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Experiment on Feedback-Loop

By: Koki Harada, M5R-No,24

Sota Fujita, M5R-No,25

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1. Abstract

In this experiment, the fundamentals of feedback control are learned from an electric circuit of a second-order delay system. First, a Bode diagram is drawn from the circuit of a second-order delay system, then the circuit is modified to enable feedback, and changes in the Bode diagram are observed. By discussing the errors between the experimental and theoretical values obtained in the experiments, students will deepen their understanding of feedback control using closed-loop circuits.

2. Introduction

Servo systems that provide feedback control of machine position and speed are used to control robots and machine tools. In this section, the basic design procedure of a servo system with feedback control is understood through experiments. In this experiment, the following tasks will be carried out:

1) Modeling of the control system

A mathematical model is created from the structure and characteristic parameters of the control object to obtain theoretical values of the response characteristics. Next, input/output data are taken from the actual control system to obtain experimental values of the response characteristics. By comparing these theoretical and experimental values, the validity of the mathematical model of the control system used in the control system design is evaluated. Here, the response characteristics are evaluated by drawing a Bode diagram using the frequency response method.

2) Design of the controller

Using the mathematical model obtained in 1), we will propose a method to calculate the necessary feedback-loop gain to achieve a control objective. The control objective will be given by the teacher. The step response of the closed-loop system will also be computed.

3) Performance evaluation of the real control system

A closed-loop system is constructed by incorporating the compensator and its response characteristics are evaluated to see whether they satisfy the specifications given in 2). If not, consider the causes and take countermeasures.

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3. Methods

3-1 Knowledge required for the experiment

1) Automatic control, Transfer function, Bode plot, Time response, Electronic circuits, Feedback control theory

The breadboard used to assemble the circuit is shown in Fig. 1. The wiring was done in the following color scheme.

Red: +15

Blue: -15

Green: Input signal

Black: GND

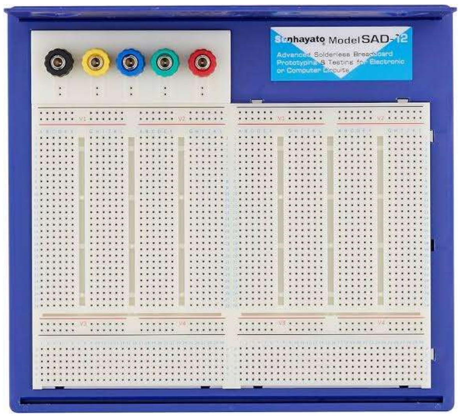

Fig. 1 Breadboard for Electronic circuit

2) OP amp transfer function and usage

The relationship between the input voltage and output voltage of an ideal OP amplifier is expressed by the following equation with the amplification ratio A.

$$A = \frac{V_0}{V_i} = \frac{-Z_0 i}{Z_i i} = -\frac{Z_0}{Z_i}$$

Point p in Fig. 2 is called a virtual short or imaginary short, and its potential is 0 [Volt], which means that no current flows into the OP amplifier. The circuit diagram is shown in Fig. 2. The pin layout of the OP amplifier (μ PC 4560) used in this study is shown in Fig. 3.

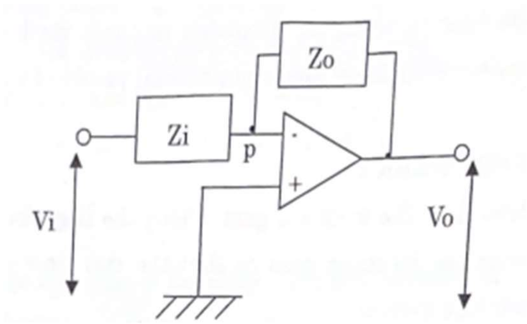

Fig. 2 The amplifier circuit with OP amp

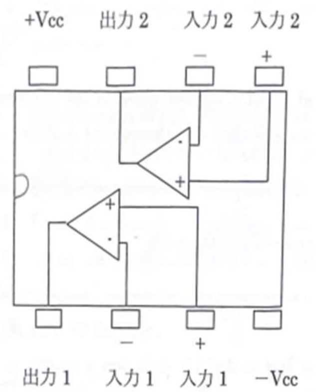

Fig. 3 The arrangement of OP

In Fig. 4, the input-output relationship is as follows.

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{V_0}{R_0}$$

From this relationship, by choosing the value of the resistor used in the circuit

By choosing the value of the resistor to be used in the circuit, an analog signal addition circuit can be realized using an OP amplifier.

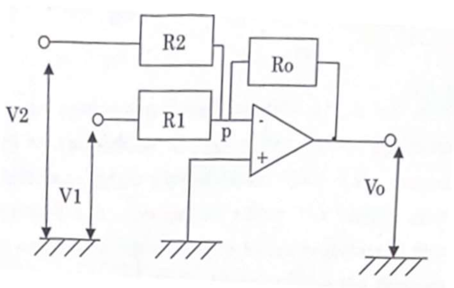

Fig.4 The adder circuit with OP amp

3) Electronic components understanding, Resistances, Capacitors

The impedance of a resistor and a capacitor can be calculated as follows.

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$$Z = R + \frac{1}{j\omega C} \qquad \frac{1}{Z} = \frac{1}{R} + j\omega C$$

$$\therefore Z = \frac{R}{1 + j\omega CR}$$

4) Usage of electronic measure devices, Oscilloscopes, Function generators

3-2 Experiment procedure

1) Plant description

The electronic circuit on Fig. 5 will be the plant studied in this experiment.

a) Find the transfer function of the plant on Fig. 5.

b) Using the mathematical model, build the Bode graph of the plant (Gain, Phase).

c) Build the physical electronic system on Fig. 5 (Be sure to apply the safety measures)

d) Use a sinusoidal (AC) input to determine the gain and phase of the plant.

e) Compare and comment the results obtained theoretically (b.) and in practice (d.).

2) Design of the control system

The objective of the control is to halve the rise time of the plant. For this, we will:

a) Calculate the theoretical gain to satisfy those condition.

b) Calculate the transfer function of the closed-loop system, and the theoretical step response.

c) Propose an electronic circuit to realize the feedback control, as well as the constant of the components.

3) Improvement of the control system

The final objective of this experiment is to improve and discuss the control system:

a) Build the physical electronic system proposed at the end of 2)

b) Using a square input for the circuit, observe the step response of the closed loop.

c) Compare the results between 2) and 3).

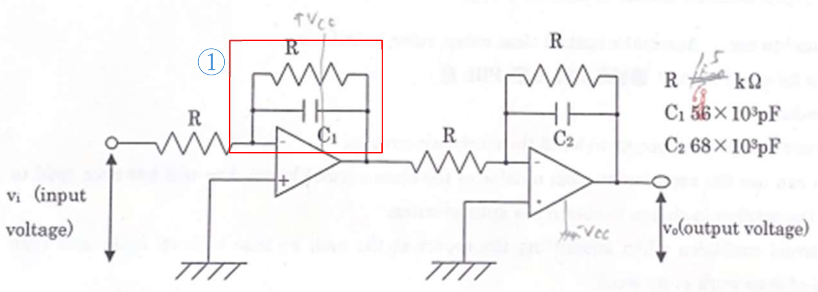

Fig. 5 Circuit diagram of Open-loop system

4. Results

4- 1 WORK FOR WEEK 1

1) Build a mathematical model of the Plant on Fig. 5, and draw the bode graph
2) Connect the circuit to the measurement equipment and plot the experimental values on the Bode diagram using the frequency response method on the Oscilloscope
3) Compare theoretical and experimental values of the Bode diagram and discuss

First, consider (1) in Fig. 5. According to the parallel circuit formula, the impedance is

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{\frac{1}{j\omega cR}}$$

$$Z = \frac{R}{1 + j\omega cR}$$

The nature of the OP amplifier then yields the following input-output relationship.

$$Let\ j\omega = s$$

$$\frac{Z^2}{R^2} = \frac{1}{(1 + \xi cR)^2}$$

$$= \frac{\frac{1}{R^2c^2}}{s^2 + \frac{2\xi}{Rc} + \frac{1}{R^2c^2}} \ldots (1)$$

$$Let\ \frac{1}{RC} = \omega_n$$

$$(1) = \frac{{\omega_n}^2}{s^2 + 2s\xi\omega_n + {\omega_n}^2}$$

$$G_1(s) = \frac{{\omega_n}^2}{s^2 + 2s\xi\omega_n + {\omega_n}^2}$$

where ω_n is the natural angular frequency, a parameter that determines the speed of motion, and ζ is the damping coefficient, a parameter that determines the presence or absence of vibration and the speed of convergence.

Fig. 6 and Fig. 7 show the experimental results. Fig. 8 shows a Bode diagram of the theoretical values obtained from the mathematical model.

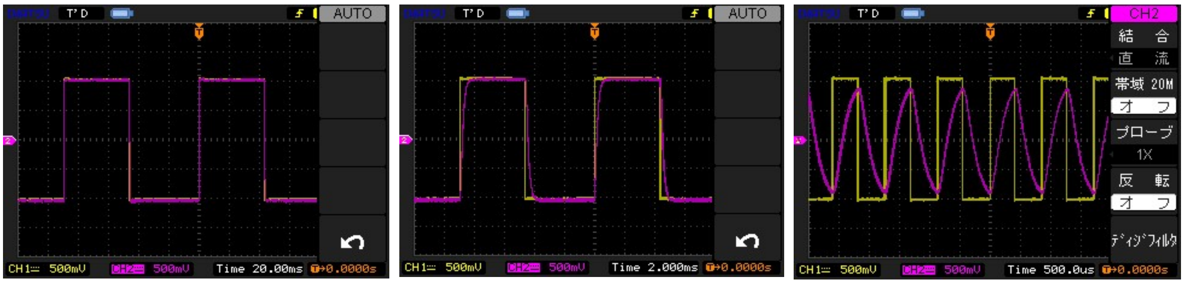

Fig. 7 Result of Experiment1(2) from left.1khz,10khz,100khz

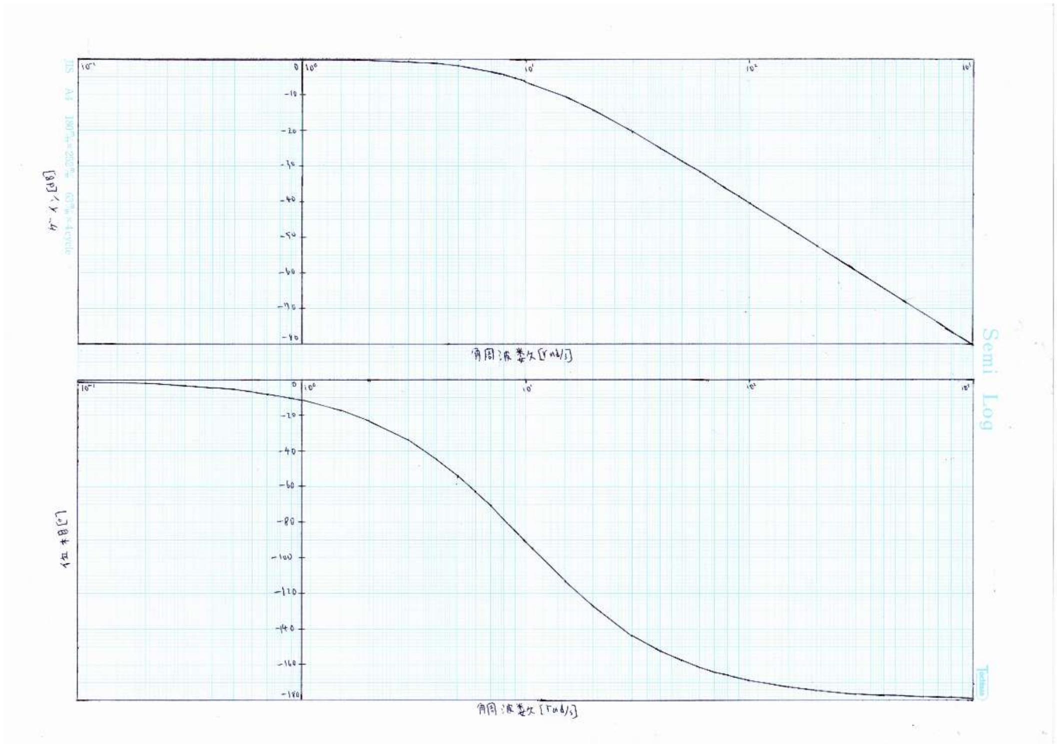

Fig. 8 Bode graph

The experimental gain was 0 dB from 1 to 10khz and -0.9151dB only at 100khz, and the phase difference was 0° from 1 to 1khz, 24° at 10khz and 180° at 100khz.

4-2 WORK FOR WEEK 2

1) Determine the feedback gain. Using the transfer function of the circuit fabricated in week 1, design the feedback gain so that the rise time of the closed-loop system is half that of the open loop system.
2) Determine the physical component necessary to achieve the objective in (1). Draw a Diagram representing the electronic circuit for the proposed closed-loop feedback system.

A block diagram of a closed-loop system is shown in Fig. 9. A closed-loop system generally consists of a controller, an actuator, a plant, and a sensor, and can optimally control a target value by adding values from the sensor to the input values. In this experiment, since a closed-loop system is created using only electrical signals, an actuator that works in the external environment and a sensor that reads the external environment are not necessary. Therefore, as shown in Fig. 10, we constructed a closed-loop system that does not take the actuator and sensor into account.

Fig. 11 shows a circuit diagram that reproduces the block diagram shown in Fig. 10.

The feedback gain is determined from the block diagram in Fig. 10. The feedback gain is obtained by the following calculation.

$$G_2(s) = \frac{KG_1(s)}{1+KG_1(s)}$$

$$= \frac{K\dfrac{{\omega_n}^2}{{\omega_n}^2+2\xi\omega_n s+{\omega_n}^2}}{1+K\dfrac{{\omega_n}^2}{{\omega_n}^2+2\xi\omega_n s+s^2}}$$

$$= \frac{K{\omega_n}^2}{s^2+2\xi\omega_n s+(1+K){\omega_n}^2}$$

$$\therefore \omega_n{}' = \sqrt{1+K}\omega_n$$

$$= \frac{\dfrac{K}{1+K}{\omega_n}'^2}{s^2+2\xi\omega_n s+{\omega_n}'^2}$$

$$\therefore \xi' = \frac{1}{\sqrt{1+K}}$$

$$G_2(s) = \frac{K}{1+K}\frac{{\omega_n}'^2}{s^2+2s\xi'\omega_n{}'+{\omega_n}'^2}$$

$$\text{Step response: } G_2(s)' = \frac{K}{1+K}\frac{{\omega_n}'^2}{s^2+2s\xi'\omega_n{}'+{\omega_n}'^2}\frac{1}{s}$$

$$= \frac{K}{1+K}\left\{\frac{1}{s}-\frac{s+2\xi'\omega_n{}'}{s^2+2s\xi'\omega_n{}'+{\omega_n}'^2}\right\}$$

The rise time is obtained from the following equation

$$\text{Rise time of } \mathrm{G}_1 : T_{1\pi} = \frac{\pi}{\omega_n}$$

$$\text{Rise time of } \mathrm{G}_2 : T_{2\pi} = \frac{\pi}{\omega_n{}'}$$

To reduce the rise time of a closed-loop system to half that of an open-loop system

$$T_{2\pi} = \frac{1}{2} T_{1\pi}$$

Since we can find K such that,

$$\omega_n = \frac{1}{RC} \quad , \quad \omega_n{}' = \sqrt{1+K}\,\omega_n$$

Substituting for

$$\frac{RC\pi}{\sqrt{1+K}} = \frac{RC\pi}{2}$$

$$\sqrt{1+K} = 2$$

$$\therefore K = 3$$

From this result, a Bode diagram of the closed-loop system can be drawn as shown in Fig. 12.

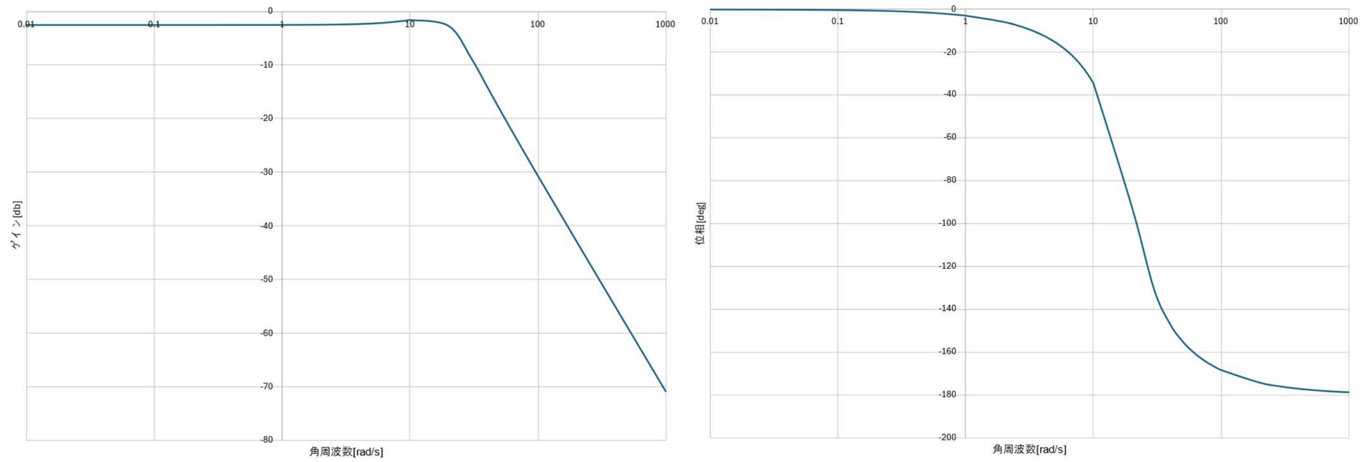

Fig. 12 Bode graph

Therefore, when the closed-loop system is made half the rise time of the open-loop system,

$\therefore R_i$ is the value of the variable resistor

$$\therefore R = 1.5[k\Omega]$$

$$K = \frac{R_i}{R}$$

$$R_i = 4.5[k\Omega]$$

The step response for this case is shown in Fig. 13. The step response is shown in Fig.13

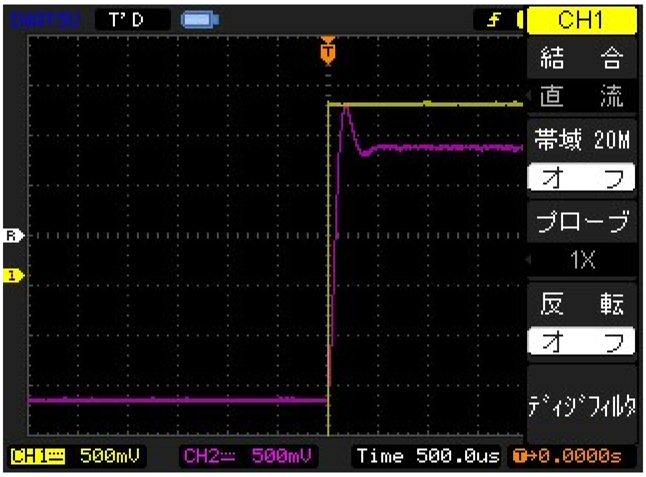

Fig. 13 Result of Experiment2

4- 3 WORK FOR WEEK 3

1) Realize the electronic circuit proposed in week 2, and input a square wave in the system to observe the step response. Discuss the differences between reality and theoretical models.

Fig. 14 shows the step response of the closed-loop system obtained as follows. Fig. 15 shows the response for each resistance value.

$$L^{-1}[A\frac{{\omega_n}'^2}{s^2+2\zeta'{\omega_n}'s+{\omega_n}'^2}\times\frac{1}{s}]$$

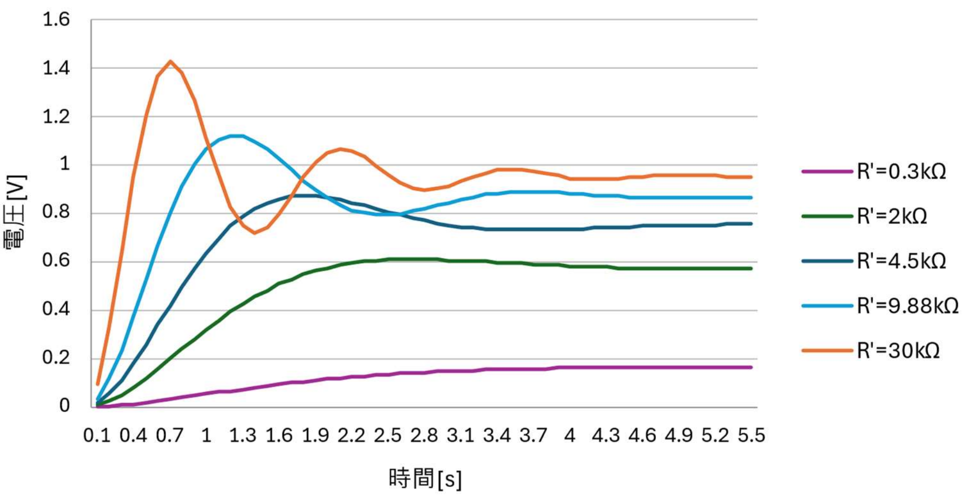

Fig. 14 Step response

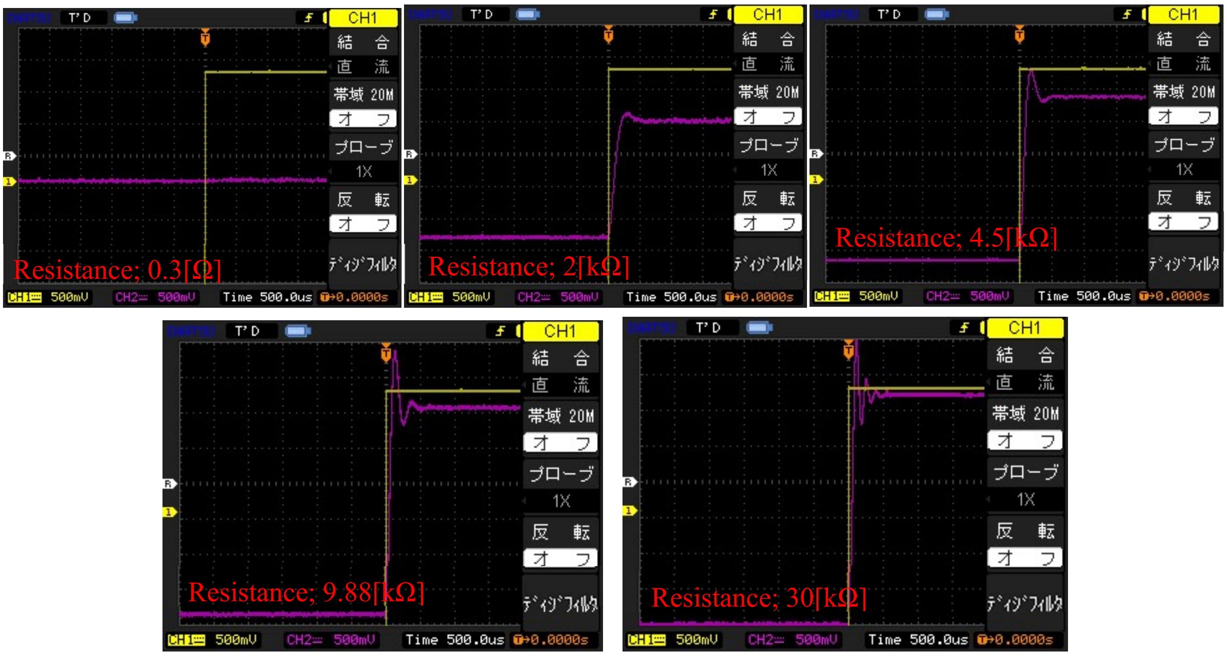

Fig. 15 Result of Experiment3

5. Discussion

5- 1 Week 1 Consideration Assignment

1) Describe the differences between the theoretical Bode diagram and the experimental Bode diagram. Also, quantitatively discuss the causes of the differences.

In the results of Experiment 1, the characteristics change drastically from 10khz to 100khz. Such a drastic change in characteristics is not observed in Fig. 8. Therefore, it is considered that there is some defect in the experimental circuit and that values that differ greatly from the theoretical values were obtained. Considering the values of 10khz and 100khz, this Bode diagram is considered to be a parallel shift of the theoretical Bode diagram to the right, and its change is considered to be abrupt. The first is that there was a problem with the circuit that had been set up, such as a resistor being disconnected, the second is that the oscilloscope was connected incorrectly, and the third is that there was a problem with the extension cable in the PBL room and the equipment was not working properly. The second and third problems occurred in the second and third week of the experiment and were corrected at that time. The first problem was that the transfer function was obtained assuming several cases, such as the case of a resistor being disconnected and the case of a capacitor being disconnected, but no Bode diagram could be found that took values close to the experimental values obtained by the experiment. However, we could not find a Bode diagram with values close to the experimental values. The cause of this problem is unknown, since it is impossible to reproduce it.

In the step response of critical damping (critical damping oscillation) of a second-order delay system treated in this experiment, it is difficult to measure the phase difference directly with an oscilloscope because there is no clear oscillation period, since the system returns to its original state without oscillation. For this reason, comparisons of theoretical and experimental Bode diagrams are often evaluated by the rise time of the response, the maximum value of the excessive response, and the stability time constant.

5-２ Week 2 Consideration Assignment

3) To investigate the phase advance compensation to improve the quick response of feedback systems, to explain its principle, and to consider how to realize the compensator using OP amplifiers.

The phase advance compensator is to improve the quick response by increasing the gain crossing frequency ω_{cg} while maintaining the gain in the low frequency band that dominates the steady-state characteristics and securing the phase margin required based on the specifications. This results in loop shaping so that ω_{cg} after compensation becomes higher frequency than before compensation, and consequently improves the quick response of the feedback control system. Fig. 16 shows the circuit diagram of the phase advance compensation using an OP amplifier.

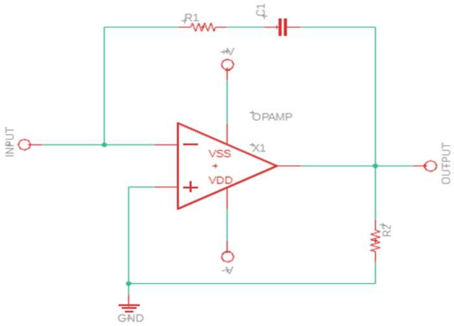

Fig. 16 Circuit diagram of phase advance compensation using OP amplifier

4) To investigate phase delay compensation to improve the steady-state characteristics of feedback systems, to explain its principle, and to consider how to realize its compensator using an OP amplifier.

The basic idea of improving the steady-state characteristics by using phase delay elements is to increase the gain in the low-frequency band and improve the steady-state characteristics without changing the transient characteristics. Fig. 17 shows a circuit diagram of phase delay compensation using an OP amplifier.

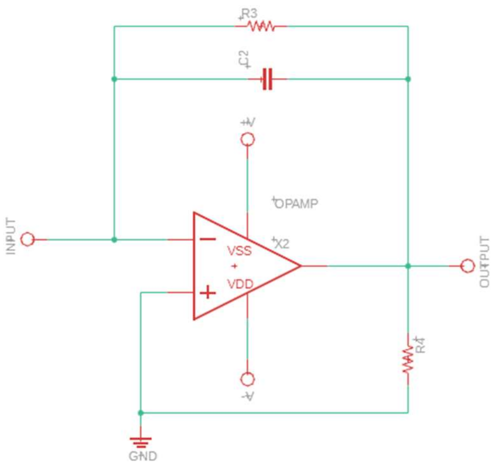

Fig. 17 Circuit diagram of phase delay compensation using OP amplifier

5- 3 Week 2 Consideration Assignment

5) Describe the differences between the theoretical and experimental values of the step response of a closed-loop system. Also, discuss quantitatively the causes of the differences.

Theoretical and experimental values were almost the same. However, the experimental values stabilized at slightly lower values than the theoretical values. The specific percentages were 0.02 at 2 kΩ, 0.05 at 4.5 kΩ, 0.013 at 9.88 kΩ, and 0.0175 at 30 kΩ, which were too small to measure at 0.3 kΩ.

The cause of this phenomenon can be considered to be errors due to temperature changes in electronic components such as resistors and capacitors, resistance of jumper wires, and so on.

6) Pick up one concrete example of a servo system and explain its function, configuration, and implementation method using diagrams.

A typical example of a servo system is the control of a robot arm. The control and configuration of a robot arm are shown in the block diagram in Fig. 18, where the angle and speed of the arm are controlled by feedback using encoders and other devices that act as sensors.

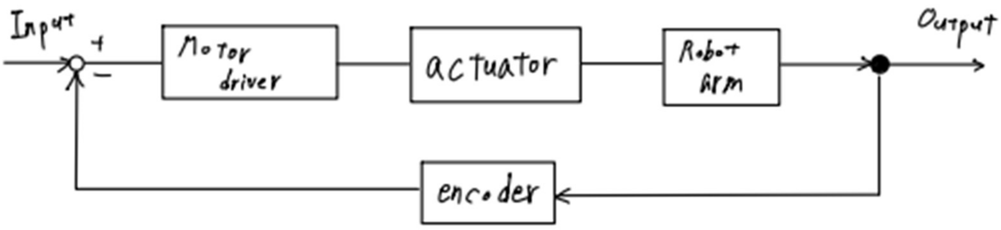

Fig. 18 Block diagram of the robot arm

Reference

1. Control engineering ~the concept of feedback control~ Author: Osami Saito collaboration: Li Xu