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Poisson regression

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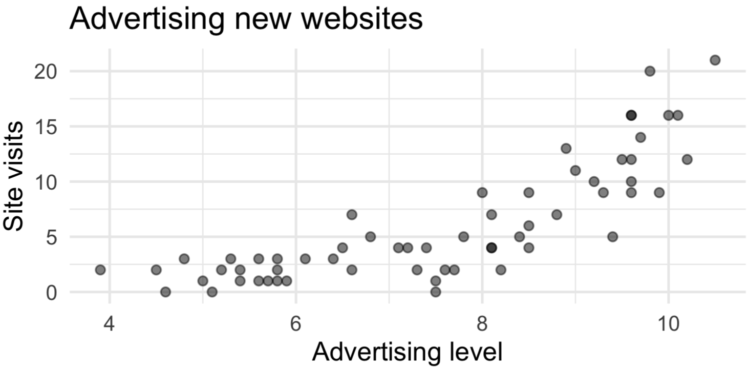
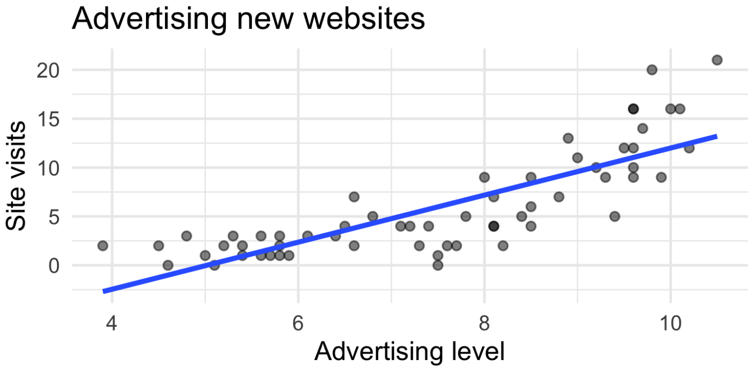


Linear regression is the wrong tool in problems like this.



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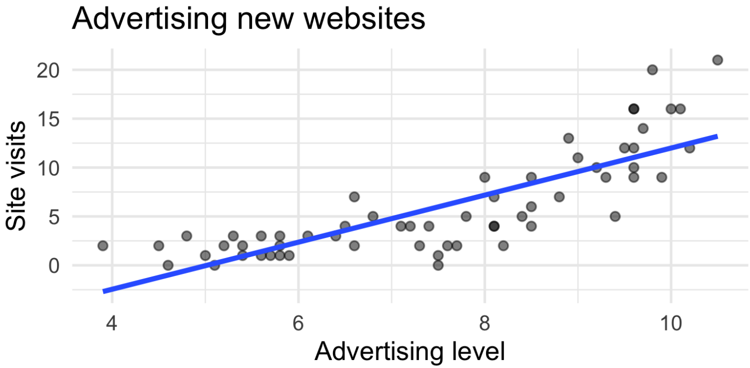

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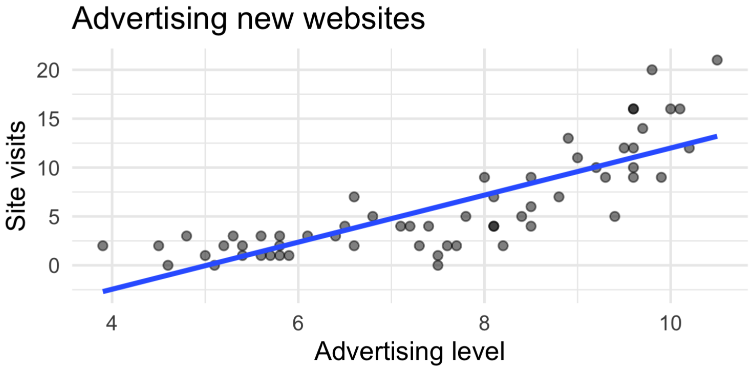

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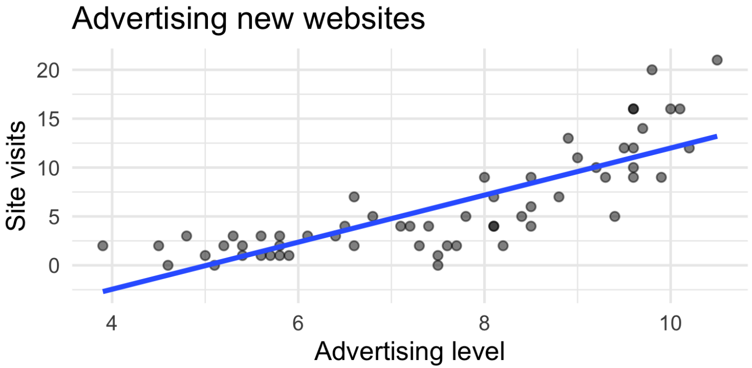

- The shape of the plot isn't linear.
- A regression line may predict negative counts for some x-values.
- The data becomes more variable as the x-value increases.

The poisson model is designed to fit this sort of data. In its simplest form (one quantitative explanatory variable), it looks like this:

$$\begin{cases} \ln(\lambda) &= \beta_0 + \beta_1 x & \text{systematic component} \\ y &\sim \text{pois}(\lambda) & \text{random component} \end{cases}$$

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Because the logarithm function is invertible, the systematic component can also be written:

$$\lambda = e^{\beta_0 + \beta_1 x}$$

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$$\texttt{glm}(\texttt{count} \sim \texttt{level}, \texttt{data} = \texttt{advertising}, \texttt{family} = \text{``}\texttt{poisson}\text{''})$$

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The resulting printout is:

```
Coefficients:
(Intercept)        level
    -2.1227       0.4778

Degrees of Freedom: 59 Total (i.e. Null);  58 Residual
Null Deviance:      264.8
Residual Deviance: 64.84        AIC: 261.8
```

In this example, the maximum likelihood estimators for β_0 and β_1 are

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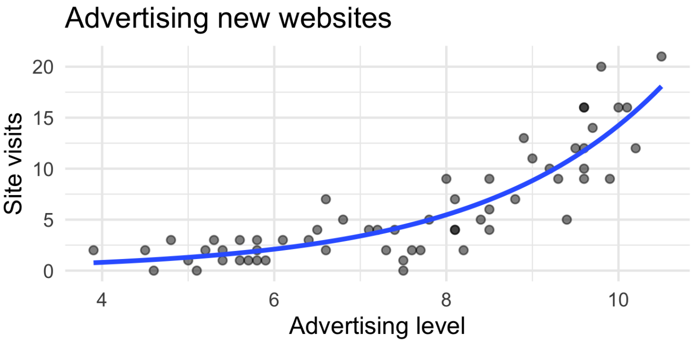
This gives us the following model,

$$\begin{cases} \ln(\hat{\lambda}) & = & -2.12 + .48x & \text{systematic component} \\ y & \sim & \text{pois}(\hat{\lambda}) & \text{random component} \end{cases}$$

This model fits the data very well.

$$\hat{\lambda} = e^{-2.12+.48x}$$



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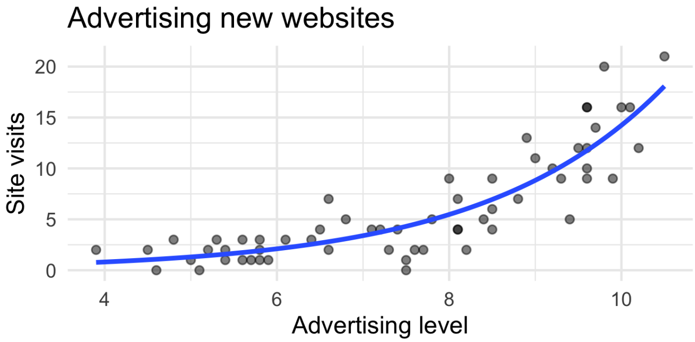

For instance, the model says that at an advertising level of $x = 8$, the expected number of site visits is $\hat{\lambda} = e^{-2.12+.48 \cdot 8} \approx 5.6$. Individual counts at that level will have a poisson distribution with this

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In the advertising example, the predicted mean count when $x = 0$ is $e^{-2.12} \approx 0.12$ and an increase of one unit in x increases the predicted mean count by a factor of $e^{.48} \approx 1.62$.

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- There is no normal distribution in a poisson regression model. At every x-value, the distribution of the response variable is *poisson*.
- The variance of the response variable isn't constant in a poisson regression model. In fact, it's larger for larger values of λ.
- Raw residuals (the differences between the observed and expected y-values) aren't a good measure of model fit since larger errors are expected for larger λ. Deviance and Pearson (standardized) residuals are typically used instead.