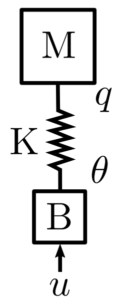
Should we really use $\bar{\boldsymbol{q}}$?

1 Spring-mass

1.1 The system



Dynamics equation:

$$u - K(\theta - q) = B\ddot{\theta} \tag{1}$$
$$K(\theta - q) = M\ddot{q} + g(q) \tag{2}$$

Defining desired motor side position according to link side position:

$$K(\theta_d - q_d) = M\ddot{q}_d \tag{3}$$

Control input:

$$u = -K_q\tilde{q} - D_q\dot{\tilde{q}} + M\ddot{q}_d + B\ddot{\theta}_d + \hat{g}(q) \tag{4}$$
$$\tilde{q} = q - q_d$$

1.2 Problem formulation

Closed loop dynamics:

$$M\ddot{q}_d + B\ddot{\theta}_d - B\ddot{\theta} - K_q\tilde{q} - D_q\dot{\tilde{q}} + \hat{g}(q) = M\ddot{q} + g(q) \tag{5}$$

Power balance equation with flow $\dot{\theta}$:

$$\dot{\theta}\underbrace{M\ddot{q}_d}_{\underbrace{K(\theta_d-q_d)}_{\dot{q}\underbrace{K(\theta_d-q_d)}_{M\ddot{q}_d}+(\dot{\theta}-\dot{q})K(\theta_d-q_d)}} +\dot{\theta}B\ddot{\theta}_d-\dot{\theta}B\ddot{\theta}-\dot{\theta}(K_q\tilde{q}+D_q\dot{\tilde{q}})+\dot{\theta}\hat{g}(q)= \dot{\theta}\underbrace{(M\ddot{q}+g(q))}_{\underbrace{K(\theta-q)}_{\dot{q}\underbrace{K(\theta-q)}_{M\ddot{q}+g(q)}+(\dot{\theta}-\dot{q})K(\theta-q)}} \tag{6}$$

Power balance equation with flow $\dot{\theta}_d$:

$$\dot{\theta}_d\underbrace{M\ddot{q}_d}_{\underbrace{K(\theta_d-q_d)}_{\dot{q}_d\underbrace{K(\theta_d-q_d)}_{M\ddot{q}_d}+(\dot{\theta}_d-\dot{q}_d)K(\theta_d-q_d)}} +\dot{\theta}_dB\ddot{\theta}_d-\dot{\theta}_dB\ddot{\theta}-\dot{\theta}_d(K_q\tilde{q}+D_q\dot{\tilde{q}})+\dot{\theta}_d\hat{g}(q)= \dot{\theta}_d\underbrace{(M\ddot{q}+g(q))}_{\underbrace{K(\theta-q)}_{\dot{q}_d\underbrace{K(\theta-q)}_{M\ddot{q}+g(q)}+(\dot{\theta}_d-\dot{q}_d)K(\theta-q)}} \tag{7}$$

Subtracting (6) from (7):

$$\dot{\tilde{q}}M\dot{\tilde{q}}+\dot{\tilde{\theta}}B\dot{\tilde{\theta}}+\underbrace{\dot{\tilde{\theta}}K_q\tilde{q}}_{\stackrel{?}{=}\dot{S}}\underbrace{-\dot{\tilde{\theta}}\hat{g}(q)+\dot{\tilde{q}}g(q)}_{\stackrel{?}{=}\dot{S}/0}=-\underbrace{\dot{\tilde{\theta}}D_q\dot{\tilde{q}}}_{\stackrel{?}{=}P_D} \tag{8}$$

1.3 Only regulation and Albu-schaeffer idea

Closed loop dynamics (8) will boil down to:

$$\dot{q}M\dot{q}+\dot{\theta}B\dot{\theta}+\dot{\theta}K_q\tilde{q}-\dot{\theta}\hat{g}(q)+\dot{q}g(q)=-\dot{\theta}D_q\dot{q} \tag{9}$$