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- The probability that a random song on the radio is less than 3 minutes long (not the probability that it is exactly 3 minutes long).

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It doesn't matter if the inequalities are strict or not.

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Visualizing $U(a, b)$

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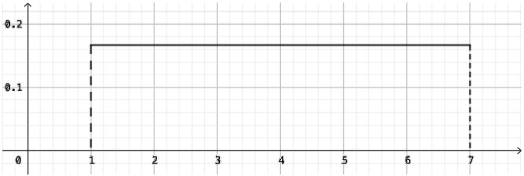

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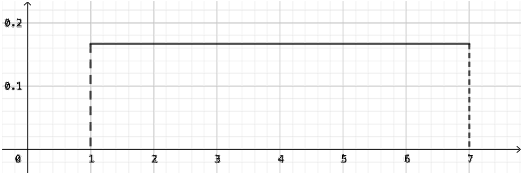

The total area enclosed must be 1. The density curve of a uniform random variable is a horizontal line, reflecting the fact that any two ranges of equal width have equal probabilities.

The height of this rectangle is $1/6$.

Computing using the density curve

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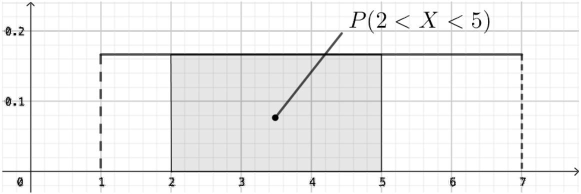

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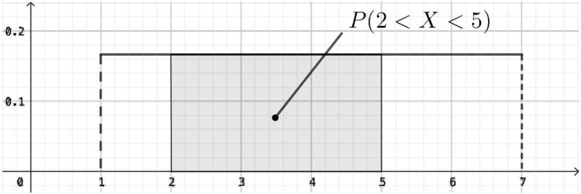

Here we have a rectangle of height 1/6 and width 3. The area is 1/2.

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An example

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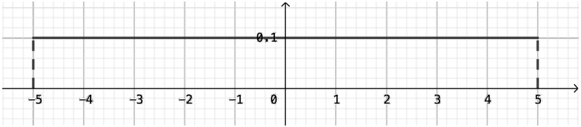

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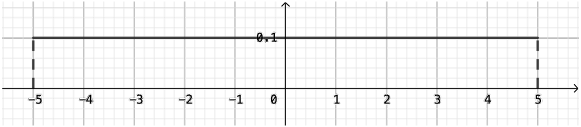

Generally, the equation of the density curve of $U(a, b)$ is

$$y = \frac{1}{b - a}$$

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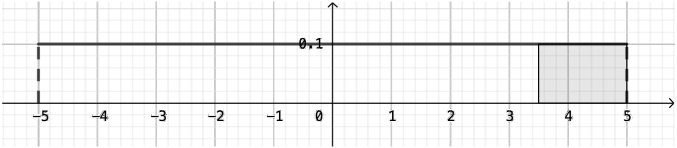

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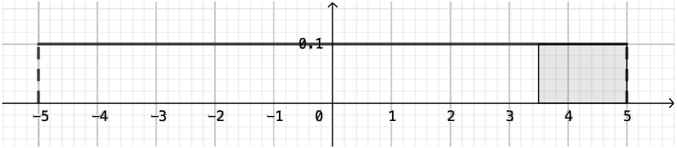

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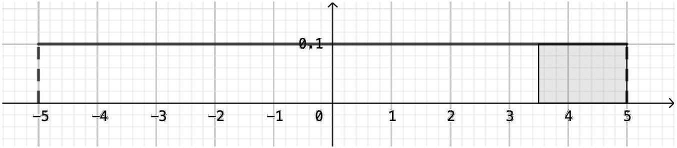

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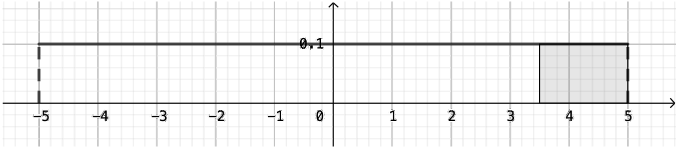

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