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Heterotic M(atrix) Strings and Their Interactions [1]

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Abstract

Following recent proposal of Dijkgraaf, Verlinde and Verlinde, we show that the M(atrix) theory compactified on $\mathbf{S}_1/\mathbf{Z}_2$ provides with a non-perturbative description of second-quantized light-cone heterotic string. This so-called heterotic M(atrix) string theory is defined by two-dimensional (8,0) supersymmetric chiral gauge theory with gauge group SO(2N) in the large N limit. We argue that at strong coupling fixed point the chiral gauge theory flows to a (8,0) superconformal field theory defined via S_N symmetric product space orbifold. We show that the leading order correction to the strong coupling expansion corresponds to a unique irrelevant operator of scaling dimension three and describes joining and splitting cubic interactions of light-cone heterotic string. We also speculate on M(atrix) description of bosonic strings via dimensional reduction of d=26 Yang-Mills theory.

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1 Introduction

The eleven-dimensional M theory [1] is a unifying theory of all known superstring theories in the strong coupling limit. Apart from the fact that massless excitations are described by the eleven-dimensional supergravity, however, little has been understood as to what the underlying fundamental constituents of the M theory are and what governs microscopic dynamics of the constituents. In an attempt to gain better understanding, one may draw a hint from the history of probing internal structure of hadrons. Partons inside strongly coupled hadrons are very fuzzy due to their characteristic high-frequency motion. If the hadrons are boosted, the motion is slowed down via Lorentz time dilation, hence, makes it possible to snapshot the parton strucutre. Indeed, morally following this idea, Banks *et.al.* [2] have proposed a partonic definition of the M theory. In the infinite momentum limit, light-front view of a strongly coupled type IIA string is infinitely many zero-branes threaded on the string itself. One thus discovers that M-theory partons consist of zero-branes and infinitely short open strings gluing them together. As such the M theory parton dynamics is accurately described by large N limit of $\mathcal{N} = 16$ supersymmetric $U(N)$ matrix quantum mechanics, the so-called M(atrix) theory. Despite clear identification of fundamental constitutent partons and underlying dynamics governing them, it has been very difficult to extract intrinsically M theoretic physics, for example, nontrivial S-matrix amplitudes among physical asymptotic states.

Very recently Dijkgraaf, Verlinde and Verlinde (DVV) [3] have offered an important new insight to the M(atrix) theory. By compactifying the M(atrix) theory one more dimension on $\mathbf{S}_1$ in addition to the $\mathbf{S}_1$ compactified 'quantum' dimension and utilizing exchange symmetry between the two directions, DVV have convincingly argued that the resulting M(atrix) string theory provides for a nonperturbative description of M theory compactified on $\mathbf{S}_1$. In particular, in the weak coupling, DVV have shown that, based on earlier basic observation by Motl [4] and identification of moduli space by Banks and Seiberg [5], the M(atrix) string theory describes second-quantized light-cone type IIA string, Virasoro projection for individual strings emerges from residual $\mathbf{Z}_N$ discrete gauge symmetry in the large N limit and, most importantly, their joining and splitting interaction vertices. The DVV observation has marked a significant progress since it has promoted the original M(atrix) theory into a firmer and calculable set-up.

On the other hand, it is not obvious that DVV proposal is robust enough and extendible to all other superstrings than type IIA. In M theory, different string theories arise from different choices of the compactification manifold. For instance, M theory compactified on an orbifold $\mathbf{S}_1/\mathbf{Z}_2$ yields heterotic and type I′ superstrings. As they have different symmetries and field content on the worldsheet, in particular, half many spacetime supersymmetries compared to type II strings, it provides a highly nontrivial check point to test whether the DVV proposal to the M(atrix) theory applies successfully to other superstrings as well.

In this paper, with the above motivation, we study the proposal of DVV for heterotic superstring. We show that heterotic M(atrix) theory [6, 7] compactified on a circle defines the heterotic M(atrix) string theory in terms of $(8, 0)$ supersymmetric chiral gauge theory coupled to twisted sector matter multiplets. In the strong coupling limit we find that the theory reproduces the spectra and interactions of light-cone Green-Schwarz heterotic superstring. We also explore the possibility of M(atrix) theory for bosonic strings in terms of dimensionally reduced d=26 Yang-Mills gauge theory. While this paper was being prepared, we have received preprints related to part of our results [8] [9].

2 Heterotic M(atrix) String Theory

In the previous work [7], we have obtained the heterotic M(atrix) theory by compactifying the M(atrix) theory on an orbifold $\mathbf{S}_1/\mathbf{Z}_2$ along the 11-th direction. Following the proposal of DVV, we compactify the heterotic M(atrix) theory further on $\mathbf{S}_1$ along, say, the 9-th direction. From the defining M(atrix) theory [2] point of view, we have compactified the theory on a cylinder $\mathbf{S}_1 \times \mathbf{S}_1/\mathbf{Z}_2$. We now interchange 9-th and 11-th directions. This results in the heterotic M(atrix) string theory, a M(atrix) theory description of second-quantized heterotic strings. Because of $11-9$ direction flip, the heterotic string is boosted longitudinally and p_+ momentum is measured in units of $1/R_{11}$, the same unit that was used before for D0-brane particle number. Likewise, string coupling is now determined by the 9-th direction radius R_9. As such, in the M(atrix) heterotic string theory, the heterotic strings are described naturally in the light-cone Green-Schwarz formulation.

The correspondence between the heterotic string and arrays of D0-branes through $11-9$ direction flip can be understood via a chain of by-now well-established $S-$ and $T-$ dualities. Consider the D-particles arrayed on 11-th direction. T-duality along 9-th direction turns the D-particles into D-strings in type I string theory. S-duality (residual $\mathbf{Z}_2$ duality of the underlying type IIB $SL(2, \mathbf{Z})$ duality) converts the D-string into heterotic string. Inverting the T-duality along 11-th direction, we have arrived at heterotic strings with longitudinal momentum around 11-th direction. The mapping is depicted in the following diagram:

$$\begin{array}{ccccc}
\mathbf{Type\ I'\ D0-brane\ no.} = \mathbf{N} & \leftarrow & (\mathtt{T-duality}) & \rightarrow & \mathbf{Type\ I\ D1-string\ no.} = \mathbf{N} \\
\uparrow & & & & \uparrow \\
(\mathtt{11-9\ flip}) & & & & (\mathtt{S-duality}) \\
\downarrow & & & & \downarrow \\
\mathbf{Heterotic\ momentum\ P_+} = \mathbf{N} & \leftarrow & (\mathtt{T-duality}) & \rightarrow & \mathbf{Heterotic\ winding\ no.} = \mathbf{N}
\end{array} \tag{1}$$

In this section, we elaborate details of the heterotic M(atrix) string theory construction along the lines sketched above. As we will see, the resulting theory is an $(1+1)$-dimensional $(8,0)$ supersymmetric gauge theory defined on $\mathbf{S}_1 \times \tilde{\mathbf{S}}_1$, in which the orbifold has turned into a circle $\tilde{\mathbf{S}}_1$ of radius $1/R_9$.

2.1 Heterotic M(atrix) Theory

We begin by reviewing aspects of the heterotic M(atrix) theory relevant for foregoing discussions. By definition, heterotic M(atrix) theory is the M(atrix) theory compactified on an orbifold $I = \mathbf{S}_1/\mathbf{Z}_2$, say, in 9-th direction [6, 7]. In the previous paper [7], we have studied constraint of the orbifold condition and have shown that N D0-brane parton dynamics is governed by $\mathcal{N} = 8$ supersymmetric SO(2N) matrix quantum mechanics. The orbifold compactification breaks the R-symmetry to Spin(8) $\subset$ Spin(9). As such, we adopt a Majorana spinor convention so that real and symmetric representations of the Spin(9) gamma matrices $\Gamma_I, (I = 1, \cdots, 9)$

are decomposed into

$$\Gamma_i = \begin{pmatrix} 0 & \sigma^i_{a\dot{a}} \\ \sigma^i_{\dot{a}a} & 0 \end{pmatrix} \quad i = 1, \cdots, 8; \qquad \Gamma_9 = \begin{pmatrix} -\delta_{ab} & 0 \\ 0 & +\delta_{\dot{a}\dot{b}} \end{pmatrix} \tag{2}$$

in terms of Spin(8) gamma matrices σ^i's. The Spin(9) spinor[2] $\mathbf{\Theta}$ is decomposed into two inequivalent chiral spinors of Spin(8):

$$\mathbf{\Theta} = \mathbf{8}_{\rm s} \oplus \mathbf{8}_{\rm c} \equiv \mathbf{S}_a \oplus \mathbf{S}_{\dot{a}} \tag{3}$$

where

$$\Gamma_9 \mathbf{S}_a = -\mathbf{S}_a; \qquad \Gamma_9 \mathbf{S}_{\dot{a}} = +\mathbf{S}_{\dot{a}}. \tag{4}$$

The field content of the heterotic M(atrix) theory consists of untwisted and twisted sectors and has been determined as follows [7]. Untwisted sector consists of a non-dynamical gauge supermultiplet $(A_0; 0)$ with purely bosonic degrees of freedom, an adjoint supermultiplet $(A_9, \mathbf{S}_a)$ and a rank-2 symmetric supermultiplet $(\mathbf{X}_i, \mathbf{S}_{\dot{a}})$.

Twisted sector states arise from $\mathbf{S}_1/\mathbf{Z}_2$ orbifold fixed points. Located at each of the two fixed points $X_9 = 0, \pi R_9$ are sixteen units of Ramond-Ramond nine-form charges. To cancel these anomalous charges, we introduce fundamental supermultiplets $(0; \chi^{(1)}_A)$ and $(0; \chi^{(2)}_B)$ $(A, B = 1, \cdots, 16)$[3] representing two sets of sixteen D8-branes. In order to cancel the Ramond-Ramond gauge flux *locally*, it is necessary to lock the D8-brane positions on top of the orbifold fixed points. This is achieved by turning on Wilson line of the spacetime gauge field $B^9_{AB} = \pi R_9 \, \mathrm{diag}(0, \cdots, 0, 1, \cdots, 1) \otimes i\sigma_2$, where $i\sigma_2$ acts on each pairs of D8-brane and its mirror image. This Wilson line configuration breaks the heterotic gauge group $E_8 \times E_8$ or Spin(32)/$\mathbf{Z}_2$ to $SO(16) \times SO(16)$. In what follows, we thus restrict our discussions to $G \equiv SO(16) \times SO(16)$ gauge group configuration.

The complete spectra and their quantum numbers of the heterotic M(atrix) theory are summarized in the following table.

Sector	Multiplet	Bosons	Fermions	Spin(8)	SO(2N)
untwisted	gauge	A_0	$\cdot$	$(\mathbf{1}\ ;\ 0)$	2N(2N–1)/2
	adjoint	A_9	$\mathbf{S}_a$	$(\mathbf{1}\ ;\ \mathbf{8}_{\rm s})$	2N (2N–1)/2
	symmetric	$\mathbf{X}^i$	$\mathbf{S}_{\dot{a}}$	$(\mathbf{8}_{\rm v}\ ;\ \mathbf{8}_{\rm c})$	2N(2N+1)/2
twisted	fundamental	$\cdot$	$\chi^{(1,2)}_M$	$(\ 0\ ;\ \mathbf{1})$	2N

Recalling that the two sets of sixteen twisted sector fermions are locked at $X_9 = 0, \pi R_9$ symmetrically by turning on the Wilson line B^9, we find that the heterotic M(atrix) theory is defined by the Lagrangian:

$$\begin{aligned} L = \mathrm{Tr} \Big(& -\frac{1}{2R_{11}}(D_\tau A_9)^2 + \frac{1}{2R_{11}}(D_\tau \mathbf{X})^2 - \frac{R_{11}}{2}[A_9, \mathbf{X}_i]^2 + \frac{R_{11}}{4}[\mathbf{X}^i, \mathbf{X}^j]^2 \\ & - \ \mathbf{S}_a D_\tau \mathbf{S}_a + R_{11}\, \mathbf{S}_a [A_9, \mathbf{S}_a] + \mathbf{S}_{\dot{a}} D_\tau \mathbf{S}_{\dot{a}} + R_{11}\, \mathbf{S}_{\dot{a}}[A_9, \mathbf{S}_{\dot{a}}] - 2R_{11}\, \mathbf{X}^i \sigma^i_{a\dot{a}} \{\mathbf{S}_a, \mathbf{S}_{\dot{a}}\} \Big) \\ & + \ \chi^{(1)}_A (D_\tau + R_{11}\, A_9) \chi^{(1)}_A + \chi^{(2)}_A (D_\tau + R_{11}\, (A_9 - i\pi R_9 \sigma_2)) \chi^{(2)}_A. \end{aligned} \tag{5}$$

[2]We adopt the conjugate spinor convention $\overline{\mathbf{\Theta}}\Gamma_- = \mathbf{\Theta}^T$.

[3]Note that purely bosonic adjoint and purely fermionic fundamental supermultiplets are compatible with $\mathcal{N} = 8$ supersymmetric matrix quantum mechanics with $Spin(8)$ R-symmetry.

Here, $D_\tau \equiv \partial_\tau - [A_0,\ ]$ defines the covariant derivative. The normalization of the twisted sector fermions to the A_9 gauge multiplet has been fixed so that the mass scale of the open string, represented by the twisted sector fermions, connecting the D0-brane parton and the D8-brane has the same mass scale as the open string among the D0-branes. The common mass scale is the prerequisite to the cancellation of induced one-loop vacuum energy [7] as well as quantum mechanical $\mathbf{Z}_2$ global anomaly [10].

The twisted sector is well-defined only if the fermion numbers are even. This is because the one-dimensional Dirac operator $iD_\tau = id/d\tau + iA_0$ is not an elliptic operator. From the D8-brane point of view, this conditions is automatically guaranteed since there exists always a mirror D8-brane for every D8-brane. From the covering space point of view, this implies that the heterotic M(atrix) theory is defined, in addition to SO(2N) gauge group, with $[\mathbf{Z}_2]^N$ bundle which acts trivially on the untwisted sector but nontrivially on the twisted sector fermions. The action of $[\mathbf{Z}_2]^N$ bundle is then identified with

$$[\mathbf{Z}_2]^N: \qquad \chi^{(1,2)}_{Ap} \rightarrow {\eta_p}^q \chi^{(1,2)}_{Aq}, \qquad {\eta_p}^q = \mathrm{diag.}(\pm, \pm, \cdots, \pm)_{N\times N}. \tag{6}$$

From the above Lagrangian, we find the light-cone Hamiltonian as:

$$\begin{aligned} H_{\rm LC} = R_{11} \Big[\mathrm{Tr} \Big(&-\frac{1}{2}\Pi_9^2 + \frac{1}{2}\mathbf{\Pi}_i^2 + \frac{1}{2}[A_9, \mathbf{X}^i]^2 - \frac{1}{4}[\mathbf{X}^i, \mathbf{X}^j]^2 \\ &- \mathbf{S}_{\dot a}[A_9, \mathbf{S}_{\dot a}] - \mathbf{S}_a[A_9, \mathbf{S}_a] + 2\mathbf{X}^i \sigma^u_{a\dot a}\{\mathbf{S}_a, \mathbf{S}_{\dot a}\}\Big) \\ &- \chi^{(1)}_A A_9 \chi^{(1)}_A - \chi^{(2)}_A (A_9 - i\pi R_9 \sigma_2)\chi^{(2)}_A \Big]. \end{aligned} \tag{7}$$

The eight kinematical generators $\mathbf{Q}^{\dot a}$ and the eight dynamical ones $\mathbf{Q}^a$ are given by:

$$\begin{aligned} \mathbf{Q}^{\dot a} &= \frac{1}{\sqrt{R_{11}}} \mathrm{Tr}\mathbf{S}^{\dot a}, \\ \mathbf{Q}^a &= \sqrt{R_{11}}\,\mathrm{Tr}\Big((\sigma^i_{a\dot a}\mathbf{S}^{\dot a}\mathbf{\Pi}^i - \mathbf{S}^a\Pi_9) + \frac{1}{2}(\sigma^{ij}_{ab}\mathbf{X}^i[\mathbf{S}^b, \mathbf{X}^j] + \sigma^i_{a\dot a}\mathbf{S}_{\dot a}[A_9, \mathbf{X}^i])\Big). \end{aligned} \tag{8}$$

It is straightforward to check that the anticommutators of the $\mathbf{Q}^{\dot a}, \mathbf{Q}^a$ supercharges give rise to the conserved longitudinal momentum $P_+ = N/R_{11}$ and the light-cone Hamiltonian $H_{\rm LC}$ up to Gauss' law constraint respectively.

2.2 Heterotic M(atrix) String Theory

We now construct the heterotic M(atrix) string theory adopting the DVV proposal. From the M-theory point of view, the heterotic and the Type I strings arise from two different degeneration limits of a cylindrical membrane stretched between the two orbifold fixed points along 9-th direction. In the limit the 11-th circular direction shrinks, the membrane is reduced to the Type I string with one-dimensional Chan-Paton fermions at each ends. In the limit the 9-th orbifold direction shrinks, the membrane becomes heterotic string with chiral fermions generating Kac-Moody current algebra. In particular, from Type I string point of view, the heterotic string is nothing but the strong-weak coupling dual D-string [11].

The heterotic string is properly described by dualizing along the squeezed, 9-th orbifold direction [7]. We first note that T-duality maps $\mathbf{S}_1/\mathbf{Z}_2$ orbifold with radius R_9 into $\tilde{\mathbf{S}}_1$ circle with dual radius $1/R_9$. This is easy to understand from the D-brane and orientifold configurations. Initial configuration is given by 8-orientifolds and sixteen D8-branes located at the two fixed points $X_9 = 0, \pi R_9$. T-duality along the 9-th direction turns them into 9-orientifold and D9-branes. Since they have co-dimensions zero in the transverse space, the 9-orientifolds and D9-branes do not give rise to any orbifolding at all. In particular, 9-th direction after T-duality becomes $\tilde{\mathbf{S}}_1$ of radius $1/R_9$. We thus introduce a worldsheet parameter $\sigma \in [0, 2\pi]$ so that the distance along the dualized 9-th orbifold direction is measured by σ/R_9. With this convention, the duality turns $A_9 \to R_9 D_\sigma$ and $\Pi_9 \to E_\sigma/R_9$ in Eqs. (5, 7). This mapping turns the untwisted sector of the $\mathcal{N} = 8$ supersymmetric SO(2N) matrix quantum mechanics Eqs. (5, 7) into $(8, 0)$ supersymmetric SO(2N) chiral gauge theory.

For twisted sector fermions, the T-duality is achieved by replacing $A_9 \to R_9 D_\sigma$. The transformation makes the twisted sector fermions propagate chirally around $\tilde{\mathbf{S}}_1$. We thus treat them symmetrically by absorbing the πR_9 dependence of $\chi^{(2)}$ and introducing redefined chiral fermions $\chi^{(P,A)}$:

$$\begin{aligned} \chi_A^{(P)}(\sigma + \tau) &\equiv \chi_A^{(1)}(\sigma + \tau), \\ \chi_A^{(A)}(\sigma + \tau) &\equiv e^{i\sigma/2}\, \chi_A^{(2)}(\sigma + \tau). \end{aligned} \tag{9}$$

Hence, $\chi^{(P,A)}$ have opposite boundary conditions each other once traversed around $\tilde{\mathbf{S}}_1$, $\sigma \to \sigma + 2\pi$. This is the M(atrix) string theory manifestatation that distinguishes the two sets of twisted sector fermions coming from each orbifold fixed points at $A_9 = 0, \pi R_9$. We adopt a convention in which $\chi^{(P)}$ is periodic and $\chi^{(A)}$ is anti-periodic. In fact, the two sets of boundary conditions, hence, twisted sector fermions mix each other. In addition, in the heterotic M(atrix) theory, we have identified the $[\mathbf{Z}_2]^N$ discrete gauge symmetry acting only on the twisted sector fermions, Eq. (6). Upon T-duality to heterotic M(atrix) string theory, the $[\mathbf{Z}_2]^N$ discrete gauge symmetry then acts together with the above boundary conditions. Altogether, this defines the action of nontrivial SO(2N)$\times[\mathbf{Z}_2]^N$ bundle via 't Hooft's twisted boundary conditions on the twisted sector fermions.

We thus find the heterotic M(atrix) string theory consisting of the following field content and quantum numbers:

Sector	Multiplet	(Components)	Spin(8)	SO(2N)	Worldsheet
untwisted	gauge	$(A_\tau, A_\sigma\ ;\ \mathbf{S}_{\dot a})$	$(\mathbf{1}\ ;\ \mathbf{8}_{\rm c})$	2N (2N−1)/2	left-moving
	symmetric	$(\mathbf{X}^i;\ \mathbf{S}_a)$	$(\mathbf{8}_{\rm v}; \mathbf{8}_{\rm s})$	2N(2N+1)/2	right-moving
twisted	fundamental	$(\ \cdot\ ;\ \chi_A^{(P,A)})$	$(0\ ;\ \mathbf{1})$	2N	left-moving

To comply with the conventional light-cone string worldsheet units, we make a few more rescalings together with the T-duality. First, we convert the M-theory time into worldsheet time by rescaling $t \to \tau/R_{11}$. Next, we normalize the kinetic terms canonically by rescaling $\mathbf{X}^i \to \mathbf{X}^i/\sqrt{R_9}$. Finally, via 11-9 flip, we identify the heterotic string coupling parameter $g_H^2 \equiv R_9^3$ in M-theory unit.

After the rescaling, the action is given by:

$$\begin{aligned} S = \int d\tau \oint_0^{2\pi} \frac{d\sigma}{2\pi} \Big[\mathrm{Tr}\Big(&+ \frac{g_H^2}{4} F_{\alpha\beta}^2 + \frac{1}{2}(D_\alpha \mathbf{X}^i)^2 + \frac{1}{4 g_H^2}[\mathbf{X}^i, \mathbf{X}^j]^2 \\ &+ \ \mathbf{S}_a \mathcal{D}_R \mathbf{S}_a + \mathbf{S}_{\dot a} \mathcal{D}_L \mathbf{S}_{\dot a} - 2\mathbf{X}^i \sigma^i_{a\dot a}\{\mathbf{S}_a, \mathbf{S}_{\dot a}\}\Big) \end{aligned}$$

$$+ \quad \chi_A^{(P)}\mathcal{D}_L\chi_A^{(P)} + \chi_B^{(A)}\mathcal{D}_L\chi_B^{(A)}\Big]. \tag{10}$$

Here, the left-, right-moving covariant derivatives are defined as $\mathcal{D}_{L,R} \equiv D_\tau \pm D_\sigma$.

The Hamiltonian of the heterotic M(atrix) string theory is given by

$$\begin{aligned} H_{\rm LC} = \oint_0^{2\pi} \frac{d\sigma}{2\pi}\Big[\,{\rm Tr}\Big(&+ \frac{1}{2g_H^2}E^2 + \frac{1}{2}\mathbf{\Pi}_i^2 + \frac{1}{2}(D_\sigma\mathbf{X}^i)^2 - \frac{1}{4g_H^2}[\mathbf{X}^i,\mathbf{X}^j]^2 \\ &- \mathbf{S}_{\dot a}D_\sigma\mathbf{S}_{\dot a} - \mathbf{S}_a D_\sigma \mathbf{S}_a + \frac{1}{g_H}\,2\mathbf{X}^i\sigma^i_{a\dot a}\{\mathbf{S}_a,\mathbf{S}_{\dot a}\}\Big) \\ &- \chi_A^{(P)}D_\sigma\chi_A^{(P)} - \chi_B^{(A)}D_\sigma\chi_B^{(A)}\Big]. \end{aligned} \tag{11}$$

In the adopted normalization convention, the longitudinal momentum is taken as $p_+ = 1$ for *all* N. That this corresponds to the heterotic string can be seen [7] by formally taking $N = 1/2$. Then, the Lagrangian is reduced precisely to the same form as the worldsheet action of a single light-cone Green-Schwarz heterotic string, the structure first identified by Polchinski and Witten [11] from the D-string worldsheet action in Type I string theory via $\mathbf{Z}_2$ heterotic – Type I duality. Therefore, for $N > 1$, it is expected that the heterotic M(atrix) theory contain N independent heterotic strings once suitably interpreted. Indeed, in the next Section, following the DVV proposal, we will show that the heterotic M(atrix) theory indeed provides for a new second-quantization description of light-cone Green-Schwarz heterotic strings.

The resulting heterotic M(atrix) string theory is a $(1+1)$-dimensional chiral gauge theory. Spectrum of the theory is tightly constrained by the requirement of anomaly cancellation. From the identified spectrum above, we find that the gauge anomaly from the right-moving symmetric multiplet fermions is cancelled by the left-moving gauge and fundamental twisted sector fermions. In fact, the anomaly cancellation requirement may be considered as the M(atrix) theory principle for identifying the twisted sector spectrum [10, 7]. The result is also consistent with the twisted sector spectrum derived from one–loop vacuum energy cancellation requirement in the heterotic M(atrix) theory, as is expected from the T-duality between the two theories [4]

A remark is in order. One may question possible higher–order corrections to the above Lagrangian and Hamiltonian Eqs. (10, 11), especially, higher-dimensional operators involving gauge field strengths. However, we believe that there are no such terms. The Hamiltonian Eq. (11) has the full structure of the light-cone Green-Schwarz heterotic string already for $N = 1/2$ as shown above. The quadratic terms in Eq. (11) saturates all we can have consistently with light-cone kinematics and supersymmetry. As will be shown in the next section, this is also the case for arbitrary N in the phase the gauge group is spontaneously broken to $S_N \times [\mathbf{Z}_2]^N$.

[4]It should be noted that the d=10, $\mathcal{N} = 1$ supersymmetric Yang-Mills theories other than SO(32) gauge group are anomalous, hence, inconsistent. The M(atrix) theory governing D0-brane parton dynamics has been derived from this theory via dimensional reduction. Given that $\mathbf{T}^d$ toroidal compactification is described by $(d+1)$-dimensional supersymmetric Yang-Mills theory, the M(atrix) theory may encounter an inconsistency once all nine dimensions are compactified. Such potential problem does not arise if higher dimensional toroidal compactifications of M(atrix) theory are instead described by some other field theories than supersymmetric Yang-Mills theories. In fact, there exist such indications from the $(2,0)$ strong coupling fixed points for $\mathbf{T}^4$ and $\mathbf{T}^5$ compactifications [13]. We expect that such a resolution continues to higher-dimensional toroidal compactifications. Alternative possibility is that the gauge group for $\mathbf{T}_9$ compactification is not enhanced to $U(N)$ but manifests $[U(1)]^N$ only.

3 Phases of Heterotic M(atrix) String Theory

In the previous section, we have defined the heterotic M(atrix) string theory in terms of (1+1)-dimensional $(8,0)$ supersymmetric chiral gauge theory. In this section, following the idea of [3] we show that the heterotic M(atrix) string defines the second-quantized heterotic strings by analyzing the phases of the supersymmetric chiral gauge theory.

The basic results may be summarized as follows. We will first show that the strong coupling Higgs phase of $SO(2N)$ chiral gauge theory defines non-interacting, light-cone multi-heterotic string states with total number of string $\leq N$ and the total longitudinal momentum $p_+ = N/R_{11}$. In addition, we show that string joining and splitting interactions are provided precisely by the strong coupling expansions of the chiral gauge theory.

3.1 Free Heterotic String Limit

The gauge coupling constant $g_{\rm YM}$ in (1+1)-dimensions has a mass dimension 1. Since $g_{\rm YM}^{-2} = \ell_s^2 g_H^2$, the dimensionless coupling governing the gauge dynamics is given by $\tilde{g}_{\rm YM}^2 = (\Sigma/\ell_s^2) g_H^{-2}$ where Σ denotes the world-sheet area.

We first consider the strong gauge coupling limit, $\tilde{g}_{\rm YM} \to \infty$. This is the limit for which $g_H \to 0$ and $\Sigma \to \infty$. The strongly coupled heterotic M(atrix) string theory flows to an infrared fixed point at which the theory is described by a nontrivial conformal field theory. Content of the conformal field theory is tightly constrained by symmetries. The (8,0) chiral supersymmetry, Spin(8) R-symmetry and $G \equiv SO(16) \times SO(16)$ affine Lie algebra on the worldsheet should be respected by a candidate conformal field theory [5]. We claim that such a conformal field theory is provided by the $(8,0)$ supersymmetric sigma model on the symmetric product space orbifold:

$$S^N(\mathbf{R}^8 \otimes G) = (\mathbf{R}^8 \otimes G)^N/(S_N \times [\mathbf{Z}_2]^N) \tag{12}$$

where $G = SO(16) \times SO(16)$. Configuration of the other N–image strings is isomorphic to Eq. (12) so we do not repeat them in what follows. It is straightforward to identify the above target space. In the free heterotic string limit $g_H \to 0$, the $\mathbf{X}^i$ fields are dynamically confined on the moduli space $\mathcal{M}_N \equiv \{\mathbf{X}^i : [\mathbf{X}^i, \mathbf{X}^j] = 0\}$. At the same time, the gauge supermultiplet $(A_0, A_9; \mathbf{S}_a)$ decouples, leaving only charge neutrality constraint for the remaining symmetric and fundamental multiplets. At generic point in the moduli space $\mathcal{M}_N$, the fields can be written as:

$$\begin{aligned}
\mathbf{X}^i &= U \cdot {\rm diag.}(X_1^i, X_2^i, \cdots, X_N^i) \cdot U^{-1} \otimes \mathbf{I}_{2\times 2} \\
\mathbf{S}^{\dot a} &= U \cdot {\rm diag.}(S_1^{\dot a},\, S_2^{\dot a}, \cdots,\, S_N^{\dot a}) \cdot U^{-1} \otimes \mathbf{I}_{2\times 2}
\end{aligned} \tag{13}$$

where $U \in$U(N)$\subset$SO(2N) is the same for all bosonic and spinor coordinate matrices. The tensor product $\mathbf{I}_{2\times 2}$ represents image strings. This means that, in the Higgs phase, the residual gauge symmetry is $S_N \times [\mathbf{Z}_2]^N$, where S_N denotes the Weyl group of U(N)$\subset$SO(2N) acting on the gauge invariant elements for a given representation and the $[\mathbf{Z}_2]^N$ denotes the discrete gauge symmetry Eq. (6) acting nontrivially on the twisted sector fermions in the fundamental representation. Together with the twisted sector fermions $(\chi_I^{(P)}, \chi_I^{(A)})$, the eigenvalues $(X_I^i, S_I^{\dot a})$

[5]This can be argued from 't Hooft anomaly matching conditions.

$(I = 1, \cdots, N)$ describe light-cone Green-Schwarz worldsheet fields of N independent heterotic strings.

The Hilbert space $\mathcal{H}_N$ of the above S_N orbifold sigma model is decomposed into twisted sectors $\mathcal{H}_g$ [12]. Each twisted sector is labelled by the conjugacy classes $[g]$ of the S_N symmetric orbifold group. Within a given twisted sector, the physical states are those invariant under the centralizer subgroup C_g of $g \subset U(N) \subset SO(2N)$. Denote this invariant subspace by $\hat{\mathcal{H}}_g$. Then, the total S_N orbifold Hilbert space is given schematically by

$$\mathcal{H}(S^N(\mathbf{R}^8 \otimes G)) = \bigoplus_{[g]} \hat{\mathcal{H}}_g. \tag{14}$$

For the S_N symmetric group, the conjugacy classes $[g]$ are characterized by all partitions of N:

$$\sum_{n \geq 1} n N_n = N \tag{15}$$

where N_n denotes the multiplicity of the irreducible cyclic permutation of n-elements in the decomposition of g as:

$$[g] = (1)^{N_1} (2)^{N_2} \cdots = \prod_{n \geq 1} (n)^{N_n}. \tag{16}$$

Associated to the above decomposition of the conjugacy classes $[g]$, each twisted sectors are decomposed into the product of N_n-fold symmetric tensor products of n-element Hilbert subspaces $\mathcal{H}_{(n)}$:

$$\mathcal{H}_{N_n} = \bigotimes_{n \geq 1} S^{N_n} \mathcal{H}_{(n)} \tag{17}$$

where

$$S^N \mathcal{H} \equiv \Big(\mathcal{H} \otimes \mathcal{H} \otimes \cdots \otimes \mathcal{H} \Big)^{S_N}. \tag{18}$$

The Hilbert subspace $\mathcal{H}_{(n)}$ denotes the $\mathbf{Z}_n$ invariant subspace of states of a single heterotic string with winding number n around $\mathbf{S}_1$. Each of the winding number n heterotic string can be represented by a $(8,0)$ supersymmetric heterotic sigma model whose n distinct worldsheet fields $(X^i_I, S^{\dot{a}}_I, \chi^{(P)}_I, \chi^{(A)}_I)$ satisfy twisted boundary conditions

$$\begin{aligned}
X^i_I(\sigma + 2\pi) &= + \, [\, V_{(\pm)} \cdot X^i(\sigma) \cdot V^{-1}_{(\pm)} \,]_I \\
S^{\dot{a}}_I(\sigma + 2\pi) &= + \, [\, V_{(\pm)} \cdot S^{\dot{a}}(\sigma) \cdot V^{-1}_{(\pm)} \,]_I \\
\chi^{(P)}_I(\sigma + 2\pi) &= + \, [\, V_{(\pm)} \cdot \chi^{(P)}(\sigma) \,]_I \\
\chi^{(A)}_I(\sigma + 2\pi) &= - \, [\, V_{(\pm)} \cdot \chi^{(A)}(\sigma) \,]_I \quad (I = 1, \cdots, n)
\end{aligned} \tag{19}$$

where $V_{(\pm)}$ denote the 't Hooft's shift operators combined with nontrivial $[\mathbf{Z}_2]^n$ bundle

$$V_{(+)} = \begin{pmatrix} & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ +1 & & & & \end{pmatrix}_{n \times n}, \qquad V_{(-)} = \begin{pmatrix} & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ -1 & & & & \end{pmatrix}_{n \times n}. \tag{20}$$

The action of $\mathbf{Z}_n$ in the definition of $\mathcal{H}_{(n)}$ is most clearly seen by patching n fields into a single field $(X^i(\sigma), S^{\dot a}(\sigma), \chi^{(P)}, \chi^{(A)})_n$ defined over $\sigma \in [0, 2\pi n]$. The single fields are defined by

$$\begin{aligned}
X^i(\sigma + 2\pi(I-1)) &\equiv X^i_I(\sigma) \ , \\
S^{\dot a}(\sigma + 2\pi(I-1)) &\equiv S^{\dot a}_I(\sigma) \ , \\
\chi^{(P)}(\sigma + 2\pi(I-1)) &\equiv (+)^{I-1}\, \chi^{(P)}_I(\sigma) \ , \\
\chi^{(A)}(\sigma + 2\pi(I-1)) &\equiv (-)^{I-1}\, \chi^{(A)}_I(\sigma) \ . \qquad (I = 1, \cdots, n)
\end{aligned} \tag{21}$$

Then the action of the cyclic group $\mathbf{Z}_n$ generated by ordered shift of the n-element sets $(X^i_I, S^{\dot a}_I, \chi^{(P)}_I, \chi^{(A)}_I)$ turns into the coordinate shift $\sigma \rightarrow \sigma + 2\pi I$ acting on the newly introduced fields $(X^i(\sigma), S^{\dot a}(\sigma), \chi^{(P)}(\sigma), \chi^{(A)}(\sigma))_n$ for $(I = 1, \cdots, n)$. Physically, these newly introduced fields are interpreted as worldsheet degrees of freedom of $\mathbf{Z}_n$ twisted, n-times long strings. These fields satisfy boundary conditions

$$\begin{aligned}
X^i(\sigma + 2\pi n) &= +\, X^i(\sigma), \\
S^{\dot a}(\sigma + 2\pi n) &= +\, S^{\dot a}(\sigma), \\
\chi^{(P)}(\sigma + 2\pi n) &= (+)^n\, \eta\, \chi^{(P)}(\sigma), \\
\chi^{(A)}(\sigma + 2\pi n) &= (-)^n\, \eta\, \chi^{(A)}(\sigma),
\end{aligned} \tag{22}$$

where $\eta \equiv \pm$ denote the action of $[\mathbf{Z}_2]^n$ bundle. Therefore, in the normalization that sets the string winding number to unity for all n, we find that the n-twisted strings have $1/n$ fractional oscillator quantum numbers. In addition, the twisted sector fermion fields give rise to various charged sectors that generate the spacetime heterotic gauge group. Denote the boundary conditions of twisted sector fermion fields as $(\chi^{(P)}, \chi^{(A)})$. From Eq. (22), one finds that $(+,+)$ and $(-,-)$ sectors arise when n is even. Similarly, when n is odd, $(+,-)$ and $(-,+)$ sectors arise.

So far, we have constructed the Hilbert space for a finite N, corresponding to N maximal number of heterotic strings [6]. The total L_0 operator of the S_N CFT is decomposed into

$$L_0^{\rm total} = \sum_{\{n_i\}} \frac{1}{n_i} L_0^{(n_i)} \tag{23}$$

in a twisted sector defined by cyclic permutations of length n_i's. The $\mathbf{Z}_n$ invariant Hilbert subspace is then described by those excitations of a single string of length n_i satisfying

$$L_0^{\rm total} - \overline{L}_0^{\rm total} = \mathbf{Z} \qquad \text{viz.} \qquad L_0^{(n_i)} - \overline{L}_0^{(n_i)} = n_i \mathbf{Z}. \tag{24}$$

To obtain the full Fock space of the second-quantized heterotic string we take the large N limit in the following manner [3]. In the light-cone formulation, length of the string is directly proportional to the total longitudinal momentum (recall that we have adopted the normalization that the n-string winding number is unity, hence, $p_+^{\rm tot} \equiv N/R_{11} = 1.$). By taking Hilbert subspaces $\mathcal{H}_{(n)}$ for which n is linearly proportional to $N \rightarrow \infty$, only those heterotic string states with *non-vanishing* longitudinal momentum p_+ are allowed:

$$0 < \ p_+ = \frac{n}{N} \leq 1. \tag{25}$$

[6] The conventional light–cone string field is then reproduced by Fourier transforming Fock space wave functions with respect to the string number

These long strings are the only surviving ones in the large N limit. Correspondingly, since we have normalized the string winding number to unity, string statets af finite massive levels constitute of very low-energy $\mathcal{O}(1/N)$ oscillator excitations.

In the N$\rightarrow \infty$ limit prescribed above, the $\mathbf{Z}_n$ invariant Hilbert subspace then consists only of individual string configurations satisfying

$$L_0^{(n_i)} - \overline{L}_0^{(n_i)} = 0. \tag{26}$$

All other string configurations with nonzero on the right-hand side become infinitely heavy in the large N limit, hence, are not relevant for low-energy dynamics.

It is now straightforward to identify the heterotic string degrees of freedom and the required GSO projections thereof. From Eq. (22) we have seen that the twisted sector fermions give rise to $(+,+)$ and $(-,-)$ sectors when n is even, and $(+,-)$ and $(-,+)$ sectors when n is odd. Since difference of the string length by one unit corresponds to difference of $p_+ = \mathcal{O}(\pm 1/N)$, hence, becomes completely negligible in N$\rightarrow \infty$ limit, we define a single heterotic string of a given $p_+ = n/N$ via a direct sum of the $(+,-)$ and $(-,+)$ sectors from n-twisted string together with the $(+,+)$ sector from $(n+1)$-twisted string and the $(-,-)$ sector from $(n-1)$-twisted string. With this definition, we then find that, in the string winding number normalization adopted above, the vacuum energy associated with $(+,-)$ and $(-,+)$ sectors are zero, while with $(+,+)$ and $(-,-)$ sectors are $+1$ and -1 respectively. This fits perfectly to the known charged spectra of the perturbative heterotic string. At the same time, the two independent GSO projection operators of the heterotic string theory are identified as follows. The first one is the diagonal $\eta = \pm$ in Eq. (22) corresponding to $(-)^{P+A}$. The second one is the diagonal cyclic permutation by ± 1, the first factor in Eq. (22). This gives rise to the projection via anti-periodic boundary condition, $(-)^A$. We therefore obtain two independent GSO projection operators, $(-)^P$ and $(-)^A$, needed to reproduce the $E_8 \times E_8$ heterotic string.

We should emphasize again the importance of N$\rightarrow \infty$ limit in identifying these two independent GSO projection operators and the resulting charged spectrum thereof. While the first GSO operator $(-)^{P+A}$ relevant for SO(32) heterotic string is well defined even for finite N, the second one $(-)^A$ arises by construction only in the large N limit. Related to this, while the $(+,-)$ and $(-,+)$ sectors are present for n odd as explained above, for finite N they are projected out by the $(-)^{P+A}$ GSO operator completely. We conclude that only SO(32) heterotic strings are present for finite N heterotic M(atrix) string theory.

3.2 Perturbative Interactions of Heterotic M(atrix) String

So far, we have shown that the strict infrared limit of the $(8,0)$ supersymmetric chiral gauge theory gives rise to free heterotic string in the light-cone Green-Schwarz form. We have seen that, in this limit, each twisted sector of the S^N orbifold is a Fock space of a definite string number and longitudinal momentum. For free heterotic string, each Fock space of multi-heterotic string states spans a superselection sector.

In this section, we consider the effect of non-zero but small heterotic string coupling constant. Recall that the height of potential barrier of $\mathbf{X}^i$ for non-abelian symmetry restoration is proportional to g_H^{-2}. Large-N counting shows that the least barrier, hence, easiest restoration is for SO(4) $\subset$ SO(2N) corresponding to two overlapping heterotic strings. Once they overlap, SO(4) gauge field fluctuations induce a non-zero intercommuting transition amplitude between the two heterotic strings. Indeed, this is precisely the elementary joining and splitting interactions between two strings since the string number between in- and out-string states differ by one.

It is also possible to represent the elementary string interaction in terms of perturbed S^N orbifold conformal field theory. The perturbation is described by a *local* operator since, being induced by non-abelian gauge field fluctuations in the Higgs phase, the transition is a *local* process [7] on the worldsheet (σ, τ) [8]. A candidate local perturbation operator should satisfy several physical conditions. First, since the operator represents elementary string joining and splitting processes, it should correspond to an appropriate twist operator that intercommutes two overlapping heterotic strings. Second, the operator should be a singlet under $(8,0)$ supersymmetry, $SO(9,1)$ Lorentz transformation and $SO(16) \otimes SO(16)$ Kac-Moody symmetry. Third, the operator should be an irrelevant operator to guarantee that the unperturbed S^N conformal field theory is an infrared stable fixed point. Fourth, the leading irrelevant operator should have scaling dimension 3 so that the perturbation coupling parameter corresponds precisely to the heterotic string coupling constant itself.

For the Type IIA M(atrix) string theory, DVV have shown the existence of a local perturbation operator satisfying all of these requirements. The corresponding S^N orbifold conformal field theory has $(8,8)$ supersymmetries. In contrast, the heterotic M(atrix) string theory corresponds to a S^N conformal field theory with chiral $(8,0)$ supersymmetry [14] and $SO(16) \times SO(16)$ Kac-Moody symmetry. As such, it is not a priori obvious that there again exists a local perturbation operator satisfying all of the above requirements. We now show that such an operator of scaling dimension $(3/2, 3/2)$ indeed exists so that the perturbed S^N conformal field theory is described by:

$$\begin{aligned} S_{(8,0)\ \mathrm{int}} &= S_{(8,0)\ \mathrm{free}} + g_H \int \frac{d^2 z}{\ell_s^2} \, [\ell_s^3 \, \mathcal{O}_{(\frac{3}{2},\frac{3}{2})}] + \cdots , \\ & \qquad \mathcal{O}_{(\frac{3}{2},\frac{3}{2})}(z, \overline{z}) = \mathcal{O}_{(\frac{3}{2},0)}(z) \cdot \mathcal{O}_{(0,\frac{3}{2})}(\overline{z}), \end{aligned} \tag{27}$$

where ellipses denote higher-order contact interactions. We now derive the operator as a tensor product of left-moving supersymmetric descendent twist operator of dimension 3/2 and right-moving bosonic twist operator of dimension 3/2.

Consider first the left-moving supersymmetric sector. This sector gives rise to all of the $\mathcal{N} = 16$ spacetime supersymmetry generators in the light-cone Green-Schwarz formulation:

$$\mathbf{Q}^{\dot a} = \frac{1}{\sqrt{N}} \oint d\sigma \sum_{I=1}^{N} S_I^{\dot a}, \qquad \mathbf{Q}^{a} = \sqrt{N} \oint d\sigma \sum_{I=1}^{N} \sigma^i_{a \dot a} S_I^{\dot a} \partial_z X_I^i. \tag{28}$$

As we have discussed, the leading order perturbation is given by intercommuting interactions between the two free heterotic strings, say, I-th and J-th. Each of them is described by the unperturbed conformal field theory. The interaction corresponds to $\mathbf{Z}_2$ twist operator associated with a discrete gauge symmetry $\mathbf{Z}_2 \in SU(2) \subset SO(4)$ acting on a product of I-th and J-th string conformal field theories. Decompose the product into a conformal field theory of center-of-mass motion and the another of relative motion. The $\mathbf{Z}_2$ acts on the conformal fields of

[7]The characteristic length scale of the massive charged gauge boson is $\sim g_H / \langle X_{IJ} \rangle$, where X_{IJ} denote separation distance between I-th and J-th strings.

[8]as well as being local in spacetime.

relative motion $X^i_{IJ} \equiv (X^i_I - X^i_J),\ S^{\dot a}_{IJ} \equiv (S^{\dot a}_I - S^{\dot a}_J)$:

$$\begin{aligned} \mathbf{Z}_2 : X^i_{IJ} &\leftrightarrow -X^i_{IJ} \\ S^{\dot a}_{IJ} &\leftrightarrow -S^{\dot a}_{IJ}. \end{aligned} \tag{29}$$

This is exactly the same as either chiral sector of Type IIA M(atrix) string theory [3] and corresponds to the well-known supersymmetric $\mathbf{R}^8/\mathbf{Z}_2$ orbifold. The twisted sector of the orbifold arises from $X^i_{IJ} = 0$, viz. when the two heterotic strings overlap.

Twist field of the above supersymmetric orbifold is a product of bosonic and fermions parts. Associated to X^i_{IJ} is the bosonic twist operator σ_{IJ} defined through the operator product expansion:

$$\begin{aligned} \partial_z X^i_{IJ}(z) \cdot \sigma_{IJ}(w) &\sim \frac{1}{(z-w)^{\frac{1}{2}}}\, \tau^i_{IJ}(w), \\ \partial_z X^i_{IJ}(z) \cdot \tau^i_{IJ}(\omega) &\sim \frac{1}{2}\delta^{ij}\frac{1}{(z-\omega)^{\frac{3}{2}}}\, \sigma_{IJ}(\omega) + 2\delta^{ij}\frac{1}{(z-\omega)^{\frac{1}{2}}}\partial_\omega \sigma_{IJ}(\omega). \end{aligned} \tag{30}$$

Here, τ^i denotes the excited twist field transforming as $\mathbf{8}_{\rm v}$ under Spin(8). Associated to $S^{\dot a}_{IJ}$ are a pair of spin fields $(\Sigma^i_{IJ}, \Sigma^a_{IJ})$ defined through the operator product expansions

$$\begin{aligned} S^{\dot a}_{IJ}(z) \cdot \Sigma^i_{IJ}(\omega) &\sim \frac{1}{(z-\omega)^{\frac{1}{2}}}\, \sigma^i_{\dot a a}\Sigma^a_{IJ}(\omega), \\ S^{\dot a}_{IJ}(z) \cdot \Sigma^a_{IJ}(\omega) &\sim \frac{1}{(z-\omega)^{\frac{1}{2}}}\, \sigma^i_{\dot a a}\Sigma^i_{IJ}(\omega). \end{aligned} \tag{31}$$

Under Spin(8), the spin fields (Σ^i, Σ^a) transform as $(\mathbf{8}_{\rm v}, \mathbf{8}_{\rm s})$. Taking operator product expansions with the energy-momentum tensor

$$T_{IJ}(z) = \frac{1}{2}\partial_z X^i_{IJ}\partial_z X^i_{IJ} + \frac{1}{2}S^{\dot a}_{IJ}\partial_z S^{\dot a}_{IJ}, \tag{32}$$

we find the conformal dimensions of the twist operators as $[\sigma] = 1/2,\ [\tau_i] = 1,\ [\Sigma^i] = [\Sigma^a] = 1/2$ respectively. The unique $(8,0)$ supersymmetric and $SO(9,1)$ Lorentz invariant operator is the supersymmetry descendent of one of the chiral primary operators $\sigma\,\Sigma^a$:

$$\begin{aligned} \mathcal{O}_{(\frac{3}{2},0)}(0) &= \oint_{z=0} dz \left(\sigma^i_{\dot a a} S^{\dot a}\partial_z X^i\right)_{IJ}(z)\left(\sigma\Sigma^a\right)_{IJ}(0) \\ &= \oint_{z=0} \frac{dz}{z^{\frac{1}{2}}} \left(\partial_z X^i \Sigma^i\right)_{IJ}(z)\, \sigma_{IJ}(0) \\ &= \left(\tau^i\Sigma^i\right)_{IJ}(0). \end{aligned} \tag{33}$$

Next, consider the right-moving bosonic sector. The sector consists of X^i's and thirty-two Majorana fermions $\chi^{(P)}$, $\chi^{(A)}$. It is convenient to bosonize the chiral fermions into compact chiral bosons Y^A, $(A = 1, \cdots, 16)$. We again analyze the action of $\mathbf{Z}_2$ discrete gauge group $\subset SO(4)$ to the right-moving sector when I-th and J-th heterotic string overlap. Again, decomposing the product of two anti-holomorphic conformal field theories into center-of-mass and relative motion conformal field theories, the $\mathbf{Z}_2$ twist acts on the chiral bosons as:

$$\begin{aligned} \mathbf{Z}_2 : \partial_{\overline z} X^i_{IJ} &\rightarrow -\partial_{\overline z} X^i_{IJ}, \\ \partial_{\overline z} Y^A_{IJ} &\rightarrow -\partial_{\overline z} Y^i_{IJ}. \end{aligned} \tag{34}$$

Hence, the relative-motion conformal field theory corresponds to $(\mathbf{R}^8 \times \mathbf{T}^{16})/\mathbf{Z}_2$ orbifold. For our purposes, however, it is sufficient to take the $Y_{IJ}^A \in \mathbf{R}$. This is because $Y_{IJ}^A \approx -Y_{IJ}^A + 2\pi$ is equivalent to $Y_I^A \to (Y_J^A + \pi)$, $Y_J^A \to (Y_I^A - \pi)$, which is part of discrete gauge symmetries in the heterotic string [15], and because the intercommuting I-th and J-th heterotic string is a purely *local* process in Y_{IJ}^A space as well as in spacetime. Therefore, the left-moving sector is given by the well-known $\overline{c} = 24\ \mathbf{R}^{24}/\mathbf{Z}_2$ orbifold conformal field theory [16]. The twist operator is again defined through the operator product expansions:

$$\begin{aligned} \partial_{\overline{z}} X^i_{IJ}(\overline{z}) \cdot \overline{\sigma}_{IJ}(\overline{\omega}) &\sim \frac{1}{(\overline{z}-\overline{\omega})^{\frac{1}{2}}}\,\overline{\tau}^i(\overline{\omega}), \\ \partial_{\overline{z}} Y^A_{IJ}(\overline{z}) \cdot \overline{\sigma}_{IJ}(\overline{\omega}) &\sim \frac{1}{(\overline{z}-\overline{\omega})^{\frac{1}{2}}}\,\overline{\tau}^A(\overline{\omega}). \end{aligned} \tag{35}$$

The energy-momentum tensor is given by

$$\overline{T}(\overline{z}) = \frac{1}{2}\partial_{\overline{z}} X^i \partial_{\overline{z}} X^i + \frac{1}{2}\partial_{\overline{z}} Y^A \partial_{\overline{z}} Y^A. \tag{36}$$

From this, we find that the twist operator $\overline{\sigma}$ has conformal dimension $24/16 = 3/2$, while the excited twist operators $\overline{\tau}^i$ have conformal dimension 2. We thus find a unique leading irrelevant twist operator on the left-moving sector as:

$$\overline{\mathcal{O}}_{(0,\frac{3}{2})}(0) = \overline{\sigma}(0). \tag{37}$$

It is straightforward to see that the above operator is unique. To see this, consider the anti-holomorphic partition function $\mathcal{Z}(\overline{q})$ of the second symmetric product of the heterotic string theory. We will explicitly label the boundary conditions along (σ, τ) directions as $\mathcal{Z}_{(\sigma,\tau)}$. Denote a single heterotic string partition function as $Z(\overline{q}) = \overline{q}^{-1} + 24 + \mathcal{O}(\overline{q})$. Then, in the $(+,+)$ sector, the second-symmetric product partition function is given by $\mathcal{Z}_{(+,+)}(\overline{q}) = (Z(\overline{q}))^2 = \overline{q}^{-2} + \cdots$. On the other hand, the partition function in the $(+,-)$ is given by $\mathcal{Z}_{(+,-)}(\overline{q}) = Z(\overline{q}^2)$. By modular invariance, the twisted sector $(-,+)$ then gives rise to the second symmetric product partition function $\mathcal{Z}_{(-,+)}(\overline{q}) = Z(\overline{q}^{\frac{1}{2}})$. Therefore, in the second symmetric space, $\overline{c}/24 = 2$ and we find the twisted sector spectrum as:

$$\mathcal{Z}_{(-,+)}(\overline{q}) = \mathrm{Tr}_{(-,+)} \overline{q}^{(\overline{\mathrm{L}}_0 - \frac{\overline{c}}{24})} = Z(\overline{q}^{\frac{1}{2}}) = 1 \cdot \overline{q}^{-\frac{1}{2}} + \cdots. \tag{38}$$

It shows that the ground-state of the twisted sector has multiplicity one and is created by the afore-mentioned conformal dimension $(0, 3/2)$ twist operator $\overline{\sigma}$ of the $\overline{c} = 24\ \mathbf{R}^{24}/\mathbf{Z}_2$ orbifold conformal field theory.

The complete form of the cubic string interaction operator is obtained by tensoring the left-moving and the right-moving twist operators for every pairs of (IJ). This yields

$$\mathcal{O}_{(\frac{3}{2},\frac{3}{2})}(0) = \mathcal{O}_{(\frac{3}{2},0)}(0) \cdot \overline{\mathcal{O}}_{(0,\frac{3}{2})}(0) = \sum_{I<J} \oint_{z=0} dz \left(\sigma^i_{\dot{a}a} S^{\dot{a}} \partial_z X^i\right)_{IJ}(z) \left(\Sigma^a \sigma \cdot \overline{\sigma}\right)_{IJ}(0) \tag{39}$$

$$= \sum_{I<J} \oint_{z=0} \frac{dz}{z^{\frac{1}{2}}} \left(\partial_z X^i \Sigma^i\right)_{IJ}(z) \left(\sigma \cdot \overline{\sigma}\right)_{IJ}(0). \tag{40}$$

The operator has a scaling dimension $(3/2, 3/2)$. As such, the corresponding worldsheet coupling constant scales linearly with g_H, a result which asserts that this operators should be identified with three string joining and splitting interactions. In the next section, we show explicitly that this is indeed the case by comparing with the heterotic string light-cone worldsheets in Mandelstam's approach. Given that the structure and field content of the conformal field theory are so different from those of type II string, the result may be viewed as a nontrivial consistency check to the heterotic M(atrix) string theory.

3.3 Comparison with Conventional Light-Cone Heterotic String

Having determined the conformal field theory and the leading irrelevant operator corresponding to joining and splitting interactions of heterotic M(atrix) string, it is of interest to compare them with the conventional light-cone heterotic string field theory and cubic interactions thereof. Below we show that the leading irrelevant operator of dimension (3/2, 3/2) in the heterotic M(atrix) string theory is precisely the cubic interaction vertex of light-cone heterotic string both in Green-Schwarz and Neveu-Schwarz-Ramond formulations.

The conventional light-cone Green-Schwarz heterotic string is described in terms of the following worldsheet fields: bosonic coordinates $X^i(\sigma, \tau)$, fermionic real coordinates $S^{\dot{a}}(\sigma, \tau)$ and 32 fermionic real coordinates $\chi^A(\sigma, \tau)$ parametrizing sixteen–dimensional even self-dual lattice of either $E_8 \times E_8$ or Spin$(32)/Z_2$ gauge groups. They have $SO(8)$ R–symmetry quantum numbers $(\mathbf{8}_{\rm v}, \mathbf{8}_{\rm c}, \mathbf{1})$ respectively. The light-cone gauge Lagrangian of a free heterotic string is

$$L_{\rm free} = \int d\sigma \left(\partial_\rho X^i \partial_{\overline{\rho}} X^i + i S^{\dot{a}} \partial_{\overline{\rho}} S^{\dot{a}} + i \chi^A \partial_\rho \chi^A \right) \tag{41}$$

where $\rho \equiv \tau + \sigma$ and $\overline{\rho} = \tau - \sigma$. We will freely use the same notation for Euclidean worldsheet $\rho \equiv \tau + i\sigma, \overline{\rho} \equiv \tau - i\sigma$, for which the fermionic fields $S^{\dot{a}}, \chi^A$ and their Hermitian conjugates should be interpreted as independent complex-valued fields. Global spacetime supersymmetry requires that both $X^i, S^{\dot{a}}$ satisfy periodic boundary conditions. The 32 fermionic fields χ^A are SO(8) singlets, hence, can take either periodic or anti-periodic boundary conditions subject to compatibility with GSO projections.

Using SO(8) triality, it can be shown [17] that the free heterotic string action Eq. (41) in light-cone Green-Schwarz formulation is equivalent to the free heterotic string action in light-cone Neveu-Schwarz-Ramond formulation. This is true in so far as both periodic and anti-periodic boundary conditions and accompanying GSO projections are assumed for the latter formulation. The light-cone worldsheet fermions Ψ^i $(i = 1, \cdots, 8)$ in the Neveu-Schwarz-Ramond formulation can be identified with the conformal dimension (1/2, 0) $\mathbf{8}_{\rm v}$ spin field Σ^i.

The light-cone cubic interaction vertex takes a simple form in the Neveu-Schwarz-Ramond formulation. In this formulation, Mandelstam [18] has found that the light-cone worldsheet Lagrangian associated with the joining and splitting cubic interaction at $\rho = \tilde{\rho}$ is given by

$$L^{\rm (NSR)}_{\rm int}(\tilde{\rho}) = \lambda_H \int d\sigma \, \mathcal{O}_{\rm NSR}(\rho, \tilde{\rho}) \prod_\sigma \delta(X^i_{\rm in} - X^i_{\rm out}) \delta(\Psi^i_{\rm in} - \Psi^i_{\rm out}) \delta(\chi^A_{\rm in} - \chi^A_{\rm out}) \tag{42}$$

where the interaction location–dependent operator insertion is

$$\mathcal{O}_{\rm NSR}(\rho, \tilde{\rho}) = {\rm Lim}_{\rho \rightarrow \tilde{\rho}} (\rho - \tilde{\rho})^{\frac{3}{4}} \, \Psi^i \partial_\rho X^i. \tag{43}$$

Here, λ_H denotes the cubic interaction coupling parameter and the overlap delta functional is between the initial and the final heterotic string wave functionals around the interaction point.

The operator insertion Eq. (43) is only for the right-moving, supersymmetric sector. On the other hand, the overlap delta functional in Eq. (42) acts on both left- and right-moving sectors, much the same way as in the bosonic string.

The light-cone cubic interaction of the Green-Schwarz heterotic string has been obtained by Green and Schwarz [19] and by Mandelstam [20]. It is most conveniently described in terms of $SU(4) \times U(1) \subset SO(8)$. Conventionally the bosonic and fermionic worldsheet fields are decomposed into irreducible representations of $SU(4) \times U(1)$,

$$\begin{aligned} \mathbf{8}_{\rm v} &= \Big(X^R \equiv \frac{1}{\sqrt{2}}(X^1 + iX^2),\;\; X^L \equiv \frac{1}{\sqrt{2}}(X^1 - iX^2),\;\; Y^{i\geq 3} \equiv X^{i\geq 3} \Big) = (\mathbf{1}_{+1}, \mathbf{6}_0, \mathbf{1}_{-1}), \\ \mathbf{8}_{\rm c} &= \Big(\tilde{S}^a \equiv \frac{1}{\sqrt{2}}(S^{\dot a} + iS^{\dot a+4}),\;\; \tilde{S}^{\dagger a} \equiv \frac{1}{\sqrt{2}}(S^{\dot a} - iS^{\dot a+4}) \Big) \;=\; (\mathbf{4}_{-\frac{1}{2}},\; \overline{\mathbf{4}}_{+\frac{1}{2}}). \end{aligned} \quad (44)$$

In the Green-Schwarz formulation, the physical ground-state form two inequivalent eight-fold degenerate multiplets that are created by acting the zero-modes of $\tilde{S}^a$. Physically they represent ground state spanned by $\mathbf{8}_{\rm v}$ massless vector boson whose polarization is either R = (1 + i 2) = $\mathbf{1}_+$ or L = (1 − i 2) = $\mathbf{1}_-$ type. With these vacuum choices light-cone worldsheet Lagrangian describing the cubic interaction at $\rho = \tilde{\rho}$ is given by [20]:

$$L^{\rm (GS)}_{\rm int}(\tilde{\rho}) = \lambda_H \int d\sigma\, \mathcal{O}^{\pm}_{\rm GS}(\rho, \overline{\rho}) \prod_{\sigma} \delta(X^i_{\rm in} - X^i_{\rm out})\, \delta(S^a_{\rm in} - S^a_{\rm out})\, \delta(\chi^A_{\rm in} - \chi^A_{\rm out}), \quad (45)$$

where

$$\mathcal{O}^{+}_{\rm GS} = {\rm Lim}_{\rho\to\tilde{\rho}} (\rho - \tilde{\rho})^{\frac{1}{2}} \Big[\partial_\rho X^L + \frac{1}{2}(\rho - \tilde{\rho})\, \lambda^i_{\dot a \dot b} \partial_\rho Y^i \tilde{S}^{\dagger \dot a} \tilde{S}^{\dagger \dot b} + (\rho - \tilde{\rho})^2\, \partial_\rho X^R \tilde{S}^{1\dagger} \cdots \tilde{S}^{4\dagger} \Big] \quad (46)$$

for L = (1 + i2) =$\mathbf{1}_+$ (Mandelstam's 'empty ground-state') boundary condition and

$$\mathcal{O}^{-}_{\rm GS} = {\rm Lim}_{\rho\to\tilde{\rho}} (\rho - \tilde{\rho})^{\frac{1}{2}} \Big[\partial_\rho X^R + \frac{1}{2}(\rho - \tilde{\rho})\, \lambda^i_{\dot a \dot b} \partial_\rho Y^i \tilde{S}^{\dot a} \tilde{S}^{\dot b} + (\rho - \tilde{\rho})^2\, \partial_\rho X^L \tilde{S}^{1} \cdots \tilde{S}^{4} \Big] \quad (47)$$

for R = (1 − i 2) =$\mathbf{1}_-$ (Mandelstam's 'full ground-state') boundary condition respectively.

The above choice of physical ground-state is by no means unique. Indeed, one can choose the ground state spanned by massless spinor $\mathbf{8}_{\rm s}$ instead of massless vector $\mathbf{8}_{\rm v}$. In this case, the SU(4)×U(1) decompositions are $\mathbf{8}_{\rm s} = (\mathbf{1}_{+1}, \mathbf{6}_0, \mathbf{1}_{-1})$, while $\mathbf{8}_{\rm v} = (\mathbf{4}_{-\frac{1}{2}}, \overline{\mathbf{4}}_{+\frac{1}{2}})$, viz. $X^i = (Z^a \equiv X^a - iX^{a+4}, Z^{\dagger a} \equiv X^a + iX^{a+4})$. Correspondingly, the required operator insertions are given by

$$\mathcal{O}^{+}_{\rm GS} = {\rm Lim}_{\rho\to\tilde{\rho}} \Big[(\rho - \tilde{\rho}) \tilde{S}^a \partial_\rho Z^a + (\rho - \tilde{\rho})^2 \partial_\rho Z^1\, \tilde{S}^2\, \tilde{S}^3\, \tilde{S}^4 + ({\rm perm.}) \Big] \quad (48)$$

for fermionic L = (1 + i 2) = $\mathbf{1}_+$ boundary condition and

$$\mathcal{O}^{-}_{\rm GS} = {\rm Lim}_{\rho\to\tilde{\rho}} \Big[(\rho - \tilde{\rho}) \tilde{S}^{\dagger a} \partial_\rho Z^a + (\rho - \tilde{\rho})^2 \partial_\rho Z^1 \tilde{S}^{\dagger 2} \tilde{S}^{\dagger 3} \tilde{S}^{\dagger 4} + ({\rm perm.}) \Big] \quad (49)$$

for fermionic R = (1 − i 2) = $\mathbf{1}_-$ boundary condition respectively.

The nonlinear spinor terms in Eqs. (48, 49) can be linearized further [21]. Instead of integrating over worldsheets with a fixed set of interaction-point boundary conditions, integrating over both types of boundary conditions at the interaction points turns cubic in $\tilde{S}^a, \tilde{S}^{\dagger a}$ into linear $\tilde{S}^{\dagger a}, \tilde{S}^a$ respectively. Note that we have still kept fixed the boundary conditions at the external string endpoints. With these proviso, the operator insertion for the Green-Schwarz cubic interaction turns into a single operator:

$$\mathcal{O}_{\rm GS} \equiv \mathcal{O}^+_{\rm GS} + \mathcal{O}^-_{\rm GS} = {\rm Lim}_{\rho\to\tilde\rho}(\rho-\tilde\rho)\Big[\tilde S^a \partial_\rho Z^{\dagger a} + \tilde S^{\dagger a}\partial_\rho Z^a\Big]. \tag{50}$$

It is now straightforward to compare the cubic interaction worldsheet Lagrangian Eqs. (42, 45) of conventional light-cone heterotic string and the unique leading irrelevant operator Eqs. (39, 40). First, near the interaction points, the light-cone worldsheet coordinates $(\rho, \overline{\rho})$ is a double–cover of the conformal field theory local coordinates $(z, \overline{z})$. Once conformal mapping is made to the latter coordinates, the factors $(\rho-\tilde\rho)^{3/4}\partial_\rho$ in Eq. (43) and $(\rho-\tilde\rho)\partial_\rho$ in Eq. (50) are mapped into $z^{-1/2}\partial_z$ in Eq. (40) and ∂_z in Eq. (39) respectively. Second, in order to represent the overlap delta functionals in Eqs. (45, 42), it is necessary to arrange the second Riemann sheet at each interaction point to cover the two joining or splitting strings. In the $(8,0)$ conformal field theory, this is achieved precisely by inserting $\mathbf{Z}_2$ bosonic twisted operator $\sigma\cdot\overline{\sigma}$. In addition, in the Green-Schwarz formulation with fermionic boundary conditions, the $\mathbf{8}_{\rm s}$ spin field Σ^a has to be inserted to allow both types of boundary conditions in Eq. (50). Finally, the conventional insertion operators Eqs. (43, 50) are identified with the holomorphic operators $\Sigma^i\partial_z X^i$ and $\sigma^I_{\dot a a}S^{\dot a}\partial_z X^i$ in Eqs. (39, 40) respectively. Putting these correspondences together we find that the leading irrelevant operator of $(8,0)$ superconformal field theory indeed matches with the light-cone cubic interaction vertices of heterotic strings either in Green-Schwarz or in Neveu-Schwarz-Ramond formulations.

In the conventional light-cone interacting string picture, it has been known that higher-order contact terms between two or more incoming and outcoming string sets have to be introduced [22] in order to ensure a stable ground state. These contact terms cancel divergences caused by two colliding cubic interaction vertices on the light-cone worldsheet. Since exact form of these contact terms either in the Green-Schwarz or in the Neveu-Schwarz-Ramond formulations are not known presently, we will not make any attempt to compare them with candidate sub-leading irrelevant operators in the S_N conformal field theory.

4 Discussions

In this paper, we have extended the DVV proposal to the heterotic M(atrix) theory. The resulting heterotic M(atrix) *string* theory, which provides with a non-perturbative description of second-quantized heterotic strings, is defined by $(8,0)$ supersymmetric chiral gauge theory with gauge group SO(2N) in large N limit. We have checked that the theory is consistent with known properties of conventional heterotic string. The leading irrelevant operator of dimension 3 in the strong coupling expansion agrees with the joining and splitting cubic interaction vertices of light-cone heterotic string either in Green-Schwarz or in Neveu-Schwarz-Ramond formulation.

We would like to conclude with a highly speculative remark on a possible M(atrix) theory description of bosonic strings. It is well-known that bosonic Yang-Mills theory in twenty-six dimensions is rather special [23]. The regularized one-loop effective action of d-dimensional Yang-Mills theory is given by

$$\Gamma_{\rm d} = \quad - \quad {\rm Tr}\int \frac{d^dx}{(4\pi)^{d/2}}\Big[\frac{2-d}{2}\frac{\Lambda^d}{d} + \frac{26-d}{24}\frac{\Lambda^{(d-4)}}{(d-4)}F^2_{MN}$$

$$- \quad \frac{\Lambda^{(d-6)}}{(d-6)}\Big(\frac{(42-d)}{120}(D_M F_{MN})^2 + \frac{(2-d)}{144}F^3_{MN}\Big) + \cdots \Big]. \tag{51}$$

For d=26, the gauge kinetic term does not receive radiative correction at all, a feature shared by the ten-dimensional super-Yang-Mills theory. We expect that this non-renormalization remains the same even after dimensional reductions. For example, the four-dimensional Yang-Mills theory with 22 pseudo-scalar fields in the adjoint representation has a vanishing beta function at one loop and the renormalization group infrared fixed points for the scalar quartic couplings $\lambda_1 \mathrm{Tr}(X^i X^j X^i X^j)$ and $\lambda_2 \mathrm{Tr}(X^i X^j X^j X^i)$ at $\lambda_1 = \lambda_2 = g^2/(2 \pm \sqrt{8/3})$. Given this non-renormalization feature unique to twenty-six dimensional Yang-Mills theory, one may wonder if it is possible to construct M(atrix) string theory following DVV proposal for bosonic string as well despite the absence of supersymmetry and BPS states.

The bosonic strings also have D-brane extended solitons ($0 \le p \le 25$) whose tension scales as $1/g_B$ for weak string coupling $g_B \ll 1$. Given the observation that the leading order string effective action of graviton, dilaton and antisymmetric tensor field may be derived from an Einstein gravity in $d = 27$, let us make an assumption that the 27-th 'quantum' dimension decompactifies as the string coupling g_B becomes large. For D0-brane, the dilaton exchange force may be interpreted as the 27-th diagonal component of $d = 27$ metric. Gravi-photon is suppressed by compactifying 27-th direction on an orbifold rather than on a circle. Likewise, its mass may be interpreted as 27-th Kaluza-Klein momentum of a massless excitation in $d = 27$. In the infinite boost limit, the light-front view of a bosonic string is that infinitely many D0-branes are threaded densely on the bosonic string. This hints that D0-branes and Yang-Mills gauge fields gluing them are the fundamental partons, the same content as the strongly coupled superstrings. This should not be surprising since the infinite momentum boost kinematics has little to do with supersymmetry.

Given the above observation, it is quite possible that large N limit of $(1+1)$-dimensional $U(N)$ gauge theory with 24 adjoint matter fields $\mathbf{X}^i$ ($i = 1, \cdots, 24$) describes second-quantized bosonic strings in light-cone formulation. As a variant of the DVV proposal, this may be taken as a definition of bosonic M(atrix) string theory. The Yang-Mills coupling $g_{\rm YM}$ scales with the bosonic string coupling g_B the same way as superstring cases: $g_{\rm YM}^{-2} = \ell_s^2 g_{\rm B}^2$. If the strong coupling limit of the Yang-Mills theory is confining and flows to a nontrivial fixed point with manifest $SO(24)$ rotational symmetry, the bosonic M(atrix) string theory is locally described by a $(c, \overline{c}) = (24, 24)$ conformal field theory defined on a symmetric product space orbifold $S^N \mathbf{R}^{24} \equiv (\mathbf{R}^{24})^N / S_N$. Because the theory is confining, only entries of diagonalized $\mathbf{X}^i$'s are observables. Since each of the left- and the right-moving sectors are the same copies as $\mathbf{R}^{24}/\mathbf{Z}_2$ orbifold conformal field theory sector of the heterotic string, the leading irrelevant operator with manifest SO(24) rotational symmetry is identified with $\mathcal{O}_{\rm B} = \sigma \cdot \overline{\sigma}$. This operator has scaling dimension (3/2, 3/2), hence, worldsheet coupling associated with the perturbation is proportional linearly to g_B, much the same as the superstring cases. Furthermore, in Section 3.3, we have shown that perturbation by the operator $\sigma \cdot \overline{\sigma}$ is equivalent to arranging the second Riemann-sheet at each interaction points so that joining and splitting string wave funcationals do overlap. The overlap delta functional is all one needs to describe the joining and splitting cubic interactions in light-cone bosonic string. These coincidences indicate that, despite lack of controlled higher-order radiative corrections, the bosonic M(atrix) string theory may offer a second-quantized description of interacting bosonic strings.

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