Formal Languages and Automata Theory
16CS52
Open Book Assignment 1

USN	Question
2GI14CS038	Use induction on the size of S to show that if S is a finite set, then $\| 2^{S} \| = 2^{\|S\|}$
2GI14CS040	Is it true that for any NFA $M = (Q, \Sigma, \delta, q_0 , F)$ the complement of $L(M)$ is equal to the set $\{w \in \Sigma^* : \delta^*(q_0 , w) \cap F = \varnothing \}$? If so, prove it. If not, give a counterexample.
2GI14CS067	Prove that for every NFA with an arbitrary number of final states there is an equivalent NFA with only one final state.
2GI14CS133	Is it true that for every NFA $M = (Q, \Sigma, \delta, q_0 , F)$ the complement of $L(M)$ is equal to the set $\{w \in \Sigma^* : \delta^*(q_0 , w) \in (Q - F) = \varnothing \}$? If so, prove it. If not, give a counterexample.
2GI15CS048	Let L be a regular language that does not contain λ. Show that there exists an NFA without λ-transitions and with a single final state that accepts L.
2GI15CS058	Show that if S_1 and S_2 are finite sets with $\| S_1 \| = n$ and $\| S_2 \| = m$, then $\| S_1 \cup S_2 \| \leq n + m$
2GI15CS117	Prove that all finite languages are regular.
2GI15CS186	If S_1 and S_2 are finite sets, show that $\| S_1 \times S_2 \| = \| S_1 \| \| S_2 \|$
2GI17CS400	Consider the relation between two sets defined by $S_1 \equiv S_2$ if and only if $\| S_1 \| = \| S_2 \|$. Show that this is an equivalence relation.
2GI17CS401	Prove DeMorgan's law: $\overline{S_1 \cup S_2} = \overline{S_1} \cap \overline{S_2}$
2GI17CS402	Prove DeMorgan's law: $\overline{S_1 \cap S_2} = \overline{S_1} \cup \overline{S_2}$
2GI17CS403	Show that, $S_1 \cup S_2 = \overline{\overline{S_1} \cap \overline{S_2}}$
2GI17CS404	Show that $S_1 = S_2$ if and only if, $(S_1 \cap \overline{S_2}) \cup (\overline{S_1} \cap S_2) = \varnothing$
2GI17CS405	Show that $S_1 \cup S_2 - (S_1 \cap \overline{S_2}) = S_2$
2GI17CS406	Show that the distributive law $S_1 \cap (S_2 \cup S_3) = (S_1 \cap S_2) \cup (S_1 \cap S_3)$ holds

	for sets.
2GI17CS407	Show that $S_1 \times (S_2 \cup S_3) = (S_1 \times S_2) \cup (S_1 \times S_3)$
2GI17CS408	Show that if $S_1 \subseteq S_2$ then $\overline{S_2} \subseteq \overline{S_1}$
2GI17CS409	Give conditions on S_1 and S_2 necessary and sufficient to ensure that $S_1 = (S_1 \cup S_2) - S_2$
2GI17CS410	Show that $2^n = O(3^n)$ but $2^n \neq \Theta(3^n)$
2GI17CS411	Show that the following order-of-magnitude results hold. $n^2 + 5 \log n = O(n^2)$
2GI17CS412	Show that the following order-of-magnitude results hold. $3^n = O(n!)$
2GI17CS413	Show that the following order-of-magnitude results hold. $n! = O(n^n)$
2GI17CS414	Prove that if $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$
2GI17CS415	Show that if $f(n) = O(n^2)$ and $g(n) = O(n^3)$, then $f(n) + g(n) = O(n^3)$
2GI17CS416	Show that if $f(n) = O(n^2)$ and $g(n) = O(n^3)$, then $f(n) + g(n) = O(n^6)$
2GI17CS417	Show that if $f(n) = \Theta(\log_2 n)$, then $f(n) = \Theta(\log_{10} n)$
2GI17CS418	Let $G = (V, E)$ be any graph. Prove the following claim: If there is any walk between $v_i \in V$ and $v_j \in V$, then there must be a path of length no larger than $\|V\| - 1$ between these two vertices.
2GI17CS419	Consider graphs in which there is at most one edge between any two vertices. Show that under this condition a graph with n vertices has at most n^2 edges.
2GI17CS420	Prove that for all $n \geq 4$ the inequality $2^n < n!$ holds.
2GI17CS421	The Fibonacci sequence is defined recursively by $f(n+2) = f(n+1) + f(n)$, $n = 1, 2, 3, \ldots$ with $f(1) = 1$, $f(2) = 1$. Show that $f(n) = O(2^n)$
2GI17CS422	The Fibonacci sequence is defined recursively by

	$f(n+2) = f(n+1) + f(n)$, $n = 1, 2, 3, \ldots$ with $f(1) = 1$, $f(2) = 1$. Show that $f(n) = \Omega\ (1.5^n)$
2GI17CS423	Show that $\sqrt{8}$ is not a rational number
2GI17CS424	Show that $2-\sqrt{2}$ is irrational.
2GI17CS425	Show that $\sqrt{3}$ is irrational.
2GI17CS426	Prove or disprove the following statement: The sum of a rational and an irrational number must be irrational.
2GI17CS427	Prove or disprove the following statement: The sum of two positive irrational numbers must be irrational.
2GI17CS428	Prove or disprove the following statement: The product of a non-zero rational and an irrational number must be irrational.
2GI17CS429	Show that every positive integer can be expressed as the product of prime numbers.
2GI17CS430	Prove that the set of all prime numbers is infinite.
2GI17CS431	A prime pair consists of two primes that differ by two. There are many prime pairs, for example, 11 and 13, 17 and 19, etc. Prime triplets are three numbers $n \geq 2$, $n + 2$, $n + 4$ that are all prime. Show that the only prime triplet is (3, 5, 7).
2GI17CS432	Use induction on n to show that $\mid u^n \mid = n \mid u \mid$ for all strings u and all n
2GI17CS433	The reverse of a string, defined by rules: $a^R = a$ and $(wa)^R = aw^R$ $\forall\ a \in \Sigma$, $w \in \Sigma^*$. Use this to prove that $(uv)^R = v^R u^R$, $\forall$ u, υ $\in \Sigma +$
2GI17CS434	Prove that $(w^R)^R = w$, $\forall\ w \in \Sigma^*$
2GI17CS435	Let L be any language on a non-empty alphabet. Show that L and $\bar{L}$ cannot both be finite.
2GI17CS436	Prove that for all languages L_1 and L_2 , $(L_1L_2)^R = L_2^R L_1^R$
2GI17CS437	Show that $(L^*)^* = L^*$ for all languages.
2GI17CS438	Prove or disprove the following claim: $(L_1 \cup L_2)^R = L_1^R \cup L_2^R$ for all languages L_1 and L_2 .

2GI17CS439	Prove or disprove the following claim: $(L^R)^* = (L^*)^R$ for all languages L .
2GI17CS440	Show that if L is regular, so is $L - \{\lambda\}$.
2GI17CS441	Show that if L is regular, so is $L \cup \{a\}$, for all $a \in \Sigma$.
2GI17CS442	Let G_M be the transition graph for some DFA M. Prove that If $L(M)$ is infinite, then G_M must have at least one cycle for which there is a path from the initial vertex to some vertex in the cycle and a path from some vertex in the cycle to some final vertex.
2GI17CS443	Let L be a regular language on some alphabet Σ, and let $\Sigma_1 \subset \Sigma$ be a smaller alphabet. Consider L_1 , the subset of L whose elements are made up only of symbols from Σ_1 , then show that L_1 is also regular.