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Goodness of fit testing - example

Example. In a random sample of 100 three-child families, the distribution of girls was found to be

# Girls	0	1	2	3
Frequency	12	31	36	21

Is it plausible that the number of girls in such families has the distribution $\text{bin}(3, .5)$, as one might expect? Test at significance level $\alpha = .05$.

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So in 100 trials, the expected values for the four possibilities are

# Girls	0	1	2	3
Expected Frequency	12.5	37.5	37.5	12.5

We compute the χ^2 statistic in the usual way:

$$\begin{aligned}\chi^2 &= \frac{(12 - 12.5)^2}{12.5} + \frac{(31 - 37.5)^2}{37.5} + \frac{(36 - 37.5)^2}{37.5} + \frac{(21 - 12.5)^2}{12.5} \\ &\approx 6.99\end{aligned}$$

In $\chi^2(3)$, $P(\chi^2 \geq 6.99) = .072$.

This data does not provide sufficient evidence to conclude that the distribution of girls in three-child families is not binomial.