Bayesian one-way ANOVA with three levels using Stan

In this program, we assume that we have a vector of N observations that can be each classified into one of three levels of a factor.

Our objective is to fit a one-way ANOVA with three levels:

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + e$$

where:

- Y is a vector of observed values
- β_1 is a parameter representing the mean at level 1, treated as unknown but constant.
- X_1 is a binary predictor that is 1 if this y has the factor at level 1, zero otherwise
- β_2 is a parameter representing the mean at level 2, treated as unknown but constant.
- X_2 is a binary predictor that is 1 if this y has the factor at level 2, zero otherwise
- β_3 is a parameter representing the mean at level 3, treated as unknown but constant.
- X_3 is a binary predictor that is 1 if this y has the factor at level 3, zero otherwise
- e is a vector of independent, identically distributed $N(0, \sigma_e)$ random variables
- σ_e is the standard deviation of the error or residual terms e, which are assumed to have mean zero.

In the Bayesian formulation, $\beta_1 - \beta_3$ and σ_e are treated as parameters, which are random variables in the Bayesian framework.

The Bayes equation is:

$$f(\beta_1, \beta_2, \beta_3, \sigma_e|Y) \propto f(Y|\beta_1, \beta_2, \beta_3, \sigma_e) \cdot f(\beta_1) \cdot f(\beta_2) \cdot f(\beta_3) \cdot f(\sigma_e)$$

where:

- $f(\beta_1, \beta_2, \beta_3, \sigma_e|Y)$ is the (joint) posterior distribution of $\beta_1, \beta_2, \beta_3$ and σ_e given Y

- $f(Y|\beta_1, \beta_2, \beta_3, \sigma_e)$ is the conditional likelihood of Y given β_1,β_2, β_3 and σ_e
- $f(\beta_1)$ is the prior distribution for β_1
- $f(\beta_2)$ is the prior distribution for β_2
- $f(\beta_3)$ is the prior distribution for β_3
- $f(\sigma_e)$ is the prior distribution for σ_e

The likelihood and priors are specified in `mean_only.stan`:

```
//single-factor ANOVA
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data {
  int N;                         //sample size - all levels
  int L;                         //number of levels
  real y[N];                     //y values
  int lvl[N];                    //level for this observation
}
parameters {
  real beta[L];                 //Parameters for each level of the single factor
  real<lower=0> sigma_e;         //common error standard deviation for all levels
}
model {

  beta ~ normal(0,100);         //normal priors for each alpha level
  sigma_e ~ cauchy(0,10);        //half-cauchy prior for error standard deviation

  for (i in 1:N){                             //loop through y values
    y[i] ~ normal(beta[lvl[i]],sigma_e);  //y[j] ~ normal(beta[grp],sigma_e)
  }
}
```

With this model file, we have the following specifications:

- The likelihood $f(Y|\beta_1, \beta_2, \beta_3, \sigma_e)$ is $N(\beta_1, \sigma_e)$ at level1
- The likelihood $f(Y|\beta_1, \beta_2, \beta_3, \sigma_e)$ is $N(\beta_2, \sigma_e)$ at level2
- The likelihood $f(Y|\beta_1, \beta_2, \beta_3, \sigma_e)$ is $N(\beta_3, \sigma_e)$ at level3
- The prior $f(\beta_i)$ is $N(0, 100)$, coded as `beta` $\sim$ `normal(0,100)`
- The prior $f(\sigma_e)$ is half-cauchy (because of $< lower = 0 >$) coded as `sigma_e` $\sim$ `cauchy(0,10)`