Bayesian mean-only linear model with Stan

In this program, we assume that we have a vector of N observations that can be reasonably modeled by a $N(\mu, \sigma)$ distribution.

Our objective is to fit the mean-only linear model:

$$Y = \beta_0 + e$$

where:

- Y is a vector of observed measurements (Sepal.Width from the iris dataset).
- β_0 is a parameter, assumed to be a random variable and given a prior distribution.
- e is a vector of independent normal random variables with mean zero and standard deviation σ_e.

In the Bayesian formulation, β_0 and σ_e are treated as parameters, which are random variables in the Bayesian framework.

The Bayes equation is:

$$f(\beta_0, \sigma_e | Y) \propto f(Y | \beta_0, \sigma_e) \cdot f(\beta_0) \cdot f(\sigma_e)$$

where:

- $f(\beta_0, \sigma_e | Y)$ is the (joint) posterior distribution of β_0 and σ_e given Y
- $f(Y | \beta_0, \sigma_e)$ is the conditional likelihood of Y given β_0 and σ_e
- $f(\beta_0)$ is the prior distribution for β_0
- $f(\sigma_e)$ is the prior distribution for σ_e

The likelihood and priors are specified in `mean_only.stan`:

```
//Estimate the parameters of a mean-only model with normal data
data {
  int N;                          //sample size is N
  real y[N];                      //y consists of N real data values
}
parameters {
  real beta_0;                    //location parameter
  real<lower=0> sigma;            //dispersion parameter constrained to be nonegative
}
```

```
model {
  beta_0  ~ normal(0,100);       //normal prior for mu: centered at zero with sd=100
  sigma   ~ cauchy(0,10);        //uniform prior for sigma

  y     ~ normal(beta_0,sigma); //normal likelihood given parameters (mu,sigma)
}
```

With this model file, we have the following specifications:

- The likelihood $f(Y|\beta_0, \sigma_e)$ is $N(\beta_0, \sigma_e)$, coded as `y` $\sim$ `normal(beta_0,sigma)`
- The prior $f(\beta_0)$ is $N(0, 100)$, coded as `beta_0` $\sim$ `normal(0,100)`
- The prior $f(\sigma_e)$ is half-cauchy (because of $< lower = 0 >$) coded as `sigma_e` $\sim$ `cauchy(0,10)`