

"A person who never made a mistake never tried anything new"

(2.1)

## ARRAYS RECORDS & POINTER

~ Albert Einstein

Data structure are classified as either linear or non-linear.

- A data structure said to be linear if its elements form a sequence.

- Operation on linear data structure.

- (a) Traversal - Processing each element in list.
- (b) Search - Finding the location of element
- (c) Insertion - Add new element into the list
- (d) Deletion - Removing an element from list
- (e) Sorting - Arrange the element in some order
- (f) Merging - combining two list into a single list.

### \* LINEAR ARRAYS

$$\text{Length} = (UB - LB) + 1$$

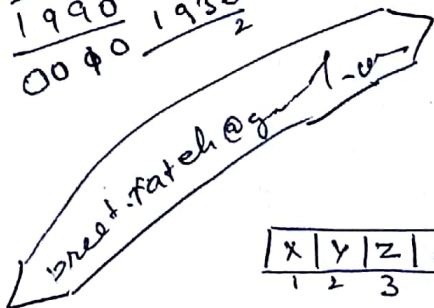
Eg: An automobile company uses an array AUTO to record the number of automobiles sold each year from 1990 to 2000. Rather than beginning the index set with 1, it is more useful to begin the index set with 1990.

Then

$$LB = 1990 \quad \& \quad UB = 2000$$

$$\begin{array}{r} 2000 \\ 1990 \\ \hline 0010 \end{array} \quad \begin{array}{r} 1984 \\ 1930 \\ \hline 54 \end{array}$$

$$\begin{aligned} \text{Length} &= (UB - LB) + 1 \\ &= (2000 - 1990) + 1 \\ &= 10 + 1 = 11 \end{aligned}$$



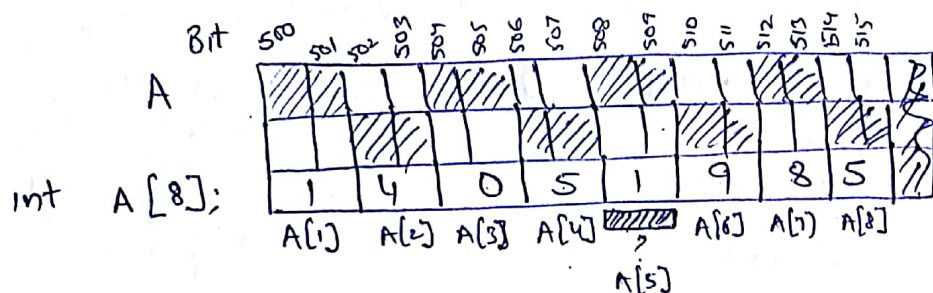
x	y	z	A	B
1	2	3	4	5

$$\begin{aligned} &= (5 - 1) + 1 \\ &= 4 + 1 \\ &= 5 \end{aligned}$$

Accordingly, the computer doesn't need to keep track of the address of every element of array A, but needs to keep track only of the address of first element. denoted as  $\text{Base}(A)$  & called the base address of A

- calculation of addresses of any element of A

$$\text{LOC}(A[K]) = \text{Base}(A) + w(K - \text{LB})$$



$$\text{LOC}(A[5]) = \text{Base}(A) + 2(5-1)$$

$$\text{LOC}(A[5]) = 500 + 2(4) = 500 + 8 = \underline{\underline{508}}$$

LOC → Memory location

Base → Starting memory location of Array

w → size of item in an array

K → index of Kth element in an array

LB → Lower Bound of an array.

TRAVERSING LINEAR ARRAYSAlgorithm: TRAVERSING

Here LA is a linear array with lower bound LB & upper bound UB. This algorithm traverses LA applying an operation PROCESS to each element of LA

1. [Initialize counter] Set  $K := LB$
2. Repeat step 3 & 4 while  $K \leq UB$
3. [Visit element] Apply PROCESS to  $LA[K]$
4. [Increase counter] Set  $K := K + 1$
- [End of step 2 loop]
5. EXIT

OR

1. Repeat for  $K = LB$  to  $UB$   
     Apply PROCESS to  $LA[K]$   
     [End of loop]
2. EXIT



### ⇒ Algorithm: INSERT (A, N, K, ITEM)

Here A is linear array with N elements and K is positive integer s.t.  $K \leq N$ . This algorithm insert ITEM into the Kth position in array A.

1. ~~2~~ Set  $J := N$  [Initialize counter]
2. Repeat step 3 & 4 while  $J \geq K$ 
  - step  $A[J+1] := A[J]$  < Move Jth element downwards >
  - Set  $J := J - 1$  < Decrease counter >

[END of step 2 loop]
3. Set  $A[K] := \text{ITEM}$  < insert element >
4. Set  $N := N + 1$  < Reset N >
5. EXIT

### ⇒ Algorithm: DELETE (A, N, K, ITEM)

Here A is a linear array with N elements and K is a positive integer s.t.  $K \leq N$ . This algo delete Kth item from A

1. Set  $\text{ITEM} := A[K]$
2. Repeat for  $J = K$  to  $N - 1$ 
  - Set  $A[J] := A[J+1]$

[END of step 2 loop]
3. Set  $N := N - 1$  < Reset N in array A >
4. EXIT

— : 0 —

# SORTING : BUBBLE SORT

Sorting array A refers to the operation of rearranging the elements of A so they are in increasing order.

		(8)	(4)	19	2	7	13	5	16
PASS 1st	4	(8)	(19)	2	7	13	5	16	
	4	8	(19)	(2)	7	13	5	16	
	4	8	2	(19)	(7)				
			2	7	(19)	(13)			
			2	7	13	(19)	(5)	16	
PASS 2nd	4	8	2	7	13	5	(19)	(16)	
	4	8	2	7	13	5	16	(19)	✓
	4	(8)	(2)	7	13	5	16	19	
	4	2	(8)	(7)					
		2	7	(8)	(13)	5	16	19	
PASS 3rd	4	2	7	8	(13)	(5)	16	19	
	4	2	7	8	5	13	16	19	
	(4)	(2)	7	8					
	2	4	(7)	(8)	(5)	13	16	19	
	2	4	7	5	8	13	16	19	
PASS 4th	2	4	(7)	(5)	8	13	16	19	
	2	4	5	7	8	13	16	19	

Algorithm: (Bubble Sort) BUBBLE (DATA, N)

Here DATA is an array with N elements

1. Repeat Step 2 and 3 for  $K=1$  to  $N-1$
  2. Set  $PTR := 1$  [Initialize pass pointer PTR]
  3. Repeat while  $PTR \leq N-K$  [Execute pass]
    - (a) If  $DATA[PTR] > DATA[PTR+1]$ , then  
Interchange  $DATA[PTR]$  and  $DATA[PTR+1]$   
[END of if structure]
    - (b) Set  $PTR := PTR + 1$   
[END of inner loop (while)]
- [END of step 1 outer loop (for)]

4. EXIT

Notes

- Pass I — Step 1 involves  $n-1$  comparison &  $A[N]$  will contain the largest element.
- Pass II — After step 2 2nd largest element will be  $A[N-1]$   
& no of comparison are  $(N-2)$

After  $n-1$  passes (steps) the list will be sorted in increasing order.

Complexity of Bubble Sort Algo

$$\begin{aligned}
 f(n) &= (n-1) + (n-2) + \dots + 2 + 1 \\
 &= \frac{n(n-1)}{2} = \frac{n^2 - n}{2} = \frac{n^2}{2} + -\frac{n}{2} \\
 &= \frac{n^2}{2} + O(n) = O(n^2) + O(n) = O(n^2)
 \end{aligned}$$



## Searching: LINEAR SEARCH

Algorithm: (Linear Search)  $\text{LINEAR}(\text{DATA}, N, \text{ITEM}, \text{LOC})$

Here DATA is a linear array with  $N$  elements, and ITEM is given item of information. This algorithm finds the LOC := 0 if search is unsuccessful.

1. Set  $\text{DATA}[N+1] := \text{ITEM}$
2. Set  $\text{LOC} := 1$
3. Repeat while  $\text{DATA}[\text{LOC}] \neq \text{ITEM}$

Set  $\text{LOC} := \text{LOC} + 1$

[End of loop]

4. If  $\text{LOC} = N+1$ , then  
Set  $\text{LOC} := 0$

5. EXIT

→ Complexity of Linear search

Suppose  $p_i$  is probability of each position to be the item & the  $q_i$  is probability of not occurrence of item

$$f(n) = (p_1 + p_2 + p_3 + \dots + p_n + q) = 1$$

Suppose element is present in our array then

$$f(n) = 1 p_1 + 2 p_2 + \dots + n p_n$$

$$= 1 \frac{1}{n} + 2 \frac{1}{n} + \dots + n \frac{1}{n}$$

(Suppose element occurs at equal probability)

$$= \frac{1}{n} (1 + 2 + \dots + n) = \frac{1}{n} \left( \frac{n \cdot (n+1)}{2} \right) = \frac{n+1}{2} = O(n)$$

Hence the complexity of Linear search algorithm is  $O(n)$

## BINARY SEARCH

Suppose DATA is an array which is sorted in increasing numerical order or equivalent alphabetically. Then there is an extremely efficient searching algorithm, called Binary search

11. 22 30 33 40 44 55 60 70  
 BEG MID END  $\frac{9}{2} = 4.5$

ITEM = 55 ; to be search.

W MID = 55 X  $33 \neq 55$

$$\frac{8}{2} = 3.5$$

— set BEG := MID + 1 = 40

MID = 44 18  $44 \neq 55$

$$\frac{4}{2} = 2$$

← set BEG = MID + 1

55 = 55 ✓

Algorithm: (Binary search) BINARY(DATA, LB, UB, ITEM, LOC)

Here DATA is a sorted array with LB & UB. & ITEM is to search. The variable BEG, END & MID denote beginning, end, middle location of segment of DATA. This also find the LOC of ITEM in DATA or ~~elements of DI~~ set LOC = NULL

1. Set BEG := LB, END := UB & MID = INT ((BEG + END) / 2)

2. Repeat step 3 & 4 while BEG ≤ END & DATA[MID] ≠ ITEM

3. If ITEM < DATA[MID], then  
     Set END := MID - 1

Else

    set BEG := MID + 1

[End of if structure]

4. Set MID := INT ((BEG + END) / 2)

[END OF STEP 2 loop]

5. If DATA[MID] = ITEM, then

    Set LOC := MID

Else

    set LOC := NULL

[END OF if structure]

6. EXIT



## Complexity of Binary Search

We require at most  $f(n)$  comparisons to locate ITEM where

$$2^{f(n)} > n \quad \text{or equivalently} \quad f(n) = \lfloor \log_2 n \rfloor + 1$$

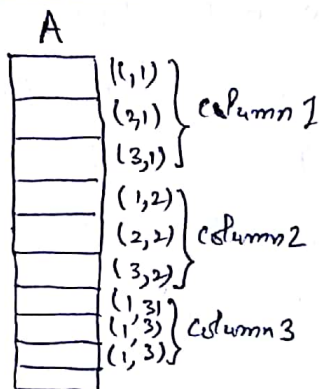
Thus the running time for the worst case is approximately equal to  $\log_2 n$ . complexity  $\Rightarrow O(\log_2 n)$

$$2^{10} = 1024 > 1000 \quad \text{Example}$$

+ Very efficient

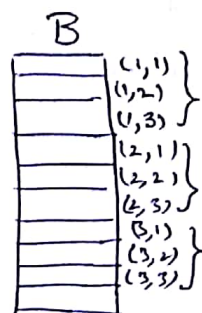
- The list must be sorted
- One must have direct access to middle element in subset.
- Keeping data in a sorted array is normally very expensive when inserting & deletions
- (In such situations, one may use different data structure i.e. linked list or binary search tree)

### 2 D ARRAY



column-major order

\* Storing array A column by column



Row-major order

\* Storing array B row-by-row

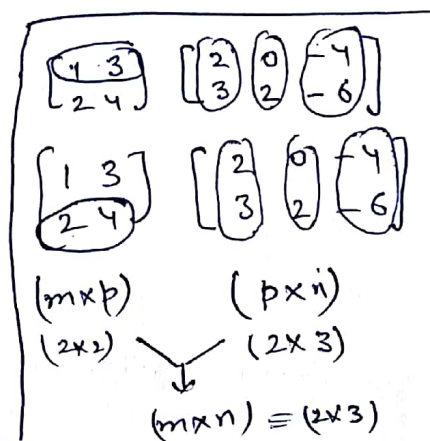
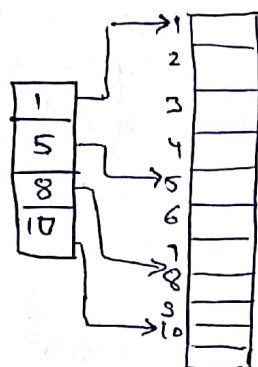
# POINTER

(2.10)

Array whose rows - or column - being with different numbers of data elements & end with unused storage locations are said to be jagged.

$$\begin{bmatrix} 1 & 2 & 3 & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & 9 & 7 & 2 & 8 \\ \vdots & \vdots & \vdots & \vdots & 5 & 5 \end{bmatrix}$$

A variable  $P$  is called a pointer if  $P$  "points" to an element in DATA. Analogously, an array PTR is called a pointer array if each element of PTR is a pointer.



Algorithm: MATMUL (A, B, C, M, P, N)

Let  $A$  be an  $M \times P$  matrix array &  $B$  be  $P \times N$  matrix array. This also stores the product of  $A$  &  $B$  in an  $M \times N$  matrix array  $C$ .

1. Repeat step 2 to 4 for  $I = 1$  to  $M$
2. Repeat step 3 & 4 for  $J = 1$  to  $N$
3. Set  $C[I, J] := 0$
4. Repeat for  $K = 1$  to  $P$

// Repeat for Row of first matrix  
or " " column of 2nd "

$$C[I, J] := C[I, J] + A[I, K] * B[K, J]$$

[End of inner loop of step 4]

[End of step 2 (middle) loop]

[End of step 1 (outer loop)]

5. EXIT

$$\begin{bmatrix} A[1,1] & B[1,1] \\ A[1,2] & B[2,1] \\ A[1,3] & B[3,1] \end{bmatrix} \begin{matrix} \times \\ \times \\ \times \end{matrix}$$

The complexity of above Algorithm is

$$C = m \cdot n \cdot p$$

## SPARSE MATRICES

Matrices with a relative high proportion of zero entries are called sparse matrices.

The matrix, where all the entries above main diagonal are zero or equivalently, where non-zero entries can only occur on or below the main diagonal, is called a (lower) triangular matrix.

$$\begin{bmatrix} 1 & & & \\ 2 & 3 & & \\ 3 & 4 & 6 & \\ 4 & 5 & 7 & 8 \end{bmatrix}$$

Tri-angular matrix

$$\begin{bmatrix} 1 & & & & \\ 2 & 1 & 2 & & \\ 3 & & 2 & 3 & \\ 4 & & 4 & 5 & 6 \\ 5 & & & 7 & 8 & 9 \\ & & & & 6 & 7 \end{bmatrix}$$

Tri-diagonal matrix