locally symmetric spaces

Generalized

Sandra Gavino Fernández

Algebraic conditions

Geometry
of the
Jacobi
operator

C-spaces
and
R-spaces

Geometry of the skewsymmetric curvature operator

D-spaces
and

T-spaces

# Einstein-like Walker manifolds and generalized locally symmetric spaces

#### Sandra Gavino Fernández



Department of Geometry and Topology University of Santiago de Compostela

July 11th, 2009

Work in progress with Miguel Brozos Vázquez

locally symmetric spaces

Generalized

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Algebraic condition

Geometry of the Jacobi operator C-spaces and \$\mathfrak{B}\$-spaces

Geometry
of the
skewsymmetric
curvature
operator
\$\D\$-spaces

Aim: To investigate some generalizations of locally symmetric spaces.

- Einstein-like manifolds (Algebraic conditions)
- ② €-spaces and ⅓-spaces (Geometry of geodesics)
- O-spaces and T-spaces (Geometry of unit circles)

locally symmetric spaces

Generalized

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Algebraic condition

Geometry
of the
Jacobi
operator

C-spaces
and

\$\partial \text{spaces} \text{spaces}

Geometry of the skewsymmetric curvature operator Aim: To investigate some generalizations of locally symmetric spaces.

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Algebraic condition

Geometr of the Jacobi operator e-spaces and \$\partial spaces

Geometry
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skewsymmetric
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Algebraic condition

Geometry
of the
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\$\partial \text{\$\pi\$-spaces}\$

Geometry of the skewsymmetric curvature operator 𝔊-spaces Aim: To investigate some generalizations of locally symmetric spaces.

- Einstein-like manifolds (Algebraic conditions)
- ② C-spaces and \$\mathfrak{P}\$-spaces (Geometry of geodesics)
- 3 D-spaces and T-spaces (Geometry of unit circles

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Algebraic conditions

Geometry of the Jacobi operator  $\mathfrak{C}$ -spaces and  $\mathfrak{P}$ -spaces

Geometry of the skew-symmetric curvature operator  $\Omega$ -spaces

Aim: To investigate some generalizations of locally symmetric spaces.

- Einstein-like manifolds (Algebraic conditions)
- 2 C-spaces and  $\mathfrak{P}$ -spaces (Geometry of geodesics)
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Algebraic conditions

Geometry of the Jacobi operator C-spaces and \$\partial \mathbb{F}\$-spaces

Geometry
of the
skewsymmetric
curvature
operator

\$\mathcal{D}\$-spaces
and

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- Einstein-like manifolds (Algebraic conditions)
- ② C-spaces and P-spaces (Geometry of geodesics)
- 3 D-spaces and T-spaces (Geometry of unit circles)

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Algebraic conditions

Geometry of the Jacobi operator e-spaces and \$\partial \mathbb{B}\$-spaces

Geometry of the skew-symmetric curvature operator  $\mathfrak{D}$ -spaces

Aim: To investigate some generalizations of locally symmetric spaces.

#### Different approaches:

- Einstein-like manifolds (Algebraic conditions)
- ${f 2}$  C-spaces and  ${f P}$ -spaces (Geometry of geodesics)
- ① D-spaces and T-spaces (Geometry of unit circles)

Three-dimensional Walker metrics

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Algebraic conditions

of the Jacobi operator

C-spaces and

Geometry of the skewsymmetric curvature

D-spaces
and
Σ-spaces

#### Definition

A manifold  $(\mathcal{M}, g)$  is Walker if it admits a null parallel distribution  $\mathcal{D}$  of dimension r.

A manifold  $(\mathcal{M}, g)$  is Walker if it admits a null parallel distribution  $\mathcal{D}$  of dimension r.

$$g_f=\left(egin{array}{ccc} 0&0&1\0&arepsilon&0\1&0&f \end{array}
ight)$$
 where  $f=f(t,x,y)$  and  $arepsilon=\pm 1.$ 

A manifold  $(\mathcal{M}, g)$  is Walker if it admits a null parallel distribution  $\mathcal{D}$  of dimension r.

• In dimension 3 there exists coordinates such that:

$$g_f = \left( egin{array}{ccc} 0 & 0 & 1 \ 0 & arepsilon & 0 \ 1 & 0 & f \end{array} 
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•  $\mathcal{D}$  is locally generated by  $\partial_t$ .

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- $\mathcal{D}$  is locally generated by  $\partial_t$ .
- $(\mathcal{M}, g)$  is strictly Walker if  $\mathcal{D}$  admits a null parallel vector field.

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$$->(\mathcal{M},g)$$
 is strictly Walker  $\Leftrightarrow Ric^2=0$ 

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- Algebraic conditions
- Q Geometry of the Jacobi operator
- 3 Geometry of the skew-symmetric curvature operator

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Algebraic conditions

Geometry of the Jacobi operator C-spaces and B-spaces

Geometry
of the
skewsymmetric
curvature
operator

D-spac and T-space

- Algebraic conditions
- 2 Geometry of the Jacobi operator
- 3 Geometry of the skew-symmetric curvature operator

# Algebraic conditions

of the Jacobi operator

e-spaces and

e-spaces and \$\psi\$-space

of the skew-symmetric curvature operator

2)-space

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold of dimension n.

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Algebraic conditions

of the Jacobi operator

C-spaces and P-spaces

of the skew-symmetric curvature operator \$\mathcal{D}\$-spaces

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold of dimension n.



Let  $(\mathcal{M}, g)$  be a Lorentzian manifold of dimension n.

- $\bullet$   $\nabla$  denotes the Levi-Civita connection.
- **2**  $R(x,y) = [\nabla_x, \nabla_y] \nabla_{[x,y]}$  is the curvature operator.

Geometry of the skew-symmetric curvature operator  $\mathfrak{D}$ -spaces

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#### Ricci tensor

$$\rho(x,y) = \sum_{i} \varepsilon_{i} R(e_{i},x,y,e_{i})$$

Geometry
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# Algebraic conditions

Geometry
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operator

&-spaces
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&-spaces

Geometry of the skew-symmetric curvature operator

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$$g(\rho(x), y) := \rho(x, y)$$
 for a unit  $x$  and  $y$ .

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#### Skew-symmetric operator

$$g(R(x,y)z, w) = R(x, y, z, w)$$
  
for  $x, y \in T_p \mathcal{M}$ .

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Algebraic conditions
Geometry

of the Jacobi operator

C-spaces and

Geometry of the skew-symmetric curvature operator

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#### Jacobi operator

$$R_x = R(\cdot, x)x$$
 for a unit x.

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# Algebraic conditions

of the Jacobi operator

C-spaces and
P-spaces

Geometry

Geometry of the skewsymmetric curvature operator \$\Darkont \cdot \spaces and \$\Tag{\text{Spaces}}\$ Let  $(\mathcal{M}, g)$  be a Lorentzian manifold of dimension n.

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#### Jacobi operator

$$R_x = R(\cdot, x)x$$
 for a unit  $x$ .

#### Szabó operator

$$R'_{x} = \nabla_{x} R(\cdot, x) x$$
 for a unit  $x$ .

#### Algebraic conditions

Generalized locally symmetric spaces

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#### Algebraic conditions

of the

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## Algebraic conditions

of the Jacobi operator

C-spaces and

B-space

Geometry of the skewsymmetric curvature operator

O-space and •  $(\mathcal{M}, g)$  is locally symmetric  $\Leftrightarrow$  local geodesic symmetries are isometries

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## Algebraic conditions

of the Jacobi operator

C-spaces and P-spaces

Geometry of the skewsymmetric curvature operator

D-space and •  $(\mathcal{M}, g)$  is locally symmetric  $\Leftrightarrow$  local geodesic symmetries are isometries

$$->(\mathcal{M},g)$$
 is locally symmetric  $\Leftrightarrow 
abla R=0$ 

of the skewsymmetric curvature operator

O-space and ullet  $(\mathcal{M},g)$  is locally symmetric  $\Leftrightarrow$  local geodesic symmetries are isometries

$$->$$
  $(\mathcal{M},g)$  is locally symmetric  $\Leftrightarrow \nabla R=0$ 

$$-> \dim \mathcal{M} = 3$$
  $(\mathcal{M}, g)$  locally symmetric  $\Leftrightarrow \nabla \rho = 0$ 

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$$W = \mathcal{A} \oplus \mathcal{B} \oplus \mathcal{C}^{\perp}$$

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### Algebraic conditions

Geometry of the Jacobi operator &-spaces and

Geometry of the skew-symmetric curvature operator  $\mathfrak{D}$ -spaces

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$$W = \mathcal{A} \oplus \mathcal{B} \oplus \mathcal{C}^{\perp}$$

- $\mathcal{A}$ :  $(\nabla_X \rho)(X,X) = 0$ ,
- $\mathcal{B}$ :  $(\nabla_X \rho)(Y, Z) = (\nabla_Y \rho)(X, Z)$ ,

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$$->(\mathcal{M},g)$$
 is locally symmetric  $\Leftrightarrow \nabla R=0$ 

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abla 
ho = 0$ 

$$W = \mathcal{A} \oplus \mathcal{B} \oplus \mathcal{C}^{\perp}$$

- $\mathcal{A}$ :  $(\nabla_X \rho)(X,X) = 0$ ,
- $\mathcal{B}$ :  $(\nabla_X \rho)(Y, Z) = (\nabla_Y \rho)(X, Z)$ ,
- $C^{\perp}$ :  $\nabla_X(\rho)(Y,Z) = \frac{1}{(n+2)(n-1)} \{ nX(\tau)g(Y,Z) + \frac{n-2}{2} [Y(\tau)g(X,Z) + Z(\tau)g(X,Y)] \}.$

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Algebraic conditions

Geometr

Jacobi operator C-spaces and

and \$\psi\$-spaces

Geometry
of the

of the skew-symmetric curvature operator

#### Theorem

For a 3-dimensional Walker manifold  $(\mathcal{M},g)$  are equivalent:

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# Algebraic conditions

of the Jacobi operator C-spaces and B-spaces

Geometry of the skewsymmetric curvature operator \$\D\(\text{-spaces}\)

#### Theorem

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 $\bullet$   $(\mathcal{M}, g)$  is locally symmetric.

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Algebraic conditions

Geometr of the Jacobi operator C-spaces and

Geometry of the skewsymmetric curvature operator

### Theorem

- $\bullet$   $(\mathcal{M}, g)$  is locally symmetric.
- **2**  $(\mathcal{M}, g)$  is in class  $\mathcal{A}$ .

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Algebraic conditions

Geometry of the Jacobi operator C-spaces and

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### Theorem

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Algebraic conditions

Geometry of the Jacobi

Jacobi operator C-spaces and P-spaces

Geometry
of the
skewsymmetric
curvature
operator
\$\D\$-spaces

#### Theorem,

- $\bullet$   $(\mathcal{M}, g)$  is locally symmetric.
- $(\mathcal{M},g)$  is in class  $\mathcal{A}$ .
- **3**  $(\mathcal{M}, g)$  is in class  $\mathcal{C}^{\perp}$ .

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# Algebraic conditions

Geometry
of the
Jacobi
operator

C-spaces
and
P-spaces

Geometry of the skew-symmetric curvature operator

#### Theorem,

For a 3-dimensional Walker manifold  $(\mathcal{M}, g)$  are equivalent:

- $\bullet$   $(\mathcal{M}, g)$  is locally symmetric.
- $(\mathcal{M}, g) \text{ is in class } \mathcal{A}.$
- **3**  $(\mathcal{M}, g)$  is in class  $\mathcal{C}^{\perp}$ .
- $\bullet \ (\mathcal{M},g) \text{ is in class } \mathcal{A} \oplus \mathcal{C}^{\perp}.$

### Theorem

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Algebraic conditions

Geometry
of the
Jacobi
operator

& Spaces
and
& Spaces

Geometry of the skew-symmetric curvature operator

#### Theorem

For a 3-dimensional Walker manifold  $(\mathcal{M}, g)$  are equivalent:

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- $(\mathcal{M},g)$  is in class  $\mathcal{A}$ .
- **3**  $(\mathcal{M}, g)$  is in class  $\mathcal{C}^{\perp}$ .
- $\bullet \ (\mathcal{M},g) \text{ is in class } \mathcal{A} \oplus \mathcal{C}^{\perp}.$

### Theorem

For a 3-dimensional Walker manifold  $(\mathcal{M}, g)$  are equivalent:

 $\bullet$   $(\mathcal{M}, g)$  is locally conformally flat.

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Algebraic conditions

Geometry of the Jacobi operator C-spaces and P-spaces

Geometry
of the
skewsymmetric
curvature
operator
\$\D\$-spaces

#### Theorem

For a 3-dimensional Walker manifold  $(\mathcal{M}, g)$  are equivalent:

- $\bullet$   $(\mathcal{M}, g)$  is locally symmetric.
- $(\mathcal{M},g)$  is in class  $\mathcal{A}$ .
- **3**  $(\mathcal{M}, g)$  is in class  $\mathcal{C}^{\perp}$ .
- $oldsymbol{0}$   $(\mathcal{M},g)$  is in class  $\mathcal{A}\oplus\mathcal{C}^{\perp}$ .

### Theorem

- $\bullet$   $(\mathcal{M}, g)$  is locally conformally flat.
- **2**  $(\mathcal{M}, g)$  is in class  $\mathcal{B}$ .

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# Algebraic conditions

Geometry of the Jacobi operator €-spaces and \$\partial \partial \

Geometry of the skew-symmetric curvature operator

#### **Theorem**

For a 3-dimensional Walker manifold  $(\mathcal{M},g)$  are equivalent:

- $\bullet$   $(\mathcal{M}, g)$  is locally symmetric.
- $(\mathcal{M},g)$  is in class  $\mathcal{A}$ .
- **3**  $(\mathcal{M}, g)$  is in class  $\mathcal{C}^{\perp}$ .
- $oldsymbol{0}$   $(\mathcal{M},g)$  is in class  $\mathcal{A}\oplus\mathcal{C}^{\perp}$ .

#### Theorem

- $(\mathcal{M},g)$  is locally conformally flat.
- $(\mathcal{M}, g)$  is in class  $\mathcal{B}$ .
- **3**  $(\mathcal{M}, g)$  is in class  $\mathcal{B} \oplus \mathcal{C}^{\perp}$ .

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Geometry of the

of the Jacobi operator C-spaces and

Geometry of the skewsymmetric curvature operator \$\Dangle\$-spaces and

- Algebraic conditions
- @ Geometry of the Jacobi operator
  - ullet C-spaces and  ${\mathfrak P}$ -spaces
- 3 Geometry of the skew-symmetric curvature operator

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Algebra

of the Jacobi operator

€-spaces and ₽-spaces

of the skew-symmetric curvature operator

D-spaces and T-spaces Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

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Geometry of the Jacobi

C-spaces and P-spaces

Geometry
of the
skewsymmetric
curvature
operator

\$\D\$-spaces
and
\$\tilde{\S}-spaces

Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

•  $(\mathcal{M}, g)$  is a C-space if the Jacobi operator has constant eigenvalues along every geodesic.

Fernandez Algebraic

Geometry of the Jacobi

operator

C-spaces
and

P-spaces

Geometry
of the

Geometry
of the
skewsymmetric
curvature
operator

\$\D\_{\text{spaces}}^{\text{spaces}}\$
and
\$\T\_{\text{spaces}}^{\text{spaces}}\$

Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

- $(\mathcal{M}, g)$  is a  $\mathfrak{C}$ -space if the Jacobi operator has constant eigenvalues along every geodesic.
- ②  $(\mathcal{M}, g)$  is a  $\mathfrak{P}$ -space if the Jacobi operator has parallel eigenspaces along every geodesic.

Generalized

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Geometry of the Jacobi operator

C-spaces and ுS-spaces

Geometry
of the
skewsymmetric
curvature
operator
①-spaces
and

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Theorem (Berndt-Vanhecke, 1992)

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of the Jacobi operator &-spaces

#-spaces
Geometry
of the
skewsymmetri

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# Theorem (Berndt-Vanhecke, 1992)

 $(\mathcal{M},g)$  is locally symmetric if and only if  $(\mathcal{M},g)$  is

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condition Geometr

of the Jacobi operator C-spaces and B-spaces

Geometry of the skew-symmetric curvature operator

Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

- $(\mathcal{M}, g)$  is a  $\mathfrak{C}$ -space if the Jacobi operator has constant eigenvalues along every geodesic.
- ②  $(\mathcal{M}, g)$  is a  $\mathfrak{P}$ -space if the Jacobi operator has parallel eigenspaces along every geodesic.

### Theorem (Berndt-Vanhecke, 1992)

 $(\mathcal{M}, g)$  is locally symmetric if and only if  $(\mathcal{M}, g)$  is simultaneously  $\mathfrak{C}$ -space and  $\mathfrak{P}$ -space.

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operator

C-spaces
and

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Geometry
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skewsymmetric
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\$\D\$-spaces
and

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### Theorem (Berndt-Vanhecke, 1992)

 $(\mathcal{M},g)$  is locally symmetric if and only if  $(\mathcal{M},g)$  is simultaneously  $\mathfrak{C}$ -space and  $\mathfrak{P}$ -space.

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold.

curvature operator ⊕-spaces and T-spaces Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

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### Theorem (Berndt-Vanhecke, 1992)

 $(\mathcal{M},g)$  is locally symmetric if and only if  $(\mathcal{M},g)$  is simultaneously  $\mathfrak{C}$ -space and  $\mathfrak{P}$ -space.

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold.

•  $(\mathcal{M}, g)$  is a timelike  $\mathfrak{C}$ -space  $(T\mathfrak{C})$  if the Jacobi operator has constant eigenvalues along every timelike geodesic.

Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

- $(\mathcal{M}, g)$  is a  $\mathfrak{C}$ -space if the Jacobi operator has constant eigenvalues along every geodesic.
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### Theorem (Berndt-Vanhecke, 1992)

 $(\mathcal{M},g)$  is locally symmetric if and only if  $(\mathcal{M},g)$  is simultaneously  $\mathfrak{C}$ -space and  $\mathfrak{P}$ -space.

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold.

- $(\mathcal{M}, g)$  is a timelike  $\mathfrak{C}$ -space  $(\mathcal{T}\mathfrak{C})$  if the Jacobi operator has constant eigenvalues along every timelike geodesic.
- ②  $(\mathcal{M}, g)$  is a timelike  $\mathfrak{P}$ -space  $(T\mathfrak{P})$  if the Jacobi operator has parallel eigenspaces along every timelike geodesic.

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Algebrai conditio

Geometry of the Jacobi operator

€-spaces and ₿-spaces

of the skew-symmetric curvature operator

\$\Damps\_{\text{-spaces}} and \text{-spaces}\$

#### **Theorem**

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold.

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Algebrai condition

Geometry of the Jacobi operator

C-spaces and P-spaces

of the skew-symmetric curvature operator

\$\D\$-spaces and \$\T\_{\text{spaces}}\$

#### **Theorem**

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold.  $(\mathcal{M}, g)$  is locally symmetric  $\Leftrightarrow (\mathcal{M}, g)$  is simultaneously a

Generalized

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Algebrai condition

Geometry
of the
Jacobi
operator
C-spaces
and
P-spaces

Geometry
of the
skewsymmetric
curvature
operator
\$\D\$-spaces
and
\$\Timespaces

#### Theorem

Let  $(\mathcal{M},g)$  be a Lorentzian manifold.  $(\mathcal{M},g)$  is locally symmetric  $\Leftrightarrow (\mathcal{M},g)$  is simultaneously a timelike  $\mathfrak{C}$ -space and a timelike  $\mathfrak{P}$ -space.

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Algebraio condition

Geometry of the Jacobi operator

C-spaces and B-spaces

Geometry of the skewsymmetric curvature operator \$\Darpoonup \cdot \spaces and

#### Theorem

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 $T\mathfrak{C} \cap T\mathfrak{P}$  is equivalent to

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condition

of the
Jacobi
operator

c-spaces
and
P-spaces

Geometry of the skew-symmetric curvature operator

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 $T\mathfrak{C} \cap T\mathfrak{P}$  is equivalent to

$$\nabla_x R_{xyyx} = 0$$
,  $\forall x$  timelike and  $y \perp x$ 

Generalized

#### Theorem

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### Remark

Previous theorem is false in other signatures.

#### **Theorem**

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### Example

A strict walker 4-dimensional manifold of signature (2,2):  $g = 2dx^1 \circ dx^3 + 2dx^2 \circ dx^4 + b(x^3)dx^4 \circ dx^4$ , is always  $T\mathfrak{C}$  and  $T\mathfrak{P}$  but not locally symmetric ( $\nabla R = 0 \Leftrightarrow b^{(3)}(x^3) = 0$ ).

Sandra Gavino Fernándo

Algebrai conditio

Geometry of the Jacobi operator &-spaces

C-spaces and P-spaces Geometry of the

of the skew-symmetric curvature operator \$\mathcal{D}\$-spaces and \$\mathcal{T}\$-spaces

 $(\mathcal{M}, \mathbf{g}) \text{ timelike $\mathfrak{C}$-space} \Leftrightarrow R_{\gamma} \text{ has constant eigenvalues along } \gamma$  for any timelike  $\gamma$ 

Generalized

C-spaces

P-spaces

 $(\mathcal{M}, g)$  timelike  $\mathfrak{C}$ -space  $\Leftrightarrow R_{\gamma}$  has constant eigenvalues along  $\gamma$ for any timelike  $\gamma$  $\Leftrightarrow \nabla_{\gamma'} Tr R_{\gamma}^{(k)} = 0$  for any timelike  $\gamma$ 

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Generalized

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Algebraic condition

of the
Jacobi
operator

C-spaces
and

P-spaces

Geometry of the skewsymmetric curvature operator  $(\mathcal{M},g)$  timelike  $\mathfrak{C}$ -space  $\Leftrightarrow R_{\gamma}$  has constant eigenvalues along  $\gamma$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{\gamma'} \operatorname{Tr} R_{\gamma}^{(k)} = 0$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{X} \operatorname{Tr} R_{X}^{(k)} = 0$ , for all X

Generalized

Sandra Gavino Fernánde

conditio

Geometry of the Jacobi operator

C-spaces and B-spaces

Geometry of the skewsymmetric curvature operator \$\Darpoonup \cdot \text{spaces}\$ and  $(\mathcal{M},g)$  timelike  $\mathfrak{C}$ -space  $\Leftrightarrow R_{\gamma}$  has constant eigenvalues along  $\gamma$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{\gamma'} \mathit{Tr} R_{\gamma}^{(k)} = 0$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{X} \mathit{Tr} R_{Y}^{(k)} = 0$ , for all X

### Lemma (Characterization)

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold of dimension 3.  $(\mathcal{M}, g)$  is a timelike  $\mathfrak{C}$ -space if and only if:

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condition

of the Jacobi operator &-spaces

\$\mathfrak{P}\$-spaces

Geometry of the skew-symmetricurvature operator

 $(\mathcal{M},g)$  timelike  $\mathfrak{C}$ -space  $\Leftrightarrow R_{\gamma}$  has constant eigenvalues along  $\gamma$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{\gamma'} \operatorname{Tr} R_{\gamma}^{(k)} = 0$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{X} \operatorname{Tr} R_{Y}^{(k)} = 0$ , for all X

### Lemma (Characterization)

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold of dimension 3.  $(\mathcal{M}, g)$  is a timelike  $\mathfrak{C}$ -space if and only if:

(1) 
$$\nabla_X \rho(X,X) = 0$$
,

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condition

of the Jacobi operator C-spaces

P-spaces

Geometry of the skew-symmetric curvature operator

 $(\mathcal{M},g)$  timelike  $\mathfrak{C}$ -space  $\Leftrightarrow R_{\gamma}$  has constant eigenvalues along  $\gamma$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{\gamma'} \operatorname{Tr} R_{\gamma}^{(k)} = 0$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{X} \operatorname{Tr} R_{Y}^{(k)} = 0$ , for all X

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Let  $(\mathcal{M}, g)$  be a Lorentzian manifold of dimension 3.  $(\mathcal{M}, g)$  is a timelike  $\mathfrak{C}$ -space if and only if:

(1) 
$$\nabla_X \rho(X, X) = 0$$
,

(2) 
$$\sum_{i} \varepsilon_{i} R(e_{i}, X, X, e_{i}) \nabla_{X} R(e_{i}, X, X, e_{i}) = 0$$
,

Generalized

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Algebraid condition

Geometry
of the
Jacobi
operator

e-spaces
and
p-spaces

Geometry of the skewsymmetri curvature  $(\mathcal{M},g)$  timelike  $\mathfrak{C}$ -space  $\Leftrightarrow R_{\gamma}$  has constant eigenvalues along  $\gamma$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{\gamma'} \mathit{Tr} R_{\gamma}^{(k)} = 0$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{X} \mathit{Tr} R_{\gamma}^{(k)} = 0$ , for all X

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,

 $\forall X \in T\mathcal{M}$ , where  $\{e_i\}$  is an orthonormal basis.

 $(\mathcal{M},g)$  timelike  $\mathfrak{C}$ -space  $\Leftrightarrow R_{\gamma}$  has constant eigenvalues along  $\gamma$  for any timelike  $\gamma$   $\Leftrightarrow \nabla_{\gamma'} \mathit{Tr} R_{\gamma}^{(k)} = 0$  for any timelike  $\gamma$ 

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 $\forall X \in T\mathcal{M}$ , where  $\{e_i\}$  is an orthonormal basis.

(1) and (2) are the Ledger conditions of order 3 and 5.

Geometry of the Jacobi operator C-spaces and B-spaces

Geometry
of the
skewsymmetric
curvature
operator
\$\Darrow\$-spaces
and

#### Lemma

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold.  $(\mathcal{M}, g)$  is a timelike  $\mathfrak{P}$ -space if and only if:

P-spaces

#### Lemma

Let  $(\mathcal{M},g)$  be a Lorentzian manifold.  $(\mathcal{M},g)$  is a timelike  $\mathfrak{P}$ -space if and only if:

$$R_{\gamma}R_{\gamma}'-R_{\gamma}'R_{\gamma}=0$$

#### Lemma

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold.  $(\mathcal{M}, g)$  is a timelike  $\mathfrak{P}$ -space if and only if:

$$R_{\gamma}R_{\gamma}'-R_{\gamma}'R_{\gamma}=0$$

for all timelike geodesic  $\gamma$ .

Generalized

#### Lemma

Let  $(\mathcal{M}, g)$  be a Lorentzian manifold.  $(\mathcal{M}, g)$  is a timelike  $\mathfrak{P}$ -space if and only if:

$$R_{\gamma}R_{\gamma}'-R_{\gamma}'R_{\gamma}=0$$

for all timelike geodesic  $\gamma$ .

The Jacobi and the Szabó operators commute for timelike geodesics if and only  $R_X R_X' - R_X' R_X = 0$  for all X.

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Algebraic conditions

of the Jacobi operator

€-spaces and ₱-spaces

of the skew-symmetric curvature operator

# Theorem.

A 3-dimensional Walker manifold  $\mathcal{M}_f$  is a timelike  $\mathfrak{C}$ -space

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Algebraic conditions

Geometry of the Jacobi operator

€-spaces and \$\$-spaces

of the skew-symmetric curvature operator

\$\infty\$-spaces and \$\infty\$-spaces

# Theorem (

A 3-dimensional Walker manifold  $\mathcal{M}_f$  is a timelike  $\mathfrak{C}$ -space if and only if it is locally symmetric.

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Algebraio condition

of the Jacobi operator C-spaces and P-spaces

Geometry of the skew-symmetricurvature operator

### Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  is a timelike  $\mathfrak{C}$ -space if and only if it is locally symmetric.

#### Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  of constant scalar curvature is a timelike  $\mathfrak{P}$ -space

Sandra Gavino Fernándo

Algebrai conditio

of the Jacobi operator C-spaces

C-spaces and P-spaces

Geometry
of the
skewsymmetri
curvature
operator

\$\Data\$-spaces
and
\$\Tampagereq\$

#### Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  is a timelike  $\mathfrak{C}$ -space if and only if it is locally symmetric.

#### Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  of constant scalar curvature is a timelike  $\mathfrak{P}$ -space if and only if it is locally symmetric or a strictly Walker metric.

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Algebraio condition

of the Jacobi operator C-spaces and B-spaces

Geometry
of the
skewsymmetric
curvature
operator

\$\Dalpha\$-spaces
and

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A 3-dimensional Walker manifold  $\mathcal{M}_f$  of constant scalar curvature is a timelike  $\mathfrak{P}$ -space if and only if it is locally symmetric or a strictly Walker metric.

-> A 3-dimensional Walker manifold  $\mathcal{M}_f$  of constant scalar curvature is a timelike  $\mathfrak{P}$ -space if and only if it is curvature recurrent.

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condition

Geometry
of the
Jacobi
operator

C-spaces
and
P-spaces

Geometry of the skewsymmetric curvature operator

D-spaces
and

∇-spaces

- Algebraic conditions
- 2 Geometry of the Jacobi operator
- Geometry of the skew-symmetric curvature operator
  - ullet D-spaces and  ${\mathfrak T}$ -spaces

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Algebra condition

of the Jacobi operator C-spaces

C-spaces and \$\P-spaces Geometry of the

skewsymmetric curvature operator \$\mathcal{D}\$-spaces and

T-spaces

Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

Generalized

Fernánde:

Algebraic condition

of the Jacobi operator

C-spaces and

P-spaces

Geometry of the skewsymmetric curvature operator \$\mathcal{D}\$-spaces

T-spaces

Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

•  $(\mathcal{M}, g)$  is a  $\mathfrak{D}$ -space if the skew-symmetric curvature operator has constant eigenvalues along each unit circle.

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Generalized

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Algebraic conditions

Geometry
of the
Jacobi
operator

e-spaces
and

p-spaces

Geometry
of the
skewsymmetric
curvature
operator
D-spaces
and

**T**-spaces

Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

- $(\mathcal{M},g)$  is a  $\mathfrak{D}$ -space if the skew-symmetric curvature operator has constant eigenvalues along each unit circle.
- ②  $(\mathcal{M}, g)$  is a  $\mathfrak{T}$ -space if there is a parallel Jordan basis for the skew-symmetric curvature operator along each unit circle.

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Algebrai conditio

Geometry of the Jacobi operator C-spaces and \$\partial \text{\$\mathcal{P}\$-spaces}\$

Geometry
of the
skewsymmetric
curvature
operator
D-spaces
and
T-spaces

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Theorem (Ivanov-Petrova, 1997)

Generalized

Sandra Gavino Fernández

conditio

of the Jacobi operator

C-spaces and

P-spaces

Geometry
of the
skewsymmetric
curvature
operator
D-spaces
and
T-spaces

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# Theorem (Ivanov-Petrova, 1997)

 $(\mathcal{M},g)$  is locally symmetric if and only if  $(\mathcal{M},g)$  is

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Geometry
of the
Jacobi
operator

C-spaces

Geometry
of the
skewsymmetric
curvature
operator
D-spaces
and
T-spaces

Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

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# Theorem (Ivanov-Petrova, 1997)

 $(\mathcal{M}, g)$  is locally symmetric if and only if  $(\mathcal{M}, g)$  is simultaneously  $\mathfrak{D}$ -space and  $\mathfrak{T}$ -space.

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Geometry
of the
Jacobi
operator

C-spaces

Geometry
of the
skewsymmetric
curvature
operator
\$\D\text{-spaces}
and
\T-spaces

Let  $(\mathcal{M}, g)$  be a Riemannian manifold.

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# Theorem (Ivanov-Petrova, 1997)

 $(\mathcal{M}, g)$  is locally symmetric if and only if  $(\mathcal{M}, g)$  is simultaneously  $\mathfrak{D}$ -space and  $\mathfrak{T}$ -space.

 $(\mathcal{M}, g)$  is a  $\mathfrak{D}$ -space if and only if  $\nabla_{c'} TrR(c', \nabla_{c'}c')^{(k)} = 0$  for all unit circle c.

 $(\mathcal{M},g)$  is a  $\mathfrak{T}$ -space if and only if  $\nabla_{c'}R(c',\nabla_{c'}c')$  and  $R(c',\nabla_{c'}c')$  commute for all unit circle c.

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Algebraic condition

Jacobi operator C-spaces

Geometry of the skewsymmetric curvature

D-spaces and T-spaces

# Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  is a  $\mathfrak{O}$ -space

Generalized

Fernández Algebraic

Geometry of the

Jacobi operator C-spaces and P-spaces

of the skew-symmetric curvature operator  $\mathcal{D}$ -spaces

T-spaces

# Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  is a  $\mathfrak{O}$ -space if and only if it is locally symmetric or a strictly Walker metric.

Generalized

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Algebraic conditions

of the Jacobi operator C-spaces and

of the skewsymmetric curvature operator

⊕-spaces and ⊊-spaces

# Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  is a  $\mathfrak{D}$ -space if and only if it is locally symmetric or a strictly Walker metric.

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condition

of the
Jacobi
operator

C-spaces
and
P-spaces

Geometry of the skewsymmetri curvature operator \$\Darrow\$-spaces

T-spaces

# Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  is a  $\mathfrak{D}$ -space if and only if it is locally symmetric or a strictly Walker metric.

#### Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  of constant scalar curvature is a  $\mathfrak{T}$ -space

#### pperator Ω-spaces and Σ-spaces

# Theorem

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Geometry of the

operator

C-spaces
and

\$\partial \text{\$\mathcal{P}\$-spaces}\$

Geometry
of the
skewsymmetric
curvature
operator
\$\mathcal{D}\$-spaces

T-spaces

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# Theorem

A 3-dimensional Walker manifold  $\mathcal{M}_f$  of constant scalar curvature is a  $\mathfrak{T}$ -space if and only if it is locally symmetric or a strictly Walker metric.

$$\{\mathfrak{O} - \operatorname{space} \cap \mathfrak{T} - \operatorname{space}\} \supseteq \{\nabla R = 0\}.$$

Generalized

Gavino Fernández

Algebraic condition:

of the Jacobi operator

C-spaces and

P-spaces

Geometry of the skewsymmetric curvature operator

 $\begin{array}{c} \mathfrak{O}\text{-spaces} \\ \text{and} \\ \mathfrak{T}\text{-spaces} \end{array}$ 

# Einstein-like Walker manifolds and generalized locally symmetric spaces

# Sandra Gavino Fernández



Department of Geometry and Topology University of Santiago de Compostela

July 11th, 2009

Work in progress with Miguel Brozos Vázquez