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## The log-normal distribution in $R$

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A log-normal distribution is completely determined by two parameters,  $\mu$  and  $\sigma$ , the mean and standard deviation of the corresponding normal distribution. These are **not** the mean and standard deviation of the log-normal distribution itself!



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The following code gives 12 values from a distribution whose logarithm is normal with mean 6.1 and standard deviation 1.0.

```
rlnorm(12, 6.1, 1.0)
```

```
## [1] 1576.48708 321.74736 1685.46920 1591.49534 674.95190 95.58826  
## [7] 176.16710 332.04801 443.29383 4937.69247 956.79912 200.53539
```





- `plnorm(q,  $\mu$ ,  $\sigma$ )` returns the probability that a random observation  $X$  is less than or equal to  $q$ . It's the cumulative distribution function of the log-normal,  $P(X \leq q)$ .



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The following code computes  $P(X \leq 200)$ ,  $P(X \leq 400)$ ,  $P(X \leq 800)$ , and  $P(X \leq 1600)$  in the distribution whose logarithm is normal with mean 6.1 and standard deviation 1.0.

```
plnorm(c(200, 400, 800, 1600), 6.1, 1.0)
```

```
## [1] 0.2113683 0.4567855 0.7205956 0.8993328
```



- `qlnorm(p,  $\mu$ ,  $\sigma$ )` returns the quantity  $q$  such that `plnorm(q,  $\mu$ ,  $\sigma$ ) =  $p$` . It's the inverse cdf of the specified log-normal distribution. Again,  $p$  can be a vector.



- `qlnorm(p,  $\mu$ ,  $\sigma$ )` returns the quantity  $q$  such that `plnorm(q,  $\mu$ ,  $\sigma$ ) = p`. It's the inverse cdf of the specified log-normal distribution. Again,  $p$  can be a vector.

The following code computes the 10<sup>th</sup>, 30<sup>th</sup>, 50<sup>th</sup>, 70<sup>th</sup>, and 90<sup>th</sup> percentiles in the distribution whose logarithm is normal with mean 6.1 and standard deviation 1.0.

```
qlnorm(c(.1, .3, .5, .7, .9), 6.1, 1.0)
```

```
## [1] 123.7729 263.9077 445.8578 753.2525 1606.0798
```

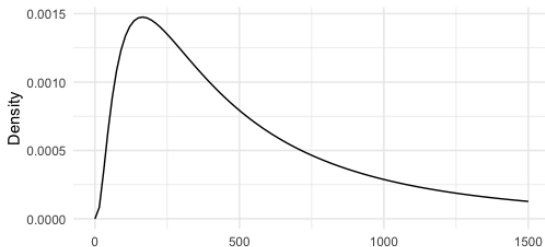


- `dlnorm(x,  $\mu$ ,  $\sigma$ )` is the probability density function (pdf) for the specified log-normal distribution.



- $\text{dlnorm}(x, \mu, \sigma)$  is the probability density function (pdf) for the specified log-normal distribution. It's mostly used for graphing and for theoretical calculations.

```
ggplot() +  
  geom_function(fun = dlnorm,  
               args = list(mean = 6.1,  
                           sd = 1.0)) +  
  xlim(0, 1500) +  
  labs(y = "Density")
```



**Example.** Freshman undergraduate enrollments at U.S. colleges have an approximate log-normal distribution with parameters  $\mu = 6.1$  and  $\sigma = 1.0$ .

1. What is the probability that a randomly-selected college enrolls between 500 and 1000 freshmen in a year?
2. What is the 99<sup>th</sup> percentile for U.S. college enrollment?
3. Simulate selecting 100 colleges at random. Plot a histogram of freshman enrollments at those colleges.

