of 10D $\mathcal{N} = 2B$ N-SG covariant derivatives we define

$$\nabla_\alpha = \mathrm{D}_\alpha + \frac{1}{2}\Psi\mathrm{D}_\alpha + \frac{1}{10}(\sigma^{\underline{ab}})_\alpha{}^\beta(\mathrm{D}_\beta\Psi)\mathcal{M}_{\underline{ab}}\,, \tag{8.92}$$

$$\overline{\nabla}_\alpha = \overline{\mathrm{D}}_\alpha + \frac{1}{2}\overline{\Psi}\overline{\mathrm{D}}_\alpha + \frac{1}{10}(\sigma^{\underline{ab}})_\alpha{}^\beta(\overline{\mathrm{D}}_\beta\overline{\Psi})\mathcal{M}_{\underline{ab}}\,, \tag{8.93}$$

$$\begin{aligned}\nabla_{\underline{a}} =& \partial_{\underline{a}} + \frac{1}{2}\Psi\partial_{\underline{a}} + \frac{1}{2}\overline{\Psi}\partial_{\underline{a}} - i\frac{1}{32}(\sigma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha\overline{\Psi})\overline{\mathrm{D}}_\beta - i\frac{1}{32}(\sigma_{\underline{a}})^{\alpha\beta}(\overline{\mathrm{D}}_\alpha\Psi)\mathrm{D}_\beta\\ &- i\frac{27}{160}(\sigma_{\underline{a}})^{\alpha\beta}(\mathrm{D}_\alpha\Psi)\overline{\mathrm{D}}_\beta - i\frac{27}{160}(\sigma_{\underline{a}})^{\alpha\beta}(\overline{\mathrm{D}}_\alpha\overline{\Psi})\mathrm{D}_\beta\\ &- \frac{1}{2}(\partial_{\underline{c}}\Psi)\mathcal{M}_{\underline{a}}{}^{\underline{c}} - \frac{1}{2}(\partial_{\underline{c}}\overline{\Psi})\mathcal{M}_{\underline{a}}{}^{\underline{c}},\end{aligned} \tag{8.94}$$

and "split" the spatial 10D $\mathcal{N} = 2B$ N-SG covariant derivative into three parts

$$\nabla_{\underline{a}}| = \mathbf{D}_{\underline{a}} + \psi_{\underline{a}}{}^\gamma\nabla_\gamma| + \overline{\psi}_{\underline{a}}{}^\gamma\overline{\nabla}_\gamma|\,. \tag{8.95}$$

On taking the $\theta \to 0$ limit the latter terms allows an identification with the gravitino and the leading term in this limit yields a component-level linearized gravitationally covariant derivative operator given by

$$\mathbf{D}_{\underline{a}} = e_{\underline{a}} + \phi_{\underline{a}}{}^\iota\mathcal{M}_\iota = \partial_{\underline{a}} + \frac{1}{2}(\Psi + \overline{\Psi})\partial_{\underline{a}} + \phi_{\underline{a}}{}^\iota\mathcal{M}_\iota\,. \tag{8.96}$$

Comparison of the l.h.s. to the r.h.s. of (8.96), we see that a linearized frame field $e_{\underline{a}}{}^{\underline{m}} = (\,1 + \frac{1}{2}(\Psi + \overline{\Psi})\,)\delta_{\underline{a}}{}^{\underline{m}}$ emerges to describe a scalar graviton. Finally, comparison of the coefficient of the Lorentz generator $\mathcal{M}_\iota$ as it appears in the latter two forms of (8.96) informs us the spin connection is given by

$$\phi_{\underline{c}}{}^{\underline{de}} = -\frac{1}{4}\delta_{\underline{c}}{}^{[\underline{d}}\big(\partial^{\underline{e}]}(\Psi + \overline{\Psi})\big)\,. \tag{8.97}$$

Comparing the result (8.94) in with the one in (8.95) the component gravitinos are identified via

$$\psi_{\underline{a}}{}^{\gamma} = -i\frac{1}{160}(\sigma_{\underline{a}})^{\gamma\delta}\big(\overline{\mathrm{D}}_{\delta}(5\Psi + 27\overline{\Psi})\big)\,, \tag{8.98}$$

$$\overline{\psi}_{\underline{a}}{}^{\gamma} = -i\frac{1}{160}(\sigma_{\underline{a}})^{\gamma\delta}\big(\mathrm{D}_{\delta}(5\overline{\Psi} + 27\Psi)\big)\,, \tag{8.99}$$

which are equivalent to

$$\overline{\mathrm{D}}_{\alpha}(5\Psi + 27\overline{\Psi}) = i16(\sigma^{\underline{a}})_{\alpha\gamma}\psi_{\underline{a}}{}^{\gamma}\,, \quad \mathrm{D}_{\alpha}(5\overline{\Psi} + 27\Psi) = i16(\sigma^{\underline{a}})_{\alpha\gamma}\overline{\psi}_{\underline{a}}{}^{\gamma}\,. \tag{8.100}$$

However, as this expression contains an explicit σ-matrix we see that it really defines the non-conformal *spin*-$\frac{1}{2}$ part of the gravitino to be

$$\psi_{\beta} \equiv (\sigma^{\underline{a}})_{\beta\gamma}\psi_{\underline{a}}{}^{\gamma}\,, \quad \overline{\psi}_{\beta} \equiv -(\sigma^{\underline{a}})_{\beta\gamma}\overline{\psi}_{\underline{a}}{}^{\gamma}\,. \tag{8.101}$$

Since the results in (8.100) are under-constrained, we are allowed to introduce a fermionic auxiliary field λ_{α} and its complex conjugate $\overline{\lambda}_{\alpha}$. So then we have

$$\overline{\mathrm{D}}_{\alpha}\Psi = i\frac{1}{2}(\sigma^{\underline{a}})_{\alpha\gamma}\psi_{\underline{a}}{}^{\gamma} - 27\overline{\lambda}_{\alpha} \equiv i\frac{1}{2}\psi_{\alpha} - 27\overline{\lambda}_{\alpha}\,, \tag{8.102}$$

$$\overline{\mathrm{D}}_{\alpha}\overline{\Psi} = i\frac{1}{2}(\sigma^{\underline{a}})_{\alpha\gamma}\psi_{\underline{a}}{}^{\gamma} + 5\overline{\lambda}_{\alpha} \equiv i\frac{1}{2}\psi_{\alpha} + 5\overline{\lambda}_{\alpha}\,, \tag{8.103}$$

$$\mathrm{D}_{\alpha}\overline{\Psi} = i\frac{1}{2}(\sigma^{\underline{a}})_{\alpha\gamma}\overline{\psi}_{\underline{a}}{}^{\gamma} - 27\lambda_{\alpha} \equiv -i\frac{1}{2}\overline{\psi}_{\alpha} - 27\lambda_{\alpha}\,, \tag{8.104}$$

$$\mathrm{D}_{\alpha}\Psi = i\frac{1}{2}(\sigma^{\underline{a}})_{\alpha\gamma}\overline{\psi}_{\underline{a}}{}^{\gamma} + 5\lambda_{\alpha} \equiv -i\frac{1}{2}\overline{\psi}_{\alpha} + 5\lambda_{\alpha}\,, \tag{8.105}$$

in the $\theta \to 0$ limit. Also observe that

$$\overline{\mathrm{D}}_{\alpha}(\overline{\Psi} - \Psi) = 32\overline{\lambda}_{\alpha}\,, \quad \mathrm{D}_{\alpha}(\Psi - \overline{\Psi}) = 32\lambda_{\alpha}\,. \tag{8.106}$$

In order to complete the specification of the geometrical superfields also requires explicit definitions of the bosonic terms to second order in D-derivatives. So we define bosonic fields:

$$U_{[3]} = (\sigma_{[3]})^{\gamma\delta}(\mathrm{D}_{\gamma}\mathrm{D}_{\delta}\Psi)\,, \qquad \overline{U}_{[3]} = -(\sigma_{[3]})^{\gamma\delta}(\overline{\mathrm{D}}_{\gamma}\overline{\mathrm{D}}_{\delta}\overline{\Psi})\,, \tag{8.107}$$

$$X_{[3]} = (\sigma_{[3]})^{\gamma\delta}(\overline{\mathrm{D}}_{\gamma}\overline{\mathrm{D}}_{\delta}\Psi)\,, \qquad \overline{X}_{[3]} = -(\sigma_{[3]})^{\gamma\delta}(\mathrm{D}_{\gamma}\mathrm{D}_{\delta}\overline{\Psi})\,, \tag{8.108}$$

$$\begin{aligned} Y_{[3]} &= (\sigma_{[3]})^{\gamma\delta}(\mathrm{D}_{\gamma}\overline{\mathrm{D}}_{\delta}\Psi) & \overline{Y}_{[3]} &= -(\sigma_{[3]})^{\gamma\delta}(\overline{\mathrm{D}}_{\gamma}\mathrm{D}_{\delta}\overline{\Psi}) \\ &= (\sigma_{[3]})^{\gamma\delta}(\overline{\mathrm{D}}_{\gamma}\mathrm{D}_{\delta}\Psi)\,, & &= -(\sigma_{[3]})^{\gamma\delta}(\mathrm{D}_{\gamma}\overline{\mathrm{D}}_{\delta}\overline{\Psi})\,. \end{aligned} \tag{8.109}$$

Ue emphasize that the component fields (the U's, X's and Y's) are defined by the $\theta \to 0$ limit of these equations.

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8.14 Adaptation to 10D, $\mathcal{N} = 2$B component/superspace results: step 2

Torsions:

$$T_{\alpha\beta}{}^{\underline{c}} = 0\,, \tag{8.110}$$

$$T_{\alpha\beta}{}^{\gamma} = -i\frac{1}{5}(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\gamma\delta}\overline{\psi}_{\delta} + 2(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\gamma\delta}\lambda_{\delta}\,, \tag{8.111}$$

$$T_{\alpha\beta}{}^{\bar{\gamma}} = 0\,, \tag{8.112}$$

$$T_{\bar{\alpha}\bar{\beta}}{}^{\underline{c}} = 0\,, \tag{8.113}$$

$$T_{\bar{\alpha}\bar{\beta}}{}^{\gamma} = 0\,, \tag{8.114}$$

$$T_{\bar{\alpha}\bar{\beta}}{}^{\bar{\gamma}} = i\frac{1}{5}(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\gamma\delta}\psi_{\delta} + 2(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\gamma\delta}\overline{\lambda}_{\delta}\,, \tag{8.115}$$

$$T_{\alpha\bar{\beta}}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\,, \tag{8.116}$$

$$T_{\alpha\bar{\beta}}{}^{\gamma} = -i\frac{1}{240}(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta}\psi_{\delta} + \frac{1}{8}\left[(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} - \frac{1}{30}(\sigma^{[5]})_{\alpha\beta}(\sigma_{[5]})^{\gamma\delta}\right]\overline{\lambda}_{\delta}\,, \tag{8.117}$$

$$T_{\alpha\bar{\beta}}{}^{\bar{\gamma}} = i\frac{1}{240}(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta}\overline{\psi}_{\delta} + \frac{1}{8}\left[(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} - \frac{1}{30}(\sigma^{[5]})_{\alpha\beta}(\sigma_{[5]})^{\gamma\delta}\right]\lambda_{\delta}\,, \tag{8.118}$$

$$T_{\alpha\underline{b}}{}^{\underline{c}} = -i\frac{1}{5}\left[2\delta_{\underline{b}}{}^{\underline{c}}\delta_{\alpha}{}^{\gamma} + (\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\gamma}\right]\overline{\psi}_{\gamma} + \left[-11\delta_{\underline{b}}{}^{\underline{c}}\delta_{\alpha}{}^{\gamma} + (\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\gamma}\right]\lambda_{\gamma}\,, \tag{8.119}$$

$$\begin{aligned} T_{\alpha\underline{b}}{}^{\gamma} = {} & \frac{1}{64}\left[-31\delta_{\underline{b}}{}^{\underline{c}}\delta_{\alpha}{}^{\gamma} + 15(\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\gamma}\right](\partial_{\underline{c}}\Psi) + \frac{1}{320}\left[27\delta_{\underline{b}}{}^{\underline{c}}\delta_{\alpha}{}^{\gamma} + 53(\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\gamma}\right](\partial_{\underline{c}}\overline{\Psi}) \\ & - i\frac{1}{2560}\left[\frac{1}{2}(\sigma^{[2]})_{\alpha}{}^{\gamma}\left(5Y_{\underline{b}[2]} - 27\overline{Y}_{\underline{b}[2]}\right) - \frac{1}{3!}(\sigma_{\underline{b}[3]})_{\alpha}{}^{\gamma}\left(5Y^{[3]} - 27\overline{Y}^{[3]}\right)\right]\,, \end{aligned} \tag{8.120}$$

$$T_{\alpha\underline{b}}{}^{\bar{\gamma}} = -i\frac{1}{2560}\left[\frac{1}{2}(\sigma^{[2]})_{\alpha}{}^{\gamma}\left(-5\overline{X}_{\underline{b}[2]} + 27U_{\underline{b}[2]}\right) - \frac{1}{3!}(\sigma_{\underline{b}[3]})_{\alpha}{}^{\gamma}\left(-5\overline{X}^{[3]} + 27U^{[3]}\right)\right]\,, \tag{8.121}$$

$$T_{\bar{\alpha}\underline{b}}{}^{\underline{c}} = i\frac{1}{5}\left[2\delta_{\underline{b}}{}^{\underline{c}}\delta_{\alpha}{}^{\gamma} + (\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\gamma}\right]\psi_{\gamma} + \left[-11\delta_{\underline{b}}{}^{\underline{c}}\delta_{\alpha}{}^{\gamma} + (\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\gamma}\right]\overline{\lambda}_{\gamma}\,, \tag{8.122}$$

$$T_{\bar{\alpha}\underline{b}}{}^{\gamma} = -i\frac{1}{2560}\left[\frac{1}{2}(\sigma^{[2]})_{\alpha}{}^{\gamma}\left(5X_{\underline{b}[2]} - 27\overline{U}_{\underline{b}[2]}\right) - \frac{1}{3!}(\sigma_{\underline{b}[3]})_{\alpha}{}^{\gamma}\left(5X^{[3]} - 27\overline{U}^{[3]}\right)\right]\,, \tag{8.123}$$

$$\begin{aligned} T_{\bar{\alpha}\underline{b}}{}^{\bar{\gamma}} = {} & \frac{1}{64}\left[-31\delta_{\underline{b}}{}^{\underline{c}}\delta_{\alpha}{}^{\gamma} + 15(\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\gamma}\right](\partial_{\underline{c}}\overline{\Psi}) + \frac{1}{320}\left[27\delta_{\underline{b}}{}^{\underline{c}}\delta_{\alpha}{}^{\gamma} + 53(\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\gamma}\right](\partial_{\underline{c}}\Psi) \\ & - i\frac{1}{2560}\left[\frac{1}{2}(\sigma^{[2]})_{\alpha}{}^{\gamma}\left(-5\overline{Y}_{\underline{b}[2]} + 27Y_{\underline{b}[2]}\right) - \frac{1}{3!}(\sigma_{\underline{b}[3]})_{\alpha}{}^{\gamma}\left(-5\overline{Y}^{[3]} + 27Y^{[3]}\right)\right]\,, \end{aligned} \tag{8.124}$$

$$T_{\underline{ab}}{}^{\underline{c}} = 0\,, \tag{8.125}$$

$$T_{\underline{ab}}{}^{\gamma} = -\frac{1}{10}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\psi_{\delta})\,, \tag{8.126}$$

$$T_{\underline{ab}}{}^{\bar{\gamma}} = \frac{1}{10}(\sigma_{[\underline{a}})^{\gamma\delta}(\partial_{\underline{b}]}\overline{\psi}_{\delta})\,. \tag{8.127}$$

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Curvatures:

$$R_{\alpha\beta}{}^{\underline{de}} = \frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}U_{[3]} - (\sigma_{[1]})_{\alpha\beta}U^{[1]\underline{de}}\right] , \tag{8.128}$$

$$R_{\bar{\alpha}\bar{\beta}}{}^{\underline{de}} = -\frac{1}{40}\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}\overline{U}_{[3]} - (\sigma_{[1]})_{\alpha\beta}\overline{U}^{[1]\underline{de}}\right] , \tag{8.129}$$

$$\begin{aligned} R_{\alpha\bar{\beta}}{}^{\underline{de}} = & -i\frac{3}{5}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}(\Psi+\overline{\Psi})) - i\frac{1}{10}(\sigma^{\underline{def}})_{\alpha\beta}(\partial_{\underline{f}}(\Psi+\overline{\Psi})) \\ & - \frac{1}{80}\Big[(\sigma_{[1]})_{\alpha\beta}\Big(Y^{[1]\underline{de}} - \overline{Y}^{[1]\underline{de}}\Big) - \frac{1}{2}(\sigma^{[2][\underline{d}})_{\alpha\beta}\Big(Y^{\underline{e}]}{}_{[2]} - \overline{Y}^{\underline{e}]}{}_{[2]}\Big) \qquad (8.130) \\ & \qquad - \frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}\Big(Y_{[3]} - \overline{Y}_{[3]}\Big)\Big], \qquad (8.131) \end{aligned}$$

$$R_{\alpha\underline{b}}{}^{\underline{de}} = -i\frac{1}{2}\left[\delta_{\underline{b}}{}^{[\underline{d}}(\partial^{\underline{e}]}\overline{\psi}_\alpha) + \frac{1}{5}(\sigma^{\underline{de}})_\alpha{}^\gamma(\partial_{\underline{b}}\overline{\psi}_\gamma)\right] - 11\delta_{\underline{b}}{}^{[\underline{d}}(\partial^{\underline{e}]}\lambda_\alpha) + (\sigma^{\underline{de}})_\alpha{}^\gamma(\partial_{\underline{b}}\lambda_\gamma) , \tag{8.132}$$

$$R_{\bar{\alpha}\underline{b}}{}^{\underline{de}} = i\frac{1}{2}\left[\delta_{\underline{b}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_\alpha) + \frac{1}{5}(\sigma^{\underline{de}})_\alpha{}^\gamma(\partial_{\underline{b}}\psi_\gamma)\right] - 11\delta_{\underline{b}}{}^{[\underline{d}}(\partial^{\underline{e}]}\overline{\lambda}_\alpha) + (\sigma^{\underline{de}})_\alpha{}^\gamma(\partial_{\underline{b}}\overline{\lambda}_\gamma) , \tag{8.133}$$

$$R_{\underline{ab}}{}^{\underline{de}} = -\frac{1}{2}\left(\partial_{[\underline{a}}\partial^{[\underline{d}}(\Psi+\overline{\Psi})\right)\delta_{\underline{b}]}{}^{\underline{e}]} . \tag{8.134}$$

8.15 Adaptation to 10D, $\mathcal{N}=2$B component/superspace results: step 3

Parameter Composition Rules:

$$\xi^{\underline{m}} = -i(\epsilon_1{}^\alpha\overline{\epsilon}_2{}^\beta + \overline{\epsilon}_1{}^\beta\epsilon_2{}^\alpha)(\sigma^{\underline{c}})_{\alpha\beta}\delta_{\underline{c}}{}^{\underline{m}}\left(1 + \frac{1}{2}(\Psi+\overline{\Psi})\right) , \tag{8.135}$$

$$\begin{aligned} \lambda^{\underline{de}} = & -(\epsilon_1{}^\alpha\overline{\epsilon}_2{}^\beta + \overline{\epsilon}_1{}^\beta\epsilon_2{}^\alpha)\Big[-i\frac{17}{20}(\sigma^{[\underline{d}})_{\alpha\beta}(\partial^{\underline{e}]}(\Psi+\overline{\Psi})) - i\frac{1}{10}(\sigma^{\underline{def}})_{\alpha\beta}(\partial_{\underline{f}}(\Psi+\overline{\Psi})) \\ & \qquad - \frac{1}{80}\Big[(\sigma_{[1]})_{\alpha\beta}\Big(Y^{[1]\underline{de}} - \overline{Y}^{[1]\underline{de}}\Big) - \frac{1}{2}(\sigma^{[2][\underline{d}})_{\alpha\beta}\Big(Y^{\underline{e}]}{}_{[2]} - \overline{Y}^{\underline{e}]}{}_{[2]}\Big) \\ & \qquad - \frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}\Big(Y_{[3]} - \overline{Y}_{[3]}\Big)\Big]\ \Big] \\ & - \frac{1}{40}\epsilon_1{}^\alpha\epsilon_2{}^\beta\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}U_{[3]} - (\sigma_{[1]})_{\alpha\beta}U^{[1]\underline{de}}\right] \\ & + \frac{1}{40}\overline{\epsilon}_1{}^\alpha\overline{\epsilon}_2{}^\beta\left[\frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}\overline{U}_{[3]} - (\sigma_{[1]})_{\alpha\beta}\overline{U}^{[1]\underline{de}}\right] , \end{aligned} \tag{8.136}$$

$$\begin{aligned} \epsilon^\delta = & -(\epsilon_1{}^\alpha\overline{\epsilon}_2{}^\beta + \overline{\epsilon}_1{}^\beta\epsilon_2{}^\alpha)\Big[i\frac{1}{10}\left[(\sigma^{[1]})_{\alpha\beta}(\sigma_{[1]})^{\delta\epsilon} - \frac{1}{24}(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\delta\epsilon}\right]\psi_\epsilon \\ & \qquad + \frac{1}{8}\left[(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\delta\epsilon} - \frac{1}{30}(\sigma^{[5]})_{\alpha\beta}(\sigma_{[5]})^{\delta\epsilon}\right]\overline{\lambda}_\epsilon\Big] \\ & - \epsilon_1{}^\alpha\epsilon_2{}^\beta\left[-i\frac{1}{5}(\sigma^{[1]})_{\alpha\beta}(\sigma_{[1]})^{\delta\epsilon}\overline{\psi}_\epsilon + 2(\sigma^{[1]})_{\alpha\beta}(\sigma_{[1]})^{\delta\epsilon}\lambda_\epsilon\right] . \end{aligned} \tag{8.137}$$

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8.16 Adaptation to 10D, $\mathcal{N}=2$B component/superspace results: step 4

$$\begin{aligned}\delta_Q e_{\underline{a}}{}^{\underline{m}} = & -\epsilon^\beta\left[-i\frac{1}{2}\left[\delta_{\underline{a}}{}^{\underline{d}}\delta_\beta{}^\gamma+\frac{1}{5}(\sigma_{\underline{a}}{}^{\underline{d}})_\beta{}^\gamma\right]\overline{\psi}_\gamma+\left[-11\delta_{\underline{a}}{}^{\underline{d}}\delta_\beta{}^\gamma+(\sigma_{\underline{a}}{}^{\underline{d}})_\beta{}^\gamma\right]\lambda_\gamma\right]\delta_{\underline{d}}{}^{\underline{m}} \\ & -\overline{\epsilon}^\beta\left[i\frac{1}{2}\left[\delta_{\underline{a}}{}^{\underline{d}}\delta_\beta{}^\gamma+\frac{1}{5}(\sigma_{\underline{a}}{}^{\underline{d}})_\beta{}^\gamma\right]\psi_\gamma+\left[-11\delta_{\underline{a}}{}^{\underline{d}}\delta_\beta{}^\gamma+(\sigma_{\underline{a}}{}^{\underline{d}})_\beta{}^\gamma\right]\overline{\lambda}_\gamma\right]\delta_{\underline{d}}{}^{\underline{m}},\end{aligned} \tag{8.138}$$

$$\begin{aligned}\delta_Q\psi_{\underline{a}}{}^\delta = & \left(1+\frac{1}{2}(\Psi+\overline{\Psi})\right)\partial_{\underline{a}}\epsilon^\delta-\frac{1}{2}\epsilon^\delta(\partial_{\underline{c}}(\Psi+\overline{\Psi}))\mathcal{M}_{\underline{a}}{}^{\underline{c}} \\ & -\frac{1}{64}\epsilon^\beta\left[-31\delta_{\underline{a}}{}^{\underline{c}}\delta_\beta{}^\delta+15(\sigma_{\underline{a}}{}^{\underline{c}})_\beta{}^\delta\right](\partial_{\underline{c}}\Psi)-\frac{1}{320}\epsilon^\beta\left[27\delta_{\underline{a}}{}^{\underline{c}}\delta_\beta{}^\delta+53(\sigma_{\underline{a}}{}^{\underline{c}})_\beta{}^\delta\right](\partial_{\underline{c}}\overline{\Psi}) \\ & +i\frac{1}{2560}\epsilon^\beta\left[\frac{1}{2}(\sigma^{[2]})_\beta{}^\delta\left(5Y_{\underline{a}[2]}-27\overline{Y}_{\underline{a}[2]}\right)-\frac{1}{3!}(\sigma_{\underline{a}[3]})_\beta{}^\delta\left(5Y^{[3]}-27\overline{Y}^{[3]}\right)\right] \\ & +i\frac{1}{2560}\overline{\epsilon}^\beta\left[\frac{1}{2}(\sigma^{[2]})_\beta{}^\delta\left(5X_{\underline{a}[2]}-27\overline{U}_{\underline{a}[2]}\right)-\frac{1}{3!}(\sigma_{\underline{a}[3]})_\beta{}^\delta\left(5X^{[3]}-27\overline{U}^{[3]}\right)\right],\end{aligned} \tag{8.139}$$

$$\begin{aligned}\delta_Q\phi_{\underline{a}}{}^{\underline{de}} = & \, i\frac{1}{2}\epsilon^\beta\left[\delta_{\underline{a}}{}^{[\underline{d}}(\partial^{\underline{e}]}\overline{\psi}_\beta)+\frac{1}{5}(\sigma^{\underline{de}})_\beta{}^\gamma(\partial_{\underline{a}}\overline{\psi}_\gamma)\right]-\epsilon^\beta\left[-11\delta_{\underline{a}}{}^{[\underline{d}}(\partial^{\underline{e}]}\lambda_\beta)+(\sigma^{\underline{de}})_\beta{}^\gamma(\partial_{\underline{a}}\lambda_\gamma)\right] \\ & -i\frac{1}{2}\overline{\epsilon}^\beta\left[\delta_{\underline{a}}{}^{[\underline{d}}(\partial^{\underline{e}]}\psi_\beta)+\frac{1}{5}(\sigma^{\underline{de}})_\beta{}^\gamma(\partial_{\underline{a}}\psi_\gamma)\right]-\overline{\epsilon}^\beta\left[-11\delta_{\underline{a}}{}^{[\underline{d}}(\partial^{\underline{e}]}\overline{\lambda}_\beta)+(\sigma^{\underline{de}})_\beta{}^\gamma(\partial_{\underline{a}}\overline{\lambda}_\gamma)\right].\end{aligned} \tag{8.140}$$

9 10D, $\mathcal{N}=2$B chiral compensator considerations

In the limits where all supergravity fields are set to zero, four sets of superalgebras emerge. These take the forms:

(a). 11D, $\mathcal{N}=1$,

$$\{\,\mathrm{D}_\alpha\,,\;\mathrm{D}_\beta\}=i\,(\gamma^{\underline{a}})_{\alpha\beta}\,\partial_{\underline{a}}\,,\quad\left[\,\mathrm{D}_\alpha\,,\;\partial_{\underline{b}}\right]=0\,,\quad\left[\,\partial_{\underline{a}}\,,\;\partial_{\underline{b}}\right]=0 \tag{9.1}$$

(b). 10D, $\mathcal{N}=1$,

$$\{\,\mathrm{D}_\alpha\,,\;\mathrm{D}_\beta\}=i\,(\sigma^{\underline{a}})_{\alpha\beta}\,\partial_{\underline{a}}\,,\quad\left[\,\mathrm{D}_\alpha\,,\;\partial_{\underline{b}}\right]=0\,,\quad\left[\,\partial_{\underline{a}}\,,\;\partial_{\underline{b}}\right]=0 \tag{9.2}$$

(c). 10D, $\mathcal{N}=2$A,

$$\begin{aligned}&\{\,\mathrm{D}_\alpha\,,\;\mathrm{D}_\beta\}=i\,(\sigma^{\underline{a}})_{\alpha\beta}\,\partial_{\underline{a}}\,, && \left\{\mathrm{D}_{\dot\alpha}\,,\;\mathrm{D}_{\dot\beta}\right\}=i\,(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}\,\partial_{\underline{a}}\,, && \left\{\mathrm{D}_\alpha\,,\;\mathrm{D}_{\dot\beta}\right\}=0\,,\\ &\left[\,\mathrm{D}_\alpha\,,\;\partial_{\underline{b}}\right]=0\,, && \left[\,\mathrm{D}_{\dot\alpha}\,,\;\partial_{\underline{b}}\right]=0\,, && \left[\,\partial_{\underline{a}}\,,\;\partial_{\underline{b}}\right]=0\,,\end{aligned} \tag{9.3}$$

(d). 10D, $\mathcal{N}=2$B,

$$\begin{aligned}&\{\,\mathrm{D}_\alpha\,,\;\mathrm{D}_\beta\}=0\,, && \{\,\overline{\mathrm{D}}_\alpha\,,\;\overline{\mathrm{D}}_\beta\}=0\,, && \{\,\mathrm{D}_\alpha\,,\;\overline{\mathrm{D}}_\beta\}=i\,(\sigma^{\underline{a}})_{\alpha\beta}\,\partial_{\underline{a}}\,,\\ &\left[\,\mathrm{D}_\alpha\,,\;\partial_{\underline{b}}\right]=0\,, && \left[\,\overline{\mathrm{D}}_\alpha\,,\;\partial_{\underline{b}}\right]=0\,, && \left[\,\partial_{\underline{a}}\,,\;\partial_{\underline{b}}\right]=0\,,\end{aligned} \tag{9.4}$$

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We next introduce a complex superfield denoted by Ω_d into each of these d-dimensional superspaces and seek to probe the implications of imposing a first order differential equation on this superfield that utilizes any of the spinorial derivatives above.

For either the 11D, $\mathcal{N}$ = 1 or 10D, $\mathcal{N}$ = 1 superspaces we have

$$\mathrm{D}_\beta\,\Omega_d = 0 \;\;\to\;\; \mathrm{D}_\alpha\mathrm{D}_\beta\,\Omega_d = 0 \;\;\to\;\; \{\,\mathrm{D}_\alpha\,,\;\mathrm{D}_\beta\}\,\Omega_d = 0 \;\;\to\;\; \partial_{\underline{c}}\,\Omega_d = 0\,, \tag{9.5}$$

and by analogy for the 10D, $\mathcal{N}$ = 2A superspace we find

$$\mathrm{D}_\beta\,\Omega_d = 0 \;\;\to\;\; \mathrm{D}_\alpha\mathrm{D}_\beta\,\Omega_d = 0 \;\;\to\;\; \{\,\mathrm{D}_\alpha\,,\;\mathrm{D}_\beta\}\,\Omega_d = 0 \;\;\to\;\; \partial_{\underline{c}}\,\Omega_d = 0\,, \tag{9.6}$$

$$\mathrm{D}_{\dot\beta}\,\Omega_d = 0 \;\;\to\;\; \mathrm{D}_{\dot\alpha}\mathrm{D}_{\dot\beta}\,\Omega_d = 0 \;\;\to\;\; \{\,\mathrm{D}_{\dot\alpha}\,,\;\mathrm{D}_{\dot\beta}\}\,\Omega_d = 0 \;\;\to\;\; \partial_{\underline{c}}\,\Omega_d = 0\,. \tag{9.7}$$

Thus, from (9.5) to (9.7) we find the superfield Ω_d in each of these d-dimensional superspaces must be a constant. However, upon repeating these considerations for the 10D, $\mathcal{N}$ = 2B superspace we find

$$\begin{aligned} &\mathrm{D}_\beta\,\Omega_d = 0 \;\;\to\;\; \mathrm{D}_\alpha\mathrm{D}_\beta\,\Omega_d = 0 \;\;\to\;\; \{\,\mathrm{D}_\alpha\,,\;\mathrm{D}_\beta\}\,\Omega_d = 0 \;\;\to\;\; 0 = 0\,,\\ &\overline{\mathrm{D}}_\beta\,\Omega_d = 0 \;\;\to\;\; \overline{\mathrm{D}}_\alpha\overline{\mathrm{D}}_\beta\,\Omega_d = 0 \;\;\to\;\; \{\,\overline{\mathrm{D}}_\alpha\,,\;\overline{\mathrm{D}}_\beta\}\,\Omega_d = 0 \;\;\to\;\; 0 = 0\,, \end{aligned} \tag{9.8}$$

which shows that the superfield Ω_d in this case can be a non-trivial representation of the translation operator.

The differential equation

$$\overline{\mathrm{D}}_\beta\,\Omega_d = 0\,, \tag{9.9}$$

in the context of four dimensions implies that Ω_d is a "chiral superfield." On the other hand the differential equation

$$\mathrm{D}_\beta\,\Omega_d = 0\,, \tag{9.10}$$

in the context of four dimensions implies that Ω_d is a "anti-chiral superfield." While it is not possible to simultaneously impose both conditions because a chiral superfield is the complex conjugate of an anti-chiral one, either one or the other can be imposed. This also means that neither the chiral nor the anti-chiral condition can be applied to a real superfield.

Let us return to the results shown in (8.104) and (8.105)

$$\begin{aligned} \mathrm{D}_\alpha\Psi &= i\frac{1}{2}(\sigma^{\underline{a}})_{\alpha\gamma}\overline{\psi}_{\underline{a}}{}^{\gamma} + 5\lambda_\alpha \equiv -i\frac{1}{2}\overline{\psi}_\alpha + 5\lambda_\alpha\,,\\ \mathrm{D}_\alpha\overline{\Psi} &= i\frac{1}{2}(\sigma^{\underline{a}})_{\alpha\gamma}\overline{\psi}_{\underline{a}}{}^{\gamma} - 27\lambda_\alpha \equiv -i\frac{1}{2}\overline{\psi}_\alpha - 27\lambda_\alpha\,, \end{aligned} \tag{9.11}$$

since the remaining equations can be obtained by complex conjugation. In all the other cases we have explored, there is no spinor field such as λ_α. Taking the difference of the two equations that appear in (9.11), we may obtain

$$i\,\frac{1}{32}\,\mathrm{D}_\alpha\big(\,\Psi - \overline{\Psi}\,\big) = i\,\lambda_\alpha\,. \tag{9.12}$$

However, the quantity $i\,(\Psi\; - \;\overline{\Psi})$ is a real superfield. The requirement that λ_α = 0 is equivalent to the imposition of an anti-chirality condition on a real superfield and this condition possesses no non-trivial solution.

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The inability to introduce such a chiral superfield distinguishes the type 2B theory from the other higher dimensional constructions we have considered. At first order in the θ-expansion of Ψ both the spin-1/2 portion of the gravitino $\overline{\psi}_{\underline{a}}{}^{\gamma}$ *and* a separate spin-1/2 auxiliary spinor λ_α must exist.

10 Conclusion

This work gives a proposal for descriptions of Nordstrom supergravity in eleven and ten dimensions, as well as the component level descriptions that follow from the superfield equations we have presented. Our work is based on the assumption that in each of the cases of 11D, $\mathcal{N}=1$, 10D, $\mathcal{N}=1$, 10D, $\mathcal{N}=2$A and 10D, $\mathcal{N}=2$B, a single scalar superfield is required to provide such a description as this was the case for both ordinary gravitation as well as 4D, $\mathcal{N}=1$ supergravity. We remark that our work is but a foundation as in future extensions of this work we plan to continue this exploration.

Having obtained the results for the theories in ten and eleven dimensional superspaces, we can compare those results with the ones seen in section 3. Looking back at (3.8), with a bit of effort, one can show that the condition $H^{\underline{a}}=0$ causes only modification in the form of the equations. Namely the terms $W_{\alpha\beta\gamma}$ will vanish under this restriction. It is thus pointedly seen all the basic superfields (i.e. R and $G_{\underline{a}}$) in the algebra of the superspace supergravity covariant derivatives are bosonic. This is to be compared to the results shown in (3.14) where a fermionic superfield T_α appears. In all of the higher dimensional theories such superfields appear ubiquitously.

In the works of [36, 41] on the basis of the study of solutions to the 11D superspace Bianchi identities up to engineering dimension one, forms for the superspace torsions and curvature supertensors were proposed. Upon comparing particularly the results in the first of these references to the result derived in the current work as seen in (4.10)–(4.18), apparent concurrence is found. In the work of [41], we have the definition

$$\nabla_\alpha J_\beta = C_{\alpha\beta}S + (\gamma^{\underline{a}})_{\alpha\beta}v_{\underline{a}} + \frac{1}{2}(\gamma^{[2]})_{\alpha\beta}t_{[2]} + \frac{1}{3!}(\gamma^{[3]})_{\alpha\beta}U_{[3]} + \frac{1}{4!}(\gamma^{[4]})_{\alpha\beta}V_{[4]} + \frac{1}{5!}(\gamma^{[5]})_{\alpha\beta}Z_{[5]}\,. \tag{10.1}$$

In this former work, we must set the 11D "on-shell" superfield $W_{\underline{abcd}}$ to zero to make comparisons. When this is done, then by a change of notation where

$$\begin{aligned} &\psi_\alpha \to J_\alpha\,, \quad K \to S\,, \quad K_{\underline{a}} \to v_{\underline{a}}\,, \quad K_{[2]} \to t_{[2]}\,, \\ &K_{[3]} \to U_{[3]}\,, \quad K_{[4]} \to V_{[4]}\,, \quad K_{[5]} \to Z_{[5]}\,, \end{aligned} \tag{10.2}$$

we then look at (10.1) in contrast to the form of (4.10) to (4.18) in this work. We find in the Nordström limit,

$$v_{\underline{a}} = \partial_{\underline{a}}\Psi\,, \quad t_{[2]} = 0\,, \quad Z_{[5]} = 0\,, \tag{10.3}$$

and thus there is significant overlap. In particular, the results in (10.3) tell us something interesting about the J_α tensor. We can decompose it into two parts

$$J_\alpha = J_\alpha^{(T)} + \mathrm{D}_\alpha\Psi \tag{10.4}$$

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which is equivalent to the usual decomposition of a gauge field into is transverse and longitudinal parts. Upon setting the $J_\alpha^{(T)} = 0$, one recovers the Nordström theory.

There is a further feature noted in the work of [36] that also is indicated as a direction to include in this new pathway of exploration for 11D superspace supergravity.

While the notation of superconformal symmetry is not presently understood in a number of approaches to the study of 11D supergravity, the superspace approach in [36] is indicative of a specific further modification. In particular, by the introduction of a scaling transformation of the supervielbein, it was found that a modification of the spinor-spinor-vector component of the supertorsion that is given by the expression

$$T_{\alpha\beta}{}^{\underline{c}} = i(\gamma^{\underline{c}})_{\alpha\beta} + i(\gamma^{[2]})_{\alpha\beta}\,\mathcal{X}_{[2]}{}^{\underline{c}} + i(\gamma^{[5]})_{\alpha\beta}\,\widehat{\mathcal{X}}_{[5]}{}^{\underline{c}} \tag{10.5}$$

is consistent with the superspace scale transformations if and only if the "$\mathcal{X}$-tensor" and "$\widehat{\mathcal{X}}$-tensor" satisfy the conditions,

$$\mathcal{X}_{\underline{ac}}{}^{\underline{c}} = 0\,, \quad \epsilon^{[8]\underline{abc}}\mathcal{X}_{\underline{abc}} = 0\,, \quad \widehat{\mathcal{X}}_{[4]\underline{c}}{}^{\underline{c}} = 0\,, \quad \epsilon^{[5]\underline{abcdef}}\widehat{\mathcal{X}}_{\underline{abcdef}} = 0\,. \tag{10.6}$$

A detailed and careful study of the 11D superspace supergravity Bianchi identities with the modifications in the current work as well as the works of [36, 41] is indicated to assess the form of any equations of motion that emerges in the presence of retaining the on-shell field strength.

In the future we will also address the very important quest of whether there exists a superspace action for the Nordström supergravity theories in higher dimensions. It is clear that in order for this to be the case, it is necessary that the scalar superfield should satisfy some superdifferential constraints. The expectation is suggested by the structure of the 4D, $\mathcal{N} = 1$ theory. We remind the reader that the irreducible theories require that the superfield X is subject to some differential constraints. So it is natural to expect this to extend into the higher dimensional theories.

Our approach also raises an interesting question about Superstring Theories, M-Theory, and F-Theory. Do these theories also possess consistent truncation limits that include Nordström supergravity theories in their low energy limits? If the answer is affirmative, such limits might provide laboratories in which to investigate these more complicated mathematical structures.

> *"Every boy in the streets of Göttingen understands more about four dimensional geometry than Einstein. Yet, in spite of that, Einstein did the work and not the mathematicians."*
>
> – David Hilbert

Note added in proof. This current paper is a combination of the previous results shown in the works of [42, 43] available on-line. In the previous work of [43], there was made a conjecture that a possible avenue to reduce the number of component fields could be possible assuming the condition $\left[\Delta^{(165)\,\underline{bcd}}\Delta^{(330)}{}_{\underline{abcd}}\,V^{(11)}\right] \neq = 0$, for a scalar superfield $V^{(11)}$ can be satisfied. In a future presentation a proof this assumption is not viable will be given.

Dedication. SJG wishes to dedicate this work to the memory of Shota Ivan Vashakidze, a valued friend and collaborator in the exploration of ten dimensional superspace geometry.

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A 11D Clifford algebra representation

In this section we briefly summerize the convention that we adopted for 11D gamma matrices. Our 32× 32 gamma matrices are defined by the Clifford algebra:

$$\{\gamma^{\underline{a}}, \gamma^{\underline{b}}\} = 2\eta^{\underline{ab}}\,\mathbb{I}\,, \tag{A.1}$$

where $\mathbb{I}$ denotes the 32×32 identity matrix and the inverse metric $\eta^{\underline{ab}}$ follows the "most plus" signature:

$$\eta^{\underline{ab}} = \mathrm{diag}(-1, +1, +1, +1, +1, +1, +1, +1, +1, +1, +1)\,. \tag{A.2}$$

It is known that D-dimensional space-time Dirac spinor has $2^{\frac{D-1}{2}}$ components when D is odd, and $2^{D/2}$ components when D is even. Hence in 11D, the spinor indices of the gamma matrices, denoted by α, β and so forth, run from 1 to 32.

One can raise and lower the spinor indices via the "spinor metric", $C_{\alpha\beta}$, which satisfies:

$$C_{\alpha\beta} = -C_{\beta\alpha}\,, \quad C_{\alpha\beta}C^{\gamma\beta} = \delta_{\alpha}{}^{\gamma}\,. \tag{A.3}$$

The gamma matrices with multiple vector indices are defined through the equations:

$$\gamma^{\underline{a}}\gamma^{\underline{b}} = \gamma^{\underline{ab}} + \eta^{\underline{ab}} \tag{A.4}$$

$$\gamma^{\underline{b}}\gamma^{\underline{a}} = -\gamma^{\underline{ab}} + \eta^{\underline{ab}} \tag{A.5}$$

$$\gamma^{\underline{a}}\gamma^{\underline{bc}} = \gamma^{\underline{abc}} + \eta^{\underline{a}[\underline{b}}\gamma^{\underline{c}]} \tag{A.6}$$

$$\gamma^{\underline{bc}}\gamma^{\underline{a}} = \gamma^{\underline{abc}} - \eta^{\underline{a}[\underline{b}}\gamma^{\underline{c}]} \tag{A.7}$$

$$\gamma^{\underline{a}}\gamma^{\underline{bcd}} = \gamma^{\underline{abcd}} + \frac{1}{2}\eta^{\underline{a}[\underline{b}}\gamma^{\underline{cd}]} \tag{A.8}$$

$$\gamma^{\underline{bcd}}\gamma^{\underline{a}} = -\gamma^{\underline{abcd}} + \frac{1}{2}\eta^{\underline{a}[\underline{b}}\gamma^{\underline{cd}]} \tag{A.9}$$

$$\gamma^{\underline{a}}\gamma^{\underline{bcde}} = \gamma^{\underline{abcde}} + \frac{1}{3!}\eta^{\underline{a}[\underline{b}}\gamma^{\underline{cde}]} \tag{A.10}$$

$$\gamma^{\underline{bcde}}\gamma^{\underline{a}} = \gamma^{\underline{abcde}} - \frac{1}{3!}\eta^{\underline{a}[\underline{b}}\gamma^{\underline{cde}]} \tag{A.11}$$

$$\gamma^{\underline{a}}\gamma^{\underline{bcdef}} = \frac{1}{5!}\epsilon^{\underline{abcdef}[5]}\gamma_{[5]} + \frac{1}{4!}\eta^{\underline{a}[\underline{b}}\gamma^{\underline{cdef}]} \tag{A.12}$$

$$\gamma^{\underline{bcdef}}\gamma^{\underline{a}} = -\frac{1}{5!}\epsilon^{\underline{abcdef}[5]}\gamma_{[5]} + \frac{1}{4!}\eta^{\underline{a}[\underline{b}}\gamma^{\underline{cdef}]} \tag{A.13}$$

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The symmetric relations of the gamma matrices are given by:

$$(\gamma^{\underline{a}})_{\alpha\beta} = (\gamma^{\underline{a}})_{\beta\alpha} \tag{A.14}$$

$$(\gamma^{\underline{ab}})_{\alpha\beta} = (\gamma^{\underline{ab}})_{\beta\alpha} \tag{A.15}$$

$$(\gamma^{\underline{abc}})_{\alpha\beta} = -(\gamma^{\underline{abc}})_{\beta\alpha} \tag{A.16}$$

$$(\gamma^{\underline{abcd}})_{\alpha\beta} = -(\gamma^{\underline{abcd}})_{\beta\alpha} \tag{A.17}$$

$$(\gamma^{\underline{abcde}})_{\alpha\beta} = (\gamma^{\underline{abcde}})_{\beta\alpha} \tag{A.18}$$

From the definitions, one can easily work out the following trace identities:

$$(\gamma_{\underline{a}})_\alpha{}^\beta (\gamma^{\underline{b}})_\beta{}^\alpha = 32\delta_{\underline{a}}^{\ \underline{b}} \tag{A.19}$$

$$(\gamma_{\underline{ab}})_\alpha{}^\beta (\gamma^{\underline{cd}})_\beta{}^\alpha = -32\delta_{[\underline{a}}^{\ \underline{c}}\delta_{\underline{b}]}^{\ \underline{d}} \tag{A.20}$$

$$(\gamma_{\underline{abc}})_\alpha{}^\beta (\gamma^{\underline{def}})_\beta{}^\alpha = -32\delta_{[\underline{a}}^{\ \underline{d}}\delta_{\underline{b}}^{\ \underline{e}}\delta_{\underline{c}]}^{\ \underline{f}} \tag{A.21}$$

$$(\gamma_{\underline{abcd}})_\alpha{}^\beta (\gamma^{\underline{efgh}})_\beta{}^\alpha = 32\delta_{[\underline{a}}^{\ \underline{e}}\delta_{\underline{b}}^{\ \underline{f}}\delta_{\underline{c}}^{\ \underline{g}}\delta_{\underline{d}]}^{\ \underline{h}} \tag{A.22}$$

$$(\gamma_{\underline{abcde}})_\alpha{}^\beta (\gamma^{\underline{fghij}})_\beta{}^\alpha = 32\delta_{[\underline{a}}^{\ \underline{f}}\delta_{\underline{b}}^{\ \underline{g}}\delta_{\underline{c}}^{\ \underline{h}}\delta_{\underline{d}}^{\ \underline{i}}\delta_{\underline{e}]}^{\ \underline{j}} \tag{A.23}$$

as well as the following Fierz identities:

$$\delta_{(\alpha}^{\ \delta}\delta_{\beta)}^{\ \gamma} = \frac{1}{16}\left\{-(\gamma^{\underline{c}})_{\alpha\beta}(\gamma_{\underline{c}})^{\delta\gamma} + \frac{1}{2}(\gamma^{[2]})_{\alpha\beta}(\gamma_{[2]})^{\delta\gamma} - \frac{1}{5!}(\gamma^{[5]})_{\alpha\beta}(\gamma_{[5]})^{\delta\gamma}\right\} \tag{A.24}$$

$$(\gamma^{[2]})_{(\alpha}^{\ \ \delta}(\gamma_{[2]})_{\beta)}^{\ \ \gamma} = \frac{1}{16}\left\{-70(\gamma^{\underline{c}})_{\alpha\beta}(\gamma_{\underline{c}})^{\delta\gamma} + 19(\gamma^{[2]})_{\alpha\beta}(\gamma_{[2]})^{\delta\gamma} - \frac{1}{12}(\gamma^{[5]})_{\alpha\beta}(\gamma_{[5]})^{\delta\gamma}\right\} \tag{A.25}$$

$$\begin{aligned}\delta_\alpha{}^{[\delta}(\gamma_{\underline{b}})^{\epsilon]\gamma} = \frac{1}{16}\Big\{&-(\gamma_{\underline{b}})_\alpha{}^\gamma C^{\delta\epsilon} + \frac{1}{2}(\gamma^{[2]})_\alpha{}^\gamma(\gamma_{\underline{b}[2]})^{\delta\epsilon} - \frac{1}{3!}(\gamma_{\underline{b}[3]})_\alpha{}^\gamma(\gamma^{[3]})^{\delta\epsilon}\\ &+\frac{1}{3!}(\gamma^{[3]})_\alpha{}^\gamma(\gamma_{\underline{b}[3]})^{\delta\epsilon} - \frac{1}{4!}(\gamma_{\underline{b}[4]})_\alpha{}^\gamma(\gamma^{[4]})^{\delta\epsilon}\Big\}\end{aligned} \tag{A.26}$$

$$\begin{aligned}(\gamma^{\underline{de}})_{(\alpha}{}^\gamma\delta_{\beta)}{}^\delta = \frac{1}{16}\Big\{&(\gamma_{[1]})_{\alpha\beta}(\gamma^{[1]\underline{de}})^{\gamma\delta} - (\gamma^{[\underline{d}})_{\alpha\beta}(\gamma^{\underline{e}]})^{\gamma\delta}\\ &-\frac{1}{2}(\gamma_{[2]})_{\alpha\beta}(\gamma^{[2]\underline{de}})^{\gamma\delta} - (\gamma^{[1][\underline{d}})_{\alpha\beta}(\gamma^{\underline{e}]}{}_{[1]})^{\gamma\delta} + (\gamma^{\underline{de}})_{\alpha\beta}C^{\gamma\delta}\\ &+\frac{1}{5!4!}\epsilon^{\underline{de}[5][4]}(\gamma_{[5]})_{\alpha\beta}(\gamma_{[4]})^{\gamma\delta} - \frac{1}{4!}(\gamma^{[4][\underline{d}})_{\alpha\beta}(\gamma^{\underline{e}]}{}_{[4]})^{\gamma\delta} - \frac{1}{3!}(\gamma^{\underline{de}[3]})_{\alpha\beta}(\gamma_{[3]})^{\gamma\delta}\Big\}\end{aligned} \tag{A.27}$$

Finally, we list the explicit representations of 11D gamma matrices in terms of tensor products of Pauli matrices.

Spinor metric:

$$C_{\alpha\beta} = -i\sigma^2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \tag{A.28}$$

Gamma matrices:

$$(\gamma^{\underline{0}})_\alpha{}^\beta = i\sigma^2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \tag{A.29}$$

$$(\gamma^{\underline{1}})_\alpha{}^\beta = \sigma^1 \otimes \sigma^2 \otimes \sigma^2 \otimes \sigma^2 \otimes \sigma^2 \tag{A.30}$$

$$(\gamma^{\underline{2}})_\alpha{}^\beta = \sigma^1 \otimes \sigma^2 \otimes \sigma^2 \otimes \mathbb{I}_2 \otimes \sigma^1 \tag{A.31}$$

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$$(\gamma^{\underline{3}})_\alpha{}^\beta = \sigma^1 \otimes \sigma^2 \otimes \sigma^2 \otimes \mathbb{I}_2 \otimes \sigma^3 \tag{A.32}$$
$$(\gamma^{\underline{4}})_\alpha{}^\beta = \sigma^1 \otimes \sigma^2 \otimes \sigma^1 \otimes \sigma^2 \otimes \mathbb{I}_2 \tag{A.33}$$
$$(\gamma^{\underline{5}})_\alpha{}^\beta = \sigma^1 \otimes \sigma^2 \otimes \sigma^3 \otimes \sigma^2 \otimes \mathbb{I}_2 \tag{A.34}$$
$$(\gamma^{\underline{6}})_\alpha{}^\beta = \sigma^1 \otimes \sigma^2 \otimes \mathbb{I}_2 \otimes \sigma^1 \otimes \sigma^2 \tag{A.35}$$
$$(\gamma^{\underline{7}})_\alpha{}^\beta = \sigma^1 \otimes \sigma^2 \otimes \mathbb{I}_2 \otimes \sigma^3 \otimes \sigma^2 \tag{A.36}$$
$$(\gamma^{\underline{8}})_\alpha{}^\beta = \sigma^1 \otimes \sigma^1 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \tag{A.37}$$
$$(\gamma^{\underline{9}})_\alpha{}^\beta = \sigma^1 \otimes \sigma^3 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \tag{A.38}$$
$$(\gamma^{\underline{10}})_\alpha{}^\beta = \sigma^3 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2 \tag{A.39}$$

B 10D Clifford algebra representation

In this section we briefly summerize the convention that we adopted for 10D sigma matrices. The Clifford algebra is

$$(\sigma^{\underline{a}})_{\alpha\beta}(\sigma^{\underline{b}})^{\beta\gamma} + (\sigma^{\underline{b}})_{\alpha\beta}(\sigma^{\underline{a}})^{\beta\gamma} = 2\eta^{\underline{ab}}\delta_\alpha{}^\gamma\,, \tag{B.1}$$

where the inverse metric $\eta^{\underline{ab}}$ is:

$$\eta^{\underline{ab}} = \mathrm{diag}(-1,+1,+1,+1,+1,+1,+1,+1,+1,+1)\,. \tag{B.2}$$

In 10D, the Dirac spinor has $2^{D/2} = 32$ components. We use undotted Greek index to denote 16 component left-handed Majorana spinor, and dotted index to denote right-handed ones,

$$(\psi^\alpha)^* = \psi^\alpha\,, \qquad (\psi^{\dot\alpha})^* = \psi^{\dot\alpha} \tag{B.3}$$

where $\alpha = 1,\ldots,16$ and $\dot\alpha = 1,\ldots,16$. We raise and lower the spinor indices by spinor metric $C_{\alpha\dot\beta}$ as follows:

$$\begin{aligned} \psi_{\dot\beta} &= \psi^\alpha C_{\alpha\dot\beta}\,, \qquad & \psi_\alpha &= \psi^{\dot\beta} C_{\alpha\dot\beta}\,, \\ C_{\alpha\dot\gamma}C^{\alpha\dot\beta} &= \delta_{\dot\gamma}{}^{\dot\beta}\,, \qquad & C_{\gamma\dot\beta}C^{\alpha\dot\beta} &= \delta_\gamma{}^\alpha\,. \end{aligned} \tag{B.4}$$

The sigma matrices are bispinors. There are three types of them: purely left-handed:

$$(\sigma^{\underline{a}})_{\alpha\beta}\,, \qquad (\sigma^{\underline{abc}})_{\alpha\beta}\,, \qquad (\sigma^{\underline{abcde}})_{\alpha\beta}\,; \tag{B.5}$$

purely right-handed (related to purely left-handed by the following):

$$(\sigma^{\underline{a}})^{\alpha\beta} = C^{\alpha\dot\alpha}C^{\beta\dot\beta}(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}\,, \quad (\sigma^{\underline{abc}})^{\alpha\beta} = C^{\alpha\dot\alpha}C^{\beta\dot\beta}(\sigma^{\underline{abc}})_{\dot\alpha\dot\beta}\,, \quad (\sigma^{\underline{abcde}})^{\alpha\beta} = C^{\alpha\dot\alpha}C^{\beta\dot\beta}(\sigma^{\underline{abcde}})_{\dot\alpha\dot\beta}\,; \tag{B.6}$$

and mixed bispinors:

$$C_{\alpha\dot\beta}\,, \qquad (\sigma^{\underline{ab}})_{\alpha\dot\beta}\,, \qquad (\sigma^{\underline{abcd}})_{\alpha\dot\beta}\,, \tag{B.7}$$

which have relations

$$\delta_\alpha{}^\beta = C^{\beta\dot\beta}C_{\alpha\dot\beta}\,, \qquad (\sigma^{\underline{ab}})_\alpha{}^\beta = C^{\beta\dot\beta}(\sigma^{\underline{ab}})_{\alpha\dot\beta}\,, \qquad (\sigma^{\underline{abcd}})_\alpha{}^\beta = C^{\beta\dot\beta}(\sigma^{\underline{abcd}})_{\alpha\dot\beta}\,. \tag{B.8}$$

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Definition of σ-matrices with more Lorentz indices:

$$(\sigma^{\underline{a}})_{\alpha\beta}(\sigma^{\underline{b}})^{\beta\gamma} = (\sigma^{\underline{ab}})_{\alpha}{}^{\gamma} + \eta^{\underline{ab}}\delta_{\alpha}{}^{\gamma} \tag{B.9}$$

$$(\sigma^{\underline{b}})_{\alpha\beta}(\sigma^{\underline{a}})^{\beta\gamma} = -(\sigma^{\underline{ab}})_{\alpha}{}^{\gamma} + \eta^{\underline{ab}}\delta_{\alpha}{}^{\gamma} \tag{B.10}$$

$$(\sigma^{\underline{a}})^{\alpha\beta}(\sigma^{\underline{bc}})_{\beta}{}^{\gamma} = (\sigma^{\underline{abc}})^{\alpha\gamma} + \eta^{\underline{a[b}}(\sigma^{\underline{c]}})^{\alpha\gamma} \tag{B.11}$$

$$(\sigma^{\underline{bc}})_{\alpha}{}^{\beta}(\sigma^{\underline{a}})_{\beta\gamma} = (\sigma^{\underline{abc}})_{\alpha\gamma} - \eta^{\underline{a[b}}(\sigma^{\underline{c]}})_{\alpha\gamma} \tag{B.12}$$

$$(\sigma^{\underline{a}})_{\alpha\beta}(\sigma^{\underline{bcd}})^{\beta\gamma} = (\sigma^{\underline{abcd}})_{\alpha}{}^{\gamma} + \frac{1}{2}\eta^{\underline{a[b}}(\sigma^{\underline{cd]}})_{\alpha}{}^{\gamma} \tag{B.13}$$

$$(\sigma^{\underline{bcd}})_{\alpha\beta}(\sigma^{\underline{a}})^{\beta\gamma} = -(\sigma^{\underline{abcd}})_{\alpha}{}^{\gamma} + \frac{1}{2}\eta^{\underline{a[b}}(\sigma^{\underline{cd]}})_{\alpha}{}^{\gamma} \tag{B.14}$$

$$(\sigma^{\underline{a}})^{\alpha\beta}(\sigma^{\underline{bcde}})_{\beta}{}^{\gamma} = (\sigma^{\underline{abcde}})^{\alpha\gamma} + \frac{1}{3!}\eta^{\underline{a[b}}(\sigma^{\underline{cde]}})^{\alpha\gamma} \tag{B.15}$$

$$(\sigma^{\underline{bcde}})_{\alpha}{}^{\beta}(\sigma^{\underline{a}})_{\beta\gamma} = (\sigma^{\underline{abcde}})_{\alpha\gamma} - \frac{1}{3!}\eta^{\underline{a[b}}(\sigma^{\underline{cde]}})_{\alpha\gamma} \tag{B.16}$$

$$(\sigma^{\underline{a}})_{\alpha\beta}(\sigma^{\underline{bcdef}})^{\beta\gamma} = \frac{1}{4!}\epsilon^{\underline{abcdef}[4]}(\sigma_{[4]})_{\alpha}{}^{\gamma} + \frac{1}{4!}\eta^{\underline{a[b}}(\sigma^{\underline{cdef]}})_{\alpha}{}^{\gamma} \tag{B.17}$$

$$(\sigma^{\underline{bcdef}})_{\alpha\beta}(\sigma^{\underline{a}})^{\beta\gamma} = -\frac{1}{4!}\epsilon^{\underline{abcdef}[4]}(\sigma_{[4]})_{\alpha}{}^{\gamma} + \frac{1}{4!}\eta^{\underline{a[b}}(\sigma^{\underline{cdef]}})_{\alpha}{}^{\gamma} \tag{B.18}$$

and

$$(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}(\sigma^{\underline{b}})^{\dot\beta\dot\gamma} = (\sigma^{\underline{ab}})_{\dot\alpha}{}^{\dot\gamma} + \eta^{\underline{ab}}\delta_{\dot\alpha}{}^{\dot\gamma} \tag{B.19}$$

$$(\sigma^{\underline{b}})_{\dot\alpha\dot\beta}(\sigma^{\underline{a}})^{\dot\beta\dot\gamma} = -(\sigma^{\underline{ab}})_{\dot\alpha}{}^{\dot\gamma} + \eta^{\underline{ab}}\delta_{\dot\alpha}{}^{\dot\gamma} \tag{B.20}$$

$$(\sigma^{\underline{a}})^{\dot\alpha\dot\beta}(\sigma^{\underline{bc}})_{\dot\beta}{}^{\dot\gamma} = (\sigma^{\underline{abc}})^{\dot\alpha\dot\gamma} + \eta^{\underline{a[b}}(\sigma^{\underline{c]}})^{\dot\alpha\dot\gamma} \tag{B.21}$$

$$(\sigma^{\underline{bc}})_{\dot\alpha}{}^{\dot\beta}(\sigma^{\underline{a}})_{\dot\beta\dot\gamma} = (\sigma^{\underline{abc}})_{\dot\alpha\dot\gamma} - \eta^{\underline{a[b}}(\sigma^{\underline{c]}})_{\dot\alpha\dot\gamma} \tag{B.22}$$

$$(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}(\sigma^{\underline{bcd}})^{\dot\beta\dot\gamma} = (\sigma^{\underline{abcd}})_{\dot\alpha}{}^{\dot\gamma} + \frac{1}{2}\eta^{\underline{a[b}}(\sigma^{\underline{cd]}})_{\dot\alpha}{}^{\dot\gamma} \tag{B.23}$$

$$(\sigma^{\underline{bcd}})_{\dot\alpha\dot\beta}(\sigma^{\underline{a}})^{\dot\beta\dot\gamma} = -(\sigma^{\underline{abcd}})_{\dot\alpha}{}^{\dot\gamma} + \frac{1}{2}\eta^{\underline{a[b}}(\sigma^{\underline{cd]}})_{\dot\alpha}{}^{\dot\gamma} \tag{B.24}$$

$$(\sigma^{\underline{a}})^{\dot\alpha\dot\beta}(\sigma^{\underline{bcde}})_{\dot\beta}{}^{\dot\gamma} = (\sigma^{\underline{abcde}})^{\dot\alpha\dot\gamma} + \frac{1}{3!}\eta^{\underline{a[b}}(\sigma^{\underline{cde]}})^{\dot\alpha\dot\gamma} \tag{B.25}$$

$$(\sigma^{\underline{bcde}})_{\dot\alpha}{}^{\dot\beta}(\sigma^{\underline{a}})_{\dot\beta\dot\gamma} = (\sigma^{\underline{abcde}})_{\dot\alpha\dot\gamma} - \frac{1}{3!}\eta^{\underline{a[b}}(\sigma^{\underline{cde]}})_{\dot\alpha\dot\gamma} \tag{B.26}$$

$$(\sigma^{\underline{a}})_{\dot\alpha\dot\beta}(\sigma^{\underline{bcdef}})^{\dot\beta\dot\gamma} = -\frac{1}{4!}\epsilon^{\underline{abcdef}[4]}(\sigma_{[4]})_{\dot\alpha}{}^{\dot\gamma} + \frac{1}{4!}\eta^{\underline{a[b}}(\sigma^{\underline{cdef]}})_{\dot\alpha}{}^{\dot\gamma} \tag{B.27}$$

$$(\sigma^{\underline{bcdef}})_{\dot\alpha\dot\beta}(\sigma^{\underline{a}})^{\dot\beta\dot\gamma} = \frac{1}{4!}\epsilon^{\underline{abcdef}[4]}(\sigma_{[4]})_{\dot\alpha}{}^{\dot\gamma} + \frac{1}{4!}\eta^{\underline{a[b}}(\sigma^{\underline{cdef]}})_{\dot\alpha}{}^{\dot\gamma} \tag{B.28}$$

The sigma matrices with five vector indices satisfy the self-dual / anti-self-dual identities:

$$(\sigma_{[5]})_{\alpha\beta} = \frac{1}{5!}\epsilon_{[5]}{}^{[\bar{5}]}(\sigma_{[\bar{5}]})_{\alpha\beta} \tag{B.29}$$

$$(\sigma_{[5]})^{\alpha\beta} = -\frac{1}{5!}\epsilon_{[5]}{}^{[\bar{5}]}(\sigma_{[\bar{5}]})^{\alpha\beta} \tag{B.30}$$

$$(\sigma_{[5]})_{\dot\alpha\dot\beta} = -\frac{1}{5!}\epsilon_{[5]}{}^{[\bar{5}]}(\sigma_{[\bar{5}]})_{\dot\alpha\dot\beta} \tag{B.31}$$

$$(\sigma_{[5]})^{\dot\alpha\dot\beta} = \frac{1}{5!}\epsilon_{[5]}{}^{[\bar{5}]}(\sigma_{[\bar{5}]})^{\dot\alpha\dot\beta} \tag{B.32}$$

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The symmetric relations of the gamma matrices are given by:

$$(\sigma^{\underline{a}})_{\alpha\beta} = (\sigma^{\underline{a}})_{\beta\alpha} \tag{B.33}$$

$$(\sigma^{\underline{abc}})_{\alpha\beta} = -(\sigma^{\underline{abc}})_{\beta\alpha} \tag{B.34}$$

$$(\sigma^{\underline{abcde}})_{\alpha\beta} = (\sigma^{\underline{abcde}})_{\beta\alpha} \tag{B.35}$$

From the definition, we can easily work out the trace identities:

$$(\sigma_{\underline{a}})_{\alpha\beta}(\sigma^{\underline{b}})^{\alpha\beta} = 16\delta_{\underline{a}}{}^{\underline{b}}, \tag{B.36}$$

$$(\sigma_{\underline{ab}})_{\alpha}{}^{\beta}(\sigma^{\underline{cd}})_{\beta}{}^{\alpha} = -16\delta_{[\underline{a}}{}^{\underline{c}}\delta_{\underline{b}]}{}^{\underline{d}}, \tag{B.37}$$

$$(\sigma_{\underline{abc}})_{\alpha\beta}(\sigma^{\underline{def}})^{\alpha\beta} = 16\delta_{[\underline{a}}{}^{\underline{d}}\delta_{\underline{b}}{}^{\underline{e}}\delta_{\underline{c}]}{}^{\underline{f}}, \tag{B.38}$$

$$(\sigma_{\underline{abcd}})_{\alpha}{}^{\beta}(\sigma^{\underline{efgh}})_{\beta}{}^{\alpha} = 16\delta_{[\underline{a}}{}^{\underline{e}}\delta_{\underline{b}}{}^{\underline{f}}\delta_{\underline{c}}{}^{\underline{g}}\delta_{\underline{d}]}{}^{\underline{h}}, \tag{B.39}$$

$$(\sigma_{\underline{abcde}})_{\alpha\beta}(\sigma^{\underline{fghij}})^{\alpha\beta} = 16\left[\delta_{[\underline{a}}{}^{\underline{f}}\delta_{\underline{b}}{}^{\underline{g}}\delta_{\underline{c}}{}^{\underline{h}}\delta_{\underline{d}}{}^{\underline{i}}\delta_{\underline{e}]}{}^{\underline{j}} + \epsilon_{\underline{abcde}}{}^{\underline{fghij}}\right], \tag{B.40}$$

and

$$(\sigma_{\underline{a}})_{\dot{\alpha}\dot{\beta}}(\sigma^{\underline{b}})^{\dot{\alpha}\dot{\beta}} = 16\delta_{\underline{a}}{}^{\underline{b}}, \tag{B.41}$$

$$(\sigma_{\underline{ab}})_{\dot{\alpha}}{}^{\dot{\beta}}(\sigma^{\underline{cd}})_{\dot{\beta}}{}^{\dot{\alpha}} = -16\delta_{[\underline{a}}{}^{\underline{c}}\delta_{\underline{b}]}{}^{\underline{d}}, \tag{B.42}$$

$$(\sigma_{\underline{abc}})_{\dot{\alpha}\dot{\beta}}(\sigma^{\underline{def}})^{\dot{\alpha}\dot{\beta}} = 16\delta_{[\underline{a}}{}^{\underline{d}}\delta_{\underline{b}}{}^{\underline{e}}\delta_{\underline{c}]}{}^{\underline{f}}, \tag{B.43}$$

$$(\sigma_{\underline{abcd}})_{\dot{\alpha}}{}^{\dot{\beta}}(\sigma^{\underline{efgh}})_{\dot{\beta}}{}^{\dot{\alpha}} = 16\delta_{[\underline{a}}{}^{\underline{e}}\delta_{\underline{b}}{}^{\underline{f}}\delta_{\underline{c}}{}^{\underline{g}}\delta_{\underline{d}]}{}^{\underline{h}}, \tag{B.44}$$

$$(\sigma_{\underline{abcde}})_{\dot{\alpha}\dot{\beta}}(\sigma^{\underline{fghij}})^{\dot{\alpha}\dot{\beta}} = 16\left[\delta_{[\underline{a}}{}^{\underline{f}}\delta_{\underline{b}}{}^{\underline{g}}\delta_{\underline{c}}{}^{\underline{h}}\delta_{\underline{d}}{}^{\underline{i}}\delta_{\underline{e}]}{}^{\underline{j}} - \epsilon_{\underline{abcde}}{}^{\underline{fghij}}\right]. \tag{B.45}$$

From the definition, we can also derive the following 10D sigma matrices identities:

$$(\sigma_{\underline{a}})^{\delta\gamma}(\sigma^{[2]})_{\gamma}{}^{\alpha}\sigma_{[2]})_{\delta}{}^{\beta} = 54(\sigma_{\underline{a}})^{\alpha\beta} \tag{B.46}$$

$$(\sigma_{\underline{a}})^{\delta\gamma}(\sigma_{\underline{c}}{}^{\underline{d}})_{\delta}{}^{\alpha}(\sigma^{\underline{ce}})_{\gamma}{}^{\beta} = 6(\sigma_{\underline{a}}{}^{\underline{de}})^{\alpha\beta} + 7\eta^{\underline{de}}(\sigma_{\underline{a}})^{\alpha\beta} - 8\delta_{\underline{a}}{}^{(\underline{d}}(\sigma^{\underline{e})})^{\alpha\beta} \tag{B.47}$$

$$(\sigma_{[5]})^{\alpha\beta}(\sigma^{[2]})_{\alpha}{}^{\delta}(\sigma_{[2]})_{\beta}{}^{\gamma} = -10(\sigma_{[5]})^{\delta\gamma} \tag{B.48}$$

$$(\sigma_{[5]})^{\dot{\alpha}\dot{\beta}}(\sigma^{[2]})_{\dot{\alpha}}{}^{\dot{\delta}}(\sigma_{[2]})_{\dot{\beta}}{}^{\dot{\gamma}} = -10(\sigma_{[5]})^{\dot{\delta}\dot{\gamma}} \tag{B.49}$$

$$(\sigma_{[5]})_{\alpha\beta}(\sigma^{\underline{c}})^{\alpha\delta}(\sigma_{\underline{c}})^{\beta\gamma} = 0 \tag{B.50}$$

as well as the following Fierz identities:

$$\delta_{\alpha}{}^{\delta}\delta_{\beta}{}^{\gamma} = \frac{1}{16}\left\{(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\delta\gamma} + \frac{1}{3!}(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\delta\gamma} + \frac{1}{2\times 5!}(\sigma^{[5]})_{\alpha\beta}(\sigma_{[5]})^{\delta\gamma}\right\} \tag{B.51}$$

$$(\sigma^{[2]})_{\alpha}{}^{\delta}(\sigma_{[2]})_{\beta}{}^{\gamma} = \frac{27}{8}(\sigma^{\underline{c}})_{\alpha\beta}(\sigma_{\underline{c}})^{\delta\gamma} + \frac{1}{16}(\sigma^{[3]})_{\alpha\beta}(\sigma_{[3]})^{\delta\gamma} - \frac{1}{384}(\sigma^{[5]})_{\alpha\beta}(\sigma_{[5]})^{\delta\gamma} \tag{B.52}$$

$$\begin{aligned}\delta_{\alpha}{}^{\beta}(\sigma_{\underline{b}})^{\gamma\delta} = \frac{1}{16}\Big\{&\delta_{\alpha}{}^{\gamma}(\sigma_{\underline{b}})^{\beta\delta} + \frac{1}{2}(\sigma^{[2]})_{\alpha}{}^{\gamma}(\sigma_{\underline{b}[2]})^{\beta\delta} - (\sigma_{\underline{b}[1]})_{\alpha}{}^{\gamma}(\sigma^{[1]})^{\beta\delta}\\ &+ \frac{1}{4!}(\sigma^{[4]})_{\alpha}{}^{\gamma}(\sigma_{\underline{b}[4]})^{\beta\delta} - \frac{1}{3!}(\sigma_{\underline{b}[3]})_{\alpha}{}^{\gamma}(\sigma^{[3]})^{\beta\delta}\Big\}\end{aligned} \tag{B.53}$$

$$\begin{aligned}(\sigma^{\underline{de}})_{\alpha}{}^{\gamma}\delta_{\beta}{}^{\delta} = \frac{1}{16}\Big\{&-(\sigma_{[1]})_{\alpha\beta}(\sigma^{[1]\underline{de}})^{\gamma\delta} + (\sigma^{[\underline{d}})_{\alpha\beta}(\sigma^{\underline{e}]})^{\gamma\delta}\\ &- \frac{1}{3!}(\sigma_{[3]})_{\alpha\beta}(\sigma^{[3]\underline{de}})^{\gamma\delta} + \frac{1}{2}(\sigma^{[2][\underline{d}})_{\alpha\beta}(\sigma^{\underline{e}]}{}_{[2]})^{\gamma\delta} + (\sigma^{\underline{de}[1]})_{\alpha\beta}(\sigma_{[1]})^{\gamma\delta}\\ &+ \frac{1}{2\times 4!}(\sigma_{[4]}{}^{[\underline{d}})_{\alpha\beta}(\sigma^{\underline{e}][4]})^{\gamma\delta} + \frac{1}{3!}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta}\Big\}\end{aligned} \tag{B.54}$$

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$$\begin{aligned}(\sigma^{\underline{de}})_{(\alpha}{}^{\gamma}\delta_{\beta)}{}^{\delta} = \frac{1}{16}\Big\{ & -2(\sigma_{[1]})_{\alpha\beta}(\sigma^{[1]\underline{de}})^{\gamma\delta} + 2(\sigma^{[\underline{d}})_{\alpha\beta}(\sigma^{\underline{e}]})^{\gamma\delta} \\ & + \frac{1}{4!}(\sigma_{[4]}{}^{[\underline{d}})_{\alpha\beta}(\sigma^{\underline{e}][4]})^{\gamma\delta} + \frac{1}{3}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta}\Big\}\end{aligned} \tag{B.55}$$

$$\begin{aligned}(\sigma_{\underline{ab}})_{\alpha}{}^{\delta}(\sigma_{\underline{c}})^{\beta\gamma} = \frac{1}{16}\Big\{ & \delta_{\alpha}{}^{\beta}(\sigma_{\underline{abc}})^{\gamma\delta} + \delta_{\alpha}{}^{\beta}\eta_{\underline{c}[\underline{a}}(\sigma_{\underline{b}]})^{\gamma\delta} \\ & - \frac{1}{2}(\sigma^{[2]})_{\alpha}{}^{\beta}(\sigma_{\underline{abc}[2]})^{\gamma\delta} - \frac{1}{2}(\sigma^{[2]})_{\alpha}{}^{\beta}\eta_{\underline{c}[\underline{a}}(\sigma_{\underline{b}][2]})^{\gamma\delta} \\ & + (\sigma^{[1]}{}_{\underline{c}})_{\alpha}{}^{\beta}(\sigma_{\underline{ab}[1]})^{\gamma\delta} - (\sigma^{[1]}{}_{[\underline{a}})_{\alpha}{}^{\beta}(\sigma_{\underline{b}]\underline{c}[1]})^{\gamma\delta} \\ & + (\sigma_{\underline{ab}})_{\alpha}{}^{\beta}(\sigma_{\underline{c}})^{\gamma\delta} - (\sigma_{\underline{c}[\underline{a}})_{\alpha}{}^{\beta}(\sigma_{\underline{b}]})^{\gamma\delta} + \eta_{\underline{c}[\underline{a}}(\sigma_{\underline{b}][1]})_{\alpha}{}^{\beta}(\sigma^{[1]})^{\gamma\delta} \\ & + \frac{1}{4!}(\sigma^{[4]})_{\alpha}{}^{\beta}\eta_{\underline{c}[\underline{a}}(\sigma_{\underline{b}][4]})^{\gamma\delta} - \frac{1}{3!}(\sigma^{[3]}{}_{\underline{c}})_{\alpha}{}^{\beta}(\sigma_{\underline{ab}[3]})^{\gamma\delta} \\ & + \frac{1}{3!}(\sigma^{[3]}{}_{[\underline{a}})_{\alpha}{}^{\beta}(\sigma_{\underline{b}]\underline{c}[3]})^{\gamma\delta} - \frac{1}{3!}\eta_{\underline{c}[\underline{a}}(\sigma_{\underline{b}][3]})_{\alpha}{}^{\beta}(\sigma^{[3]})^{\gamma\delta} \\ & - \frac{1}{2}(\sigma^{[2]}{}_{\underline{ab}})_{\alpha}{}^{\beta}(\sigma_{\underline{c}[2]})^{\gamma\delta} + \frac{1}{2}(\sigma^{[2]}{}_{\underline{c}[\underline{a}})_{\alpha}{}^{\beta}(\sigma_{\underline{b}][2]})^{\gamma\delta} \\ & + \frac{1}{4!3!}\epsilon_{\underline{abc}[4][3]}(\sigma^{[4]})_{\alpha}{}^{\beta}(\sigma^{[3]})^{\gamma\delta} - (\sigma_{\underline{abc}[1]})_{\alpha}{}^{\beta}(\sigma^{[1]})^{\gamma\delta}\Big\}\end{aligned} \tag{B.56}$$

$$\begin{aligned}(\sigma_{\underline{b}}{}^{\underline{c}})_{\alpha}{}^{\beta}(\sigma_{\underline{c}})^{\delta\gamma} = \frac{1}{16}\Big\{ & -9\delta_{\alpha}{}^{\gamma}(\sigma_{\underline{b}})^{\beta\delta} - \frac{5}{2}(\sigma^{[2]})_{\alpha}{}^{\gamma}(\sigma_{\underline{b}[2]})^{\beta\delta} - 7(\sigma_{\underline{b}[1]})_{\alpha}{}^{\gamma}(\sigma^{[1]})^{\beta\delta} \\ & - \frac{1}{4!}(\sigma^{[4]})_{\alpha}{}^{\gamma}(\sigma_{\underline{b}[4]})^{\beta\delta} - \frac{1}{2}(\sigma_{\underline{b}[3]})_{\alpha}{}^{\gamma}(\sigma^{[3]})^{\beta\delta}\Big\}\end{aligned} \tag{B.57}$$

$$\begin{aligned}(\sigma_{[\underline{a}})^{\alpha\gamma}(\sigma_{\underline{b}]}{}^{\underline{de}})^{\beta\delta} = \frac{1}{16}\Big\{ & -2(\sigma_{[1]})^{\alpha\beta}(\sigma_{\underline{ab}}{}^{\underline{de}[1]})^{\gamma\delta} \\ & + (\sigma_{[1]})^{\alpha\beta}\delta_{[\underline{a}}{}^{[\underline{d}}(\sigma_{\underline{b}]}{}^{\underline{e}][1]})^{\gamma\delta} - 2(\sigma^{[\underline{d}})^{\alpha\beta}(\sigma^{\underline{e}]}{}_{\underline{ab}})^{\gamma\delta} \\ & + \delta_{[\underline{a}}{}^{[\underline{d}}(\sigma_{\underline{b}]})^{\alpha\beta}(\sigma^{\underline{e}]})^{\gamma\delta} - \delta_{[\underline{a}}{}^{[\underline{d}}(\sigma^{\underline{e}]})^{\alpha\beta}(\sigma_{\underline{b}]})^{\gamma\delta} \\ & + \frac{1}{3!}(\sigma_{[3]})^{\alpha\beta}\delta_{[\underline{a}}{}^{[\underline{d}}(\sigma_{\underline{b}]}{}^{\underline{e}][3]})^{\gamma\delta} \\ & + (\sigma_{[2][\underline{a}})^{\alpha\beta}(\sigma_{\underline{b}]}{}^{\underline{de}[2]})^{\gamma\delta} - (\sigma^{[2][\underline{d}})^{\alpha\beta}(\sigma^{\underline{e}]}{}_{\underline{ab}[2]})^{\gamma\delta} \\ & - \frac{1}{18}\epsilon_{\underline{ab}}{}^{\underline{de}[3][\bar{3}]}(\sigma_{[3]})^{\alpha\beta}(\sigma_{[\bar{3}]})^{\gamma\delta} \\ & - 2(\sigma_{[1]\underline{ab}})^{\alpha\beta}(\sigma^{\underline{de}[1]})^{\gamma\delta} + 2(\sigma^{[1]\underline{de}})^{\alpha\beta}(\sigma_{\underline{ab}[1]})^{\gamma\delta} \\ & + \frac{1}{2}\delta_{[\underline{a}}{}^{[\underline{d}}(\sigma_{\underline{b}][2]})^{\alpha\beta}(\sigma^{\underline{e}][2]})^{\gamma\delta} - \frac{1}{2}\delta_{[\underline{a}}{}^{[\underline{d}}(\sigma^{\underline{e}][2]})^{\alpha\beta}(\sigma_{\underline{b}][2]})^{\gamma\delta} \\ & - \delta_{[\underline{a}}{}^{[\underline{d}}(\sigma_{\underline{b}]}{}^{\underline{e}][1]})^{\alpha\beta}(\sigma_{[1]})^{\gamma\delta} - 2(\sigma_{\underline{ab}}{}^{[\underline{d}})^{\alpha\beta}(\sigma^{\underline{e}]})^{\gamma\delta} \\ & - \frac{1}{3}(\sigma_{[3]\underline{ab}})^{\alpha\beta}(\sigma^{\underline{de}[3]})^{\gamma\delta} + \frac{1}{3!}(\sigma_{[3][\underline{a}}{}^{[\underline{d}})^{\alpha\beta}(\sigma_{\underline{b}]}{}^{\underline{e}][3]})^{\gamma\delta} \\ & - \frac{1}{3!}\delta_{[\underline{a}}{}^{[\underline{d}}(\sigma_{\underline{b}]}{}^{\underline{e}][3]})^{\alpha\beta}(\sigma_{[3]})^{\gamma\delta} - (\sigma_{[2]\underline{ab}}{}^{[\underline{d}})^{\alpha\beta}(\sigma^{\underline{e}][2]})^{\gamma\delta} \\ & + 2(\sigma_{\underline{ab}}{}^{\underline{de}[1]})^{\alpha\beta}(\sigma_{[1]})^{\gamma\delta}\Big\}\end{aligned} \tag{B.58}$$

$$\begin{aligned}(\sigma^{[2]})_{\alpha}{}^{\beta}(\sigma_{\underline{b}[2]})^{\delta\gamma} = & -\frac{9}{2}\delta_{\alpha}{}^{\gamma}(\sigma_{\underline{b}})^{\beta\delta} - \frac{1}{2}(\sigma^{[2]})_{\alpha}{}^{\gamma}(\sigma_{\underline{b}[2]})^{\beta\delta} + \frac{5}{2}(\sigma_{\underline{b}[1]})_{\alpha}{}^{\gamma}(\sigma^{[1]})^{\beta\delta} \\ & + \frac{1}{48}(\sigma^{[4]})_{\alpha}{}^{\gamma}(\sigma_{\underline{b}[4]})^{\beta\delta}\end{aligned} \tag{B.59}$$

$$\begin{aligned}(\sigma^{\underline{a}[\underline{d}})_{\alpha}{}^{\gamma}(\sigma_{\underline{a}}{}^{\underline{e}]})_{\beta}{}^{\delta} = \frac{1}{16}\Big\{ & 12(\sigma_{[1]})_{\alpha\beta}(\sigma^{\underline{de}[1]})^{\gamma\delta} + 12(\sigma^{\underline{de}[1]})_{\alpha\beta}(\sigma_{[1]})^{\gamma\delta} \\ & + \frac{2}{3}(\sigma_{[3]})_{\alpha\beta}(\sigma^{\underline{de}[3]})^{\gamma\delta} + \frac{2}{3}(\sigma^{\underline{de}[3]})_{\alpha\beta}(\sigma_{[3]})^{\gamma\delta}\Big\}\end{aligned} \tag{B.60}$$

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$$\begin{aligned}\delta_\alpha{}^\beta(\sigma_{\underline{a}})^{\dot\gamma\dot\delta}=\frac{1}{16}\Big\{&(\sigma_{\underline{a}})_\alpha{}^{\dot\gamma}C^{\beta\dot\delta}-(\sigma^{[1]})_\alpha{}^{\dot\gamma}(\sigma_{\underline{a}[1]})^{\beta\dot\delta}-\frac{1}{3!}(\sigma^{[3]})_\alpha{}^{\dot\gamma}(\sigma_{\underline{a}[3]})^{\beta\dot\delta}\\&+\frac{1}{2}(\sigma_{\underline{a}[2]})_\alpha{}^{\dot\gamma}(\sigma^{[2]})^{\beta\dot\delta}+\frac{1}{4!}(\sigma_{\underline{a}[4]})_\alpha{}^{\dot\gamma}(\sigma^{[4]})^{\beta\dot\delta}\Big\}\end{aligned}\tag{B.61}$$

$$\begin{aligned}\delta_{\dot\alpha}{}^{\dot\beta}(\sigma_{\underline{a}})^{\gamma\delta}=\frac{1}{16}\Big\{&(\sigma_{\underline{a}})_{\dot\alpha}{}^{\gamma}C^{\delta\dot\beta}+(\sigma^{[1]})_{\dot\alpha}{}^{\gamma}(\sigma_{\underline{a}[1]})^{\delta\dot\beta}-\frac{1}{3!}(\sigma^{[3]})_{\dot\alpha}{}^{\gamma}(\sigma_{\underline{a}[3]})^{\delta\dot\beta}\\&-\frac{1}{2}(\sigma_{\underline{a}[2]})_{\dot\alpha}{}^{\gamma}(\sigma^{[2]})^{\delta\dot\beta}+\frac{1}{4!}(\sigma_{\underline{a}[4]})_{\dot\alpha}{}^{\gamma}(\sigma^{[4]})^{\delta\dot\beta}\Big\}\end{aligned}\tag{B.62}$$

$$\begin{aligned}(\sigma^{\underline{de}})_\alpha{}^\gamma\delta_{\dot\beta}{}^{\dot\delta}=\frac{1}{16}\Big\{&-C_{\alpha\dot\beta}(\sigma^{\underline{de}})^{\gamma\dot\delta}+(\sigma^{\underline{de}})_{\alpha\dot\beta}C^{\gamma\dot\delta}-(\sigma^{[1][\underline{d}})_{\alpha\dot\beta}(\sigma^{\underline{e}]}{}_{[1]})^{\gamma\dot\delta}-\frac{1}{2}(\sigma_{[2]})_{\alpha\dot\beta}(\sigma^{\underline{de}[2]})^{\gamma\dot\delta}\\&+\frac{1}{2}(\sigma^{\underline{de}[2]})_{\alpha\dot\beta}(\sigma_{[2]})^{\gamma\dot\delta}-\frac{1}{3!}(\sigma^{[3][\underline{d}})_{\alpha\dot\beta}(\sigma^{\underline{e}]}{}_{[3]})^{\gamma\dot\delta}+\frac{1}{4!4!}\epsilon^{\underline{de}[4][\bar 4]}(\sigma_{[4]})_{\alpha\dot\beta}(\sigma_{[\bar 4]})^{\gamma\dot\delta}\Big\}\end{aligned}\tag{B.63}$$

$$\begin{aligned}(\sigma^{\underline{de}})_{\dot\beta}{}^{\dot\delta}\delta_\alpha{}^\gamma=\frac{1}{16}\Big\{&C_{\alpha\dot\beta}(\sigma^{\underline{de}})^{\gamma\dot\delta}-(\sigma^{\underline{de}})_{\alpha\dot\beta}C^{\gamma\dot\delta}-(\sigma^{[1][\underline{d}})_{\alpha\dot\beta}(\sigma^{\underline{e}]}{}_{[1]})^{\gamma\dot\delta}+\frac{1}{2}(\sigma_{[2]})_{\alpha\dot\beta}(\sigma^{\underline{de}[2]})^{\gamma\dot\delta}\\&-\frac{1}{2}(\sigma^{\underline{de}[2]})_{\alpha\dot\beta}(\sigma_{[2]})^{\gamma\dot\delta}-\frac{1}{3!}(\sigma^{[3][\underline{d}})_{\alpha\dot\beta}(\sigma^{\underline{e}]}{}_{[3]})^{\gamma\dot\delta}-\frac{1}{4!4!}\epsilon^{\underline{de}[4][\bar 4]}(\sigma_{[4]})_{\alpha\dot\beta}(\sigma_{[\bar 4]})^{\gamma\dot\delta}\Big\}\end{aligned}\tag{B.64}$$

Finally, we list the explicit (real) representations of the sigma matrices in terms of tensor products of Pauli matrices:

$$(\sigma^0)_{\alpha\beta}=\mathbb{I}_2\otimes\mathbb{I}_2\otimes\mathbb{I}_2\otimes\mathbb{I}_2\tag{B.65}$$
$$(\sigma^1)_{\alpha\beta}=\sigma^2\otimes\sigma^2\otimes\sigma^2\otimes\sigma^2\tag{B.66}$$
$$(\sigma^2)_{\alpha\beta}=\sigma^2\otimes\sigma^2\otimes\mathbb{I}_2\otimes\sigma^1\tag{B.67}$$
$$(\sigma^3)_{\alpha\beta}=\sigma^2\otimes\sigma^2\otimes\mathbb{I}_2\otimes\sigma^3\tag{B.68}$$
$$(\sigma^4)_{\alpha\beta}=\sigma^2\otimes\sigma^1\otimes\sigma^2\otimes\mathbb{I}_2\tag{B.69}$$
$$(\sigma^5)_{\alpha\beta}=\sigma^2\otimes\sigma^3\otimes\sigma^2\otimes\mathbb{I}_2\tag{B.70}$$
$$(\sigma^6)_{\alpha\beta}=\sigma^2\otimes\mathbb{I}_2\otimes\sigma^1\otimes\sigma^2\tag{B.71}$$
$$(\sigma^7)_{\alpha\beta}=\sigma^2\otimes\mathbb{I}_2\otimes\sigma^3\otimes\sigma^2\tag{B.72}$$
$$(\sigma^8)_{\alpha\beta}=\sigma^1\otimes\mathbb{I}_2\otimes\mathbb{I}_2\otimes\mathbb{I}_2\tag{B.73}$$
$$(\sigma^9)_{\alpha\beta}=\sigma^3\otimes\mathbb{I}_2\otimes\mathbb{I}_2\otimes\mathbb{I}_2\tag{B.74}$$

and

$$(\sigma^0)^{\alpha\beta}=-\mathbb{I}_2\otimes\mathbb{I}_2\otimes\mathbb{I}_2\otimes\mathbb{I}_2\tag{B.75}$$
$$(\sigma^1)^{\alpha\beta}=\sigma^2\otimes\sigma^2\otimes\sigma^2\otimes\sigma^2\tag{B.76}$$
$$(\sigma^2)^{\alpha\beta}=\sigma^2\otimes\sigma^2\otimes\mathbb{I}_2\otimes\sigma^1\tag{B.77}$$
$$(\sigma^3)^{\alpha\beta}=\sigma^2\otimes\sigma^2\otimes\mathbb{I}_2\otimes\sigma^3\tag{B.78}$$
$$(\sigma^4)^{\alpha\beta}=\sigma^2\otimes\sigma^1\otimes\sigma^2\otimes\mathbb{I}_2\tag{B.79}$$
$$(\sigma^5)^{\alpha\beta}=\sigma^2\otimes\sigma^3\otimes\sigma^2\otimes\mathbb{I}_2\tag{B.80}$$
$$(\sigma^6)^{\alpha\beta}=\sigma^2\otimes\mathbb{I}_2\otimes\sigma^1\otimes\sigma^2\tag{B.81}$$
$$(\sigma^7)^{\alpha\beta}=\sigma^2\otimes\mathbb{I}_2\otimes\sigma^3\otimes\sigma^2\tag{B.82}$$
$$(\sigma^8)^{\alpha\beta}=\sigma^1\otimes\mathbb{I}_2\otimes\mathbb{I}_2\otimes\mathbb{I}_2\tag{B.83}$$
$$(\sigma^9)^{\alpha\beta}=\sigma^3\otimes\mathbb{I}_2\otimes\mathbb{I}_2\otimes\mathbb{I}_2\tag{B.84}$$

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C Superspace perspective on component results

The technology developed in *Superspace* [34] allows a presentation of component results in all superspaces... by an appropriate adaptation of notations. In particular, the equations indicated in section (5.6) in this book can be applied to the cases of eleven and ten dimensions. This is true even though the sole focus of the book is the case of 4D, $\mathcal{N} = 1$ supersymmetry. The discussion in the book can be easily modified for use in 11D and 10D superspace theories. The relevant equations were designated as (5.6.13), (5.6.16)–(5.6.18), (5.6.21), (5.6.22)–(5.6.24), (5.6.28), (5.6.33), and (5.6.34). For the convenience of the reader, we bring these results all together in the text to follow. In the text, these are all appropriately modified for the cases of 11D, $\mathcal{N} = 1$, 10D, $\mathcal{N} = 1$, 10D, $\mathcal{N} = 2$A, and 10D, $\mathcal{N} = 2$B superspaces, respectively.

C.1 Recollection of 4D, $\mathcal{N} = 1$ component/superspace results

In the context of 4D, $\mathcal{N} = 1$ superspace supergravity, we may distinguish among three types of symmetries:

(a). space time translations with generator $iK_{GC}(\xi^{\underline{m}})$, dependent on local parameters $\xi^{\underline{m}}(x)$,

(b). SUSY transformations with generator $iK_Q(\epsilon^{\underline{\alpha}})$ dependent on local parameters $\epsilon^{\underline{\alpha}}(x)$, and

(c). tangent space transformations with generator $iK_{TS}(\lambda^{\iota})$ dependent on local parameters $\lambda^{\iota}(x)$.

The tangent space transformations act as "internal angular momentum," chirality, etc. on all "flat indices" associated with the superspace quantities.

The commutator algebra of two SUSY transformations generated by $iK_Q(\epsilon_1{}^{\underline{\alpha}})$, and $iK_Q(\epsilon_2{}^{\underline{\alpha}})$, respectively takes the form

$$\big[\, iK_Q(\epsilon_1)\,,\; iK_Q(\epsilon_2)\,\big] = iK_{GC}(\xi^{\underline{m}}) + iK_Q(\epsilon) + iK_{TS}(\lambda^{\iota})\,, \tag{C.1}$$

where the parameters $\xi^{\underline{m}}$, $\epsilon^{\underline{\delta}}$, and λ^{ι} on the r.h.s. of this equation are quadratic in ϵ_1 and ϵ_2, dependent on linear and quadratic terms in the gravitino, and linear terms in the superspace torsions and curvature supertensors according to:

$$\xi^{\underline{m}} = -\left[(\epsilon_1{}^{\underline{\alpha}}\bar\epsilon_2{}^{\underline{\dot\beta}} + \bar\epsilon_1{}^{\underline{\dot\beta}}\epsilon_2{}^{\underline{\alpha}})T_{\underline{\alpha}\underline{\dot\beta}}{}^{\underline{c}} + \epsilon_1{}^{\underline{\alpha}}\epsilon_2{}^{\underline{\beta}}T_{\underline{\alpha}\underline{\beta}}{}^{\underline{c}} + \bar\epsilon_1{}^{\underline{\dot\alpha}}\bar\epsilon_2{}^{\underline{\dot\beta}}T_{\underline{\dot\alpha}\underline{\dot\beta}}{}^{\underline{c}}\right] e_{\underline{c}}{}^{\underline{m}}\,, \tag{C.2}$$

$$\epsilon^{\underline{\delta}} = -\left[(\epsilon_1{}^{\underline{\alpha}}\bar\epsilon_2{}^{\underline{\dot\beta}} + \bar\epsilon_1{}^{\underline{\dot\beta}}\epsilon_2{}^{\underline{\alpha}})(T_{\underline{\alpha}\underline{\dot\beta}}{}^{\underline{\delta}} + T_{\underline{\alpha}\underline{\dot\beta}}{}^{\underline{c}}\psi_{\underline{c}}{}^{\underline{\delta}}) + \epsilon_1{}^{\underline{\alpha}}\epsilon_2{}^{\underline{\beta}}(T_{\underline{\alpha}\underline{\beta}}{}^{\underline{\delta}} + T_{\underline{\alpha}\underline{\beta}}{}^{\underline{c}}\psi_{\underline{c}}{}^{\underline{\delta}}) + \bar\epsilon_1{}^{\underline{\dot\alpha}}\bar\epsilon_2{}^{\underline{\dot\beta}}(T_{\underline{\dot\alpha}\underline{\dot\beta}}{}^{\underline{\delta}} + T_{\underline{\dot\alpha}\underline{\dot\beta}}{}^{\underline{c}}\psi_{\underline{c}}{}^{\underline{\delta}})\right], \tag{C.3}$$

$$\lambda^{\iota} = -\left[(\epsilon_1{}^{\underline{\alpha}}\bar\epsilon_2{}^{\underline{\dot\beta}} + \bar\epsilon_1{}^{\underline{\dot\beta}}\epsilon_2{}^{\underline{\alpha}})(R_{\underline{\alpha}\underline{\dot\beta}}{}^{\iota} + T_{\underline{\alpha}\underline{\dot\beta}}{}^{\underline{c}}\Phi_{\underline{c}}{}^{\iota}) + \epsilon_1{}^{\underline{\alpha}}\epsilon_2{}^{\underline{\beta}}(R_{\underline{\alpha}\underline{\beta}}{}^{\iota} + T_{\underline{\alpha}\underline{\beta}}{}^{\underline{c}}\Phi_{\underline{c}}{}^{\iota}) + \bar\epsilon_1{}^{\underline{\dot\alpha}}\bar\epsilon_2{}^{\underline{\dot\beta}}(R_{\underline{\dot\alpha}\underline{\dot\beta}}{}^{\iota} + T_{\underline{\dot\alpha}\underline{\dot\beta}}{}^{\underline{c}}\Phi_{\underline{c}}{}^{\iota})\right]. \tag{C.4}$$

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The supersymmetry variations of the inverse frame field $e_{\underline{a}}{}^{\underline{m}}(x)$, gravitino $\psi_{\underline{a}}{}^{\underline{\delta}}(x)$, and connection fields for the tangent space symmetries $\phi_{\underline{a}}{}^{\iota}(x)$ take the forms below and are expressed in terms dependent on linear and quadratic in the gravitino, and linear in the superspace torsions and curvature supertensors.

$$\delta_Q e_{\underline{a}}{}^{\underline{m}} = -\left[\epsilon^{\underline{\beta}} T_{\underline{\beta}\underline{a}}{}^{\underline{d}} + \bar{\epsilon}^{\dot{\underline{\beta}}} T_{\dot{\underline{\beta}}\underline{a}}{}^{\underline{d}} + (\bar{\epsilon}^{\dot{\underline{\beta}}} \psi_{\underline{a}}{}^{\underline{\gamma}} + \epsilon^{\underline{\gamma}} \bar{\psi}_{\underline{a}}{}^{\dot{\underline{\beta}}}) T_{\dot{\underline{\beta}}\underline{\gamma}}{}^{\underline{d}} + \epsilon^{\underline{\beta}} \psi_{\underline{a}}{}^{\underline{\gamma}} T_{\underline{\gamma}\underline{\beta}}{}^{\underline{d}} + \bar{\epsilon}^{\dot{\underline{\beta}}} \bar{\psi}_{\underline{a}}{}^{\dot{\underline{\gamma}}} T_{\dot{\underline{\beta}}\dot{\underline{\gamma}}}{}^{\underline{d}} \right] e_{\underline{d}}{}^{\underline{m}} , \tag{C.5}$$

$$\begin{aligned} \delta_Q \psi_{\underline{a}}{}^{\underline{\delta}} = {} & \mathrm{D}_{\underline{a}} \epsilon^{\underline{\delta}} - \epsilon^{\underline{\beta}} (T_{\underline{\beta}\underline{a}}{}^{\underline{\delta}} + T_{\underline{\beta}\underline{a}}{}^{\underline{e}} \psi_{\underline{e}}{}^{\underline{\delta}}) - \bar{\epsilon}^{\dot{\underline{\beta}}} (T_{\dot{\underline{\beta}}\underline{a}}{}^{\underline{\delta}} + T_{\dot{\underline{\beta}}\underline{a}}{}^{\underline{e}} \psi_{\underline{e}}{}^{\underline{\delta}}) - (\bar{\epsilon}^{\dot{\underline{\beta}}} \psi_{\underline{a}}{}^{\underline{\gamma}} + \epsilon^{\underline{\gamma}} \bar{\psi}_{\underline{a}}{}^{\dot{\underline{\beta}}}) (T_{\underline{\gamma}\dot{\underline{\beta}}}{}^{\underline{\delta}} + T_{\underline{\gamma}\dot{\underline{\beta}}}{}^{\underline{e}} \psi_{\underline{e}}{}^{\underline{\delta}}) \\ & - \epsilon^{\underline{\beta}} \psi_{\underline{a}}{}^{\underline{\gamma}} (T_{\underline{\beta}\underline{\gamma}}{}^{\underline{\delta}} + T_{\underline{\beta}\underline{\gamma}}{}^{\underline{e}} \psi_{\underline{e}}{}^{\underline{\delta}}) - \bar{\epsilon}^{\dot{\underline{\beta}}} \bar{\psi}_{\underline{a}}{}^{\dot{\underline{\gamma}}} (T_{\dot{\underline{\beta}}\dot{\underline{\gamma}}}{}^{\underline{\delta}} + T_{\dot{\underline{\beta}}\dot{\underline{\gamma}}}{}^{\underline{e}} \psi_{\underline{e}}{}^{\underline{\delta}}) , \end{aligned} \tag{C.6}$$

$$\begin{aligned} \delta_Q \phi_{\underline{a}}{}^{\iota} = {} & -\epsilon^{\underline{\beta}} (R_{\underline{\beta}\underline{a}}{}^{\iota} + T_{\underline{\beta}\underline{a}}{}^{\underline{e}} \phi_{\underline{e}}{}^{\iota}) - \bar{\epsilon}^{\dot{\underline{\beta}}} (R_{\dot{\underline{\beta}}\underline{a}}{}^{\iota} + T_{\dot{\underline{\beta}}\underline{a}}{}^{\underline{e}} \phi_{\underline{e}}{}^{\iota}) - (\bar{\epsilon}^{\dot{\underline{\beta}}} \psi_{\underline{a}}{}^{\underline{\gamma}} + \epsilon^{\underline{\gamma}} \bar{\psi}_{\underline{a}}{}^{\dot{\underline{\beta}}}) (R_{\underline{\gamma}\dot{\underline{\beta}}}{}^{\iota} + T_{\underline{\gamma}\dot{\underline{\beta}}}{}^{\underline{e}} \phi_{\underline{e}}{}^{\iota}) \\ & - \epsilon^{\underline{\beta}} \psi_{\underline{a}}{}^{\underline{\gamma}} (R_{\underline{\beta}\underline{\gamma}}{}^{\iota} + T_{\underline{\beta}\underline{\gamma}}{}^{\underline{e}} \phi_{\underline{e}}{}^{\iota}) - \bar{\epsilon}^{\dot{\underline{\beta}}} \bar{\psi}_{\underline{a}}{}^{\dot{\underline{\gamma}}} (R_{\dot{\underline{\beta}}\dot{\underline{\gamma}}}{}^{\iota} + T_{\dot{\underline{\beta}}\dot{\underline{\gamma}}}{}^{\underline{e}} \phi_{\underline{e}}{}^{\iota}) . \end{aligned} \tag{C.7}$$

The supersymmetry covariantized versions of the torsions, gravitino field strength and field strengths associated respective with the inverse frame field $e_{\underline{a}}{}^{\underline{m}}(x)$, gravitino $\psi_{\underline{a}}{}^{\underline{\delta}}(x)$, and connection fields for the tangent space symmetries $\phi_{\underline{a}}{}^{\iota}(x)$ take the forms below and are expressed in terms dependent on linear and quadratic in the gravitino, and linear in the superspace torsions and curvature supertensors.

$$T_{\underline{a}\underline{b}}{}^{\underline{c}} = t_{\underline{a}\underline{b}}{}^{\underline{c}} + \psi_{[\underline{a}}{}^{\underline{\delta}} T_{\underline{\delta}\underline{b}]}{}^{\underline{c}} + \bar{\psi}_{[\underline{a}}{}^{\dot{\underline{\delta}}} T_{\dot{\underline{\delta}}\underline{b}]}{}^{\underline{c}} + \psi_{[\underline{a}}{}^{\underline{\delta}} \bar{\psi}_{\underline{b}]}{}^{\dot{\underline{\epsilon}}} T_{\underline{\delta}\dot{\underline{\epsilon}}}{}^{\underline{c}} + \psi_{\underline{a}}{}^{\underline{\delta}} \psi_{\underline{b}}{}^{\underline{\epsilon}} T_{\underline{\delta}\underline{\epsilon}}{}^{\underline{c}} + \bar{\psi}_{\underline{a}}{}^{\dot{\underline{\delta}}} \bar{\psi}_{\underline{b}}{}^{\dot{\underline{\epsilon}}} T_{\dot{\underline{\delta}}\dot{\underline{\epsilon}}}{}^{\underline{c}} , \tag{C.8}$$

$$T_{\underline{a}\underline{b}}{}^{\gamma} = t_{\underline{a}\underline{b}}{}^{\gamma} + \psi_{[\underline{a}}{}^{\underline{\delta}} T_{\underline{\delta}\underline{b}]}{}^{\gamma} + \bar{\psi}_{[\underline{a}}{}^{\dot{\underline{\delta}}} T_{\dot{\underline{\delta}}\underline{b}]}{}^{\gamma} + \psi_{[\underline{a}}{}^{\underline{\delta}} \bar{\psi}_{\underline{b}]}{}^{\dot{\underline{\epsilon}}} T_{\underline{\delta}\dot{\underline{\epsilon}}}{}^{\gamma} + \psi_{\underline{a}}{}^{\underline{\delta}} \psi_{\underline{b}}{}^{\underline{\epsilon}} T_{\underline{\delta}\underline{\epsilon}}{}^{\gamma} + \bar{\psi}_{\underline{a}}{}^{\dot{\underline{\delta}}} \bar{\psi}_{\underline{b}}{}^{\dot{\underline{\epsilon}}} T_{\dot{\underline{\delta}}\dot{\underline{\epsilon}}}{}^{\gamma} , \tag{C.9}$$

$$R_{\underline{a}\underline{b}}{}^{\iota} = r_{\underline{a}\underline{b}}{}^{\iota} + \psi_{[\underline{a}}{}^{\underline{\delta}} R_{\underline{\delta}\underline{b}]}{}^{\iota} + \bar{\psi}_{[\underline{a}}{}^{\dot{\underline{\delta}}} R_{\dot{\underline{\delta}}\underline{b}]}{}^{\iota} + \psi_{[\underline{a}}{}^{\underline{\delta}} \bar{\psi}_{\underline{b}]}{}^{\dot{\underline{\epsilon}}} R_{\underline{\delta}\dot{\underline{\epsilon}}}{}^{\iota} + \psi_{\underline{a}}{}^{\underline{\delta}} \psi_{\underline{b}}{}^{\underline{\epsilon}} R_{\underline{\delta}\underline{\epsilon}}{}^{\iota} + \bar{\psi}_{\underline{a}}{}^{\dot{\underline{\delta}}} \bar{\psi}_{\underline{b}}{}^{\dot{\underline{\epsilon}}} R_{\dot{\underline{\delta}}\dot{\underline{\epsilon}}}{}^{\iota} . \tag{C.10}$$

In the linearized limit of these theories, not all of the terms in (C.2)–(C.10) appear. Instead these equations take the forms

$$\xi^{\underline{m}} = -\left[(\epsilon_1{}^{\underline{\alpha}} \bar{\epsilon}_2{}^{\dot{\underline{\beta}}} + \bar{\epsilon}_1{}^{\dot{\underline{\beta}}} \epsilon_2{}^{\underline{\alpha}}) T_{\underline{\alpha}\dot{\underline{\beta}}}{}^{\underline{c}} + \epsilon_1{}^{\underline{\alpha}} \epsilon_2{}^{\underline{\beta}} T_{\underline{\alpha}\underline{\beta}}{}^{\underline{c}} + \bar{\epsilon}_1{}^{\dot{\underline{\alpha}}} \bar{\epsilon}_2{}^{\dot{\underline{\beta}}} T_{\dot{\underline{\alpha}}\dot{\underline{\beta}}}{}^{\underline{c}} \right] e_{\underline{c}}{}^{\underline{m}} , \tag{C.11}$$

$$\epsilon^{\underline{\delta}} = -\left[(\epsilon_1{}^{\underline{\alpha}} \bar{\epsilon}_2{}^{\dot{\underline{\beta}}} + \bar{\epsilon}_1{}^{\dot{\underline{\beta}}} \epsilon_2{}^{\underline{\alpha}}) (T_{\underline{\alpha}\dot{\underline{\beta}}}{}^{\underline{\delta}}) + \epsilon_1{}^{\underline{\alpha}} \epsilon_2{}^{\underline{\beta}} (T_{\underline{\alpha}\underline{\beta}}{}^{\underline{\delta}}) + \bar{\epsilon}_1{}^{\dot{\underline{\alpha}}} \bar{\epsilon}_2{}^{\dot{\underline{\beta}}} (T_{\dot{\underline{\alpha}}\dot{\underline{\beta}}}{}^{\underline{\delta}}) \right] , \tag{C.12}$$

$$\lambda^{\iota} = -\left[(\epsilon_1{}^{\underline{\alpha}} \bar{\epsilon}_2{}^{\dot{\underline{\beta}}} + \bar{\epsilon}_1{}^{\dot{\underline{\beta}}} \epsilon_2{}^{\underline{\alpha}}) (R_{\underline{\alpha}\dot{\underline{\beta}}}{}^{\iota}) + \epsilon_1{}^{\underline{\alpha}} \epsilon_2{}^{\underline{\beta}} (R_{\underline{\alpha}\underline{\beta}}{}^{\iota}) + \bar{\epsilon}_1{}^{\dot{\underline{\alpha}}} \bar{\epsilon}_2{}^{\dot{\underline{\beta}}} (R_{\dot{\underline{\alpha}}\dot{\underline{\beta}}}{}^{\iota}) \right] , \tag{C.13}$$

$$\delta_Q \psi = -\left[\epsilon^{\underline{\beta}} T_{\underline{\beta}\underline{d}}{}^{\underline{d}} + \bar{\epsilon}^{\dot{\underline{\beta}}} T_{\dot{\underline{\beta}}\underline{d}}{}^{\underline{d}} \right] , \tag{C.14}$$

$$\delta_Q \psi_{\underline{a}}{}^{\underline{\delta}} = \mathrm{D}_{\underline{a}} \epsilon^{\underline{\delta}} - \epsilon^{\underline{\beta}} (T_{\underline{\beta}\underline{a}}{}^{\underline{\delta}}) - \bar{\epsilon}^{\dot{\underline{\beta}}} (T_{\dot{\underline{\beta}}\underline{a}}{}^{\underline{\delta}}) , \tag{C.15}$$

$$\delta_Q \phi_{\underline{a}}{}^{\iota} = -\epsilon^{\underline{\beta}} (R_{\underline{\beta}\underline{a}}{}^{\iota}) - \bar{\epsilon}^{\dot{\underline{\beta}}} (R_{\dot{\underline{\beta}}\underline{a}}{}^{\iota}) , \tag{C.16}$$

$$T_{\underline{a}\underline{b}}{}^{\gamma} = t_{\underline{a}\underline{b}}{}^{\gamma} , \tag{C.17}$$

$$T_{\underline{a}\underline{b}}{}^{\underline{c}} = t_{\underline{a}\underline{b}}{}^{\underline{c}} , \tag{C.18}$$

$$R_{\underline{a}\underline{b}}{}^{\iota} = r_{\underline{a}\underline{b}}{}^{\iota} \quad . \tag{C.19}$$

The terms on the r.h.s. of the final three equation correspond to the non-supercovariantized versions of the respective torsions, gravitino field strengths, and connection field strengths.

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