HW#12 Optimization

Q1. 다음과 같은 이변수 스칼라 함수 f에 대해 아래의 내용을 수행 하시오.

1) -2<=x<=2, -2<=y<=2 구간에서 이 함수의 그래프를 도시 하시오

In[310]:=
```
Function[{x, y}, (x + y) * (x * y + x * y^2)][x, y]
```

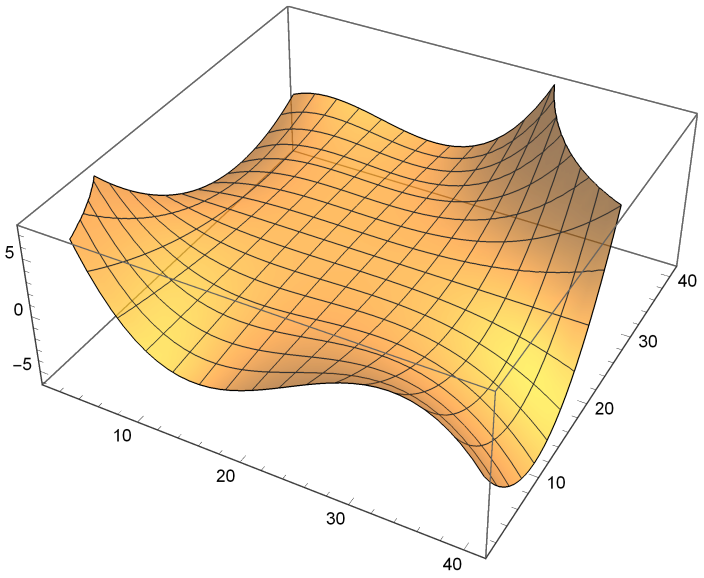
Out[310]=
```
0.
```

In[363]:=
```
f[x_, y_] = (x + y) * (x * y + x * y^2);
Function[{x, y}, (x + y) * (x * y + x * y^2)][x, y]
l = Table[Function[{x, y}, (x + y) * (x * y + x * y^2)][x, y],
   {x, -2, 2, 0.1}, {y, -2, 2, 0.1}];
ListPlot3D[l, ClippingStyle → None, PlotStyle → Opacity[.65]]
Plot3D[Function[{x, y}, (x + y) * (x * y + x * y^2)][x, y], {x, -2, 2}, {y, -2, 2},
 PlotStyle → Opacity[.65], ClippingStyle → None, PlotRange → {{-2, 2}, {-2, 2}, {-2, 2}}]
```

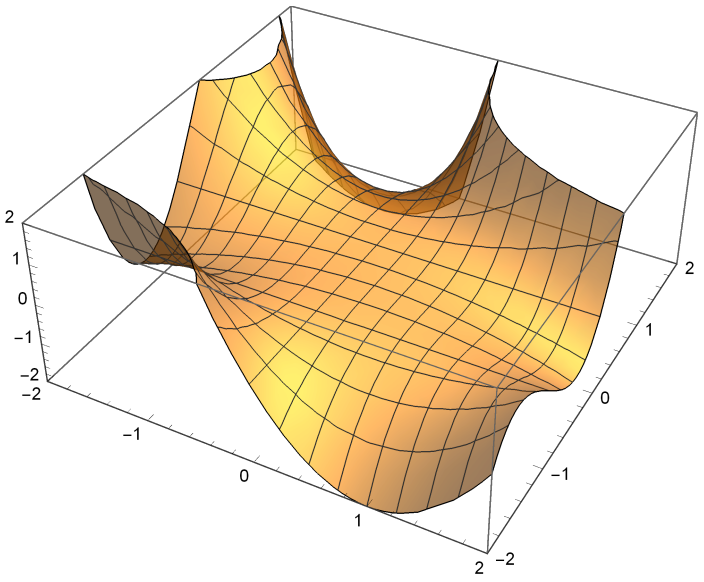
… Set: Tag Real in 0.[x_, y_] is Protected.

Out[364]= $(x + y)\left(x y + x y^2\right)$

Out[366]=

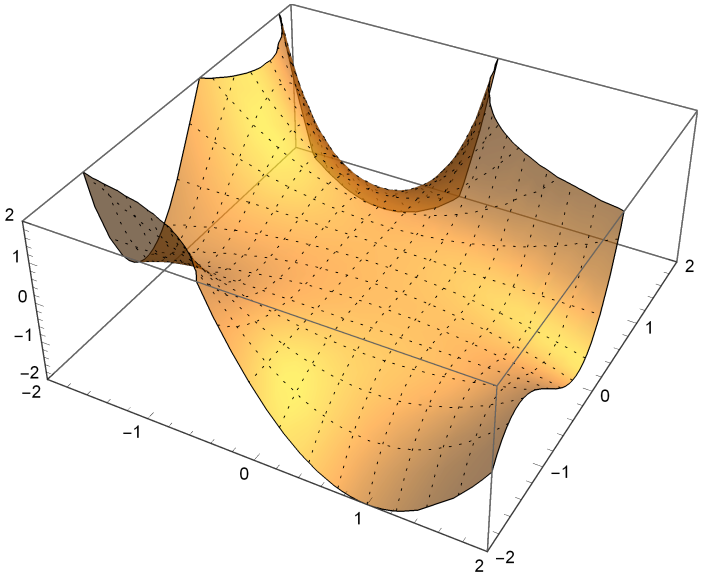


Out[367]=



In[368]:=
```
Plot3D[Function[{x, y}, (x + y) * (x * y + x * y^2)][x, y], {x, -2, 2},
 {y, -2, 2}, Mesh → Automatic, MeshStyle → Dotted, PlotStyle → Opacity[0.65],
 ClippingStyle → None, PlotRange → {{-2, 2}, {-2, 2}, {-2, 2}}]
```

Out[368]=



In[352]:=
```
Clear[x]
Clear[y]
```

In[369]:=
```
f[x_, y_] = (x + y) * (x * y + x * y^2);
```

Set: Tag Real in 0.[x_, y_] is Protected.

In[370]:=
```
grad = Grad[(x + y) * (x * y + x * y^2), {x, y}]
hessian = Grad[grad, {x, y}]
```

Out[370]= $\left\{x\,y + x\,y^2 + (x + y)\,\left(y + y^2\right),\ x\,y + x\,y^2 + (x + y)\,\left(x + 2\,x\,y\right)\right\}$

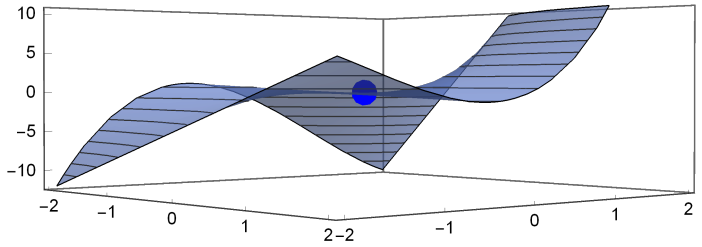
In[356]:=
```
df1 = grad[[1]]
df2 = grad[[2]]
```

Out[356]= $x\,y + x\,y^2 + (x + y)\,\left(y + y^2\right)$

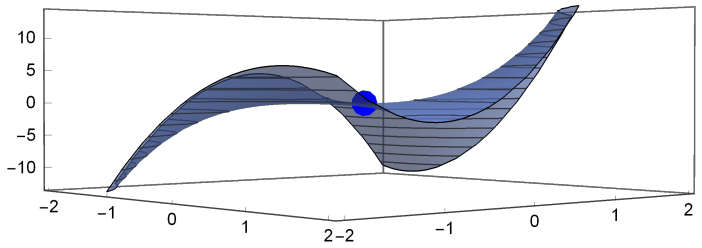
Out[357]= $x\,y + x\,y^2 + (x + y)\,\left(x + 2\,x\,y\right)$

In[358]:=
```
Show[Plot3D[df1, {x, -2, 2}, {y, -2, 2}, ClippingStyle → None,
  PlotStyle → Opacity[.65], PlotTheme → "Business"], Graphics3D[
  {PointSize[.04], Riffle[{Red, Blue, Green}, Point[{x, y, df1}] /. {x → 0, y → 0}]}]]
Show[Plot3D[df2, {x, -2, 2}, {y, -2, 2}, ClippingStyle → None,
  PlotStyle → Opacity[.65], PlotTheme → "Business"], Graphics3D[
  {PointSize[.04], Riffle[{Red, Blue, Green}, Point[{x, y, df2}] /. {x → 0, y → 0}]}]]
```

Out[358]=



Out[359]=



In[382]:=
```
fxy = (x + y) * (x * y + x * y^2)
```

Out[382]= $(x + y)\left(x\,y + x\,y^2\right)$

In[388]:=
```
criticalPoints = NSolve[grad == {0, 0}, {x, y}, Reals]
Map[MatrixForm, criticalPoints]
p = Table[{x, y, fxy} /. criticalPoints[[i]], {i, 1, 5, 1}]
```

Out[388]= $\{\{x \to 0., y \to -1.\}, \{x \to 1., y \to -1.\},$
$\{x \to 0.375, y \to -0.75\}, \{x \to 2.46371 \times 10^{-158}, y \to 5.70055 \times 10^{-159}\},$
$\{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}\}$

Out[389]= $\left\{\begin{pmatrix} x \to 0. \\ y \to -1. \end{pmatrix}, \begin{pmatrix} x \to 1. \\ y \to -1. \end{pmatrix}, \begin{pmatrix} x \to 0.375 \\ y \to -0.75 \end{pmatrix},\right.$
$\left.\begin{pmatrix} x \to 2.46371 \times 10^{-158} \\ y \to 5.70055 \times 10^{-159} \end{pmatrix}, \begin{pmatrix} x \to 0. \\ y \to 0. \end{pmatrix}, \begin{pmatrix} x \to 0. \\ y \to 0. \end{pmatrix}, \begin{pmatrix} x \to 0. \\ y \to 0. \end{pmatrix}\right\}$

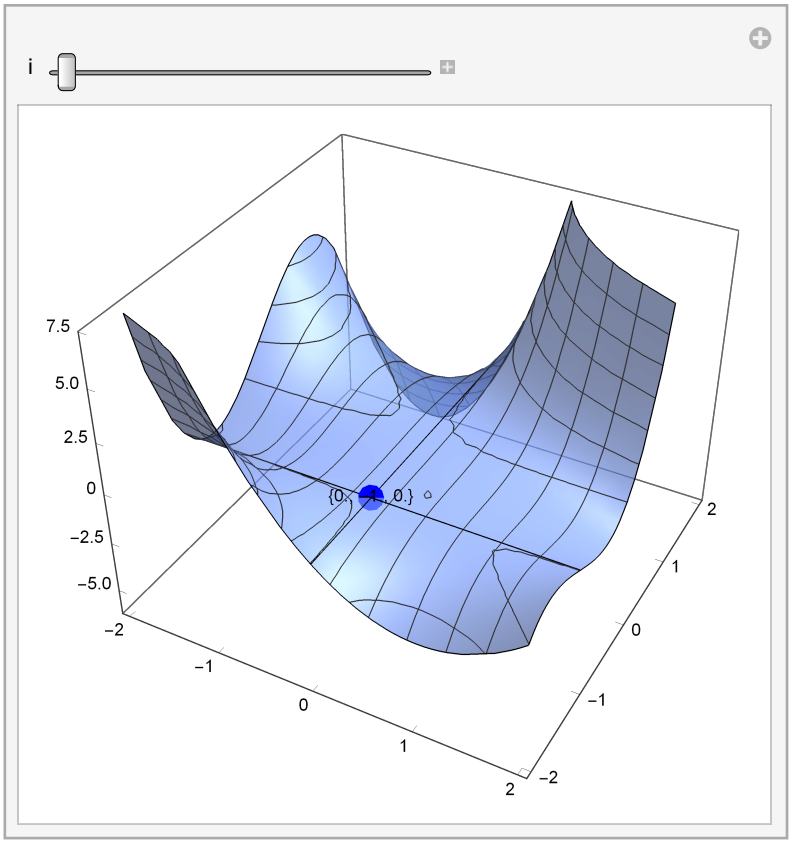
... General: 2.46371×10^{-158} 5.70055×10^{-159} is too small to represent as a normalized machine number; precision may be lost.

... General: $5.70055 \times 10^{-159^2}$ is too small to represent as a normalized machine number; precision may be lost.

Out[390]= $\{\{0., -1., 0.\}, \{1., -1., 0.\}, \{0.375, -0.75, 0.0263672\},$
$\{2.46371 \times 10^{-158}, 5.70055 \times 10^{-159}, 0.\}, \{0., 0., 0.\}\}$

In[419]:=
```
Manipulate[
 Show[g0 = Plot3D[fxy, {x, -2, 2}, {y, -2, 2}, ClippingStyle → None, Mesh → {10, 10},
    PlotStyle → Opacity[.65], BoxRatios → {1, 1, 3/4}, PlotTheme -> "Business"],
  g1 = Plot3D[fxy, {x, -2, 2}, {y, p[[i, 2]], p[[i, 2]] + 0.001}, PlotStyle → Red],
  g2 = Plot3D[fxy, {x, p[[i, 1]], p[[i, 1]] + 0.001}, {y, -2, 2}, PlotStyle → Red],
  Graphics3D[{PointSize[.04],
    Riffle[{Red, Blue, Green}, Point[{p[[i]]}]], Text[p[[i]], p[[i]]]}], {i, 1, 5, 1}]
```

Out[419]=



In[429]:=
```
criticalPoints = NSolve[grad == {0, 0}, {x, y}, Reals]
```

Out[429]=
$$\{\{x \to 0., y \to -1.\}, \{x \to 1., y \to -1.\}, \{x \to 0.375, y \to -0.75\}, \{x \to 2.46371 \times 10^{-158}, y \to 5.70055 \times 10^{-159}\}, \{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}, \{x \to 0., y \to 0.\}\}$$

In[468]:=
```
Map[MatrixForm, hessian /. criticalPoints]
```

... General: $5.70055 \times 10^{-159^2}$ is too small to represent as a normalized machine number; precision may be lost.

... General: 4.92743×10^{-158} 5.70055×10^{-159} is too small to represent as a normalized machine number; precision may be lost.

... General: $5.70055 \times 10^{-159^2}$ is too small to represent as a normalized machine number; precision may be lost.

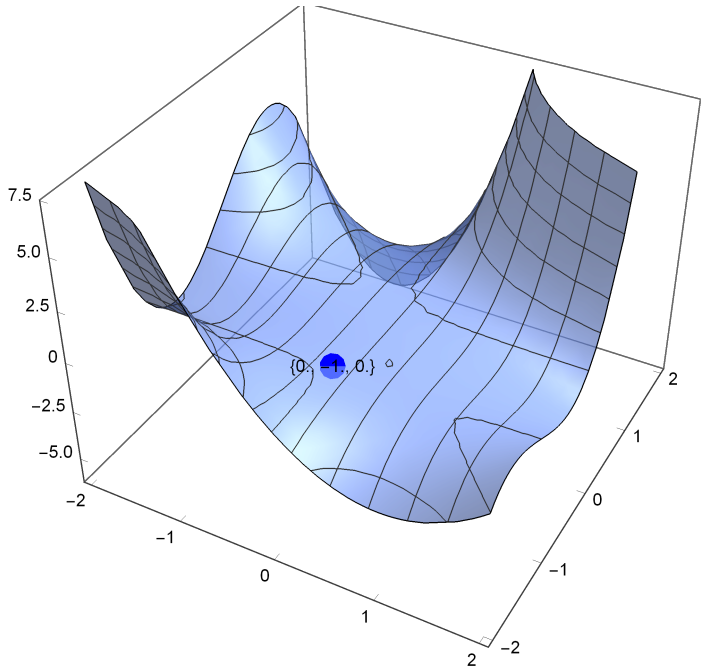
... General: Further output of General::munfl will be suppressed during this calculation.

Out[468]=
$$\left\{\begin{pmatrix} 0. & 1. \\ 1. & 0. \end{pmatrix}, \begin{pmatrix} 0. & -1. \\ -1. & -2. \end{pmatrix}, \begin{pmatrix} -0.375 & -0.1875 \\ -0.1875 & -0.65625 \end{pmatrix}, \begin{pmatrix} 1.14011 \times 10^{-158} & 6.06754 \times 10^{-158} \\ 6.06754 \times 10^{-158} & 4.92743 \times 10^{-158} \end{pmatrix}, \begin{pmatrix} 0. & 0. \\ 0. & 0. \end{pmatrix}, \begin{pmatrix} 0. & 0. \\ 0. & 0. \end{pmatrix}, \begin{pmatrix} 0. & 0. \\ 0. & 0. \end{pmatrix}\right\}$$

In[457]:=
```
Eigenvalues[Grad[grad, {x, y}] /. criticalPoints[[1]]]
Show[g0 = Plot3D[fxy, {x, -2, 2}, {y, -2, 2}, ClippingStyle → None, Mesh → {10, 10},
   PlotStyle → Opacity[.65], BoxRatios → {1, 1, 3/4}, PlotTheme -> "Business"],
 Graphics3D[{PointSize[.04], Riffle[{Red, Blue, Green},
    Point[{x, y, fxy} /. criticalPoints[[1]]]], Text[p[[1]], p[[1]]]}]]
```

Out[457]= $\{-1., 1.\}$

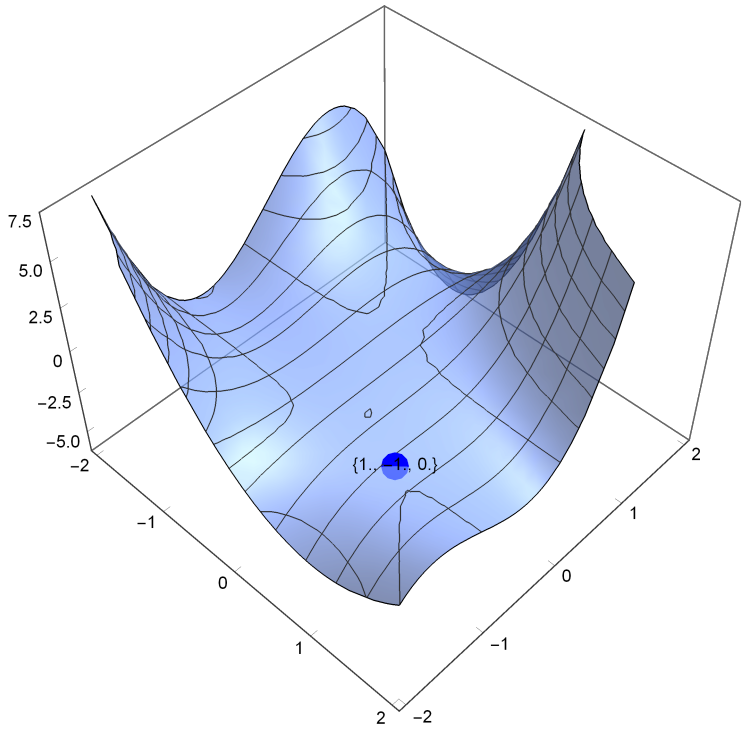
Out[458]=



In[445]:=
```
Eigenvalues[Grad[grad, {x, y}] /. criticalPoints[[2]]]
Show[g0 = Plot3D[fxy, {x, -2, 2}, {y, -2, 2}, ClippingStyle → None, Mesh → {10, 10},
   PlotStyle → Opacity[.65], BoxRatios → {1, 1, 3/4}, PlotTheme -> "Business"],
 Graphics3D[{PointSize[.04], Riffle[{Red, Blue, Green},
    Point[{x, y, fxy} /. criticalPoints[[2]]]], Text[p[[2]], p[[2]]]}]]
```

Out[445]= $\{-2.41421, 0.414214\}$

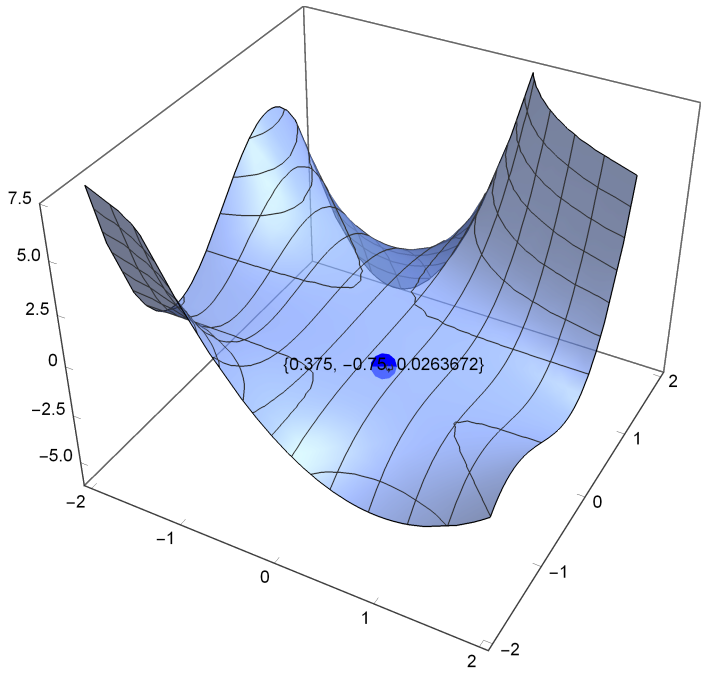
Out[446]=



In[449]:=
```
Eigenvalues[Grad[grad, {x, y}] /. criticalPoints[[3]]]
Show[g0 = Plot3D[fxy, {x, -2, 2}, {y, -2, 2}, ClippingStyle → None, Mesh → {10, 10},
   PlotStyle → Opacity[.65], BoxRatios → {1, 1, 3/4}, PlotTheme -> "Business"],
 Graphics3D[{PointSize[.04], Riffle[{Red, Blue, Green},
    Point[{x, y, fxy} /. criticalPoints[[3]]]], Text[p[[3]], p[[3]]]}]]
```

Out[449]= $\{-0.75, -0.28125\}$

Out[450]=



In[451]:=
```
Eigenvalues[Grad[grad, {x, y}] /. criticalPoints[[4]]]
Show[g0 = Plot3D[fxy, {x, -2, 2}, {y, -2, 2}, ClippingStyle → None, Mesh → {10, 10},
   PlotStyle → Opacity[.65], BoxRatios → {1, 1, 3/4}, PlotTheme -> "Business"],
 Graphics3D[{PointSize[.04], Riffle[{Red, Blue, Green},
    Point[{x, y, fxy} /. criticalPoints[[4]]]], Text[p[[4]], p[[4]]]}]]
```

... General: $5.70055\times10^{-159^2}$ is too small to represent as a normalized machine number; precision may be lost.

... General: 4.92743×10^{-158} 5.70055×10^{-159} is too small to represent as a normalized machine number; precision may be lost.

... General: $5.70055\times10^{-159^2}$ is too small to represent as a normalized machine number; precision may be lost.

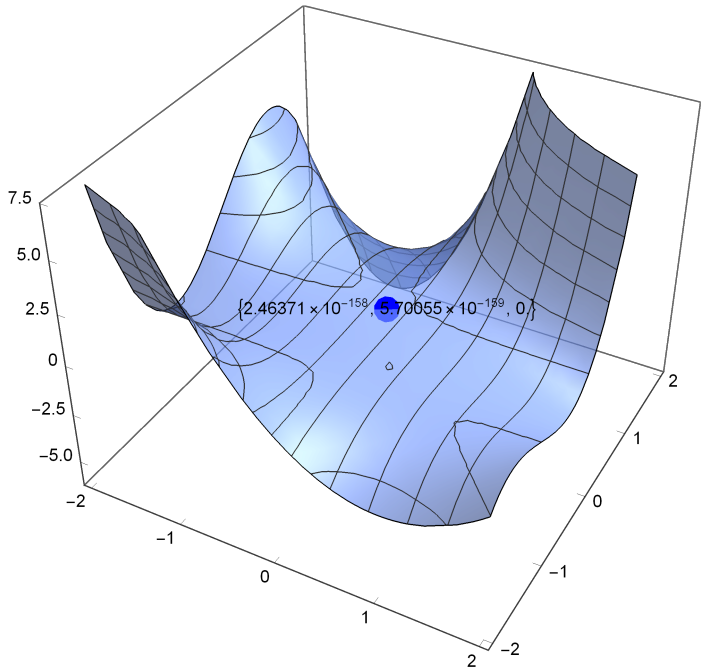
... General: Further output of General::munfl will be suppressed during this calculation.

Out[451]= $\{9.38994\times10^{-158}, -3.32241\times10^{-158}\}$

... General: 2.46371×10^{-158} 5.70055×10^{-159} is too small to represent as a normalized machine number; precision may be lost.

... General: $5.70055\times10^{-159^2}$ is too small to represent as a normalized machine number; precision may be lost.

Out[452]=



In[453]:=
```
Eigenvalues[Grad[grad, {x, y}] /. criticalPoints[[5]]]
Show[g0 = Plot3D[fxy, {x, -2, 2}, {y, -2, 2}, ClippingStyle → None, Mesh → {10, 10},
   PlotStyle → Opacity[.65], BoxRatios → {1, 1, 3/4}, PlotTheme -> "Business"],
 Graphics3D[{PointSize[.04], Riffle[{Red, Blue, Green},
    Point[{x, y, fxy} /. criticalPoints[[5]]], Text[p[[5]], p[[5]]]}]]
```

Out[453]= {0., 0.}

Out[454]=