Exercise 1 — K means Clustering

1) Points: $(1,4)$; $(3,5)$; $(2,-1)$; $(-3,8)$

Centers: $(1,2)$; $(3,-2)$

$k=2$

1) Center initialize

2) $d_{11} = \sqrt{(1-1)^2 + (2-4)^2} = 2$ $d_{21} = \sqrt{4+9} = \sqrt{13}$

$d_{12} = \sqrt{4+36} = 2\sqrt{10}$ $d_{22} = \sqrt{48} = 7$

$d_{31} = \sqrt{1+9} = \sqrt{10}$ $d_{41} = \sqrt{16+49} = \sqrt{65}$

$d_{32} = \sqrt{1+1} = \sqrt{2}$ $d_{42} = \sqrt{36+121} = \sqrt{157}$

Points	c_1	c_2	Cluster
$(1,4)$	2	6.3	C_1
$(3,5)$	3.6	7	C_1
$(2,-1)$	3.16	1.41	C_2
$(-3,8)$	8.06	12.52	C_1

→ New clusters:

$x = \dfrac{1+3+(-3)}{3} = \dfrac{1}{3}$

$y = \dfrac{4+5+9}{3} = 6$

$C_1 = (\tfrac{1}{3}; 6)$

$C_2 = (2, -1)$

Points	C_1	C_2	Clusters
$(1,4)$	2.1	5.0	C_1
$(3,5)$	2.8	6.0	C_1
$(2,-1)$	7.19	0	C_2
$(-3,8)$	4.48	11.1	C_1

Answer: $C_1 \in [(1,4); (3;5); (-3;8)]$

$C_2 \in [(2;-1)]$

No cluster is changed, we stop iterating

Exercise 2

2) Points $(-1,1,2)$; $(1,-2,1)$; $(2,5,3)$; $(3,4,4)$

Centers: $(1,1,1)$; $(3,3,3)$

$k=2$

$d_{11} = \sqrt{4+1} = \sqrt{5}$ $d_{21} = \sqrt{0+9+0} = 3$

$d_{12} = \sqrt{16+4+1} = \sqrt{21}$ $d_{22} = \sqrt{4+25+4} = \sqrt{33}$

$d_{41} = \sqrt{4+9+9} = \sqrt{22}$ $d_{31} = \sqrt{21}$

$d_{42} = \sqrt{0+1+1} = \sqrt{2}$ $d_{32} = \sqrt{5}$

Points	C_1	C_2	Clusters
$(-1,1,2)$	$\sqrt{5}$	$\sqrt{21}$	C_1
$(1,-2,1)$	3	$\sqrt{33}$	C_1
$(2,5,3)$	$\sqrt{21}$	$\sqrt{5}$	C_2
$(3,4,4)$	$\sqrt{22}$	$\sqrt{2}$	C_2

New centroids:

$C_1 = \left(\dfrac{-1+1}{2}; \dfrac{1-2}{2}; \dfrac{2+1}{2}\right)$ $C_2 = \left(\dfrac{2+3}{2}; \dfrac{5+4}{2}; \dfrac{3+4}{2}\right)$

$C_1 = \left(0; -\dfrac{1}{2}; \dfrac{3}{2}\right)$ $C_2 = \left(\dfrac{5}{2}; \dfrac{9}{2}; \dfrac{7}{2}\right)$

$d_{c11} = \sqrt{1 + \dfrac{9}{4} + \dfrac{1}{4}} = \sqrt{\dfrac{11}{4}}$ $d_{c12} = \sqrt{1 + \dfrac{9}{4} + \dfrac{1}{4}} = \sqrt{\dfrac{18}{4}}$ $d_{c13} = \sqrt{4 + \dfrac{121}{4} + \dfrac{9}{4}} = \sqrt{\dfrac{146}{4}}$

$d_{c22} = \sqrt{\dfrac{49}{4} + \dfrac{49}{4} + \dfrac{9}{4}} = \sqrt{\dfrac{107}{4}}$ $d_{c22} = \sqrt{\dfrac{9}{4} + \dfrac{169}{4} + \dfrac{25}{4}} = \sqrt{\dfrac{203}{4}}$ $d_{c23} = \sqrt{\dfrac{1}{4} + \dfrac{1}{4} + \dfrac{1}{4}} = \dfrac{\sqrt{3}}{2}$

$$d_{C_14} = \sqrt{9 + \frac{81}{4} + \frac{25}{4}} = \sqrt{\frac{142}{4}}$$

$$d_{C_24} = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

Points:	C_1	C_2	Cluster
$(-1, 1, 2)$	$\sqrt{\frac{11}{4}}$	$\sqrt{\frac{102}{4}}$	C_1
$(1, -2, 1)$	$\sqrt{\frac{54}{4}}$	$\sqrt{\frac{203}{4}}$	C_1
$(2, 5, 3)$	$\sqrt{\frac{146}{4}}$	$\sqrt{\frac{3}{4}}$	C_2
$(3, 4, 4)$	$\sqrt{\frac{142}{4}}$	$\sqrt{\frac{3}{4}}$	C_2

No cluster is changed, we stop iterating.

Answer: $C_1 \in \left[(-1; 1, 2) \text{ and } (1, -2, 1) \right]$

$C_2 \in \left[(2, 5, 3) \text{ and } (3, 4, 4) \right]$