Vortex Lattice Method Analysis of a Symmetrical, Untwisted Swept Wing

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Terminology

$\boldsymbol{V}_{\infty}$: Freestream velocity

$\frac{d\boldsymbol{S}}{d\boldsymbol{x}}$: The slope of the airfoil with respect to x, the chord-length

$\boldsymbol{w}'$: Symbol for downwash

$\overrightarrow{\boldsymbol{r_1}}$: Directional vector denoting the distance and direction between a point and an arbitrarily determined 'first' edge of a vortex segment

$\overrightarrow{\boldsymbol{r_2}}$: Directional vector denoting the distance and direction between a point and the other edge of a vortex segment

$\boldsymbol{p}$: Generally, a point of interest – specifically in this report, the control point(s)

$\boldsymbol{\Gamma}$: Vortex strength

$\boldsymbol{A_{ij}}$: The matrix containing the distance from each ith control point to the horseshoe vortex in the panel j

$\boldsymbol{\rho}$: Fluid density of the freestream

I: Problem Statement

The problem at hand involves the use of the vortex lattice method in computing the entire-wing lift coefficient of an untampered, untwisted, symmetrical swept wing defined in a 1958 study[1] conducted by Weber and Brebner, at angles of attack 2.1, 4.2, 6.3, 8.4, and 10.5 degrees. This task is divided into two relative tasks – the writing of a computer script that will produce such analytical results (and thereby producing them), and subsequent comparison to the results found by Weber and Brebner[1], in Table X. Coordinate information is found in Table 1 and displayed in Figure 1. Further information about the setup of this study is found in Table 2. Details on the methodology used in this report are found in Section II.

Table 1: Relative geometry of the airfoil

Chordwise Location (% Chord, c)	Relative Thickness (%c)
0	0
1.25	1.64
2.5	2.3
5	3.19
7.5	3.83
10	4.33
15	5.06
20	5.56
30	6
40	5.76
50	5.12
60	4.24
70	3.22
80	2.15
90	1.07
100	0

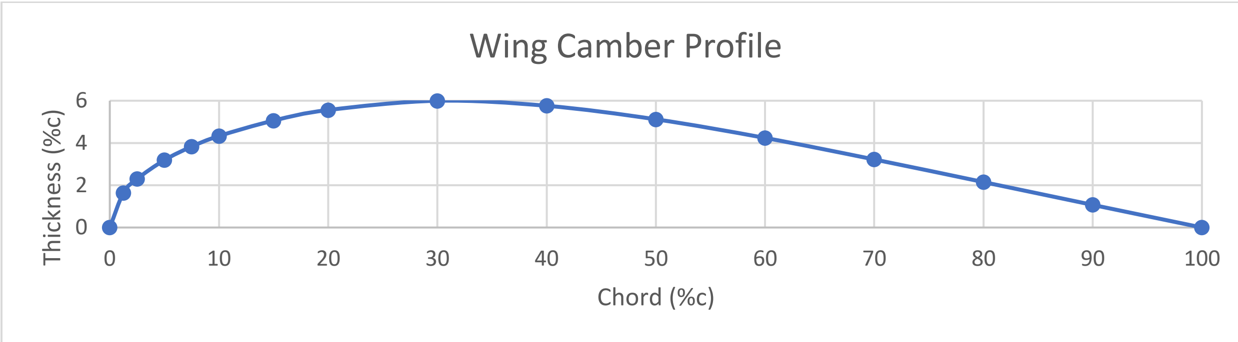

Figure 1

Table 2: Wing characteristics

Aspect Ratio	5
Sweep Angle	45 deg
Freestream Velocity	163 ft/s
Chord Length	20 inches
Flow Density	0.0765 lb/ft^3

II: Mathematical Formulation and Methodology

As stated, the objective of the problem is to implement the vortex lattice method, which is an extension of Prandtl lifting line theory – all the same assumptions apply, especially the specific characteristic that the wing is thin. A short summary of the formulas and theory used will be given here. For an in-depth explanation, please see the reference material[2].

The wing planform of the wing described in Section I is given in Figure 2. In the vortex lattice method, the wing is subdivided into several panels, with each panel having its own unique control point which lies on the surface of that panel. For reference, there are eight panels chosen for this study, and each control point lies at ¾ chord at the spanwise center of each panel.

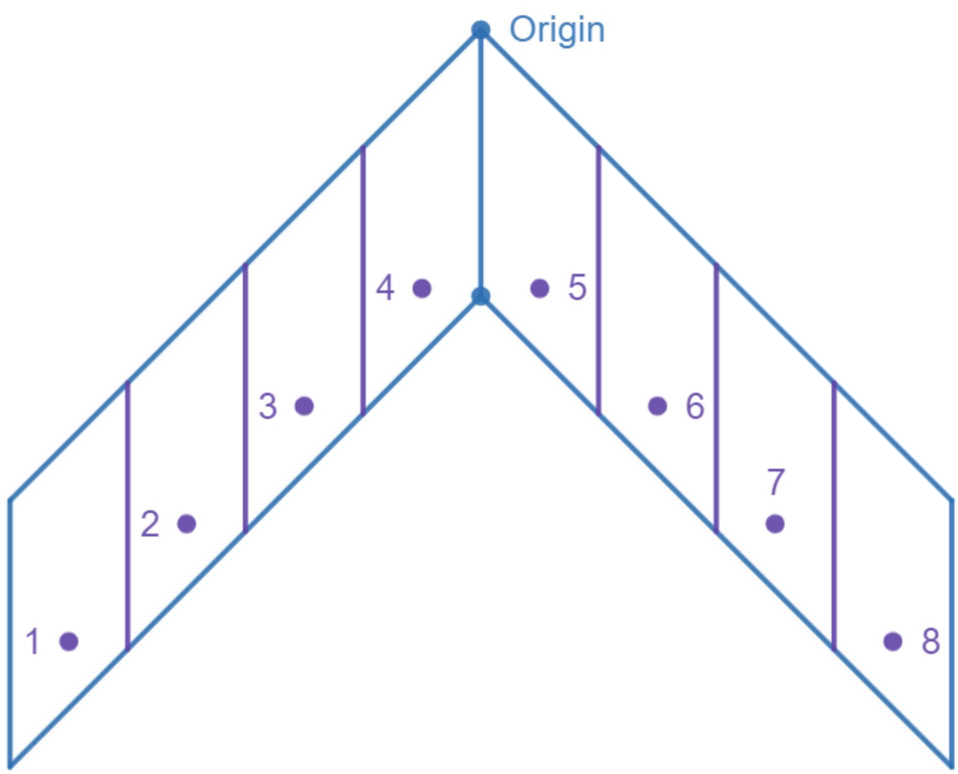

Figure 2: Graphical display of the wing planform, with the nine control points and eight panels clearly marked.

The purpose of the control point becomes clear with Eq. 1 below – the flow at the control point is known due to the flow tangency condition. Now, the horseshoe vortices that develop across each panel (see Figure 3) each have unknown vortex strength Γ_j – this unknown quantity can be determined with an application (Eq. 3) of the Biot-Savart Law (Eq. 2). Note that the orange line in Figure 3 denotes the location of the finite segment of the horseshoe vortex. For this study's purposes, that segment is said to develop at the quarter-chord.

$$Eq. 1: w' = V_\infty \frac{\partial S}{\partial x}$$

$$Eq. 2: \vec{V}_{induced} = \frac{\Gamma}{4\pi}\vec{r_1} \times \vec{r_2} \frac{(\vec{r_1} + \vec{r_2})(1 - (\vec{r_1} \cdot \vec{r_2})/r_1 r_2)}{(r_1 r_2)^2 - (\vec{r_1} \cdot \vec{r_2})^2 + (r_1^2 + r_2^2 - 2(\vec{r_1} \cdot \vec{r_2}))}$$

$$Eq. 3: \sum_{J=1}^{N} A_{ij}\Gamma_j = -V_\infty \frac{\partial S}{\partial x}$$

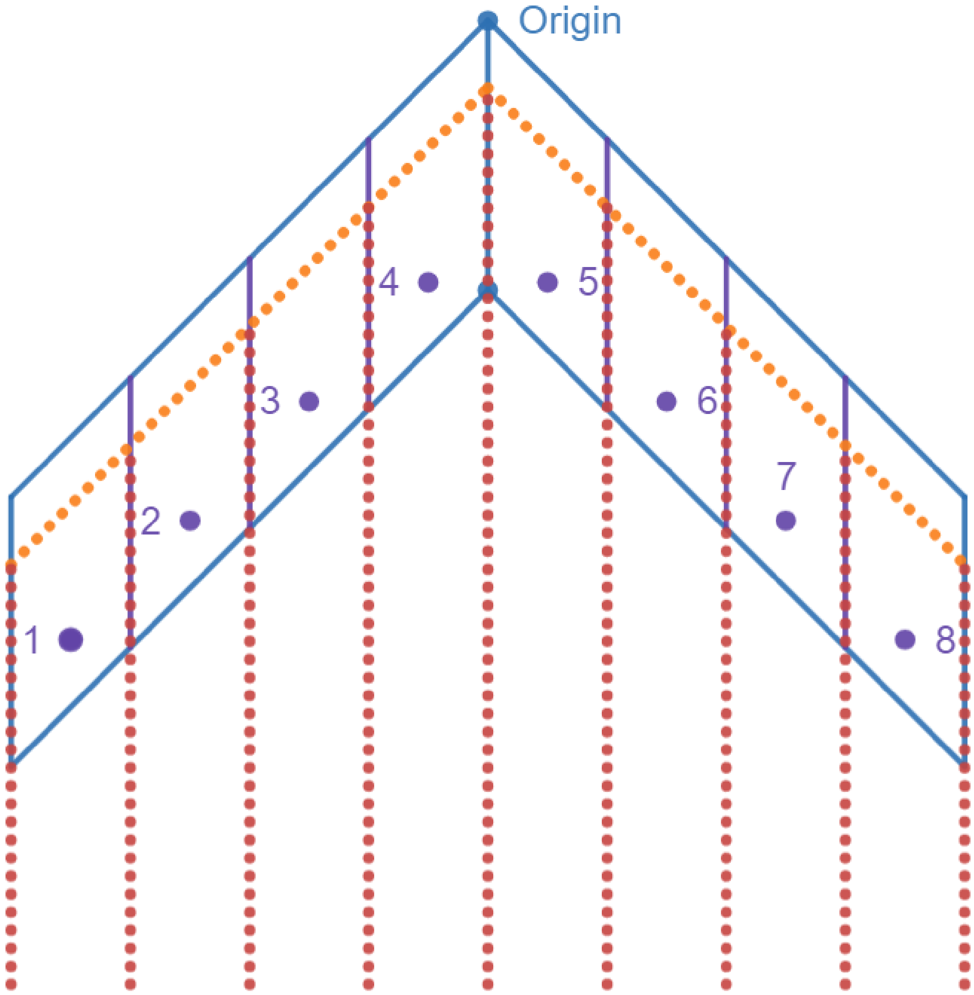

Figure 3: Illustration of the horseshoe vortices. The trailing vortices (red) go to infinity, the center vortices (orange) are finite and occur at the quarter-chord location.

Since the panels only divide the wing in the spanwise direction and not in the chordwise direction, the vortex lattice method implemented is actually a simplified version, called Weissinger's L Method. The use of Eqs. 1-3 remain the same, only that because the chord is not subdivided, the trailing vortices (Figure 3) are the same for the entire length of the chord, whereas with the presence of chordwise panels they would differ from such panel to panel.

With the vortex strengths computed, it then becomes a simple substitution task to determine the lift across the entire wing (Eq. 4), and subsequently the coefficient of lift (Eq. 5).

$$Eq.4: \sum_{j=1}^{N} \rho V_\infty \Gamma_J \, \Delta X_j = L$$

$$Eq.5: C_l = \frac{2L}{\rho V^2 (AR * c^2)}$$

III: Results and Analysis

Successful implementation of the process described in Section II (code attached separately) resulted in the numerical arguments presented in Table 3. The results obtained from testing in the reference material[1] is also found in Table 3.

Table 3: Lift coefficients at significant angles of attack, theoretical and actual

$\alpha(\mathrm{deg})$	$C_l\ (theoretical)$	$C_l(actual)$
2.1	0.119	0.121
4.2	0.238	0.238
6.3	0.356	0.350
8.4	0.475	0.456
10.5	0.594	0.559

As observed, the theoretical lift coefficient value is about equal to the experimental value at $\alpha = 4.2$ degrees and greater than the experimental values at $\alpha = 6.3, 8.4, 10.5$ degrees. Conversely, the theoretical value was less than the actual value at $\alpha = 2.1$ degrees.

This effect may be at least partially attributed to the development of boundary layers, which effectively add 'extra' camber to the airfoil and thus reduce the lift produced at higher angles of attack. This appears to be consistent as the difference between the theoretical and actual values only grows as angle of attack increases.

While no drag data was computed through the use of the computer script, an interesting further point of study could be the drag polar of the wing, especially in comparison to data found in the reference material[1] as well as data readily available by analyzing other airfoils, of which the coordinates are already known (NACA series).

IV: References

1. Weber, J., and Brebner, G.G. (1958). *Low-Speed Tests on 45-deg Swept-back Wings. Part I: Pressure Measurements on Wings of Aspect Ratio 5.* Aeronautical Research Council.
2. Sankar, Lakshmi N. (2021). *Vortex Lattice Methods for Incompressible Potential Flow Over Thin Wings.* Georgia Institute of Technology, Guggenheim School of Aerospace Engineering.