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Statistical Power and Sample Size

Specifying both α and β

It's common to want a statistical test to be both powerful and significant. That it, we might want to limit the chances of both a type I error and a type II error. All other things be equal, we can accomplish this by carefully choosing the size of the sample we collect.

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For instance, suppose we want to test the hypotheses

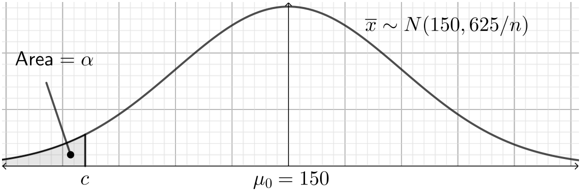
$$\begin{aligned} H_0 &: \quad \mu = 150 \\ H_a &: \quad \mu < 150 \end{aligned}$$

when we know that the population is approximately normal with standard deviation $\sigma = 25$. Suppose we require significance level $\alpha = .05$ and a maximum probability of a type II error of $\beta = .10$ when $\mu = 140$.

What is the minimum sample size required?

Considering α

Under H_0, the sample mean $\overline{x}$ is normal with mean 150 and standard deviation $25/\sqrt{n}$. The critical region will be one sided, and will depend on n. Let's write it as $\overline{x} < c$.



Algebraically, we have

$$\Phi\left(\frac{c - 150}{25/\sqrt{n}}\right) = .05$$

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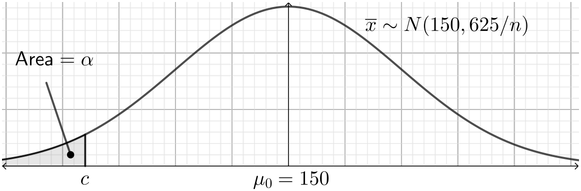

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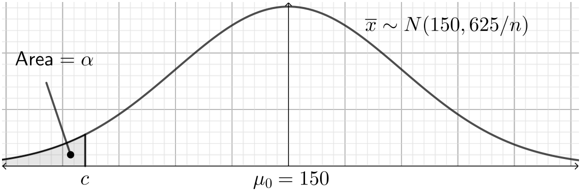
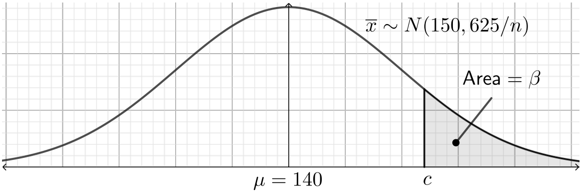


Algebraically, we have

$$\begin{aligned}
\Phi\left(\frac{c-150}{25/\sqrt{n}}\right) &= .05 \\
\frac{c-150}{25/\sqrt{n}} &= -1.645 \\
c &= 150 - 41.13/\sqrt{n}
\end{aligned}$$

Considering β

When the true mean μ is equal to 140, there is a 10% chance that the null hypothesis will not be rejected, that is, that $\overline{x} \geq c$.



Algebraically, we have

$$1 - \Phi\left(\frac{c - 140}{25/\sqrt{n}}\right) \quad = \quad .10$$

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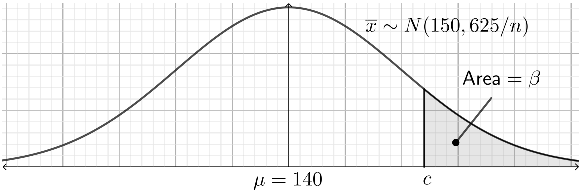

Algebraically, we have

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\begin{aligned}
1 - \Phi\left(\frac{c - 140}{25/\sqrt{n}}\right) &= .10 \\
\frac{c - 140}{25/\sqrt{n}} &= 1.28
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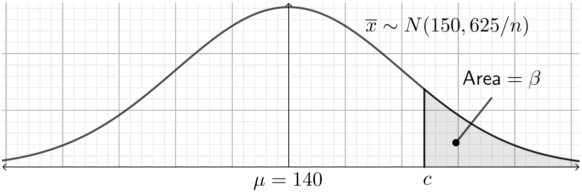

Algebraically, we have

$$\begin{aligned} 1 - \Phi\left(\frac{c - 140}{25/\sqrt{n}}\right) &= .10 \\ \frac{c - 140}{25/\sqrt{n}} &= 1.28 \\ c &= 140 + 32/\sqrt{n} \end{aligned}$$

Solving for n and c

Overall, we have two equations using c and n:

$$
\begin{aligned}
c &= 150 - 41.13/\sqrt{n} \\
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Solving these equations simultaneously gives us $n = 53.5$ and $c = 144.4$. As always, we should round the sample size up, regardless of how close it is to its floor.

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We conclude that a sample of size 54 will ensure that the probability of a false positive is less than $\alpha = .05$ and the probability of a false negative when $\mu = 140$ is less than $\beta = .10$.