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Curriculum Vitae

Personal Details

Date of birth February 15, 1971
Place of birth Gioia del Colle, Bari, Italy
Citizenship Italian

Academic Position

07/2015–today Associate Professor of Mathematical Analysis, Polytechnic University of Bari, Italy
10/2002–06/2015 Ricercatore Universitario of Mathematical Analysis, Polytechnic University of Bari, Italy
30/12/2013 Italian National Scientific Qualification as associate professor

Education

03/2000–10/2002 Dottorato di ricerca (PhD) in Mathematics, University of Florence, Italy.
Ph.D. thesis: “Global Properties of the Relativistic Lorentz Force Equation”.
Advisor: Prof. Antonio Masiello (Polytechnic University of Bari).
Date of defence of the PhD thesis: 30.09.2003.
10/1993–03/1999 Laurea in Mathematics (BS+MS), University of Bari, Italy
1985–1990 Liceo Scientifico Statale “R. Canudo”, Gioia del Colle, Italy

Teaching Assignments

2020-2023	Advanced Mathematical Methods (20 hours)
2018-2023	Calculus
2016-2017	Advanced Calculus Calculus
2015	Advanced Calculus
2014	Calculus
2009-2013	Advanced Calculus
2008-2009	Calculus
2007	Calculus Ordinary Differential Equations (graduate course)
2005-2007	Calculus Differential Equations in Mathematical Physics (graduate course)
2002-2004	Calculus

Supervision of PhD Students

2013-2016	G. Stanciarone, PhD thesis: "Finsler spacetimes with a Killing vector field", University of Bari, Italy.
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Memberships	GNAMPA (Italian National Group for Mathematical Analysis, Probability and their Applications)
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Grants

09/2022	GNAMPA financial support for organizing Meeting, School and Workshop.
2022-today	Participation to the project INdAM-GNAMPA 2022 "Variational methods for Lagrangians of low regularity".
2019-today	Participation to the project PRIN 2017JPCAPN "Qualitative and quantitative aspects of nonlinear PDEs".
03/2017	Erasmus+ KA103 SMT grant for the academic year 2016/2017.

- 18/03/2016–18/03/2017 Principal investigator of the INdAM–GNAMPA Project 2016 “Finslerian type structures and Killing vector fields”.
- 01/01/2014–28/02/2018 Participation to the project “Semi-Riemannian Geometry and Variational Problems in Mathematical Physics”, supported by the Ministry of Economía y Competitividad (Spain); coordinator of the project Prof. M. Sánchez (University of Granada, Spain). ID Number Grant MTM-2013-47828-C2-1-P.
- 12/2013 GNAMPA financial support for participation to Meeting, School and Workshop.
- 06/2013 GNAMPA financial support for participation to Meeting, School and Workshop.
- 24/09/2012–31/12/2015 Principal investigator of the project “Finsler spacetimes”, FRA2011, Polytechnic University of Bari, Italy.
- 09/05/2012–09/05/2013 Participation to the project INdAM–GNAMPA Project 2012 “Geometric Analysis on Lorentz Manifolds and Applications to General Relativity”; head of the project Prof. A. Masiello (Polytechnic University of Bari, Italy).
- 27/04/2011–27/04/2012 Participation to the INdAM–GNAMPA Project 2011 “Geometric Analysis on Lorentz Manifolds and Applications to General Relativity”; head of the project Prof. A. Masiello (Polytechnic University of Bari, Italy).
- 26/01/2009–31/03/2011 Participation to the project Azione Integrata Italia–Spagna 2009 “Variational and Topological Methods in Nonlinear Analysis and Geometry and Applications”; coordinators: Prof. G. Cerami (Polytechnic University of Bari, Italy), Prof. M. Sanchez (University of Granada, Spain).
- 17/10/2011–17/10/2013 Participation to the PRIN 2009 “Variational Methods and Applications to the Study of Nonlinear Differential Equations”; head of the project Prof. V. Benci (University of Pisa, Italy).
- 22/09/2008–22/09/2010 Participation to the PRIN 2007 “Variational Methods and Applications to the Study of Nonlinear Differential Equations”; head of the project Prof. V. Benci (University of Pisa, Italy).
- 30/01/2006–30/01/2008 Participation to the PRIN 2005 “Variational Methods and Applications to the Study of Nonlinear Differential Equations”; head

of the project Prof. V. Benci (University of Pisa, Italy).

20/11/2003–20/11/2005 Participation to the PRIN 2003 “Variational Methods and Applications to the Study of Nonlinear Differential Equations”; head of the project Prof. V. Benci (University of Pisa, Italy).

03/2000–10/2002 PhD grant, University of Florence, Italy.

Publications

Papers

R. Bartolo, E. Caponio, “Trajectories of affine control systems and geodesics of a spacetime with a causal Killing vector field”, *J. Dyn. Control Syst.* (2023), <https://doi.org/10.1007/s10883-023-09639-z>.

E. Caponio, M. A. Javaloyes, and A. Masiello, “Multiple connecting geodesics of a Randers-Kropina metric via homotopy theory for solutions of an affine control system”, *Topol. Methods Nonlinear Anal.* (2023), DOI 10.12775/TMNA.2022.066.

E. Caponio, D. Corona, “A variational setting for an indefinite Lagrangian with an affine Noether charge”, *Calc. Var. Partial Differential Equations*, 62 (2023), 39.

E. Caponio, M.A. Javaloyes, M. Sánchez, “Wind Finslerian structures: from Zermelo’s navigation to the causality of spacetimes”, *Memoirs AMS* in press, [arXiv:1407.5494 \[math.DG\]](https://arxiv.org/abs/1407.5494).

E. Caponio, A. Masiello, “A note on the Sagnac effect in general relativity as a Finslerian effect”, *Foundations of Physics*, 52 (2022), 5.

E. Caponio, F. Giannoni, A. Masiello, S. Sühr “Connecting and closed geodesics of a Kropina metric”, *Advanced Nonlinear Studies*, 21 (2021), 683–695.

R. Bartolo, E. Caponio, A. Pomponio, “Spacelike graphs with prescribed mean curvature on exterior domains in the Minkowski spacetime”, *Proceedings of the American Mathematical Society*, 149 (2021), 5139–5151.

E. Caponio, A. Masiello, “On the Analyticity of Static Solutions of a Field Equation in Finsler Gravity”, *Universe* 6 (2020), 59.

- E. Caponio, A. Masiello, "Harmonic coordinates for the nonlinear Finsler Laplacian and some regularity results for Berwald metrics", *Axioms*, 8 (2019), 83.
- E. Caponio, G. Stanciarone, "On Finsler spacetimes with a time-like Killing vector field", *Classical and Quantum Gravity*, 35 (2018), 085007.
- E. Caponio, G. Stanciarone "Standard static Finsler spacetimes", *International Journal of Geometric Methods in Modern Physics*, 13 (2016), 1650040.
- E. Caponio, A. V. Germinario, M. Sánchez, "Convex regions of stationary spacetimes and Randers spaces. Applications to lensing and asymptotic flatness", *The Journal of Geometric Analysis*, 26 (2016), 791–836.
- E. Caponio, M. A. Javaloyes, A. Masiello, "Addendum to *Morse theory of causal geodesics in a stationary spacetime via Morse theory of geodesics of a Finsler metric*, Ann. Inst. H. Poincaré Anal. Non Linéaire, 27 (2010), 857–876 ", *Annales de l'Institut Henri Poincaré. Analyse non Linéaire*, 30 (2013), 961–968.
- R. Bartolo, E. Caponio, A.V. Germinario, M. Sanchez "Convex domains of Finsler and Riemannian manifolds", *Calculus of Variations and Partial Differential Equations*, 40 (2011), 335–356.
- E. Caponio, M.A. Javaloyes, M. Sanchez, "On the interplay between Lorentzian Causality and Finsler metrics of Randers type", *Revista Matemática Iberoamericana*, 27 (2011), 919–952.
- E. Caponio, M. A. Javaloyes, A. Masiello, "On the energy functional on Finsler manifolds and applications to stationary spacetimes", *Mathematische Annalen*, 351 (2011), 365–392.
- R. Bartolo, A.M. Candela, E. Caponio "Normal geodesics connecting two non-necessarily spacelike submanifolds in a stationary spacetime" *Advanced Nonlinear Studies*, 10 (2010), 851–866.
- E. Caponio, "The index of a geodesic in a Randers space and some remarks about the lack of regularity of the energy functional of a Finsler metric" *Acta Mathematica Academiae Paedagogicae Nyíregyháziensis*, 26 (2010), 265–274.

- E. Caponio, M. A. Javaloyes, A. Masiello, "Morse theory of causal geodesics in a stationary spacetime via Morse theory of geodesics of a Finsler metric", *Annales de l'Institut Henri Poincaré. Analyse non Linéaire*, 27 (2010), 857–876.
- E. Caponio, M.A. Javaloyes, A. Masiello "Finsler geodesics in the presence of a convex function and their applications", *Journal of Physics A, Mathematical and Theoretical*, 43 (2010), 135207–135222.
- E. Caponio, M. A. Javaloyes, P. Piccione, "Maslov index in semi-Riemannian submersions", *Annals of Global Analysis and Geometry*, 38 (2010), 57–75.
- E. Caponio, A. Masiello, P. Piccione, "Maslov index and Morse theory for the relativistic Lorentz force equation", *Manuscripta Mathematica*, 113 (2004), 471–506.
- E. Caponio, E. Minguzzi, "Solutions to the Lorentz force equation with fixed charge-to-mass ratio in globally hyperbolic space-times", *Journal of Geometry and Physics*, 49 (2004), 176–186.
- E. Caponio, "Timelike solutions to the Lorentz force equation in time-dependent electromagnetic and gravitational fields", *Journal of Differential Equations*, 199 (2004), 115–142.
- E. Caponio, A. Masiello, "Causal properties of Kaluza-Klein metrics", *Applied Mathematics Letters*, 17 (2004), 1371–1374.
- E. Caponio, A. Masiello, "The Avez-Seifert theorem for the relativistic Lorentz force equation", *Journal of Mathematical Physics*, 45 (2004), 4134–4140.
- E. Caponio, A. Masiello, P. Piccione, "Some global properties of static spacetimes", *Mathematische Zeitschrift*, 244/3 (2003), 457–468.
- E. Caponio, A. Masiello, "Trajectories of charged particles in a region of a stationary spacetime", *Classical and Quantum Gravity*, 19 (2002), 2229–2256.
- E. Caponio, A. Masiello, "Trajectories for relativistic particles under the action of an electromagnetic field in a stationary space-time", *Nonlinear Analysis*, 50 (2002), 71–89.

E. Caponio, “An intrinsic Fermat principle on stationary Lorentzian manifolds and applications”, *Differential Geometry and its Applications*, 16 (2002), 245–265.

Preprints

Book chapters E. Caponio, “Infinitesimal and local convexity of a hypersurface in a semi-Riemannian manifold”, in *Recent Trends in Lorentzian Geometry*, Springer Proceedings in Mathematics & Statistics, 26, M. Sánchez, M. Ortega, A. Romero Eds., Springer Science + Business Media, New York, USA, 2013, pp. 163–177, ISBN:978-1-4614-4896-9.

Proceedings E. Caponio, G. Stanciarone “Causality properties of static Finsler spacetimes”, proceedings of the “1st Workshop on the State of the Art and Challenges of Research Efforts at POLIBA, 03–05 dicembre 2014, Bari, Italy, track C2 – Research Contributions”. Cangemi Editore, Roma, 2014, pp. 21–25, ISBN: 978-88-492-2967-7.

E. Caponio, M. A. Javaloyes, “A remark on the Morse Theorem about infinitely many geodesics between two points” proceedings of the “International Meeting on Differential Geometry, Córdoba 2010”, Córdoba, Spain, November 15–17, 2010. Ediciones Don Folio, Córdoba, 2013, pp. 39–48, ISBN:978-84-15105-91-6.

[arXiv:1105.3923v2](https://arxiv.org/abs/1105.3923v2) [[math.DG](#)].

R. Bartolo, A.M. Candela, E. Caponio, “An Avez-Seifert type theorem for orthogonal geodesics on a stationary spacetime” Proceedings of the conference “New developments in Lorentzian geometry” Berlin, Germany, November 18–20, 2009, *Advances in Lorentzian Geometry*, AMS/IP Studies in Advanced Mathematics, vol. 49, Amer. Math. Soc. and International Press, Providence, RI, 2011, pp. 1–9, ISBN: 978-0-8218-5352-8.

E. Caponio, “Trajectories of charged particles in the Reissner-Nordström spacetime”, *Recent Developments in Gravitational Physics: Proceedings of the 15th SIGRAV Conference on General Relativity and Gravitational Physics*, Villa Mondragone, Monte Porzio Catone, Rome, Italy, September 9–12, 2002. Eds I. Ciufolini, E. Coccia, M. Colpi, V. Gorini, and R. Peron. Institute of Physics, Conference Series Nr. 176, Taylor & Francis, New York, NY USA, 2006, pp. 341–347,

ISBN-13: 978-0750309431.

E. Caponio, “Null geodesics for Kaluza-Klein metrics and world-lines of charged particles”, II International Meeting of Lorentzian Geometry, Murcia, Spain, November 12–14, 2003. Eds. L. J. Alias Linares, A. Ferrandez Izquierdo, M. A. Hernandez Cifre, P. Lucas Saorin, J. A. Pastor Gonzalez. Publicaciones de la Real Sociedad Matematica Española, vol. 8, 2004, pp 69–75, ISBN: 84-933610-5-4.

E. Caponio, A. Masiello, “Trajectories for relativistic particles in an electromagnetic field”, XIV SIGRAV Congress on General Relativity and Gravitational Physics, Genova, September 18–22, 2000. Recent Developments in General Relativity, Genova 2000. Eds. R. Cianci, R. Collina, M. Francaviglia, P. Fre. Springer Verlag, 2002, pp. 369–373, ISBN: 88-470-0162-5.

Invited Talks

“A Lagrangian generalisation of the notion of a stationary Lorentzian metric”, Workshop on Astro-particles and Gravity, Cairo University, September 20–22, 2022, Cairo, Egypt (on-line).

“Some remarks on Rutz’s vacuum field equation in standard static Finsler spacetimes”, one-hour talk at the conference *Interactions between Contact and Lorentzian Geometry*, Ruhr-Universität Bochum, May 20–24, 2019, Bochum, Germany.

“On a Finslerian generalisation of standard static and stationary spacetimes”, one-hour talk at the conference *IX International Meeting on Lorentzian Geometry*, Institute of Mathematics of the Polish Academy of Sciences, Warsaw, June 17 – 24, 2018, Warsaw, Poland.

“Finsler spacetimes with a timelike Killing vector field” *IUCSS Workshop on Finsler Geometry and Lorentz Violation (LVF'17)*, Indiana University Center for Spacetime Symmetries (IUCSS), May 12 – 13, 2017, Indiana University, Bloomington, USA.

“Some applications of Finsler geometry to topological lensing” one-hour talk at the conference *Symposium on Gravity and Light*, Kavli Institute for the Physics and mathematics of the Universe, Todai Institutes for Advanced Study, The University of Tokyo, September

30 – October 3, 2013, Kashiwa, Japan.

“Stationary-to-Randers correspondence and convexity” one-hour talk at the conference *VI International Meeting in Lorentzian Geometry – Gelogra 2011*, September 6–9, 2011, Granada, Spain.

“On the Morse Theorem about infinitely many geodesics between two points”, one-hour talk at the conference *Geometry and Relativity Day*, April 13, 2011, Malaga, Spain.

“The index of a geodesics in a Randers space and some remarks about Morse theory on Finsler manifolds”, main talk at the *Workshop on Finsler Geometry and its Applications*, May 24–29, 2009, Debrecen, Hungary.

“Some results about causality in conformally stationary spacetimes via Finsler Geometry”, main talk, *Variational and Topological Methods in Nonlinear Phenomena*, May 1–5, 2008, Otranto, Italy.

Mini-courses

“Finsler structures, Zermelo navigation and spacetimes”, PhD and master class in the framework of Erasmus+ KA103 SMT, Mathematics Department of the Technical University of Cluj-Napoca, Romania, June 12th, 15th, 16th, 2017

“Finsler structures and spacetimes”, mini-course at the School of Mathematical Sciences, Beijing Normal University, September 8th, 15th, 18th, 23th 2015, Beijing, China.

“Morse theory for the energy factional of a Finsler metric” mini-course at Institute of Mathematics and Statistics of the University of São Paulo from 30 July to 1st August 2013, São Paulo, Brazil.

“Finsler geometry: a selection of introductory topics”, two seminars for the PhD lectures *Elements of Calculus of Variations with Applications to the Study of Geodesics* at the Mathematics Department of the University of Bari, March 25 and 27, 2013, Bari, Italy.

Seminars

“On the existence of harmonic coordinates in Finsler manifolds”, Fakultät für Mathematik, Ruhr-Universität Bochum, April 10, 2018,

Bochum, Germany.

“Stationary and static metrics: from Lorentz to Finsler and vice versa”, workshop *Geometry and Pde's*, Department of Mathematics of the West University of Timisoara, June 13 – 14, 2017, Timisoara, Romania.

“Variations on the theme ‘Finsler manifold’ ”, “*Colloqui Matematici*” seminars, Mathematics Department of the University of Bari, October 22, 2014, Bari, Italy.

“On the Zermelo navigation problem with a strong wind” “*P(n) Seminars*” of the Mathematics Department of the “Sapienza” University of Rome, January 30, 2014, Rome, Italy.

“Stationary spacetimes and the Fermat principle”, Mathematics Department of the University of Bari, December 12, 2010, Bari, Italy.

“Critical groups of the action functional of a Lagrangian function”, “*Lorentz*” seminar of the Department of Algebra, Geometry and Topology of the University of Málaga, November 24, 2010, Málaga, Spain.

“Morse theory of causal geodesics in a stationary spacetime via Morse theory of geodesics of a Randers metric”, *Geometry and Topology seminar* of the Department of Geometry and Topology of the University of Granada, September 23, 2008, Granada, Spain.

“Interactions between Randers metrics and some variational and causal issues concerning conformally stationary spacetimes”, “*Rey Pastor*” seminar of the Mathematics Department of the University of Murcia, January 24, 2008, Murcia, Spain.

“Risultati di esistenza e di molteplicità per le soluzioni di tipo tempo dell'equazione di Lorentz in Relatività Generale”, Mathematics and Information Technology Department of the University of Camerino, November 21, 2001, Camerino, Italy.

Contributed talks

“On the Sagnac effect in general relativity”, *Nonlinear Analysis in the context of the General Relativity*, February 09–10, 2023, Politecnico di Bari, Bari (Italy)

“Geodesics for some singular Finsler metrics arising in Zermelo's navigation”, *Third International Conference on “New Methods in Finsler Geometry”*, August 24–26, 2022, Eindhoven (Netherland).

“An extension of Bartnik and Simon's existence result for spacelike graphs with prescribed mean curvature to exterior domains”, *X International Meeting on Lorentzian Geometry*, February 1—5, 2021, Córdoba (Spain). (On-line meeting).

“Finsler spacetimes and Killing vector fields”, *Second International Conference on “New Methods in Finsler Geometry”*, Centro di Ricerca Matematica Ennio De Giorgi, May 21–25, 2018, Pisa, Italy.

“Some convexity properties of Finsler and wind Finsler manifolds”, international conference *New Methods in Finsler Geometry*, July 5–9, 2016, Leipzig, Germany.

“Causality in static Finsler spacetimes”, *1st SCORE@POLIBA Workshop*, December 03–05, 2014, Bari, Italy.

“Some remarks about the generalized Morse Lemma for non- C^2 functionals”, *International Meeting on Differential Geometry*, November 15–17, 2010, Córdoba, Spain.

“On the global hyperbolicity of a conformally stationary spacetime”, *XVIII Congresso UMI*, September 24–29, 2007, Bari, Italy.

“Completeness of non-reversible Finsler metrics and geodesics in standard stationary spacetimes”, *IV International Meeting on Lorentzian Geometry*, February 5–8, 2007, Santiago de Compostela, Spain.

“Null geodesics for Kaluza-Klein metrics and world-lines of charged particles”, *II International Meeting of Lorentzian Geometry*, Murcia, Spain, November 12–14, 2003.

“An Avez-Seifert type result for the Lorentz force equation”, main talk at the *IV Turin Fortnight on Nonlinear Analysis*, October 1–3, 2003, Turin, Italy.

“Soluzioni di tipo tempo per l'equazione della forza di Lorentz con campi dipendenti dal tempo”, *XVII UMI Congress*, September 8–13, 2003, Milan, Italy.

“Trajectories of charged particles in the Reissner-Nordström space-time”, *XV SIGRAV Conference on General Relativity and Gravitational Physics*, September 9–12, 2002, Villa Mondragone, Monte Porzio Catone, Rome, Italy.

Schools and Visiting Periods

August 29 – September 3, 2022 and April 09-14, 2018, visiting professor at the Fakultät für Mathematik, Ruhr-Universität Bochum, Bochum, Germany.

June 11–21, 2017, visiting professor at the Mathematics Department of the Technical University of Cluj-Napoca, Cluj-Napoca, Romania.

May 6–10, 2017, visiting professor at the Mathematics and Computer Science Department, of Clark University, Worcester, MA, USA.

September 1–30, 2015, visiting professor at the School of Mathematical Sciences, Beijing Normal University, Beijing, China.

July 22–August 4, 2013, visiting researcher at the Institute of Mathematics and Statistics of the University of São Paulo, Brazil.

March 6–12, 2011, visiting researcher at the Mathematics Department of the University of Murcia, Spain.

November 18–27, 2010, visiting researcher at the Geometry and Topology Department of the University of Granada, Spain.

September 22–27, 2008, visiting researcher at the Geometry and Topology Department of the University of Granada, Spain.

January 20–25, 2008, visiting researcher at the Mathematics Department of the University of Murcia, Spain.

“School on Lorentzian geometry” held at Santiago de Compostela, Spain, February 5–8, 2007.

“Two weeks on Global Analysis” held at Centro di Ricerca Matematica Ennio De Giorgi, Pisa, Italy, February 21–25, 2005.

“Villa Mondragone International School of Gravitation and Cosmology”, held in Villa Mondragone, University of Rome “Tor Vergata”, Rome, Italy, September 6–10, 2002.

“Nonlinear Analysis and Applications to Physical Sciences”, school organized by the Italian research group “Variational and Topological Methods in the Study of Nonlinear Phenomena” at San Mommè, Pistoia, Italy, May 2–8, 2002.

“Summer School of Mathematics”, organized by SMI (Mathematical Inter-university School) at Perugia, Italy, 25/07–27/08 1999.

Other Academic Activities

Reviewer for the Mathematical Review of the AMS

Member of the scientific and organizing committee of the workshop *Nonlinear Analysis in the court of the General Relativity*, Bari (Italy), February 9–10, 2023.

Member of the organizing committee of the workshop *Qualitative and quantitative aspects of nonlinear PDEs*, Bari, Italy, September 05–09, 2022.

Member of the scientific and organizing committee of the winter school *Mathematics for Engineering Applications*, held at the Department of Mechanics, Mathematics and Management of the Polytechnic University of Bari, Bari, January 27–31, 2020, Italy.

Member of the scientific and organizing committee of the “Seminars on Analysis and Geometry”, Polytechnic University of Bari, Bari, February 22, 2017.

Member of the scientific committee of the conference *VII International Meeting on Lorentzian Geometry*, July 22–26, 2013, São Paulo, Brazil, 2013.

Member of the organizing committee of the workshop *V International Meeting on Lorentzian Geometry*, Martina Franca, Taranto, Italy, July 8–11, 2009.

09/2014–09/2018 Member of the committee for the revision of the statute of the Polytechnic University of Bari.

2014–2016 Member of the working group of a project about promotion, tutoring and placement for students of the Polytechnic University of Bari.

2014–2018 Member of the committee for the peer tutoring programme of the

	Polytechnic University of Bari.
2013–2014	Member of the working group for the admission test (TAI) at the Polytechnic University of Bari.
2020–today	Member of the “Polo di Ingegneria del Sistema Bibliotecario di Ateneo” .
	Member of the committee for the admission to the 2007 PhD program in Mathematics of the University of Bari.
	Member of a PhD committee in Mathematics of the University of Malaga, Spain, April 14, 2011.
	Member of a PhD committee in Mathematics of the University of Murcia, Spain, February 21, 2023.

Other Qualifications

Italian national teaching qualification for Mathematics and Physics, MIUR 1999.

Bari, February 27, 2023