Breaking

longest piece $\rightarrow$ $2 \leq x \leq 4$

1. $CDF_x(X \leq x) = \frac{x-2}{4-2} = \frac{1}{2}x - 1$

2. $PDF_x(x) = \frac{1}{2}$

3. $E(x) = \int_{-\infty}^{+\infty} \frac{1}{2}x\,dx = \int_{2}^{4} \frac{1}{2}x\,dx = \frac{1}{4} \cdot 16 - \frac{1}{4} \cdot 4 = 3$

Stronium:

1. Express h in terms of λ:

$$CDF(x) = 1 - e^{-\lambda x} \qquad x \in [0, \infty)$$

$$\frac{1}{2} = P(T > h) = 1 - P(T \leq h) = 1 - CDF(h) = \cancel{1} - \cancel{1} + e^{-\lambda h} =$$

$$= e^{-\lambda h}$$

$$\Rightarrow -\lambda h = \ln\left(\frac{1}{2}\right) \quad \Rightarrow h = \frac{\ln(2)}{\lambda}$$

2. find λ: $\lambda = \frac{\ln(2)}{28}$ (based on $h = \frac{\ln(2)}{\lambda}$)

3. $P(T > 50) = 1 - P(T \leq 50) = 1 - CDF(50) = 1 - (1 - e^{-\lambda 50}) =$

$$= e^{-\lambda 50} = e^{-\frac{\ln(2) \cdot 50}{28}} = \left(e^{\ln(2)}\right)^{-\frac{50}{28}} = 2^{-\frac{50}{28}}$$

4. $CDF(x) = 0.99 \Rightarrow 1 - e^{-\lambda x} = 0.99$

$$\Rightarrow e^{-\lambda x} = 0.01 \Rightarrow -\lambda x = \ln(0.01)$$

$$\Rightarrow x = \frac{28 \ln(0.01)}{\ln(2)} \quad \text{(years)}$$

Portfolio:

1. $2000\$$ 10% per year $\Rightarrow$ 2200

$1000\$$ Norm$(\mu=1160,\ \sigma=80) \Rightarrow 1160$

$\Rightarrow$ Total after 1 year: 3360

2. $\mu=3360$ $\quad Y=X+2200$ $\quad \mu=E[Y]$

Y - total amount after year $\quad E[X]=1160$

$$Var(Y)=E[Y^2]-E[Y]^2=E[X^2+4400X+2200^2]-3360^2=$$
$$=E[X^2]+4400E[X]+2200^2-3360=$$
$$=E[X^2]+4400\cdot 1160+2200^2-3360 \qquad \#1$$

$$Var(X)=E[X^2]-E[X]^2=80^2 \Rightarrow E[X^2]=80^2+1160^2$$

Lets return to #1 after we extracted $E[X^2]$:

$$Var(Y)=80^2+1160^2+4400\cdot 1160+2200^2-3360^2=6400$$

$$\sigma_Y=\sqrt{Var(Y)}=80$$

$$\Rightarrow \frac{80}{3360}=0.0238 \Rightarrow \text{std of } 2.38\%$$

3. 1. Loss (fund only):

$$Loss=160=-2\sigma_X \Rightarrow P(Loss_X)=P(X<\mu-2\sigma_X)=$$
$$=\frac{1-P(\mu-2\sigma_X\le X\le \mu+2\sigma_X)}{2}=2.5\%$$

2. Loss (fund + deposit account):

$$Loss=360=-4.5\sigma_X \Rightarrow P(X<1160-360)=$$
$$=P(X<800)=P\left(Z\le\frac{800-1160}{80}\right)=$$
$$=\Phi(-4.5)=1-\Phi(4.5)=0.0001\%$$

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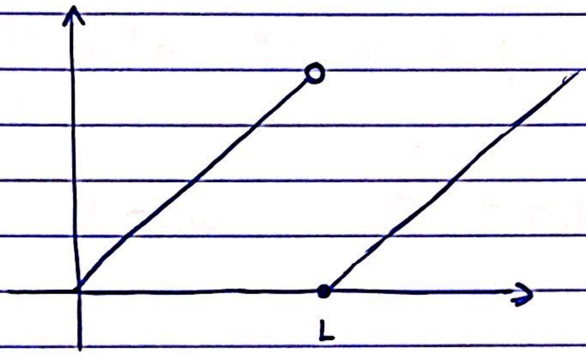
Waste:

1. $PDF_x(x) = f_x(x)$ X - Pole length

Y - Pole part lost

$$g_x(x) = \begin{cases} x & x < L \\ x - L & x \geq L \end{cases}$$



$$\mu_y = E(Y) = \int_0^\infty g_x(x) f_x(x) dx = \int_0^L x f(x) dx + \int_L^\infty (x-L) \cdot f_x(x) dx =$$

$$= \int_0^\infty x f(x) dx - L \int_L^\infty f_x(x) dx = \mu_x - L \int_L^\infty f_x(x) dx$$

2. We need to solve $\frac{\partial \mu_y}{\partial \mu_x} = 0$

$$\frac{\partial \mu_y}{\partial \mu_x} = 1 - \frac{L}{\sigma_x \sqrt{2\pi}} \cdot \int_L^\infty \left(\frac{x - \mu_x}{\sigma_x^2} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} \right) dx =$$

$$= 1 - \frac{L}{\sigma_x \sqrt{2\pi}} \cdot e^{-\frac{(L-\mu_x)^2}{2\sigma_x^2}} = 0$$

$$e^{-\frac{(L-\mu_x)^2}{2\sigma_x^2}} = \frac{\sigma_x \sqrt{2\pi}}{L}$$

$$\frac{(L-\mu_x)^2}{2\sigma_x^2} = -\ln\left(\frac{\sigma_x \sqrt{2\pi}}{L} \right)$$

$$(L-\mu_x)^2 = -2\sigma_x^2 \ln\left(\frac{\sigma_x\sqrt{2\pi}}{L}\right)$$

$$\Rightarrow \mu_x = L \pm \sqrt{-2\sigma_x^2 \ln\left(\frac{\sigma_x\sqrt{2\pi}}{L}\right)}$$

+ is min

− is max

$$\Rightarrow \mu^* = L + \sqrt{-2\sigma_x^2 \ln\left(\frac{\sigma_x\sqrt{2\pi}}{L}\right)}$$

3. $L = 2m$ $\sigma_x = 0.02$

$$\Rightarrow \mu^* = 2 + 0.0543 = 2.0543\ m$$