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The Gamma Distribution in *R*

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Base *R* functions use the rate parameter λ by default.



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Here and elsewhere, the scale parameter θ may be used instead, but it must be named explicitly.

```
rgamma(6, 2, scale = 2)
```

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- `qgamma(p, alpha, lambda)` is the inverse distribution function. It returns the value x such that $\text{pgamma}(x, \alpha, \lambda) = p$. In other words, it computes quantiles in the specified gamma distribution. Again, p can be a vector. For instance,

```
qgamma(c(.2, .4, .6, .8), 2, .5)
```

```
## [1] 1.648777 2.752843 4.044626 5.988617
```

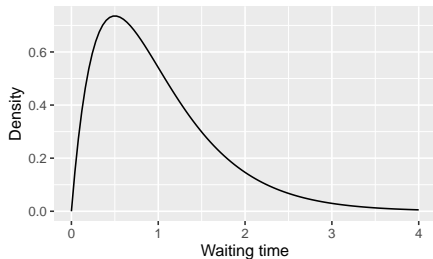


- `dgamma(x, alpha, lambda)` is the probability density function (pdf) of the gamma distribution with shape parameter `alpha` and rate parameter `lambda`.



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```
library(tidyverse)
ggplot() +
  xlim(c(0, 4)) +
  geom_function(fun = dgamma,
               args = list(shape = 2, scale = .5)) +
  labs(x = "Waiting time", y = "Density")
```



Example. Calls to a customer-service line come at an average rate of 1 every 3 minutes.

1. What is the probability that more than an hour elapses before 25 calls come in?
2. What is the 95th percentile for time needed for 5 calls to come in?
3. Simulate waiting times for 5 calls 1000 times. Plot the results.

