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Extras

Maths

Common Sums

$\sum_{k=0}^n k = \frac{n(n+1)}{2}$	$\sum_{k=0}^n k^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{k=0}^n k^3 = \frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^n k^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$		$\sum_{k=0}^n a^k = \frac{1-a^{n+1}}{1-a}$
$\sum_{k=0}^n ka^k = \frac{a[1-(n+1)a^n+na^{n+1}]}{(1-a)^2}$	$\sum_{k=0}^n k^2a^k = \frac{a[(1+a)-(n+1)^2a^n+(2n^2+2n-1)a^{n+1}-n^2a^{n+2}]}{(1-a)^3}$	
$\sum_{k=0}^\infty a^k = \frac{1}{1-a}, a < 1$	$\sum_{k=0}^\infty ka^k = \frac{a}{(1-a)^2}, a < 1$	$\sum_{k=0}^\infty k^2a^k = \frac{a^2+a}{(1-a)^3}, a < 1$
$\sum_{k=0}^\infty \frac{1}{a^k} = \frac{a}{a-1}, a > 1$	$\sum_{k=0}^\infty \frac{k}{a^k} = \frac{a}{(a-1)^2}, a > 1$	$\sum_{k=0}^\infty \frac{k^2}{a^k} = \frac{a^2+a}{(a-1)^3}, a > 1$
$\sum_{k=0}^\infty \frac{a^k}{k!} = e^a$	$\sum_{k=0}^n \binom{n}{k} = 2^n$	$\sum_{k=0}^n \binom{k}{m} = \binom{n+1}{m+1}$

Logarithm Rules

$\log_b(b^k) = k$	$\log_b(1) = 0$	$\log_b(X) = \frac{\log_c(X)}{\log_c(b)}$
$\log_b(X \cdot Y) = \log_b(X) + \log_b(Y)$	$\log_b(\frac{X}{Y}) = \log_b(X) - \log_b(Y)$	$\log_b(X^k) = k \cdot \log_b(X)$

Graphs

Data Structures

Union Find

```
struct UnionFind {
    int n;
    vector<int> dad, size;

    UnionFind(int N) : n(N), dad(N), size(N, 1) {
        while (N--) dad[N] = N;
```

```
}

// O(lg*(N))
int root(int u) {
    if (dad[u] == u) return u;
    return dad[u] = root(dad[u]);
}

// O(1)
void join(int u, int v) {
    int Ru = root(u), Rv = root(v);
    if (Ru == Rv) return;
    if (size[Ru] > size[Rv]) swap(Ru, Rv);
    --n, dad[Ru] = Rv;
    size[Rv] += size[Ru];
}
```

```
// O(lg*(N))
bool areConnected(int u, int v) {
    return root(u) == root(v);
}

int getSize(int u) { return size[root(u)]; }

int numberOfSets() { return n; }
};
```

Articulation Points And Bridges

```
// APB = articulation points and bridges
// Ap = Articulation Point
// br = bridges, p = parent
// disc = discovery time
// low = lowTime, ch = children
// nup = number of edges from u to p

typedef pair<int, int> Edge;
int Time;
vector<vector<int>> adj;
vector<int> disc, low, isAp;
vector<Edge> br;

void init(int N) { adj.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}

int dfsAPB(int u, int p) {
    int ch = 0, nup = 0;
    low[u] = disc[u] = ++Time;
    for (int &v : adj[u]) {
        if (v == p && !nup++) continue;
        if (!disc[v]) {
            ch++, dfsAPB(v, u);
            if (disc[u] <= low[v]) isAp[u]++;
            if (disc[u] < low[v]) br.push_back({u, v});
            low[u] = min(low[u], low[v]);
        } else
            low[u] = min(low[u], disc[v]);
    }
    return ch;
}

// O(N)
void APB() {
    br.clear();
    isAp = low = disc = vector<int>(adj.size());
    Time = 0;
    for (int u = 0; u < adj.size(); u++)
        if (!disc[u]) isAp[u] = dfsAPB(u, u) > 1;
}
```

Topological Sort

```
// vis = visited
vector<vector<int>> adj;
vector<int> vis, toposorted;

void init(int N) {
    adj.assign(N, vector<int>());
    vis.assign(N, 0), toposorted.clear();
}

void addEdge(int u, int v) { adj[u].push_back(v); }

// returns false if there is a cycle
// O(E)
bool toposort(int u) {
    vis[u] = 1;
    for (auto &v : adj[u])
        if (v != u && vis[v] != 2 &&
            (vis[v] || !toposort(v)))
            return false;
    vis[u] = 2;
    toposorted.push_back(u);
    return true;
}

// O(V + E)
bool toposort() {
    for (int u = 0; u < adj.size(); u++)
        if (!vis[u] && !toposort(u)) return false;
    return true;
}
```

Flow

Max Flow Min Cut Edmonds-Karp

```
// cap[a][b] = Capacity left from a to b
// iflow = initial flow, icap = initial capacity
// pathMinCap = capacity bottleneck for a path (s->t)

typedef int T;
vector<int> level;
vector<vector<int>> adj, cap;
T inf = 1 << 30;

void init(int N) {
    adj.assign(N, vector<int>());
    cap.assign(N, vector<int>(N));
}

void addEdge(int u, int v, T icap, T iflow = 0) {
    if (!cap[u][v])
        adj[u].push_back(v), adj[v].push_back(u);
    cap[u][v] = icap - iflow;
    // cap[v][u] = cap[u][v]; // if graph is undirected
}
```

```
// O(N)
T bfs(int s, int t, vector<int> &dad) {
    dad.assign(adj.size(), -1);
    queue<pair<int, T>> q;
    dad[s] = s, q.push(s);
    while (q.size()) {
        int u = q.front().first;
        T pathMinCap = q.front().second;
        q.pop();
        for (int v : adj[u])
            if (dad[v] == -1 && cap[u][v]) {
                dad[v] = u;
                T flow = min(pathMinCap, cap[u][v]);
                if (v == t) return flow;
                q.push({v, flow});
            }
    }
    return 0;
}

// O(E^2 * V)
T maxFlowMinCut(int s, int t) {
    T maxFlow = 0;
    vector<int> dad;
    while (T flow = bfs(s, t, dad)) {
        maxFlow += flow;
        int u = t;
        while (u != s) {
            cap[dad[u]][u] -= flow, cap[u][dad[u]] += flow;
            u = dad[u];
        }
    }
    return maxFlow;
}
```

Shortest Paths

Dijkstra

```
// s = source
typedef int T;
typedef pair<T, int> DistNode;
T inf = 1 << 30;
vector<vector<int>> adj;
unordered_map<int, unordered_map<int, T>> weight;

void init(int N) {
    adj.assign(N, vector<int>());
    weight.clear();
}

void addEdge(int u, int v, T w, bool isDirected = 0) {
    adj[u].push_back(v);
    weight[u][v] = w;
    if (isDirected) return;
    adj[v].push_back(u);
    weight[v][u] = w;
}
```

```
// ~ O(E * lg(V))
vector<T> dijkstra(int s) {
    vector<long long int> dist(adj.size(), inf);
    priority_queue<DistNode> q;
    q.push({0, s}), dist[s] = 0;
    while (q.size()) {
        DistNode top = q.top();
        q.pop();
        int u = top.second;
        if (dist[u] < -top.first) continue;
        for (int &v : adj[u]) {
            T d = dist[u] + weight[u][v];
            if (d < dist[v]) q.push({-(dist[v] = d), v});
        }
    }
    return dist;
}
```

Maths

Data Structures

Matrix

```
template <class T>
struct Matrix {
    int rows, cols;
    vector<vector<T>> m;

    Matrix(int r, int c) : rows(r), cols(c) {
        m.assign(r, vector<T>(c));
    }

    Matrix(const vector<vector<T>>& b)
        : rows(b.size()), cols(b[0].size()), m(b) {}

    Matrix(int n) {
        m.assign(n, vector<T>(n));
        while (n--) m[n][n] = 1;
    }

    vector<T>& operator[](int i) const {
        return const_cast<Matrix*>(this)->m[i];
    }

    // O(N * M)
    Matrix operator+(const Matrix& b) {
        Matrix ans(rows, cols);
        for (int i = 0; i < rows; i++)
            for (int j = 0; j < m[i].size(); j++)
                ans[i][j] = m[i][j] + b[i][j];
        return ans;
    }
}
```

```
// O(N * M)
Matrix operator-(const Matrix& b) {
    Matrix ans(rows, cols);
    for (int i = 0; i < rows; i++)
        for (int j = 0; j < m[i].size(); j++)
            ans[i][j] = m[i][j] - b[i][j];
    return ans;
}

// O(N^3)
Matrix operator*(const Matrix& b) {
    if (cols != b.rows) return Matrix(0, 0);
    Matrix ans(rows, b.cols);
    for (int i = 0; i < rows; i++)
        for (int j = 0; j < b[i].size(); j++)
            for (int k = 0; k < b.rows; k++)
                ans[i][j] += m[i][k] * b[k][j];
    return ans;
}

Matrix& operator+=(const Matrix& b) {
    return *this = *this + b;
}

Matrix& operator-=(const Matrix& b) {
    return *this = *this - b;
}

Matrix& operator*=(const Matrix& b) {
    return *this = *this * b;
}
};
```

Number Theory

Binary Exponentiation

```
typedef long long int li;

li binPow(li a, li p) {
    li ans = 1LL;
    while (p) {
        if (p & 1LL) ans *= a;
        a *= a, p >>= 1LL;
    }
    return ans;
}
```

Divisibility Criterion

```
def divisorCriteria(n, lim):
    results = []
    tenElevated = 1
    for i in range(lim):
        # remainder = pow(10, i, n)
        remainder = tenElevated % n
        negremainder = remainder - n
        if (remainder <= abs(negremainder)):
            results.append(remainder)
        else:
            results.append(negremainder)
        tenElevated *= 10
    return results

def testDivisibility(dividend, divisor,
    ↪ divisor_criteria):
    dividend = str(dividend)
    addition = 0
    dividendSize = len(dividend)
    i = dividendSize - 1
    j = 0
    while j < dividendSize:
        addition += int(dividend[i]) *
            ↪ divisor_criteria[j]
        i -= 1
        j += 1
    return addition % divisor == 0

if __name__ == '__main__':
    dividend, divisor = map(int, input().split())
    divisor_criteria = divisorCriteria(divisor,
    ↪ len(str(dividend)))
    print(divisor_criteria)
    print(testDivisibility(dividend, divisor,
    ↪ divisor_criteria))
```

Primes

Is Prime Miller Rabin

```
#include "../Number Theory/Modular Exponentiation.cpp"

bool isPrime(lli p, int k = 20) {
    if (p == 2 || p == 3) return 1;
    if ((~p & 1) || p == 1) return 0;
    lli d = p - 1, phi = d, r = 0;
    while (~d & 1) d >>= 1, r++;
    while (k--) {
        // set seed with: int main() { srand(time(0)); }
        lli a = 2 + rand() % (p - 3); // [2, p - 2]
        lli e = pow(a, d, p), r2 = r;
        if (e == 1 || e == phi) continue;
        bool flag = 1;
        while (--r2) {
            e = multiply(e, e, p);
            if (e == 1) return 0;
            if (e == phi) {
                flag = 0;
                break;
            }
        }
        if (flag) return 0;
    }
    return 1;
}
```

Primes Sieve

```
vector<int> sieve, primes;

// ~O(N * lg(lg(N)))
void primeSieve(int n) {
    sieve.assign(n + 1, 0);
    primes = {};
    for (int i = 3; i * i <= n; i += 2)
        if (!sieve[i])
            for (int j = i * i; j <= n; j += 2 * i)
                if (!sieve[j]) sieve[j] = i;
    primes.push_back(2);
    for (int i = 3; i < n; i++)
        if (!sieve[i] && (i & 1)) primes.push_back(i);
}
```