Andrew Gard - equitable.equations@gmail.com

Percentiles of Continuous Distributions

The p$^{\text{th}}$ **percentile** of a continuous random variable X is the value π_p such that

$$P(X \leq \pi_p) \; = \; p\%$$

The p$^{\text{th}}$ **percentile** of a continuous random variable X is the value π_p such that

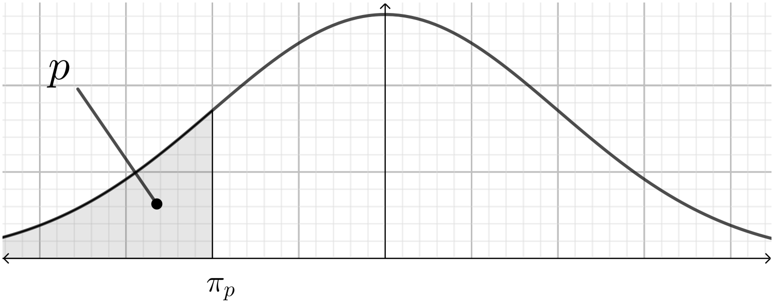
$$P(X \le \pi_p) = p\% = \int_{-\infty}^{\pi_p} f(x)\,dx = F(\pi_p)$$

where $f(x)$ is the probability density function (pdf) and $F(x)$ is the cumulative distribution function (cdf) of X.

The p$^{\text{th}}$ **percentile** of a continuous random variable X is the value π_p such that

$$P(X \leq \pi_p) \; = \; p\% \; = \; \int_{-\infty}^{\pi_p} f(x)\,dx \; = \; F(\pi_p)$$

where $f(x)$ is the probability density function (pdf) and $F(x)$ is the cumulative distribution function (cdf) of X. In other words, there is a $p\%$ chance that X is less than or equal to π_p.



In most of this vid, p represents a percentage between 0 and 100. It's also common to use decimal equivalents between 0 and 1. Those are sometimes referred to as **quantiles**. Other times the terms are just used interchangeably.

Example. Consider a random variable X with the following pdf:

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the 10^{th} percentile.

Example. Consider a random variable X with the following pdf:

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the 10^{th} percentile.

$$10\% = \int_{-\infty}^{\pi_{10}} f(x)\, dx$$

Example. Consider a random variable X with the following pdf:

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the 10^{th} percentile.

$$10\% = \int_{-\infty}^{\pi_{10}} f(x)\,dx = \int_0^{\pi_{10}} \left(1 - \frac{1}{2}x\right)\,dx$$

Example. Consider a random variable X with the following pdf:

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the 10^{th} percentile.

$$10\% = \int_{-\infty}^{\pi_{10}} f(x)\,dx = \int_0^{\pi_{10}} \left(1 - \frac{1}{2}x\right)\,dx$$

$$.10 = \left(x - \frac{1}{4}x^2\right)\Big|_0^{\pi_{10}}$$

Example. Consider a random variable X with the following pdf:

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the 10^{th} percentile.

$$10\% = \int_{-\infty}^{\pi_{10}} f(x)\,dx = \int_{0}^{\pi_{10}} \left(1 - \frac{1}{2}x\right)\,dx$$

$$.10 = \left(x - \frac{1}{4}x^2\right)\Big|_0^{\pi_{10}} = \pi_{10} - \frac{1}{4}\pi_{10}^2$$

Example. Consider a random variable X with the following pdf:

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the 10^{th} percentile.

$$\begin{aligned} 10\% &= \int_{-\infty}^{\pi_{10}} f(x)\,dx = \int_0^{\pi_{10}} \left(1 - \frac{1}{2}x\right) dx \\ .10 &= \left(x - \frac{1}{4}x^2\right)\Big|_0^{\pi_{10}} = \pi_{10} - \frac{1}{4}\pi_{10}^2 \\ 0 &= \frac{1}{4}\pi_{10}^2 - \pi_{10} + .10 \end{aligned}$$

Example. Consider a random variable X with the following pdf:

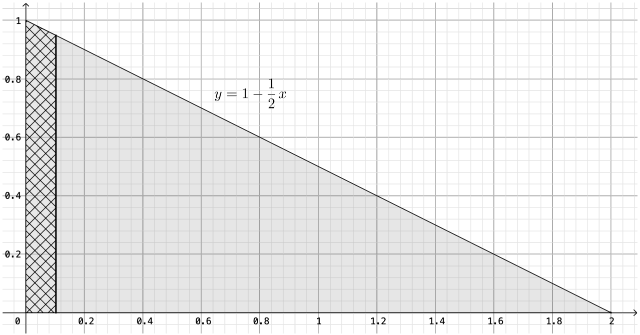
$$f(x) = \begin{cases} 1 - \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Compute the 10^{th} percentile.

$$\begin{aligned} 10\% &= \int_{-\infty}^{\pi_{10}} f(x)\,dx = \int_0^{\pi_{10}} \left(1 - \frac{1}{2}x\right)\,dx \\ .10 &= \left(x - \frac{1}{4}x^2\right)\Big|_0^{\pi_{10}} = \pi_{10} - \frac{1}{4}\pi_{10}^2 \\ 0 &= \frac{1}{4}\pi_{10}^2 - \pi_{10} + .10 \end{aligned}$$

This quadratic equation has solutions .103 and 3.90, but only the first value is in the support of X. That must be the answer. $\pi_{10} = .103$.

Represented visually,



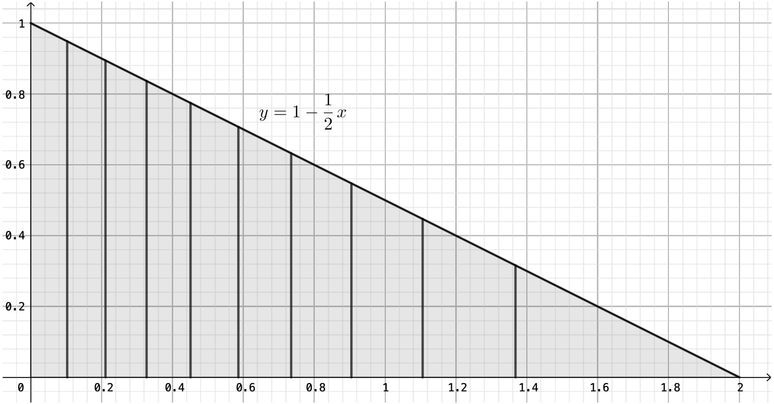
So 10% of the area under $y = 1 - \frac{1}{2}x$ lies to the left of $x = .103$ and

$$P(X < .103) = 10\%$$

The **quartiles** of a continuous random variable are the $25^{th}, 50^{th}$ and 75^{th} percentiles. The **deciles** are the $10^{th}, 20^{th}, 30^{th}$, etc. percentiles.

The **quartiles** of a continuous random variable are the $25^{th}, 50^{th}$ and 75^{th} percentiles. The **deciles** are the $10^{th}, 20^{th}, 30^{th}$, etc. percentiles.

The random variable X from the last example has deciles .103, .211, 327, etc.



All of these shaded regions have equal areas.

The **median** M of a continuous random variable is the 50^{th} percentile, which is also the second quartile and the fifth decile.

The **median** M of a continuous random variable is the 50^{th} percentile, which is also the second quartile and the fifth decile. In the last example, $M = .586$. Note that this is **not** the same as the expected value, which in this case is $\mu = .667$.