ResearchGate

See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/291437506

NEUTRINOS: Mysterious Particles with Fascinating Features, which led to the Physics Nobel Prize 2015

Article *in* Revista Cubana de Fisica · December 2015

Source: arXiv

CITATIONS

2

READS

588

2 authors:

Alexis Aguilar-Arevalo

Instituto de Ciencias Nucleares. Universidad Nacional Autónoma de México

182 PUBLICATIONS **10,576** CITATIONS

SEE PROFILE

Wolfgang Bietenholz

Universidad Nacional Autónoma de México

221 PUBLICATIONS **3,045** CITATIONS

SEE PROFILE

All content following this page was uploaded by Wolfgang Bietenholz on 06 March 2017.

The user has requested enhancement of the downloaded file.

arXiv:1601.04747v1 [physics.pop-ph] 18 Jan 2016

NEUTRINOS:
Mysterious Particles with Fascinating Features, which led to the Physics Nobel Prize 2015

Alexis Aguilar-Arevalo and Wolfgang Bietenholz

Instituto de Ciencias Nucleares
Universidad Nacional Autónoma de México
A.P. 70-543, C.P. 04510 Distrito Federal, Mexico

The most abundant particles in the Universe are photons and neutrinos. Both types of particles are whirling around everywhere, since the early Universe. Hence the neutrinos are all around us, and permanently pass through our planet and our bodies, but we do not notice: they are extremely elusive. They were suggested as a theoretical hypothesis in 1930, and discovered experimentally in 1956. Ever since their properties keep on surprising us; for instance, they are key players in the violation of parity symmetry. In the Standard Model of particle physics they appear in three types, known as "flavors", and since 1998/9 we know that they keep on transmuting among these flavors. This "neutrino oscillation" implies that they are massive, contrary to the previous picture, with far-reaching consequences. This discovery was awarded the Physics Nobel Prize 2015.

1 A desperate remedy

ETH Zürich, the Swiss Federal Institute of Technology, has a long tradition of excellence in physics and other sciences. In addition, it has a tradition (dating back to the 19th century) to celebrate each year a large dance event, the Polyball. This also happened in 1930, when Wolfgang Pauli, one of the most renowned theoretical physicists, was working at ETH. The Polyball prevented him from attending a workshop in Tübingen (Germany), where leading scientists met to discuss aspects of radioactivity. Instead Pauli sent a letter to the participants, whom he addressed as "Liebe Radioaktive Damen und Herren" ("Dear Radioactive Ladies and Gentlemen") [1]. This letter of one page was of groundbreaking importance: it was the first document where a new type of particle was suggested, which we now denote as the *neutrino*.

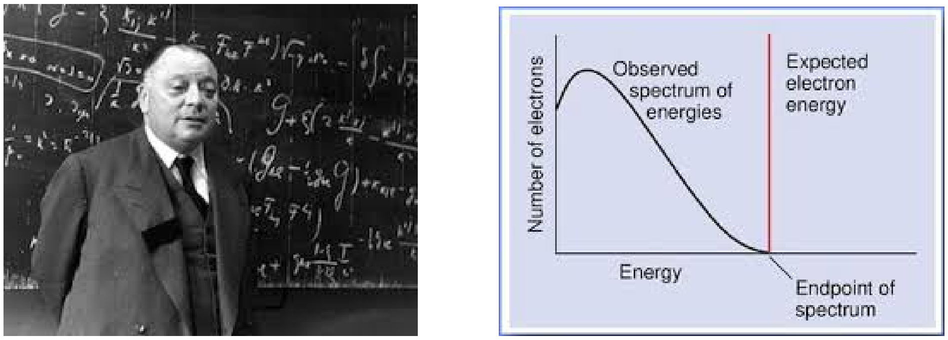


Figure 1: *On the left: Wolfgang Pauli (1900-1958), Austrian physicist working in Zürich, Switzerland. On the right: the energy spectrum of the electron, which is emitted in a β-decay; the observation does not match the original expectation of a sharp peak. Pauli solved this puzzle by postulating the emission of an additional particle, which was hypothetical at that time.*

Pauli was referring to the energy spectrum of electrons emitted in the β-decay: from a modern perspective (not known in 1930), a neutron is transformed into a slightly lighter proton, emitting an electron. This *β-radiation* was observed, but the puzzling point was the following: there is some energy reduction in a nucleus where this decay happens, and if we subtract the electron mass, we should obtain the electron's kinetic energy, which ought to be the same for all electrons emitted. In fact, the α- and γ-radiation spectra do exhibit such a sharp peak. For the β-radiation, however, one observed instead a broad spectrum of electron energies [2], with a maximum at this value. In particular, in 1927 C.D. Ellis and W.A. Wooster had studied the decay ${}^{210}_{83}\mathrm{Bi} \to {}^{210}_{84}\mathrm{Po}$ and identified a maximal electron energy of 1050 keV, but a mean value of only 390 keV [3].

This seemed confusing indeed, and prominent people like Niels Bohr even considered giving up the law of energy conservation. Pauli, however, made an effort to save it: as a "desperate remedy" he postulated that yet another particle could be emitted in this decay, which would carry away the energy, which seemed to be missing. He estimated its mass to be of the same order as the electron mass. He also knew that some nuclei change their spin by 1 unit under β-decay, so he specified that this new particle should carry spin 1/2, just like the electron; thus also angular momentum conservation is saved. To further conserve the electric charge, it must be electrically neutral, therefore

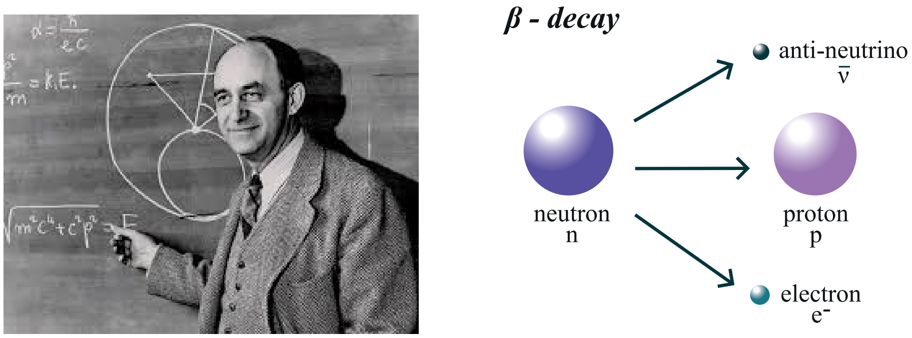


Figure 2: *On the left: Enrico Fermi (1901-1954), famous for his achievements both in theoretical and experimental physics. On the right: scheme of the β-decay, which transforms a neutron into a proton, while emitting an electron and an anti-neutrino.*

he wanted to call it a “neutron”. That would explain why this particle had not been observed, thus completing a hypothetical but consistent picture.[1]

2 Fermi’s theory

Two years later, James Chadwick discovered the far more massive particle, which we now call the neutron [4]. In 1933/4 Enrico Fermi, who was working in Rome, elaborated a theory for the interaction of Pauli’s elusive particle [5]. He introduced the name “neutrino”,[2] and suggested that it might be *massless.*

In our modern terminology, the emitted particle is actually an *anti-neutrino, $\bar{\nu}$.* This $\bar{\nu}$-emission is, in some sense, equivalent to an incoming neutrino, ν, so the β-decay can be written in its usual scheme, or as a related variant,

$$\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu} \qquad \text{or} \qquad \mathrm{n} + \nu \to \mathrm{p} + \mathrm{e}^- \ .$$

Referring to the latter scheme, Fermi made an ansatz for the transition amplitude M, where the wave functions of all four fermions interact in one

---

[1] Hence Pauli suggested one new particle, for truly compelling reasons like the conservation of energy and angular momentum. This can be contrasted with the modern literature, where a plethora of hypothetical particles are suggested, often based on rather weak arguments.

[2] Since “neutrino” is a diminutive in Italian, its plural should actually be “neutrini”, but we adopt here the commonly used plural.

space-time point x (to be integrated over),

$$M(x) = G_{\rm F} \left(\bar{\Psi}_{\rm p}(x)\Gamma\Psi_{\rm n}(x)\right) \left(\bar{\Psi}_{\rm e}(x)\Gamma'\Psi_{\nu}(x)\right) , \quad G_{\rm F} \simeq 1.2\cdot 10^{-5} \frac{(\hbar c)^3}{{\rm GeV}^2} \ . \quad (1)$$

This *4-fermi term* describes the simultaneous transformations $\rm n \to p$ and $\nu \to {\rm e}^-$, with factors $G_{\rm F}$ (Fermi's constant),[3] and Γ, Γ' (to be addressed below). In Heisenberg's formalism, these are just transitions between the two isospin states of the same particle.[4]

This process is a prototype of the *weak interaction*, which is nowadays described by the exchange of W- and Z-bosons (Fermi's constant can be expressed as $G_{\rm F} = g^2/(2^{5/2}M_W)$, where g is the weak coupling constant and M_W the W-mass). Fermi's simple theory works well up to moderate energy. The refined picture — with an intermediate W-boson instead of the 4-fermi interaction in one point — prevents a divergent cross-section at high energy.

3 Neutrinos exist!

Pauli is often quoted as saying "I have done a terrible thing, I have postulated a particle that cannot be detected" (although it is not clear where this statement is really documented). In any case, it turned out to be wrong: in 1956 Clyde Cowan and Frederick Reines observed that anti-neutrinos, produced in a nuclear reactor in South Carolina, did occasionally interact with protons, which leads to a neutron and a positron (the positively charged anti-particle of an electron), $\rm p + \bar{\nu} \to n + e^+$. This is an *inverse β-decay,* which they observed in two large water tanks [6].[5] They sent a telegram to Pauli, alerting him that his particle really exists!

3.1 . . . and they are all around!

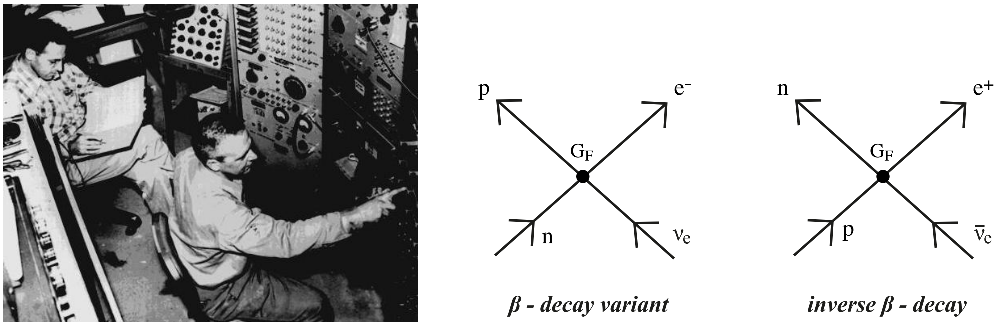
Of course, neutrinos had existed long before, since the Big Bang: just 2 seconds later they decoupled and ever since they are flying around all over

---

[3] It is remarkable that Fermi already estimated its magnitude correctly, his value was $G_{\rm F} = 0.3 \cdot 10^{-5} (\hbar c)^3/{\rm GeV}^2$.

[4] The nucleons, *i.e.* the proton and the neutron, were assumed to be elementary particles at that time.

[5] Even today, reactor neutrinos are still detected with a variant of the technique employed by Cowan, Reines and collaborators.



β - decay variant ***inverse β - decay***

Figure 3: *On the left: Fred Reines (left) and Clyde Cowan (right), the pioneers who first succeeded in detecting anti-neutrinos. On the right: diagrams of a* β*-decay variant (compatible with Fermi's formula (1)), and of the inverse* β*-decay (observed by Cowan and Reines in 1956).*

the Universe. This is the *Cosmic Neutrino Background,* CνB. It has gradually cooled down, from $\approx 10^{10}$ K to its temperature today of 1.95 K. It can be compared to the (better known) Cosmic Microwave Background, which was formed about 380 000 years later by photons, and which is somewhat warmer, 2.73 K.

In contrast to the Cosmic Microwave Background, which is being monitored intensively, the CνB has not been observed directly — neutrino detection is very difficult in general, and at such low energies it seems hardly possible. Still, the arguments for its existence are compelling and generally accepted. New indirect evidence has been provided in 2015 by Planck satellite data for details of the temperature fluctuations in the Cosmic Microwave Background [7]. A *direct* detection of the CνB, however, is still a long-term challenge. The density throughout the Universe is about 336 neutrinos (and 411 photons) per cm^3; in our galaxy it might be higher due to gravitational effects.

Neutrinos of higher energies are generated in stars — like the Sun — by nuclear fusion, in Active Galactic Nuclei, Gamma Ray Bursts, supernova explosions, etc. They are also produced inside the Earth (by decays), in our atmosphere (when cosmic rays hit it and trigger an air shower of secondary particles), and on the Earth, in particular in nuclear reactors. The latter provide $\bar{\nu}$-energies around 1 MeV, with a typical cross section of about 10^{-44} cm^2. The probability of an interaction in a solid detector of 1 m length is of order 10^{-18}, so their chance of scattering while crossing the Earth is around 10^{-11}.

This shows why it took a while to discover them; the search for neutrinos is sometimes described as "ghost hunting". For instance, in our daily life we never feel that we are exposed to a neutrino flux originating from the Sun, although some $6 \cdot 10^{14}$ solar neutrinos cross our body every second. If we could install a detector that fills all the space between the Sun and the Earth, it would capture only 1 out of 10 million neutrinos. In Section 7 we will come back to the solar and atmospheric neutrinos; this is what the 2015 Nobel Prize experiments were about.

4 Parity violation: a stunning surprise

4.1 Theory

A parity transformation, P, is simply a sign change of the spatial coordinates, P: $x = (t, \vec{r}) \to (t, -\vec{r})$. For a long time, people assumed it to a basic principle that the Laws of Nature are parity invariant. This seems obvious by common sense, and in fact it holds for gravity, electromagnetic and strong interactions. How about the weak interaction? The neutrinos are the only particles that only interact weakly (if we neglect gravity), so it is promising to focus on them to investigate this question.

At this point, we come back to the factors Γ and Γ' between the fermionic 4-component Dirac spinors $\bar{\Psi}$, Ψ in eq. (1). They characterize the structure of the weak interaction, which arranges for these particle transformations. *A priori* one could imagine any Dirac structure: scalar, pseudo-scalar, vector, pseudo-vector or tensor ($\mathbb{1}$, γ_5, $\gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}$). Under parity transformation, the "pseudo"-quantities (which involve a factor γ_5) pick up a sign opposite to (ordinary) scalar and vector terms.

If Γ and Γ' were both parity even, or both parity odd, then also this weak interaction process would be parity symmetric. However, in 1956 Tsung-Dao Lee and Chen-Ning Yang suggested that this might not be the case [8]. Their scenario is reflected by a structure of the form

$$M(x) = \frac{G_{\rm F}}{\sqrt{2}} \Big(\bar{\Psi}_{\rm p}(x) \gamma^\mu (1 - \frac{g_{\rm A}}{g_{\rm V}} \gamma_5) \Psi_{\rm n}(x) \Big) \Big(\bar{\Psi}_{\rm e}(x) \gamma_\mu (1 - \gamma_5) \Psi_\nu(x) \Big) \ , \qquad (2)$$

which mixes vector currents — which Fermi had in mind — with pseudo-vector (or axial) currents. The ratio $g_{\rm A}/g_{\rm V}$ is a constant; its value is now

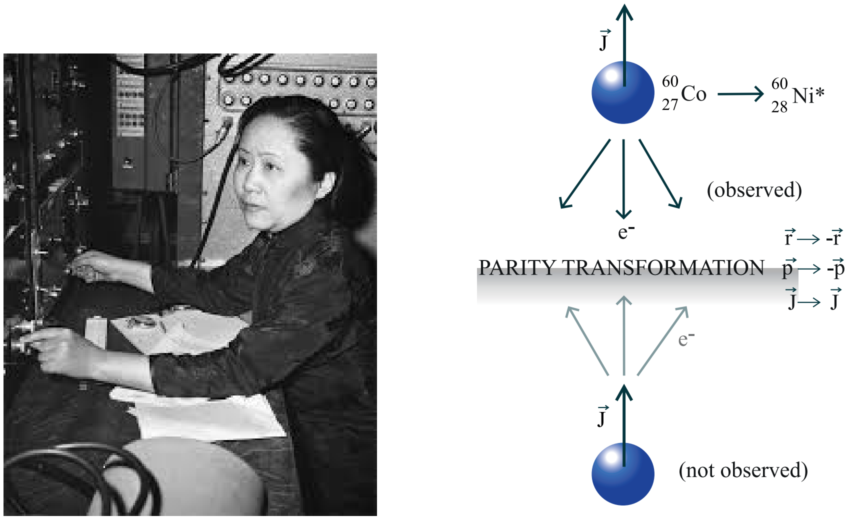


Figure 4: *On the left: Chien-Shiung Wu (1912-1997), leader of the experiment that demonstrated the violation of parity invariance in 1957. On the right: the concept of her experiment, as described in the text.*

determined as $\simeq 1.26$. Hence vector and axial vector currents are strongly mixed, which breaks P invariance. But how was the violation of parity symmetry verified?

4.2 Experiment

In fact, it was confirmed only one year after Lee and Yang's suggestion in an experiment, which was led by another brilliant Chinese researcher, Chien-Shiung Wu [9]. Her experiment dealt with the β-decay, which transforms a cobalt nucleus into nickel,

$$^{60}_{27}\mathrm{Co} \ \to \ ^{60}_{28}\mathrm{Ni}^* + \mathrm{e}^- + \bar{\nu} \ ,$$

a process, which lowers the nuclear spin from $J = 5 \to 4$. A magnetic moment is attached to the nuclear spin, hence a strong magnetic field can align the spins in a set of Co nuclei. (This was not easy in practice: only after cooling the sample down to 0.003 K, a polarization of 60% could be attained.)

How can the nuclear spin change be compensated by the leptons, *i.e.* by the electron e^- and the anti-neutrino $\bar{\nu}$? They are both spin-1/2 particles, as

Pauli had predicted, and they could be right-handed (spin in the direction of motion) or left-handed (spin opposite to the direction of motion).[6] Clearly, the compensation requires a right-handed particle flying away in the direction of the nuclear spin $\vec{J}$, and a left-handed one being emitted in the opposite direction.

The electrons are much easier to detect, and one observed their preference in the $-\vec{J}$ direction. Under a parity transformation, the spin $\vec{J}$ behaves like an angular momentum $\vec{L} = \vec{p} \times \vec{r}$; it remains invariant. The direction of flight of the leptons, however, is exchanged. Hence this dominance of electrons in one direction demonstrates the violation of parity invariance. The reason is that the anti-neutrino only occurs right-handed (and the neutrino only left-handed),[7] so the $\bar{\nu}$ has to move in the $\vec{J}$-direction.

This came a great surprise, *Nature does distinguish between left and right!* An example for the consternation that this result caused is Pauli's first reaction, who exclaimed "This is total nonsense!". It is a striking example for the fascinating features of the neutrinos. This sequence of surprises is still going on, and it embraces the 2015 Nobel Prize. Long before, in 1957 Lee and Yang received the Nobel Prize for their discovery; unfortunately Wu was left out.

As a *Gedankenexperiment*, one could also perform a C transformation ("charge conjugation"), which transforms all particles into their anti-particles and vice versa, thus flipping the signs of all charges. This shows that the Wu experiment also demonstrated the violation of C symmetry, but invariance is recovered under the combined transformation CP. In particular for the *chirality* (handedness) of ν and $\bar{\nu}$, CP invariance holds. Lev Landau suggested that this might be a true symmetry of Nature [10].

In 1964, however, an experiment directed by James Cronin and Val Fitch demonstrated that — in even more subtle decays, also due to the weak interaction — CP symmetry is violated as well. Now we are left with the CPT Theorem:[8] if we still add a simultaneous T transformation (a flip of the di-

---

[6] Strictly speaking this is the helicity, which coincides with the handedness, or chirality, in the relativistic limit; we are a bit sloppy about this distinction.

[7] This can be seen from eq. (2), which includes a projection of Ψ_ν to its left-handed component, $\psi_{\nu;L} = \frac{1}{2}(1-\gamma_5)\Psi_\nu$, but no right-handed component $\psi_{\nu;R} = \frac{1}{2}(1+\gamma_5)\Psi_\nu$ is involved. In fact, a right-handed neutrino, or a left-handed anti-neutrino, has never been observed. We will comment on their possible existence in the appendix.

[8] A rigorous proof for this theorem was given in 1957 by Res Jost [11], previously Pauli's assistant. It is one of the most important and elegant results in Quantum Field Theory,

rection of time), then invariance must hold, if our world is described by a relativistic and local quantum field theory — that seems to be the case, so far a huge number of high precision experiments support it.

5 Neutrinos occur in distinct flavors

What distinguishes a neutrino from an anti-neutrino? We have mentioned the different chirality. In the Standard Model — to be addressed below — left-handed neutrinos ν_L (right-handed anti-neutrinos $\bar{\nu}_R$) occur, and they carry a weak hypercharge Y $(-Y)$, which characterizes their coupling to a W or Z gauge boson (like the electric charge of other particles represents the coupling to a photon). Thus also the sign of Y distinguishes ν from $\bar{\nu}$. However, their distinction was introduced much earlier, even before either of them had been detected.

In 1953, E.J. Konopinski and H.M. Mahmoud studied the decays involving the light particles that we call leptons [12]. At that time, they knew the electron, the neutrino (as a hypothesis) and the muon, μ^-, which had been discovered in 1936. The latter is similar to an electron, but 207 times heavier. Konopinski and Mahmoud introduced a new quantum number: they assigned to the particles ν, e^-, μ^- the *lepton number* $L = 1$, their anti-particles $\bar{\nu}$, e^+, μ^+ carry $L = -1$, and all the (non-leptonic) rest has $L = 0$.

The role of the lepton number should simply be its conservation, which holds indeed *e.g.* in the β-decay, or inverse β-decay, or in decays of charged pions,

$$\pi^- \to \mu^- + \bar{\nu} \ , \quad \pi^+ \to \mu^+ + \nu \ , \tag{3}$$

but it rules out a process like $\mathrm{n} + \bar{\nu} \to \mathrm{p} + \mathrm{e}^-$, which is not observed.

This rule is still incomplete, however, since it allows for a decay like $\mu^- \to \mathrm{e}^- + \gamma$ (γ represents a photon), which is not observed either.

This led to the insight that leptons occur in distinct *generations,* with their own lepton numbers, like the electron number $L_\mathrm{e} = \pm 1$ for $\mathrm{e}^\mp$, and the muon number $L_\mu = \pm 1$ for $\mu^\mp$. This suggested that there are also distinct neutrinos, as Bruno Pontecorvo — an Italian physicist who had emigrated to the Soviet Union — pointed out in 1960: an electron-neutrino ν_e with $L_\mathrm{e} = 1$ and a muon-neutrino ν_μ with $L_\mu = 1$ (while $\bar{\nu}_\mathrm{e}, \bar{\nu}_\mu$ have $L_\mathrm{e} = -1$ and

---

but it is not easily accessible: Jost wrote his paper in German and published it in the Swiss journal *Helvetica Physica Acta,* which does not exist anymore.

	Fermion generations		
	1	***2***	***3***
leptons	$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}$	$\begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix}$	$\begin{pmatrix}\nu_\tau \\ \tau^-\end{pmatrix}$
quarks	$\begin{pmatrix}u \\ d\end{pmatrix}$	$\begin{pmatrix}c \\ s\end{pmatrix}$	$\begin{pmatrix}t \\ b\end{pmatrix}$

Figure 5: *Table of the fermions in the Standard Model.*

$L_\mu = -1$, respectively, and the rest is zero) [13]. The stronger assumption that L_e and L_μ are *separately conserved* explains observed decays such as

$$\mu^- \to \mathrm{e}^- + \nu_\mu + \bar{\nu}_\mathrm{e} \ , \quad \mu^+ \to \mathrm{e}^+ + \bar{\nu}_\mu + \nu_\mathrm{e} \ , \tag{4}$$

which take $2.2 \cdot 10^{-6}$ s. It also distinguishes transitions like

$$\mathrm{n} + \nu_\mathrm{e} \to p + \mathrm{e}^- \ , \quad \mathrm{n} + \nu_\mu \to p + \mu^- \ , \tag{5}$$

which require an intermediate charged boson $W^\pm$. These transitions do not occur if we exchange ν_e and ν_μ, or replace them by anti-neutrinos. This distinction enabled the experimental discovery of ν_μ in 1962, by Lederman, Schwartz, Steinberger and collaborators [14]. Now we can write the inverse β-decay, observed by Cowan and Reines, in a more precise form: $\mathrm{p}+\bar{\nu}_\mathrm{e} \to \mathrm{n}+\mathrm{e}^+$.

The *Standard Model of particle physics* takes into account that later (in 1975) yet another cousin of the electron was found [15]: the tauon τ, which is 3477 times heavier than the electron (hence its life time is only $2.9 \cdot 10^{-13}$ s). It is also accompanied by its own type of neutrino [16], ν_τ, so we are actually dealing with *three* distinct lepton numbers, L_e, L_μ and L_τ.

Similarly the Standard Model incorporates three generations of quarks, so its fermionic content can be summarized as shown in Table 5. In addition, the Standard Model involves gauge bosons (photons for the electromagnetic interaction, W and Z for the weak interaction, and 8 gluons for the strong interaction), plus the (scalar) Higgs particle. This is what all known matter in the Universe consists of.[9]

---

[9] The graviton might still be added to this list. We also have indirect evidence for Dark Matter, which must be of a different kind.

From a conceptual point of view, the Standard Model is only consistent for entire *fermion generations,* composed of a lepton doublet and a quark doublet (otherwise quantum effects break gauge invariance [17]). On the other hand, there is no theoretical constraint on the number of generations. The higher generations involve heavier fermions, so they were discovered later. Hence one could wonder if this sequence is going on, and further generations will be discovered step by step.

This cannot be rigorously excluded, but there are good reasons to assume that there are not more than these 3 generations. The Z-boson is one of the heaviest elementary particles that we know, with a mass of 91 GeV, and it can decay into a neutrino–anti-neutrino pair of the same flavor,

$$Z \to \nu_x + \bar{\nu}_x \ , \quad x \in \{\mathrm{e},\, \mu,\, \tau\} \ .$$

It can also decay into $\mathrm{e}^- + \mathrm{e}^+$, $\mu^- + \mu^+$ or $\tau^- + \tau^+$, or into a quark–anti-quark pair. If we sum up all these decay channels (which were measured very precisely in the Large Electron-Positron Collider at CERN [18]), we obtain — to a good precision — the full decay rate of the Z-boson. This is an argument against a 4th generation: if the Z-boson could decay into yet another ν–$\bar{\nu}$ pair, we should have noticed the missing part in this sum of decay channels.[10]

6 The mixing of quark and of lepton flavors

6.1 A look at the quark sector

We follow the historical evolution and first discuss mixing in the quark sector: we saw that the quarks occur in 6 *flavors,* such as the "strange" s quark. Also here quantum numbers were introduced, which indicate the quark contents of a specific flavor. For instance, the *strangeness* of a hadron[11] counts the number of its $\bar{s}$ minus s (valence) quarks.

As a general trend, also the quarks can easily be transformed within one generation; that is analogous to the conservation of the generation specific

---

[10] A loophole in this argument are neutrinos, with a very heavy mass $> m_Z/2 \simeq 46$ GeV, which are, however, considered unlikely.

[11] Hadrons are observable particles, composed of quarks and gluons. One distinguishes baryons (with 3 valence quarks, (qqq)) and mesons (with a valence quark–anti-quark pair, $(q\bar{q})$).

lepton numbers. This encompasses for instance the β-decay, n $\sim (udd) \to$ p $\sim (uud)+$ leptons.

However, transformations between different generations happen as well: for instance, the strangeness of a hadron changes when an s quark decays into the much lighter quarks u and d. An example is the decay of the baryon Λ^0 into a nucleon and a pion,

$$\begin{array}{ccc} \Lambda^0 \to p + \pi^- & \text{or} & \Lambda^0 \to n + \pi^0 \\ (uds) \to (uud) + (\bar{u}d) & & (uds) \to (udd) + (\bar{u}u - \bar{d}d)/\sqrt{2} \ , \end{array}$$

(the lower line indicates the valence quark contents of the hadrons involved). Based on the heavy Λ^0-mass of 1.1 GeV, one could expect this decay to happen within $O(10^{-23})$ s, but since it proceeds only by the weak interaction it takes as long as $2.6 \cdot 10^{-10}$ s.

The evolution is driven by the Hamiltonian, and from examples like these strangeness changing decays we infer that the upper, or the lower, doublet partners are *not* (exactly) its eigenstates. Hence we have to distinguish the mass eigenstates (u, c, t), or (d, s, b), from the slightly different eigenstates of the weak interaction, (u', c', t') or (d', s', b'), respectively.

At this point, we recall that Dirac's 4-component spinor Ψ actually describes a left-handed and a right-handed fermion; the corresponding spinors are obtained by chiral projection $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\Psi$, cf. footnote 7. The kinetic term in the Lagrangian keeps them apart, but the mass term involves both, $m\bar{\Psi}\Psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$, so $m > 0$ breaks the chiral symmetry.

In terms of upper and lower quark doublet components, the mass term takes the form

$$-\mathcal{L}_{\text{quark masses}} = (\bar{d}'_L, \bar{s}'_L, \bar{b}'_L) M_d \begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix} + (\bar{u}'_L, \bar{c}'_L, \bar{t}'_L) M_u \begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix} \ . \tag{6}$$

A transformation to the mass base diagonalizes the matrices M_d and M_u, $U^\dagger_{d;L} M_d U_{d;R} = \text{diag}(m_d, m_s, m_b)$, $U^\dagger_{u;L} M_u U_{u;R} = \text{diag}(m_u, m_c, m_t)$. Thus the weak interaction eigenstates and the mass eigenstates are related by unitary transformations,

$$\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_{L,R} = U_{u;L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \ , \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{L,R} = U_{d;L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R} \ , \tag{7}$$

$U_{u;L,R}$, $U_{d;L,R} \in U(3)$. The Standard Model describes the flavor changing due to the weak interaction by charged currents $J^{\pm}_{\mu}$, such as

$$J^{+}_{\mu} = (\bar{u}', \bar{c}', \bar{t}')_L \gamma_\mu \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu \underbrace{U^{\dagger}_{u;L} U_{d;L}}_{V \in U(3)} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \quad . \tag{8}$$

Hence flavor changes are parameterized by a unitary matrix V, known as the *Cabbibo-Kobayashi-Maskawa (CKM) matrix.*

For N_g fermion generations it would be a matrix $V \in U(N_g)$, with N_g^2 real parameters. However, the diagonalization still works if we vary any diagonal phase factor in $U_{u;L}$ and $U_{d;L}$, so if we count the physical parameters, we should subtract these $2N_g$ phases. On the other hand, one common phase in $U_{u;L}$ and $U_{d;L}$ leaves V invariant, so that phase cannot be subtracted. We end up with

$$N_g^2 - (2N_g - 1) = (N_g - 1)^2$$

physical mixing parameters.

This formula obviously works for one generation (nothing to be mixed). For $N_g = 2$ there is only one rotation angle, hence an SO(2) matrix is sufficient [19]; this is the Cabbibo angle, $\theta_c \approx 13^\circ$. For $N_g = 3$ we obtain the 3 rotation angles (*e.g.* the Euler angles) plus one complex phase. Kobayashi and Maskawa noticed that this phase *breaks CP symmetry* [20] (if it doesn't vanish), so the aforementioned CP violation does naturally emerge in the Standard Model with $N_g \geq 3$ generations.

The CKM matrix is well explored now by numerous experiments — its unitarity was a theoretical prediction, which is compatible with the data. This is another argument why more than 3 fermion generations seem unlikely. Actually V is quite close to a unit matrix, with diagonal elements $|V_{ii}| > 0.97$. Hence the off-diagonal elements, which enable the generation changes, are suppressed, but the complex phase is clearly non-zero.

6.2 . . . and how about the leptons?

The way the Standard Model was traditionally formulated, it does not include right-handed neutrinos (as we mentioned before), and all neutrino masses vanish. Still, there are flavor changing lepton currents, in analogy to the quark current (8),

$$j^+_\mu = (\bar{\nu}'_{\rm e}, \bar{\nu}'_\mu, \bar{\nu}'_\tau)_L \gamma_\mu \left(\begin{array}{c} e' \\ \mu' \\ \tau' \end{array} \right)_L = (\bar{\nu}_{\rm e}, \bar{\nu}_\mu, \bar{\nu}_\tau)_L \gamma_\mu U^\dagger_{n;L} U_{e;L} \left(\begin{array}{c} e^- \\ \mu^- \\ \tau^- \end{array} \right)_L . \qquad (9)$$

However, in this case the choice of the matrix $U_{n;L}$ is completely free — if all neutrino masses vanish, there is no condition for the diagonalization of their mass matrix. In particular we are free to choose $U_{n;L} = U_{e;L}$, so the matrix, which would correspond to the CKM matrix, can be set to $\mathbb{1}$. This shows that no physical mixing effects — analogous to the quark sector — can be expected, *in this original form of the Standard Model.*

We can turn this statement the other way round: if a transmutation of ν-flavors is observed, we can conclude that also for neutrinos the flavor and mass eigenstates differ, and therefore they cannot be all massless. We now know that this is Nature's choice, as we are going to review next.

7 Neutrino oscillation: a chameleon-like metamorphosis

In 1957 Pontecorvo formulated a first idea that neutrinos could somehow transform into each other [21]. This early suggestion was an oscillation between neutrino and anti-neutrino, $\nu \leftrightarrow \bar{\nu}$, which would violate the conservation of the lepton number L. In 1962, the year when the neutrino ν_μ was discovered, Ziro Maki, Masami Nakagawa and Shoichi Sakata at Nagoya University (Japan) considered the possibility of massive neutrinos, and suggested that their mass eigenstates could be superpositions of $\nu_{\rm e}$ and ν_μ [22]. In 1968 it was again Pontecorvo who elaborated a full-fledge theory for this scenario [23], and for the resulting $\nu_{\rm e} \leftrightarrow \nu_\mu$ oscillation, which changes the generation-specific lepton numbers $L_{\rm e}$ and L_μ, but not L.

This 2-flavor setting is convenient for illustration: we denote the mass eigenstates as ν_1, ν_2. As we saw in the discussion of the CKM quark mixing matrix, this case only involves one physical mixing parameter, namely the rotation angle of an SO(2) matrix,

$$\left(\begin{array}{c} \nu_{\rm e} \\ \nu_\mu \end{array} \right) = \left(\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right) \left(\begin{array}{c} \nu_1 \\ \nu_2 \end{array} \right) .$$

Let us assume a plane wave dynamics for the mass eigenstates, which we write as *kets* (in Dirac's notation),

$$|\nu_i(t)\rangle = \exp(-\mathrm{i}(E_i\, t - \vec{p}_i \cdot \vec{r}))\ |\nu_i(0)\rangle \ , \quad (i = 1, 2) \ .$$

The distance that the neutrino has travelled — after its start at time $t = 0$ — is (in natural units) $L \simeq t$; the mass is so small that it is ultra-relativistic even at modest energy. This also implies $m_i \ll |\vec{p}_i| = p_i \approx E_i$, and we obtain

$$E_i - p_i = \sqrt{p_i^2 + m_i^2} - p_i \approx m_i^2/(2p_i) \approx m_i^2/(2E_i) \ ,$$

which simplifies the propagation to

$$|\nu_i(t)\rangle = \exp(-\mathrm{i} m_i^2 L/(2E_i))\ |\nu_i(0)\rangle \ .$$

In the framework of this approximation, an initial state $|\nu_{\mathrm{e}}\rangle$ is converted into $|\nu_\mu\rangle$ (or vice versa), after flight distance L, with probability (for a derivation, see *e.g.* Ref. [24])

$$P_{\mathrm{e}\leftrightarrow\mu} = |\langle\nu_\mu|\nu_{\mathrm{e}}\rangle|^2 = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right) \ , \quad \Delta m_{12}^2 = m_2^2 - m_1^2 \ . \qquad (10)$$

Intuitively, the initial state $|\nu_{\mathrm{e}}\rangle$ consists of a peculiar superposition of $|\nu_1\rangle$ and $|\nu_2\rangle$, but these components propagate with different speed. Therefore the composition changes to new states, which mix $|\nu_{\mathrm{e}}\rangle$ and $|\nu_\mu\rangle$.

It is straightforward to extend this approach to the case of 3 flavors and 3 mass eigenstates $|\nu_i\rangle$,

$$\begin{pmatrix} \nu_{\mathrm{e}} \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\mathrm{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \ , \quad U_{\mathrm{PMNS}} \in U(3) \ , \qquad (11)$$

where U_{PMNS} is the *Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.* As we saw in the case of the CKM matrix, there are now 3 mixing angles plus one complex phase, which implies additional CP symmetry breaking, now in the lepton sector.

In this case, the oscillation probability is $\propto \sin^2(\Delta m_{ij}^2 L/(4E))$, so we can determine $|\Delta m_{12}^2|$, $|\Delta m_{23}^2|$ and $|\Delta m_{13}^2|$ (they are not independent, hence one can focus on two of them).

Experiments are built with a given average neutrino energy E and a fixed baseline L. If two $|\Delta m_{ij}^2|$ are sufficiently different, an appropriate ratio L/E selects to which one the experiment is most sensitive. Initially this separability was uncertain, but fortunately for the experimentalists it turned out that $|\Delta m_{12}^2| \approx 30\ |\Delta m_{23}^2|$. The former (latter) was crucial for the observation of solar (atmospheric) neutrinos, see below.

So this can be tested experimentally, but in practice it is a delicate task: many attempts to probe this behavior ended up with results that were not fully conclusive. This changed at the dawn of the new millennium, with the experiments that were awarded the 2015 Nobel Prize.

7.1 Atmospheric neutrinos viewed by Super-Kamiokande

In 1996 the experiment *Super-Kamiokande* was launched, as an extension of the previous Kamiokande. It is located in the Mozumi zinc mine, near the town Kamioka (now part of Hida) in central Japan, about 1000 m underground. Such locations deep underground are standard for neutrino experiments (and also for Dark Matter search), because of the shielding from the background radiation, which is a major challenge for the experimentalists.

Super-Kamiokande used 50 000 t of water as a Cherenkov detector. It focused on *atmospheric neutrinos,* which we briefly mentioned in Section 3: high energy cosmic rays hit our atmosphere and generate a shower of secondary particles, in particular light mesons (pions and kaons), which subsequently decay into leptons, including neutrinos. Examples are the charged pion decays,

$$\pi^+ \to \mu^+ + \nu_\mu,\ \mu^+ \to \mathrm{e}^+ + \nu_\mathrm{e} + \bar{\nu}_\mu \quad \text{or} \quad \pi^- \to \mu^- + \bar{\nu}_\mu,\ \mu^- \to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu\ ,$$

i.e. successions of the decays (3) and (4). The flux of cosmic rays is well-known, so also the resulting neutrino flux could be predicted: the ratio between the number of μ-(anti-)neutrinos and e-(anti-)neutrinos should be about 2:1, as in our example. Cosmic rays arrive isotropically, and — as we mentioned in Section 3 — crossing the Earth reduces the neutrino flux only by a negligible fraction of $O(10^{-18})$. Does this mean that the neutrino flux observed in the Mozumi mine is isotropic as well?

Super-Kamiokande monitored neutrino reactions, which involve charged currents and emit $\mathrm{e}^\pm$ or $\mu^\pm$; examples are given in scheme (5). This causes water Cherenkov radiation, which indicates the neutrino direction and energy; the high energies — up to several GeV — distinguish them from the background neutrinos. The profile of the Cherenkov cone further reveals whether it was triggered by an $e^{\pm}$ or by a $\mu^{\pm}$, and therefore whether its origin was an atmospheric e- or μ-neutrino (though ν and $\bar{\nu}$ could not be distinguished).

For the ν_e and $\bar{\nu}_e$ flux, the prediction was well confirmed, and its isotropy too. This was *not* the case for the ν_μ and $\bar{\nu}_\mu$ flux: here part of the expected neutrinos were missing, and the flux from above was significantly larger than the one from below (after passing through the Earth). This was announced in 1998, after two years of operation, based on 5000 neutrino signals [25].

In light of this section, the explanation is clear: part of the missing μ-neutrinos were transformed into τ-neutrinos! This oscillation takes a while, this is why it happens mostly along the extended path across the Earth. The precise angular distribution reveals the oscillation rate as a function of the travelling distance L, divided by the ν_μ energy E. This determines the difference $|\Delta m_{23}^2| = |m_3^2 - m_2^2| \approx 2.4 \cdot 10^{-3}$ eV^2. That has been confirmed later by experiments with accelerator neutrinos, which attain $O(1)$ GeV.

7.2 The solar neutrino puzzle and its solution by SNO

Almost all our activities are driven by solar energy. For $4.5 \cdot 10^9$ years the Sun has been shining with a luminosity of $3.8 \cdot 10^{26}$ W, and it is expected to continue doing so for another $4.5 \cdot 10^9$ years. Until the 19th century the origin of all this energy seemed mysterious: a chemical process was assumed, but estimates showed that the Sun could only burn for 6000 years, even under the "most optimistic assumption" that it consisted of coal.

In the 20th century *nuclear fusion* was identified as the energy source of the Sun, in particular the "pp chain reaction", which amounts to

$$\mathrm{p} + \mathrm{p} + \mathrm{p} + \mathrm{p} \to \ \dots \ \to \ {}^4\mathrm{He} + 2\mathrm{e}^+ + 2\nu_\mathrm{e} \ .$$

If we divide the solar luminosity by the energy, which is released by this chain reaction (26.7 MeV), we obtain the fusion rate, as well as an estimate for the ν_e production ($\approx 2 \cdot 10^{38}$ s^{-1}). In addition there are sub-dominant processes, which emit electron neutrinos of higher energies.

The entire spectrum ranges from about $E_{\nu_e} \approx (0.1 \dots 10)$ MeV. Since the 1960s the ν_e-flux arriving at the Earth was quite well predicted [26], and also

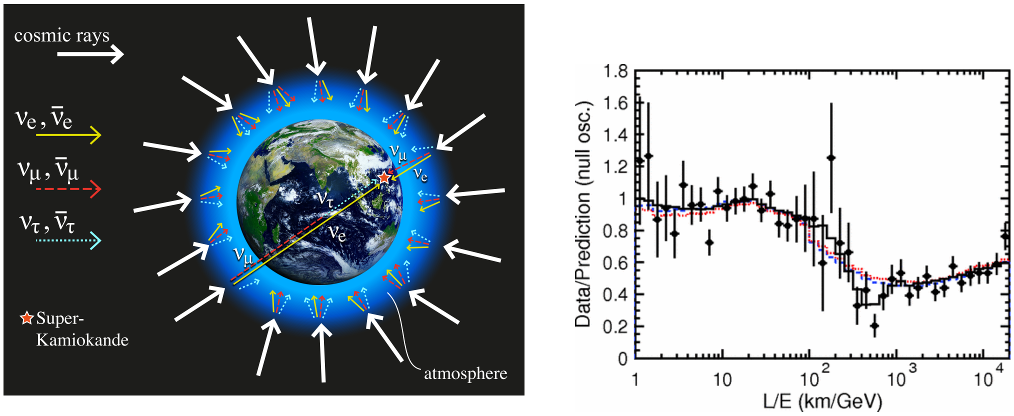


Figure 6: *On the left: Illustration of the Super-Kamiokande experiment on atmospheric neutrinos. Cosmic rays generate air showers of secondary particles, including neutrinos. The e-neutrino flux arrives as predicted, but for a long path part of the* μ*-neutrinos are converted into* τ*-neutrinos. On the right: The atmospheric* ν_μ *plus* $\bar{\nu}_\mu$ *flux, as observed by Super-Kamiokande, as a function of the travelling distance* L *divided by the neutrino energy* E*. The vertical axis is the ratio between measured flux and the prediction without neutrino oscillation.*

measured — first in the Homestake gold mine in South Dakota [27] — but the data confirmed only about 1/3 of the expected flux. This *solar neutrino puzzle* (see *e.g.* Ref. [28]) persisted for more than 30 years.

Various solutions were discussed, such as corrections to the solar model, but the latter was constantly improved, in particular by John Bahcall and collaborators, which led to the Standard Solar Model [29]. This model was refined to a point that made it truly difficult to still raise objections which could reduce the ν_e-flux that much. Another explanation, which had been discussed for decades, was finally confirmed in 2001: the solution to this puzzle is *neutrino oscillation* — this scenario had been suggested first by V.N. Gribov and B. Pontecorvo in 1969 [30].

The breakthrough was due to the *Sudbury Neutrino Observatory (SNO)* in Ontario, Canada, 2000 m underground [31]. In its crucial experiment, 9500 photomultipliers monitored a sphere with 6 m radius, which contained 1000 t of *heavy water,* D_2O (compared to ordinary water, a neutron is added to each proton, thus forming deuterium, $D \sim (np)$). This offered several options for the detection of neutrino events:

- The variant of the β-decay shown in Figure 3, with an incoming $\nu_{\rm e}$ and an outgoing electron; this measures exclusively the $\nu_{\rm e}$ flux.
- A deuterium dissolution, $\mathrm{D} + \nu_x \to \mathrm{n} + \mathrm{p} + \nu_x \ , \ x \in \{\mathrm{e}, \mu, \tau\}$. That process measures the total neutrino flux without flavor distinction, *i.e.* the sum of $\nu_{\rm e}$, ν_μ and ν_τ neutrinos.
- Elastic $\nu_x \, \mathrm{e}^-$ scattering enables a good identification of the direction, which affirmed that the observed neutrino flux indeed originates from the Sun. (Only for $\nu_{\rm e}$ the scattered particles can also be exchanged.)

The total flux is well compatible with the prediction by the Standard Solar Model. On the other hand, this model predicts solely $\nu_{\rm e}$-production, but the first process accounts for only $\approx 1/3$ of the expected $\nu_{\rm e}$-flux, in agreement with earlier experiments. Taken together, these results imply that 2/3 of the solar $\nu_{\rm e}$ have been transformed into other flavors before they reach us.

If neutrinos can oscillate, we can expect all flavors to be equally frequent after a long path, like the $1.5 \cdot 10^{11}$ m that separate us from the Sun, which yields a $\nu_{\rm e}$ survival probability of 1/3. Moreover, neutrino oscillation takes place already inside the Sun, before the neutrinos even leave it, enhanced by the medium [32].

This is the ultimate demonstration that neutrino oscillation *is* the solution to the long-standing solar neutrino puzzle, as Gribov and Pontecorvo had conjectured.

8 Status today: PMNS matrix and open questions

Meanwhile a host of experiments confirmed these observations on atmospheric and solar neutrinos: some detected reactor neutrinos at distances of $O(100)$ km, confirming the atmospheric $\nu_\mu \leftrightarrow \nu_\tau$ oscillation, while accelerator neutrinos are consistent with the solar $\nu_{\rm e} \leftrightarrow \nu_\mu$, ν_τ transmutation. By global fits, the absolute values of the PMNS matrix elements in eq. (11) are quite well determined,

$$\begin{pmatrix} |U_{\rm e1}| & |U_{\rm e2}| & |U_{\rm e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} = \begin{pmatrix} 0.82(2) & 0.55(3) & 0.15(1) \\ 0.37(15) & 0.57(13) & 0.70(9) \\ 0.39(14) & 0.59(12) & 0.68(9) \end{pmatrix} .$$

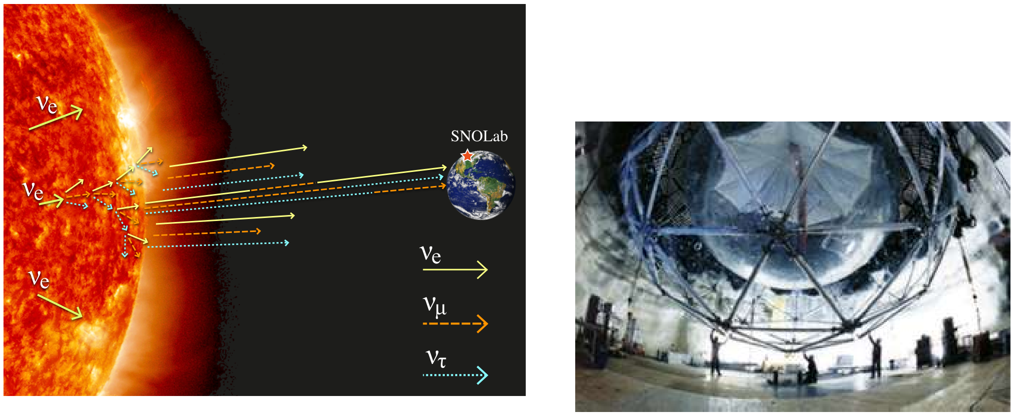


Figure 7: *The Standard Solar Model predicts the generation of numerous electron neutrinos* $\nu_{\rm e}$ *inside the Sun* ($\approx 2 \cdot 10^{38}\ {\rm s}^{-1}$)*, such that a flux of* $\approx 6 \cdot 10^{10}\ \nu_{\rm e}/(cm^2{\cdot}s)$ *was expected at the Earth. Only 1/3 of them arrive as* $\nu_{\rm e}$*, the rest is transmuted into* ν_μ *or* ν_τ *by means of neutrino oscillation, as illustrated on the left. This was demonstrated by the SNO Laboratory, which used a spherical detector filled with 1000 t of heavy water, shown on the right.*

The reduction of the uncertainties is in progress.

The dark horse is the *complex phase:* it depends on the parameterization how it occurs in this matrix, but the physically interesting aspect of a leptonic CP violation is still highly uncertain.

This could be relevant for the famous puzzle about the *matter–anti-matter imbalance* in the Universe: the Big Bang should have generated the same amount of both, so how comes that today there is an enormous dominance of matter? One of the three conditions for a possible explanation (formulated by Andrei Sakharov in 1967 [33]) is CP violation. We have mentioned that this was indeed observed in weak decays, and that the complex phase in the CKM matrix breaks CP invariance, but this violation is not sufficient to account for the striking matter–anti-matter imbalance. In this regard, an additional CP violation in the lepton sector could be helpful.

Regarding the neutrino masses, it seems natural to assume that the flavors follow the same hierarchy as the charged leptons, $m_1 < m_2 < m_3$. However, since the neutrino oscillation between any two flavors in vacuum only determines $|\Delta m^2|$, cf. eq. (10), an "inverse hierarchy" with $m_3 < m_1 < m_2$ cannot be ruled out either (so far only $m_1 < m_2$ is considered safe, based on processes inside the Sun).

In any case, we see that this mixing matrix is much more animated than its counterpart in the quark sector; neutrinos mix strongly! The element with the least absolute value is U_{e3}; for quite a while it seemed to be compatible with 0, and people invented theories to explain its possible vanishing — until 2012, when the Chinese reactor experiment Daya Bay, as well as RENO in South Korea and Double Chooz in France, showed that it differs from 0, with more than 5σ significance (here the baseline was just $O(1)$ km) [34].

Generally, the attempts to search for a systematic "texture" in the PMNS matrix were not that fruitful — it seems that we just have to accept the values of its physical parameters as experimental input.

Moreover, this still leaves the question open how large the neutrino masses really are — the PMNS matrix only contains information about their mass squared *differences.* The masses themselves are even more difficult to determine, and alternative techniques are required: one approach is the study of the β-decay to an extreme precision — in particular the electron spectrum near the endpoint is slightly sensitive to the neutrino mass. Such a study is ongoing in the KArlsruhe TRItium Neutrino (KATRIN) experiment in Germany [35], which has the potential to improve the current bound of $m_{\nu_{\bar{e}}} < 2.3$ eV (by the experiments Mainz in Mainz [36] and Troitsk in Russia [37]) by an order of magnitude.

There are also cosmological estimates and bounds for the neutrino masses (an overview is given in Ref. [38]), though they necessarily involve some model dependence. In any case, the absolute values will be relevant for cosmology. Even if the neutrino masses are tiny, their sum — all over the Universe one estimates $O(10^{89})$ neutrinos — could well be powerful: for instance, the exact masses could, along with the amount of Dark Matter, be crucial for our long-term future, regarding the question if the Universe will keep on expanding for ever, or if it will end in a Big Crunch — let's see …

Acknowledgements This work was supported by UNAM-PAPIIT through grants IN107915, IN112213 and IB100413.

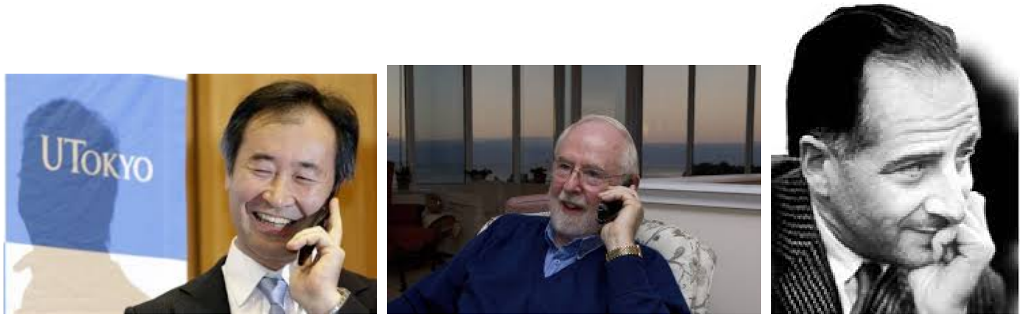


Figure 8: *Left and center: Takaaki Kajita and Arthur McDonald, Nobel Prize laureates 2015. On the right: Bruno Pontecorvo.*

- *Kajita (born 1959) studied at Saitama University and completed his Ph.D. 1986 at Tokyo University, where he later worked in the Institute for Cosmic Radiation Research. He led the group at Super Kamiokande, which found evidence for the oscillation of atmospheric neutrinos. In 1999 he became director of the Research Center for Cosmic Neutrinos in Tokyo.*
- *McDonald (born 1943) studied at Dalhousie University (Halifax, Canada) and did his Ph.D. at the California Institute of Technology. He worked from 1970 to 1982 at the Chalk River Laboratories near Ottawa, from 1982 to 1989 at Princeton University, then he became director of the Sudbury Neutrino Observatory (SNO), which solved the solar neutrino puzzle.*
- *If he were still alive, then Pontecorvo (1913-1993) should be another 2015 Nobel Prize winner, as the leading theorist involved. He worked in Rome with Enrico Fermi, and later in Paris, Montreal and Liverpool. In 1950 he moved to the Joint Institute for Nuclear Research (JINR) in Dubna (near Moscow), where he elaborated the theory of neutrino oscillation. On this basis, he and Vladimir Gribov predicted in 1969 the correct solution to the solar neutrino puzzle.*

A Neutrino masses are still puzzling

In the traditional form of the Standard Model, the first fermion generation contains the following leptons and quarks,

$$\begin{pmatrix} \nu_{\mathrm{e};L} \\ \mathrm{e}_L \end{pmatrix},\ \mathrm{e}_R\ ,\quad \begin{pmatrix} u_L \\ d_L \end{pmatrix},\ u_R,\ d_R\ ,$$

where we now keep track of left- and right-handed fermions separately. For instance the term for the electron mass m_{e} takes the form $m_e(\bar{\mathrm{e}}_R \mathrm{e}_L + \bar{\mathrm{e}}_L \mathrm{e}_R)$. However, this explicit mass term must not appear in the Lagrangian: e_L and e_R couple differently to the electroweak gauge fields, so this term would break gauge invariance.

Instead the Higgs field

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \in \mathbb{C}^2$$

comes to the rescue and endows the gauge invariant Yukawa term

$$-\mathcal{L}_{\mathrm{Yukawa}} = f_e \left[\bar{\mathrm{e}}_R\ \Phi^\dagger \cdot \begin{pmatrix} \nu_{\mathrm{e};L} \\ \mathrm{e}_L \end{pmatrix} + (\bar{\nu}_{\mathrm{e};L}, \bar{\mathrm{e}}_L) \cdot \Phi\ \mathrm{e}_R \right] ,$$

where f_e is a (dimensionless) Yukawa coupling. The Higgs potential arranges for spontaneous symmetry breaking. If the Higgs field takes the classical ground state configuration

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix},\ v \simeq 246\ \mathrm{GeV}\ \Rightarrow\ \mathcal{L}_{\mathrm{Yukawa}} = -f_e v\, [\bar{\mathrm{e}}_R \mathrm{e}_L + \bar{\mathrm{e}}_L \mathrm{e}_R],\ m_e = f_e v\ ,$$

while the neutrino remains massless.

The analogous term for the quark doublet (with a Yukawa coupling f_d) leads to the d-quark mass $m_d = f_d v$. But how do we give mass to the u-quark? One could introduce an additional Higgs field, but the Standard Model is economic and recycles Φ: another quark Yukawa term is added, with $\tilde{\Phi} = \begin{pmatrix} -\phi_0^* \\ \phi_+^* \end{pmatrix}$ instead of Φ, and we obtain $m_u = -f_u v$ ($f_u < 0$ is allowed).

If we want to construct a *neutrino mass,* we can do exactly the same, *if* we add a right-handed neutrino, $\nu_{\mathrm{e};R}$. It turns out that $\nu_{\mathrm{e};R}$ is “sterile”; it does not have any charge, so it does not couple to any gauge field. It could have hidden from our detectors, and it is a Dark Matter candidate.

One often hears the statement that the neutrino mass is "beyond the Standard Model". While this is ultimately a matter of semantics, we would like to emphasize that neutrino masses can be constructed in the same way as it is done for the u, c, and t-quark, so this does not necessarily require a conceptual extension of the Standard Model.

Alternative approaches do speculate about conceptual novelties, like a dimension 5 mass term,[12] or even scenarios in higher space-time dimensions, but we are not going to discuss them.

We just add that the presence of ν_R opens the door to new scenarios (we do not specify the generation anymore). In general, the C transformation (charge conjugation) of a fermion field Ψ reads

$$\mathrm{C} \ : \ \Psi \to \Psi^C = C \bar{\Psi}^T \ ,$$

where T means "transposed", and C is a matrix that fulfills suitable conditions. Therefore the *Majorana spinors*

$$\nu_1^{\mathrm{M}} = \nu_R + C \bar{\nu}_R^T \doteq \nu_R + \nu_L^C \ , \quad \nu_2^{\mathrm{M}} = \nu_L + C \bar{\nu}_L^T \doteq \nu_L + \nu_R^C$$

are C-invariant; each of them represents a *Majorana neutrino,* which is its own anti-particle. In one generation we obtain one Majorana neutrino with the chirality components ν_R and $\bar{\nu}_L$, and another one with ν_L and $\bar{\nu}_R$.

This construction yields real, *i.e.* neutral spinor fields. In Dirac's and Weyl's original approaches, the γ-matrices are chosen such that the Dirac operator $(\mathrm{i}\gamma^\mu \partial_\mu - m)$ contains complex elements, which was considered as an argument that fermions should have some charge, and the corresponding operators generate distinct particles and anti-particles.

However, in the 1930s Ettore Majorana found a way to fulfill the conditions of the Dirac algebra $(\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu})$ with purely imaginary γ-matrices, such that the Dirac operator becomes entirely real, which disproved this argument, and showed that neutral fermions are another option.[13]

---

[12] A term of this kind is $\propto \Big[(\bar{\nu}_L, \bar{\mathrm{e}}_L) \cdot \tilde{\Phi} \Big] \Big[\tilde{\Phi}^\dagger \cdot \begin{pmatrix} \nu_L \\ \mathrm{e}_L \end{pmatrix} \Big]$, which is not renomalizable, but it does not require any sterile neutrino.

[13] Majorana did not publish the work with this insight himself, but he told Fermi about it, and allowed him to do so in his name. This paper appeared in 1937 [39], one year before Majorana mysteriously disappeared. For a semi-popular review, see Ref. [40].

In fact, it is conceivable that the neutrinos are Majorana particles, and not "Dirac neutrinos" as we assumed in the main part of this article. Then the counting of the physical parameters in the mixing matrix has to be reconsidered: roughly speaking, we argued before that the $U(3)$ matrix in eq. (9) has 9 parameters, but — with massive neutrinos — each fermion field in the current j^+_μ can absorb one phase (but one common phase cancels), so we are left with $9-(6-1)=4$ physical parameters. If we insert Majorana neutrinos instead, these three fields cannot absorb any phase, and there is no common phase either. So in that case there are $9-3=6$ physical parameters, which include *3 complex phases.*

For Majorana fermions, an explicit mass term

$$\mathcal{L}_{\text{Majorana mass}} = -\frac{\mathcal{M}}{2}\bar{\nu}^{\text{M}}\nu^{\text{M}}$$

can be incorporated directly in the Lagrangian. Then the theory contains another dimensional parameter, the Majorana mass $\mathcal{M}$ (not related to the Higgs mechanism), in addition to v, without breaking gauge symmetry. It does, however, break the conservation of the total lepton number $L = L_{\text{e}} + L_\mu + L_\tau$.[14] After the observation that neutrino oscillation violates the separate L_{e}, L_μ and L_τ conservation, could it be that not even L is on safe ground?

Back in 1939, Wendell Furry pointed that a *neutrinoless double β-decay* $2n \to 2p + 2\text{e}^-$ would confirm this scenario [41]; it changes $L \to L+2$. This is a way how experiment could confirm that neutrinos are of Majorana type, and — by means of the decay rate — also explore their masses [42]. The ordinary double β-decay (with $2\bar{\nu}_{\text{e}}$ emission) has been observed since 1987 [43], but the hunt for its *neutrinoless* counterpart is still going on: some events were reported, but the community is not convinced.[15] The consensus so far is a lower bound of $\approx 2\cdot 10^{25}$ years for the corresponding life time.

Last but not least, Majorana neutrinos enable the *seesaw mechanism,* which is popular as a possible explanation why neutrinos are so light (a "hierarchy problem"). It was suggested by Peter Minkowski in 1977 [44] and others; here we illustrate its simplest form ("type 1") in one generation.

---

[14] It also changes the difference between baryon and lepton number, $B-L$. This is the quantity, which is strictly conserved in the Standard Model. There combined B and L anomalies are conceivable, but not observed.

[15] A drama began in 2001, when part of the Heidelberg-Moscow Collaboration claimed evidence for the decay ${}^{76}_{32}\text{Ge} \to {}^{76}_{34}\text{Se} + 2\text{e}^-$, but it was refuted by other experts, including members of the same collaboration.

We endow the Majorana spinor fields $\nu_1^{\rm M}$, $\nu_2^{\rm M}$ with a "Dirac mass" im (a coupling between components of distinct Majorana fields with different chirality; for later convenience we choose it imaginary), and a "Majorana mass" M (it would be the Majorana mass of $\nu_1^{\rm M}$, in the absence of $\nu_2^{\rm M}$),

$$-\mathcal{L}_{\rm neutrino\ masses} = \frac{1}{2}(\bar{\nu}_L, \bar{\nu}_L^C) \left(\begin{array}{cc} 0 & {\rm i}m \\ {\rm i}m & M \end{array} \right) \left(\begin{array}{c} \nu_R^C \\ \nu_R \end{array} \right) + \mbox{Hermitian conjugate} \ .$$

Really physical are the Majorana masses for the eigenstates, *i.e.* the eigenvalues of this matrix. In particular, for $M \gg m$ we obtain

$$\mathcal{M}_{\rm large} \simeq M \ , \quad \mathcal{M}_{\rm small} \simeq \frac{m^2}{M} \ll \mathcal{M}_{\rm large} \ .$$

The more we amplify $\mathcal{M}_{\rm large}$ (by increasing M), the more we suppress $\mathcal{M}_{\rm small}$. This setting of injustice inspired the term "seesaw mechanism".

If we choose m somewhat above the vacuum expectation value of the Higgs field, $v \lesssim m = O(1)$ TeV, and insert a huge $M \approx 10^{24} \dots 10^{25}$ eV, we obtain a very light neutrino, with a realistic mass $\mathcal{M}_{\rm small} \approx 0.1 \dots 1$ eV. In this scenario, $\mathcal{M}_{\rm large}$ has the magnitude of the energy, where a Grand Unification of the electroweak and strong interactions is expected ("GUT scale", somewhat below the Planck scale $\approx 10^{28}$ eV), which many theorists find appealing.

Hence in some sense history is repetitive: as in 1930, there are strong theoretical reasons for postulating a hypothetical particle, now it is the sterile neutrino ν_R. It is even more elusive than the known, weakly interacting neutrinos, but it could possibly fix several short-coming in the traditional form of the Standard Model, while preserving its elegant and successful concepts: it provides a sensible Dark Matter candidate, and neutrino masses appear in an natural way. We can even explain why they are so light, if we assume the seesaw mechanism. Then primordial heavy Majorana neutrinos should have decayed in the very early Universe, generating slightly more anti-leptons than leptons ("leptogenesis"). A cascade of further decays would generate an extreme excess of baryons over anti-baryons ("baryogenesis"), and thus the dominance of matter that we still experience today.[16]

---

[16] The possible impact of sterile neutrinos in astrophysics and cosmology is reviewed in Ref. [45].

So postulating ν_R (in 3 generations) seems to be a good deal, but its experimental search is a tremendous issue: *e.g.* the Chandra X-Ray Observatory searches for faint pulses from their possible decay into lighter neutrinos, while the Wilkinson Microwave Anisotropy Probe (WMAP) measures tiny fluctuations in the Cosmic Microwave Background, which could indicate the likelihood of 4 neutrino types. Clear evidence is still missing, although some hints of its existence may be hiding in a few anomalous experimental results [46]. So the ghost hunt for the sterile neutrino ν_R is going on, and neutrino physics will continue to be exciting in the future.

References

[1] W. Pauli, Offener Brief an die Gruppe de Radioaktiven bei der Gauvereins-Tagung zu Tübingen (1930), unpublished; *reproduced in* L.M. Brown, The idea of the neutrino, Physics Today 31 (1978) 23-28.

[2] J. Chadwick, Intensitätsverteilung im magnetischen Spektrum von β-Strahlen von Radium B+C, Verh. Deutsch. Phys. Ges. 16 (1914) 383-391.

[3] C.D. Ellis and W.A. Wooster, The Continuous Spectrum of β-Rays, Nature 119 (1927) 563-564.

[4] J. Chadwick, The Existence of a Neutron, Proc. Roy. Soc. A 136 (1932) 692-708.

[5] E. Fermi, Versuch einer Theorie der β-Strahlen, Z. Phys. 88 (1934) 161-177.

[6] F. Reines and C.L. Cowan Jr., The Neutrino, Nature 178 (1956) 446-449.
C.L. Cowan Jr., F. Reines, F.B. Harrison, H.W. Kruse and A.D. McGuire, Detection of the Free Neutrino: a Confirmation, Science 124 (1956) 103-4.

[7] B. Follin, L. Knox, M. Millea and Z. Pan, First Detection of the Acoustic Oscillation Phase Shift Expected from the Cosmic Neutrino Background, Phys. Rev. Lett. 115 (2015) 091301.

[8] T.D. Lee and C.N. Yang, Question of Parity Conservation in Weak Interactions, Phys. Rev. 104 (1956) 254-258. [Erratum: Phys. Rev. 106 (1957) 1371].

[9] C.S. Wu, E. Ambler, R.W. Hayward, D.D. Hoppes and R.P. Hudson, Experimental Test of Parity Conservation in Beta Decay, Phys. Rev. 105 (1957) 1413-1415.

[10] L. Landau, On the conservation laws for weak interactions, Nucl. Phys. 3 (1957) 127-131.

[11] R. Jost, Eine Bemerkung zum CPT Theorem, Helv. Phys. Acta 30 (1957) 409-416.

[12] E.J. Konopinski and H. Mahmoud, The universal Fermi interaction, Phys. Rev. 92 (1953) 1045-1049.

[13] B. Pontecorvo, Electron and Muon Neutrinos, Sov. Phys. JETP 10 (1960) 1236-1240.

[14] G. Danby, J.-M. Gaillard, K. Goulianos, L.M. Lederman, N.B. Mistry, M. Schwartz and J. Steinberger, Observation of high-energy neutrino reactions and the existence of two kinds of neutrinos, Phys. Rev. Lett. 9 (1962) 36-44.

[15] M.L. Perl *et al.,* Evidence for Anomalous Lepton Production in $e^+ - e^-$ Annihilation, Phys. Rev. Lett. 35 (1975) 1489-1492.

[16] K. Kodama *et al.* (DONUT Collaboration), Observation of tau neutrino interactions, Phys. Lett. B 504 (2001) 218-224.

[17] S. Adler, Axial-Vector Vertex in Spinor Electrodynamics, Phys. Rev. 177 (1969) 2426-2438.
J.S. Bell and R. Jackiw, A PCAC puzzle: $\pi^0 \to \gamma\gamma$ in the σ-Model, Nuovo Cimento A 60 (1969) 47-61.

[18] D. DeCamp *et al.* (ALEPH Collaboration), Determination of the number of light neutrino species, Phys. Lett. B 231 (1989) 519-529.

[19] N. Cabibbo, Unitary Symmetry and Leptonic Decays, Phys. Rev. Lett. 10 (1963) 531-533.

[20] M. Kobayashi and T. Maskawa, CP-Violation in the Renormalizable Theory of Weak Interaction, Prog. Theor. Phys. 49 (1973) 652657.

[21] B. Pontecorvo, Mesonium and anti-mesonium, Sov. Phys. JETP 6 (1957) 429.
B. Pontecorvo, Inverse beta processes and nonconservation of lepton charge, Sov. Phys. JETP 7 (1958) 172-173.

[22] Z. Maki, M. Nakagawa and S. Sakata, Remarks on the Unified Model of Elementary Particles, Prog. Theor. Phys. 28 (1962) 870-880.

[23] B. Pontecorvo, Neutrino Experiments and the Problem of Conservation of Leptonic Charge, Sov. Phys. JETP 26 (1968) 984-988.

[24] P. Hernández, Neutrino physics, CERN Yellow Report, CERN-2010-001 (2010) 229-278.

[25] Y. Fukuda *et al.*, Measurements of the Solar Neutrino Flux from Super-Kamiokande's First 300 Days, Phys. Rev. Lett. 81 (1998) 1158. [Erratum: Phys. Rev. Lett. 81 (1998) 4279].

[26] M. Schwarzschild, R. Howard and R. Härm, Inhomogeneous Stellar Models. V. a. Solar Model with Convective Envelope and Inhomogeneous Interior, Astrophys. J. 125 (1957) 233-241.

[27] R. Davis Jr., D.S. Harmer and K.C. Hoffman, Search for Neutrinos from the Sun, Phys. Rev. Lett. 20 (1968) 1205.

[28] J.N. Bahcall and R. Davis Jr., Solar Neutrinos: A Scientic Puzzle, Science 191 (1976) 264-267.

[29] J.N. Bahcall, M.H. Pinsonneault and S. Basu, Solar Models: current epoch and time dependences, neutrinos, and helioseismological properties, Astrophys. J. 555 (2001) 990-1012.

[30] V.N. Gribov and B. Pontecorvo, Neutrino astronomy and lepton charge, Phys. Lett. B 28 (1969) 493-496.

[31] Q.R. Ahmad *et al.* (SNO Collaboration), Measurement of the Rate of $\nu_{\mathrm{e}} + d \rightarrow p + p + e^{-}$ Interactions Produced by ^{8}B Solar Neutrinos at the Sudbury Neutrino Observatory, Phys. Rev. Lett. 87 (2001) 071301.

Q.R. Ahmad *et al.* (SNO Collaboration), Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory, Phys. Rev. Lett. 89 (2002) 011301.

[32] L. Wolfenstein, Neutrino Oscillations in Matter, Phys. Rev. D17 (1978) 2369-2374.
S.P. Mikheev and A.Y. Smirnov, Resonance enhancement of oscillations in matter and solar neutrino spectroscopy, Sov. J. Nucl. Phys. 42 (1985) 913-917.

[33] A.D. Sakharov, Violation of CP Invariance, C Asymmetry, and Baryon Asymmetry of the Universe, JETP Lett. 5 (1967) 24-27.

[34] F.P. An *et al.* (Daya Bay Collaboration), Observation of electron-antineutrino disappearance at Daya Bay, Phys. Rev. Lett. 108 (2012) 171803.
J.K. Ahn *et al.* (RENO Collaboration), Observation of Reactor Electron Antineutrinos Disappearance in the RENO Experiment, Phys. Rev. Lett. 108 (2012) 191802.
Y. Abe *et al.* (Double Chooz Collaboration), Reactor $\bar{\nu}_{\rm e}$ disappearance in the Double Chooz experiment, Phys. Rev. D86 (2012) 052008.

[35] http://www.katrin.kit.edu/

[36] Ch. Kraus *et al.*, Final results from phase II of the Mainz neutrino mass search in tritium β decay, Eur. Phys. J. C 40 (2005) 447-468.

[37] V.N. Aseev *et al.*, An upper limit on electron antineutrino mass from Troitsk experiment, Phys. Rev. D 84 (2011) 112003.

[38] J.W.F. Valle and J. Romao, Neutrinos in High Energy and Astroparticle Physics, Wiley-VCH, 2015.

[39] E. Majorana, Teoria simmetrica dell'elettrone e del positrone, Nuovo Cimento 5 (1937) 171-184.

[40] F. Wilczek, Majorana returns, Nat. Phys. 5 (2009) 614-618.

[41] W.H. Furry, Note on the Theory of the Neutral Particle, Phys. Rev. 54 (1939) 56-67.

[42] W. Rodejohann, Neutrino-less Double Beta Decay and Particle Physics, Int. J. Mod. Phys. E 20 (2011) 1833-1930, and references therein.

[43] S. Elliott, A. Hahn and M.K. Moe, Direct evidence for two-neutrino double-beta decay in ^{82}Se, Phys. Rev. Lett. 59 (1987) 2220-2223.

[44] P. Minkowski, $\mu \to$ e γ at a rate of one out of 10^9 muon decays?, Phys. Lett. B 67 (1977) 421-428.

[45] A. Boyarsky, O. Ruchayskiy and M. Shaposhnikov, The Role of Sterile Neutrinos in Cosmology and Astrophysics, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191-214.

[46] K.N. Abazajian *et al.*, Light Sterile Neutrinos: A White Paper, arXiv:1204.5379 [hep-ph].

View publication stats