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What is the χ^2 Distribution?

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The sampling distribution of the random variable χ^2 is called the **χ^2-distribution with r degrees of freedom**, or $\chi^2(r)$ for short.

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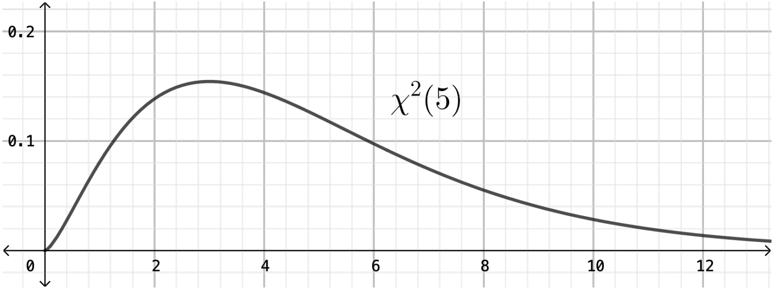

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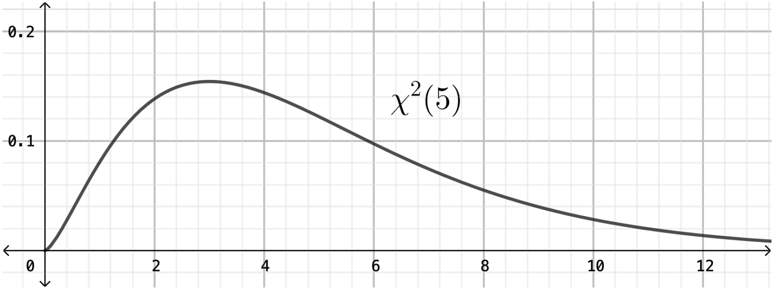

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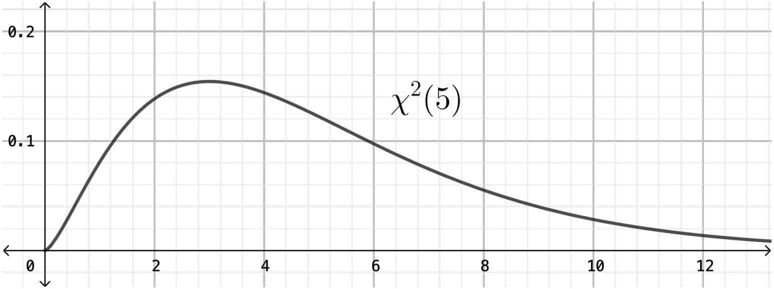

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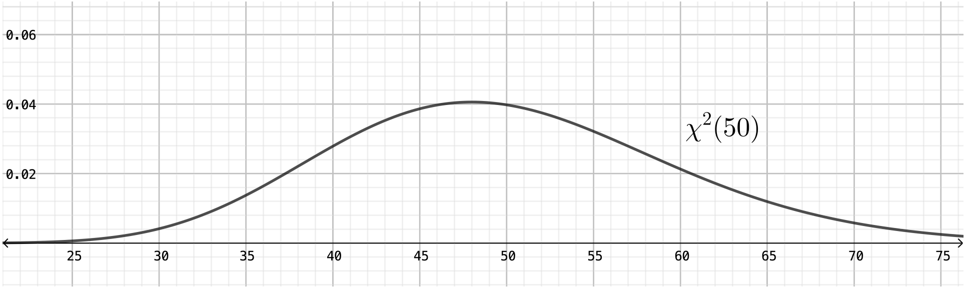
Fact 2. The mode (peak) of $\chi^2(r)$ is $r-2$ if $r \geq 2$ and zero otherwise.

When r is large, the distribution $\chi^2(r)$ is approximately normal. This is already visible when $r = 50$, as pictured below.



If you look closely you can still see the skew in this plot, however.

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- **Significance testing for variance.** When drawing samples of size n from a normal distribution with hypothesized variance σ^2, the sampling distribution of $(n-1)S^2/\sigma^2$ is $\chi^2(n-1)$, where S^2 is the sample variance.

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- **Goodness-of-fit testing.** When a categorical variable is hypothesized to have a certain distribution, the sampling distribution of $\sum \frac{(O-E)^2}{E}$ is approximately $\chi^2(n-1)$, where n is the number of categories, O is the observed count in each category, and E is the expected count under the hypothesized distribution.

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- **The χ^2-test for independence.** A similar test statistic can be used when testing whether two categorical variables are independent of one another.

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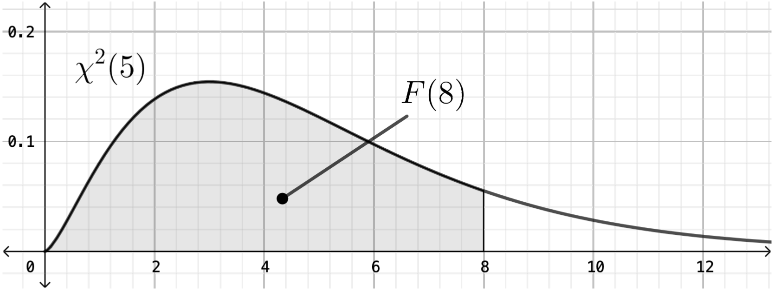
For instance, $F(8) = P(X \leq 8)$ in $\chi^2(5)$ is pictured below.



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```r
pchisq(8, 5)
```

```
## [1] 0.8437644
```