Optimization HW 5

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1 Question 1.1

Proof. $B_{k+1} = B_k + \frac{(y_k - B_k s_k)v^T}{v^T s_k}$

- [1] Multiply by s_k on the right on both sides:
  $B_{k+1}s_k = B_k s_k + \frac{(y_k - B_k s_k)v^T s_k}{v^T s_k}$
- $\Rightarrow \quad B_{k+1}s_k = B_k s_k + y_k - B_k s_k$
- $\Rightarrow \quad B_{k+1}s_k = y_k$
- Thus the secant condition is met.

$\square$

2 Question 1.2

Proof. $B_{k+1} = B_k + \frac{y_k y_k^T}{y_k^T s_k} - \frac{B_k s_k (B_k s_k)^T}{s_k^T B_k s_k}$

- [1] Multiply by s_k on the right on both sides:
  $B_{k+1}s_k = B_k s_k + \frac{y_k y_k^T s_k}{y_k^T s_k} - \frac{B_k s_k s_k^T B_k^T s_k}{s_k^T B_k s_k}$
- [2] $\Rightarrow \quad B_{k+1}s_k = B_k s_k + y_k - \frac{B_k s_k \cdot (s_k^T B_k s_k)}{(s_k^T B_k s_k)}$
- $\Rightarrow \quad B_{k+1}s_k = B_k s_k + y_k - B_k s_k$
- $\Rightarrow \quad B_{k+1}s_k = y_k$.
- Thus the secant condition is met.

$\square$

[1] $v^T s_k$ or $y_k^T s_k$ is a scalar and cancels in the fraction.

[2] Since B_k approximates the Hessian, it is symmetric $\implies B_k^T = B_k$. Therefore we get: $s_k^T B_k^T s_k = s_k^T B_k s_k$.