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Understanding ANOVA

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$$
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H_0 &: \quad \mu_1 \;=\; \mu_2 \;=\; \cdots \;=\; \mu_m \\
H_a &: \quad \mu_i \neq \mu_j \;\; \text{for some} \;\; i \neq j.
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For instance, doctors may wonder whether people in different age groups respond equally to a new blood pressure medicine, or whether several different medications are all equally effective.

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The corresponding group means are the same in each plot. *The differences between group means is more significant in the first case because of the lower variability within groups.*

More specifically, ANOVA looks at the ratio of the between-groups variance and within-groups variance, both appropriately weighted.

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$$F = \frac{SS_T/(m-1)}{SS_E/(n-m)}$$

where m is the number of groups, n is the total sample size, and

$$SS_T = \sum_i n_i(\overline{Y}_i - \overline{Y})^2 = \text{Sum of squared errors between groups}$$

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If the null hypothesis is true, this ratio has a known distribution (an F-distribution) which we can use to compute a p-value and make a decision.

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ANOVA is fairly robust when it comes to the second two assumption (particularly normality). Independence is more serious as it's usually a consequence to the way the study was designed.

Example. Researchers measure the petal lengths (in mm) of a certain species of flower at three different latitudes. The sample data is shown below.

$$\begin{array}{lccccccc} A: & 6.4, & 6.0, & 4.8, & 5.0, & 4.4 & & \\ B: & 4.5, & 4.0, & 2.3, & 2.6, & 3.2, & 6.0 & \\ C: & 7.5, & 6.6, & 4.8, & 1.0, & 2.6, & 6.0, & 4.5 \end{array}$$

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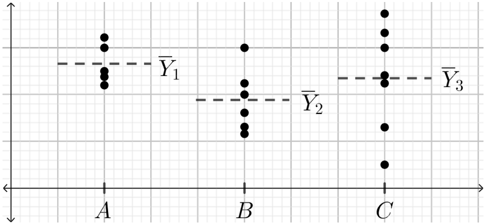
A simple plot illustrates the variability within and between groups in this sample.



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Source	df	SS	Mean Square	F	p-value
Treatment	2	6.83	3.42	1.18	.335
Error	15	43.55	2.90		
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In practice, you probably only care about the p-value, which represents the probability of data like that found in the sample occurring by chance when the null hypothesis is true. Lower p-values indicate stronger evidence against the claim that the quantitative variable is independent from the categorical one.

Even when H_0 is rejected, the conclusion of an analysis of variation is rather weak. The alternative hypothesis

$$H_a: \quad \mu_i \neq \mu_j \quad \text{for some} \quad i \neq j$$

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The most common post-hoc test is the **Tukey honest significant differences test**. Roughly speaking, this test performs multiple t-tests between groups, limiting the total probability of a type I error by demanding smaller p-values in those pairwise tests.