The Standard Model of Particle Physics, Lecture 4

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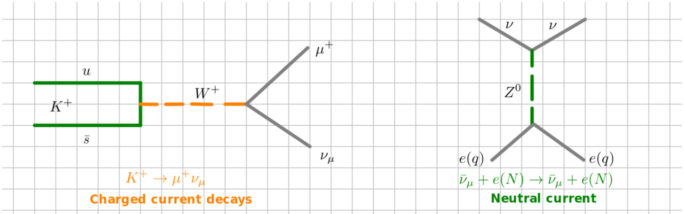
The weak currents

- The charged current interactions can be written in terms of lowering operator $\tau^- = \frac{\sigma_1 - i\sigma_2}{2}$ as

$$j_\mu^- = \bar{e}_L \gamma_\mu \nu_L = \bar{\chi}_L \gamma_\mu \tau^- \chi_L \quad , \quad \chi_L = \begin{pmatrix} \nu_L \\ e \end{pmatrix}$$



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- If there were another current $j_3^\mu = \bar{\chi}_L \gamma^\mu \tau_3 \chi_L$ then one can show that the charges constructed out of these current are invariant under $SU_L(2)$

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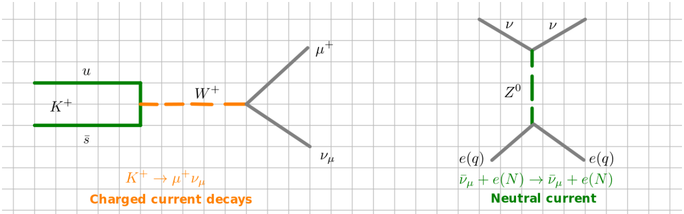

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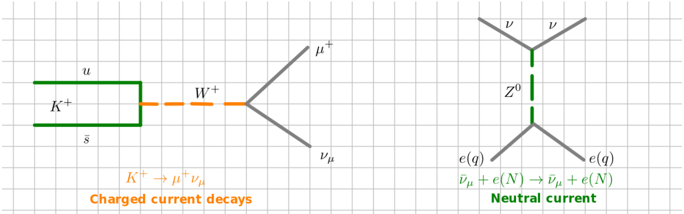
- However the neutral current is not equal to $j_3^\mu \rightarrow$ breaks $SU_L(2)$



Weak Isospin and hypercharge

- The standard electromagnetic current also mixes the left and right-handed states, for e.g.

$$j_\mu^{em} = -\bar{e}\gamma_\mu e = -\bar{e_R}\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L \ .$$

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- Can we combine the EM and neutral currents in a way that their linear superpositions are invariant under $SU(2)_L$? Yes!

$$\text{Using } Q = I_3 + \frac{Y}{2} \text{ formulate } J_\mu^Y = 2J_\mu^{em} - 2J_\mu^3 \ .$$

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- Thus the enlarged symmetry group is $SU(2)_L \times U_Y(1)$.

$SU(2)_L \times U_Y(1)$

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$$\mathcal{L}_{EW} = -ig\, j_i^\mu W_\mu^i - \frac{ig'}{2}\, j_Y^\mu B_\mu \quad , \quad i = 1, 2, 3 \; .$$

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$$A_\mu = B_\mu\ \cos\theta_W + W_\mu^3\ \sin\theta_W \quad , \ Z_\mu = -B_\mu\ \sin\theta_W + W_\mu^3\ \cos\theta_W \ .$$

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- NC are not maximally V-A except for neutrinos!

Electroweak sector of SM

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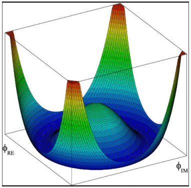
- This can be explained via spontaneous symmetry breaking also known as Higgs mechanism.
- The standard model has the $SU(2)_L \times U_Y(1)$ gauge symmetry however when we are sitting in the ground state this gauge invariance is not apparent. Same can be understood in Ising model!

Higgs mechanism in U(1) theory

- Consider a simple qft of a charged scalar field in presence of a Abelian $U(1)$ field described by the Lagrangian

$$\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) + \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F^2 \ .$$

- Here the covariant derivative $D_\mu = \partial_\mu - ieA_\mu$ ensures gauge invariance of the Lagrangian.
- Scalar fields are subjected to the Mexican hat potential $V(|\phi|) = -\mu^2\phi^*\phi + \lambda(\phi^*\phi)^2$. If we write $\phi = \phi_1 + i\phi_2$ then we can show that the ground state of the potential is given by $\phi_1^2 + \phi_2^2 = \frac{\mu^2}{\lambda} = v$ [image courtesy: www.wikipedia.org].



Higgs mechanism in U(1) theory

- We can choose the ground state to be characterized by $\phi_1 = v$, $\phi_2 = 0$. If the system is very slightly perturbed from its vacuum with tiny perturbations $\eta(x), \zeta(x) \ll v$ such that

$$\phi_1 = v + \eta(x)\ ,\ \phi_2 = \zeta(x)\ ,\ \ \phi = v + \eta + i\zeta = (v + \eta)\mathrm{e}^{\mathrm{i}\zeta/\mathrm{v}}$$

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- Here $\eta(x)$ is the Higgs field. Since the scalar field acquires a phase, in order to preserve local gauge invariance one has to ensure that $A_\mu \to A_\mu + \frac{1}{ev}\partial_\mu\zeta$. Substituting this the original Lagrangian very close to the vacuum state looks like

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta)^2 - \lambda v^2\eta^2 + \frac{e^2v^2}{2}A_\mu^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 + \frac{e^2}{2}A_\mu^2\eta^2 + ve^2A_\mu^2\eta - \frac{1}{4}F^2\ .$$

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- The gauge boson acquires a mass and interacts with the Higgs field $\eta(x)$.

Spontaneous breaking of $SU(2)_L \times U_Y(1)$

- If we couple a complex scalar field ϕ which is a doublet under $SU(2)_L$ to the W_μ and B_μ in presence of a double well potential, where,

$$\phi = \frac{1}{2}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix} \quad , \quad \text{vacuum state is } \phi_0 = \frac{1}{2}\begin{pmatrix}0 \\ v\end{pmatrix} \; .$$

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- We want our vacuum to be $Q = 0$ state to preserve gauge invariance. Hence $Y = 2(Q - I_3) = 2(0 - (-\frac{1}{2})) = 1$ for the vacuum state. To quantify the masses of the gauge bosons it is sufficient to look at their coupling to the vacuum state in the kinetic term of the Lagrangian $|D_\mu \phi|^2 = |\left(\partial_\mu - i\frac{g}{2}\tau_i W_\mu^i - i\frac{g'}{2}YB_\mu\right)\phi|^2$ which is

$$|-\frac{ig}{2}\begin{pmatrix}W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3\end{pmatrix}\begin{pmatrix}0 \\ v\end{pmatrix} - \frac{ig'}{2}B_\mu\begin{pmatrix}0 \\ v\end{pmatrix}|^2 \ .$$

Salam-Weinberg Model

- This can be simplified to give

$$|-iv\begin{pmatrix} \frac{g}{\sqrt{2}}W_\mu^- \\ \frac{g^{'}}{2}B_\mu - \frac{g}{2}W_\mu^3 \end{pmatrix}|^2$$

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- Indeed if we look at the upper term then we get an expression $\frac{g^2v^2}{2}W^{\mu+}W_\mu^-$ which tells us that mass of $(W^\pm) = gv$. The lower part of the column vector written in terms of Z, A fields gives us

$$-ivA_\mu\left[g\sin\theta_W - g^{'}\cos\theta_W\right] - ivZ_\mu\left[g\cos\theta_W + g^{'}\sin\theta_W\right] \ .$$

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- Using $g\sin\theta_W = g'\cos\theta_W$ indeed shows that mass term of photon is identically zero. Also gives $M_Z = \frac{gv}{\cos\theta_W}$[Assignment: Verify this]. Experimental measurements give $M_Z = 91$ GeV and $M_W = 80$ GeV.

Fermion masses in Salam-Weinberg model

- The fermion masses also arise from spontaneous symmetry breaking from the terms in the Lagrangian which arise from coupling of fermions with the Higgs field,

$$-Y_e \begin{pmatrix} \bar{\nu}_L^e \\ \bar{e}_L \end{pmatrix} \phi \, e_R - Y_d \begin{pmatrix} \bar{u}_L \\ \bar{d}_L \end{pmatrix} \phi \, d_R - Y_u \begin{pmatrix} \bar{u}_L \\ \bar{d}_L \end{pmatrix} \phi_c \, u_R + h.c. -$$

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- One has to assign the Yukawa couplings to recover the masses of fermions. For e.g. $v = 246$ GeV and to recover the electron mass $m_e \sim Y_e v$ one needs $Y_e \sim 10^{-6}$. Similarly $Y_{\text{top}} \gg Y_u > Y_e$.

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- The electroweak theory is renormalizable! [t'Hooft and Veltman] There are no divergences in WW scatterings.

Standard Model and beyond?

- The Standard model Lagrangian consists of the Salam-Weinberg model+ strong color interactions mediated by gluons (Quantum Chromodynamics)

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- Is there any larger symmetry group G whose subgroup is $SU_c(3) \times SU(2)_L \times U_1^Y \rightarrow$ Grand Unified Theory?
- Some of the candidate theories are $G = SO(10), SU(5)$. There is no consensus yet. Cannot explain many important observations like the proton decay rates! However with exhaustive experimental tests the Standard model is now established as a very successful effective theory of particle physics.

References

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- T-P Cheng, L-F Li, "Gauge Theory of Elementary Particle Physics", Oxford University Press (1984).