MTH225 Spring2017 Final Problem 5

A measurement instrument that has a tendency to either consistently overestimate or consistently underestimate the true value is said to be *biased*.

The accuracy of a biased measurement is usually expressed as its **mean square error**, which is defined as the sum of its variance, a measure of precision, and the square of its bias. The variance of a random variable is the square of its standard deviation.

If θ represents the true value of the quantity being measured, and $\hat{\theta}$ is the output of the measurement instrument, then:

$$\mathrm{MSE}(\hat{\theta}) = \mathrm{V}(\hat{\theta}) + (\mathrm{bias}(\hat{\theta}))^2$$

In this exercise, we assume that we have a series of calibration measurements for a digital flow meter. The data consists of the actual flow in gallons per hour, and the corresponding meter reading.

The data is in `MTH225_Spring2017_Final_Problem5.csv`. The variables are:

- `gph` Actual flow in gallons per hour.
- `sensor_reading` Reading on the digital flow meter.

There is a STAN model file called `mean_square_error.stan` on the `example_models.html` web page that you can use for this exercise.

- 2 points: Write R code to read the data and convert it to an R data frame.
- 1 point: Write the data block of a STAN model file that extracts the data from the R workspace.
- 1 point: Write the parameters block of a STAN model file that declares the paramter(s) of your model.
- 2 points: Write the model block of a STAN model file that specifies the priors and likelihood for your model.
- 1 point: Write R code to apply the `extract` function to the data structure output from the `stan` function.
- 1 point: Use the `extract()` function of the RSTAN package to obtain the values for the parameters from the posterior draw.
- 1 point: Use the posterior draw to construct a **90%** confidence interval for the bias. (note: this will require slightly different coding of the `quantile` function)

- 1 point: Use the posterior draw to construct a **90%** confidence interval for the mean square error.

(10 points possible)