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Index

1. NATURAL NUMBERS & WHOLE NUMBERS

1.1 NATURAL NUMBERS (REVISION)

The children are screaming in a classroom. Shall we visit that class?

"Hundred", "Hundred and ten", "Two hundred and ten", "Two hundred and twenty", "Two hundred and fifty", "Three hundred", "Five hundred" and "Thousand".

Why are they calling out numbers? What order is this?

It is a game. If one student calls out a number, another student calls out a number bigger than that number. A student who calls out the biggest number wins the game. Shall we listen again?

"Ten thousand", "Twenty thousand", "Fifty thousand", "One lakh", "Ten lakhs". You can also play.

"One crore", "Thousand crores", "One lakh crores", "One crore crores", "One crore crore crores", "One crore crore crore crore crores ...".

All the students screamed "Crore crore crore ...". It was announced that all have won the game. Can anyone lose this game? Can anyone claim that he will be the winner?

In Ascending order of numbers there is no end.

It is easy to tell a number bigger than a given number. If you say twenty, I can say twenty one. If I say hundred, you can say two hundred.

We know about Predecessor and Successor

PREDECESSOR	NUMBER	SUCCESSOR
999	1000	1001
54	55	56

A successor of a number is bigger than that number. It takes longer time to complete the game counting by successor method. We can do it faster through addition and multiplication.

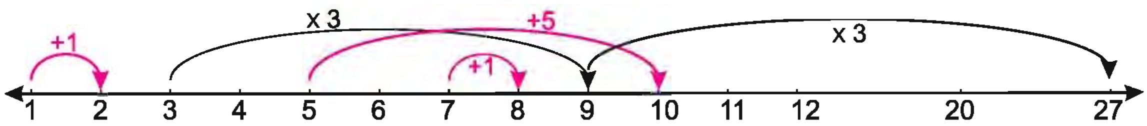
"Hundred", "Hundred and ten", "Hundred and fifty – Addition

"Hundred", "Two hundred", "Five hundred" – Multiplication

One natural number + Another natural number = A bigger natural number.

One natural number x Another natural number = A bigger natural number.

Let us see this in the following Number Line.



Exercise 1.1

1. Give a number bigger and smaller than the following numbers.

i) Ten thousand. **ii)** Twenty three. **iii)** Twenty lakhs. **iv)** Three crores. **v)** Hundred.

2. Write the following in ascending and descending order.

i) Ten lakhs, Twenty crores, Thirty thousand, Four hundred, Eight thousand.

ii)8888, 55555, 23456, 99, 1111111.

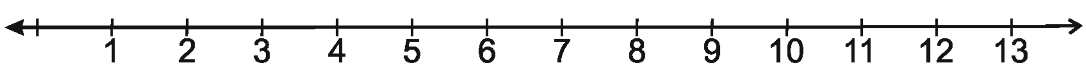
1.2 Small Numbers

Shall we play with small numbers now? Give a number smaller than the given number. The winner of the game is the one who gives the smallest number.

"Thousand", "Five hundred", "Hundred", "Fifty", "Forty".

"Zero", "Zero", "Zero".

It is very easy to win this game. When zero comes the game gets over.



Except zero, all the numbers have a predecessor. Predecessor of any number is smaller than that number.

Given number – Smaller number = A number smaller than the given number.

The counting numbers 1,2,3,... are called Natural numbers.

When there is nothing to count, it is zero. Zero is included with counting numbers to enable subtraction. Since they appear repeatedly in Mathematics they are given a specific name and symbol.

Numbers with more digits are seen not only in games, but also in many places around us. If anyone denote these numbers as "countless", it is wrong. These numbers are definitely countable. Since they are very large it is difficult to count.

Natural numbers are called counting numbers or positive integers.
We denote natural numbers as N={1,2,3,4,...}.

Similarly we denote the whole numbers as W={0, 1,2,3,4,...}.
The other name for the whole numbers is non-negative integers.

Exercise 1.2

1. Complete the following sequence

Crore, Ten lakhs, Lakhs, ...

2. Is there an end to the following sequence?

Thousand, Ten thousand, Lakh, ...

3. Is there any end to the following sequence?

i) Ten thousand, Twenty thousand, ...

ii) Ninety thousand, Lakh, ...

iii) Ninety thousand, Eighty thousand, ...

1.3 Numbers with more digits

There is a Neem tree near your house. Can you count the number of leaves? Is it in thousands or in lakhs? You cannot count accurately. It is easy to say in thousands and lakhs approximately.

Look at the tree. Assume that there are 9 big branches and in each big branch there are 5 small branches. Let us take a small branch and count the number of leaves. Assume that there are 48 leaves.

Total number of small branches = 9x 5 = 45.

There may be more than 5 small branches in few big branches. If approximately 50 small branches of 48 leaves are there, then the total number of leaves = 50 x 48 = 2400. Hence there may be more than 2000 leaves in the tree. It can be 4000 or 8000 but not in lakhs.

			Number of Zeros
10 ones	= 1 ten	= 10	1
10 tens	= 1 hundred	= 100	2
10 hundreds	= 1 thousand	= 1,000	3
10 thousands	= 1 ten thousand	= 10,000	4
10 ten thousands	= 1 lakh	= 1,00,000	5
10 lakhs	= 1 million	= 10,00,000	6
100 lakhs	= 1 crore (10 million)	= 1,00,00,000	7

1 is followed by 5 zeros in 1 lakh, 7 zeros in 1 crore, 8 zeros in 10 crores, 10 zeros in 1000 crores.

There are many digits in a big number. How many digits are there in a crore? 8 digits. In one lakh there are 6 digits and in one thousand? Four digits.

It is difficult to count the number of zeros if 1 lakh is written as 100000. We use comma to group the number of zeros and write it as follows.

Indian System		International System		
Ten thousand	= 10,000	Ten thousand	= 10,000	
One lakh	= 1,00,000	One lakh	= Hundred thousand	= 100,000
10 lakhs	= 10,00,000	10 lakhs	= One million	= 1,000,000
1 crore	= 1,00,00,000	One crore	= 10 millions	= 10,000,000
100 crores	= 1,00,00,00,000	Hundred crores	=One billion	= 1,000,000,000

Exercise 1.3

1. Discuss in groups how many leaves are there in a mango tree, a neem tree and a tamarind tree near your place.

2. How many thousands, hundreds, tens and ones are there in one lakh?

3. How many lakhs and thousands are there in one crore?

4. There are more than thousand labourers in a factory. Find the minimum amount needed if each gets Rs.1000 as bonus?

5. Find the value of

i) 6 x 6 = ; 6 x 6 x6 = ; 6 x 6 x 6 x 6 =

ii) 10 x 10 = ; 100 x 100 = ; 10,000 x 10,000 =

6. Show which is greater or smaller using the signs > or < for the following;

Eighty thousand, Ten thousand, Twenty thousand.

1.4 Method of Writing Numbers

How do we read numbers with more digits?

When 1234567 is written as 12, 34, 567 it is easy to read as 12 lakhs, 34 thousand and 567. Similarly the number 12345678 can be read easily when commas are added. (i.e) 1,23,45,678 can be read as 1 crore, 23 lakhs, 45 thousand and 678.

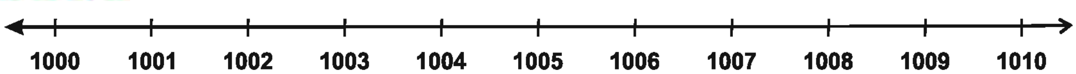
Numbers	Word Form (Number Name)
6	Six
66	Sixty six
666	Six hundred and sixty six
6,666	Six thousand, six hundred and sixty six
66,666	Sixty six thousand, six hundred and sixty six
6,66,666	Six lakhs, sixty six thousand, six hundred and sixty six
1,001	One thousand and one
10,011	Ten thousand and eleven
1,10,101	One lakh, ten thousand , One hundred and one

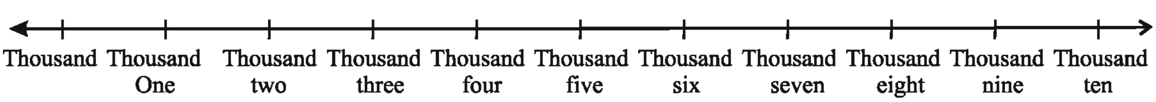
1.5 Activities of the Numbers

We know many concepts about numbers. Is it applicable to all numbers? Yes, whether a number has more or few digits, it is a number. It has the same property as any number.

Predecessor	Number	Successor
99,999	1,00,000	1,00,001
1,10,004	1,10,005	1,10,006
2,27,226	2,27,227	2,27,228
5,55,499	5,55,500	5,55,501

Number Line





1.5.1 Addition

1,10,110	1,10,110	1,10,110	1,10,110
+ 90	+ 990	+ 9,990	+ 99,990
1,10,200	1,11,100	1,20,100	2,10,100

1.5.2 Subtraction

1,10,110	1,10,110	1,10,110	1,10,110
− 90	− 990	− 9,990	− 99,990
1,10,020	1,09,120	1,00,120	10,120

1.5.3 Multiplication

5 lakhs x 6 = 30 lakhs
22 lakhs x 12 = (22 x 12) lakhs = 264 lakhs
1,00,005 x 5 = (1 lakh + 5) x 5 = 5 lakhs and twenty five

1,23,456 x 5 = ?

$$\begin{array}{rr} & 1,23,456 \\ \times & 5 \\ \hline & 6,17,280 \\ \hline \end{array}$$

1,23,456 x 15 = ?

$$\begin{array}{rr} & 1,23,456 \\ \times & 15 \\ \hline & 617280 \\ & 123456 \\ \hline & 18,51,840 \\ \hline \end{array}$$

We can multiply in the usual method. It is little difficult to check if we have written all the digits correctly. We have to be careful while writing and adding many numbers in order.

Importance should be given for the place value of numbers.

Example :

Number	Face Value (digit)	Place Value
456	4	Hundreds
23, 456	2	Ten thousand
1, 23, 456	1	Lakh

When 1, 23, 456 is multiplied by 5, the result is definitely more than 5 lakhs.

1.5.4 DIVISION

98,76,543 ÷ 3 = ?

By the method of Continuous subtraction we can write the answer as 32,92,181. Division is possible for all numbers. But when the number of digits is more there is a possibility of making mistakes.

```
          3 2 9 2 1 8 1
3)        9 8, 7 6, 5 4 3
          9
            8
            6
            2 7
            2 7
              0 6
                6
                0 5
                  3
                  2 4
                  2 4
                    0 3
                      3
                      0
```

Let us see the method of division using numbers having more digits.

32,32,032 ÷ 16 = ?

Take it as (32 lakhs + 32 thousand + 32) ÷ 16.

Separate the number and divide as follows

32 lakhs ÷ 16 = 2 lakhs

32 thousand ÷ 16 = 2 thousand

32 ÷ 16 = 2

and write the answer as 2 lakhs, 2 thousand and two = 2,02,002.

18 lakhs ÷ 9 = 2 lakhs

18 lakhs ÷ 9 lakhs = 2

18 lakhs ÷ 9000 = 200

18 lakhs ÷ 90 = 20,000

Why to stop with crores?
Why the numbers bigger than crores are not named in our country?

How do you read this? 1234567891011

We can read it as one lakh twenty three thousand 456 crores 78 lakh 91 thousand and eleven which is not useful.

It is important to know that the value of this number is more than one lakh crore. 10 digit number is used only in cell phones. No one reads 98404 36985 as 984 crores, 4 lakhs, 36 thousand 985.

In postal address, no one reads pincode number 600113 as 6 lakhs one hundred and thirteen because it is not a number, but a number sequence. So 600113 is considered as a number sequence which is read as six, zero, zero, one, one, three.

Hence we don't add, subtract or multiply the pincode numbers, telephone numbers or bus numbers.

Exercise 1.4

1. The population of Nilgiri district is approximately 7 lakhs and five thousand. In Kanyakumari it is approximately sixteen lakhs. My friend says the population in Kanyakumari is twice as Nilgiri's. Is it true?

2. There are 462 students in a school. It was decided that each one gets a pen costing Rs. 18 as a gift. Is it enough to have Rs.7200 or Rs.10,000 ?

3. 52 students need Rs.5184 to go for an excursion. How much should be collected from each student?

4. i. 28,760 + 38,530 = ______ ii. 22,760 + 40,530 = ______ iii. 20,760 + 40,530 = ______ iv. 119,800 - 88,565 = ______ v. 1,19,800 - 89,565 = ______ vi. 1,19,500 - 89,565 = ______

5. i. 1,00,000 ÷ 100 = iii. 10,000 ÷ 25 = v. 5,55,555 ÷ 11 =
ii. 1,00,000 ÷ 50 = iv. 1,00,000 ÷ 200 = vi. 90,909 ÷ 9 =

Points to remember

- N = {1,2,3,4,...} Natural numbers.
- W = {0,1,2,3,4,...} Whole numbers.
- There is no end if you extend a number line from zero.
- There is a successor for every whole number.
- You can add and multiply all the whole numbers.
- There is a predecessor for every whole number except zero.
- From any natural number we can subtract a smaller natural number or the same number.
- We can find the remainder, dividing a bigger number by a smaller number.
- All these are possible for any number having more digits.
- When we read 1,23,546 it is important to know that it is greater than one lakh twenty thousand and less than one lakh twenty five thousand.

2. DIVISORS AND FACTORS

2.1 ADDITION AND MULTIPLICATION

One day in 1784, a German teacher in his primary school felt tired as he entered the class. So he decided to give a difficult addition problem to his students and rest for a while. He asked them to find the total of "1 to 100".

Within a few seconds one particular student called out the answer 5050. The teacher was astonished and asked for the explanation. The explanation was as follows

$$\underbrace{1 + \underbrace{2 + \underbrace{3 + \ldots\ldots + 98}_{101} + 99}_{101} + 100}_{101}$$

100 numbers are equal to 50 pairs
$100 \div 2 = 50$

The above representation shows 50 pairs. The value of each pair is 101.
So altogether $50 \times 101 = 5050$.

The student who gave the above explanation was Gauss. He lived in the period 1777 to 1855 A.D. and was titled 'Emperor of mathematicians'. How is it possible to change an addition to multiplication? Is it possible always? Basically this is what Gauss understood.

$$
\begin{aligned}
1+2+3+\ldots+99+100 &= (1+100)+(2+99)+(3+98)\ldots+(50+51)\\
&= 101 \times 50\\
&= 5050
\end{aligned}
$$

Here the arrangement of numbers in a particular sequence is very important. Addition becomes easier when we rearrange them. This is possible for any sequence of natural numbers.

At the age of three Gauss was able to find mistakes in his father's office accounts.

Verify:

$$
\begin{aligned}
35 + 65 &= 65+35 = 100\\
33 + 34 + 35 &= 33+35+34 = 35 + 34 + 33\\
&= 34+33+35 = 35 + 33 + 34\\
&= 102\\
1777 + 1784 + 1855 &= 1855 + 1777 + 1784 = 5416\\
5050 + 50 + 1050 &= 50 + 1050 + 5050 = 6150
\end{aligned}
$$

A sequence of numbers added in any order gives the same answer.

This will help us in many ways.

$$\begin{aligned}32 + 2057 + 68 &= 2057 + (32 + 68)\\ &= 2057 + 100\\ &= 2157\\ 125 + 250 + 125 + 250 &= (2\times 250) + 125 + 125\\ &= (2\times 250) + 250\\ &= 3\times 250\\ &= 750\end{aligned}$$

If you want to add many numbers we have to do the following

- **i)** Split the suitable numbers first.
- **ii)** Add them separately.
- **iii)** Finally add them all.

The above property is true for multiplication also.

Check them:

$$\begin{aligned}5 \times 7 \times 20 &= (20 \times 5) \times 7\\ &= 100 \times 7 = 700\\ 125 \times 20 \times 8 \times 50 &= (125 \times 8) \text{ X } (20 \times 50)\\ &= 1000 \times 1000 = 10{,}00{,}000\end{aligned}$$

A sequence of numbers multiplied in any order gives the same result.

We must be careful while addition and multiplication are involved together.

What is the answer for $5 \times 8 + 3$?

If we multiply, $5 \times 8 = 40$ and add 3 we get $40 + 3 = 43$ as the answer. If we add $8 + 3 = 11$ and then multiply we get $5 \times 11 = 55$ as the answer.
There is no two different answers for one problem. Therefore $(5 \times 8) + 3$ or $5 \times (8+3)$ are correct.

You can see (.....) brackets used in the above examples.

Check them.

When both the operations addition and multiplication are involved it is important to use these ( ) brackets.

2.1.1 PROBLEMS INVOLED IN SUBTRACTION AND DIVISION

- Whole number + Whole number = Whole number
- Whole number x Whole number = Whole number
- This is called as closure property of addition and multiplication.
- Is there any closure property for subtraction and division?
- We should be careful about this.
- Is it possible to subtract a number from any number?

$5050 - 50 = 5000$

$5050 - 5050 = 0$

$50 - 5050 = ?$

While subtracting it is not always necessary to get the answer as a natural number, zero (or whole number). This is applicable for division also.

$5050 \div 50 = 101$

$5050 \div 5050 = 1$

$50 \div 5050 = ?$

- ☐ There is no closure property for division.
- ☐ Arrangement is very important for subtraction and division.

$(23 - 12) - 5 = 6$

$23 - (12 - 5) = 16$

The above given statements are not the same.

$23 - 12 = 11$ but

$12 - 23 = ?$

Arrangement is important for division.

$120 \div 12 = 10$

$12 \div 120 = ?$

Exercise 2.1

1. Add the following by simple method.

(i) $25 + 69 + 75$ (ii) $119 + 64 + 1 + 80$ (iii) $750 + 60 + 240 + 250$

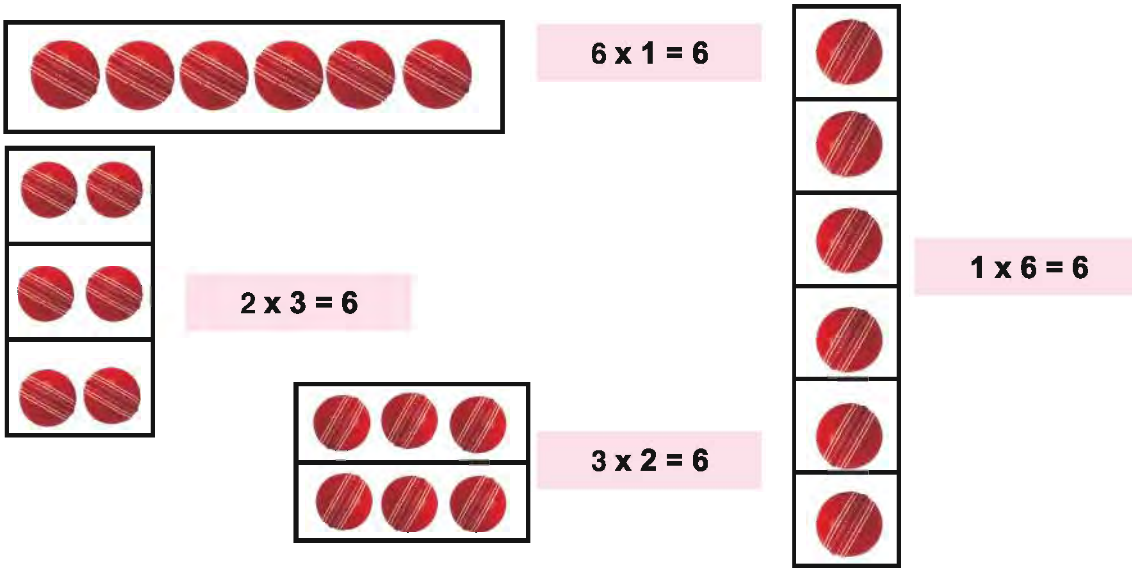
2. Answer the following $51 + 52 + \ldots + 99 + 100$

3. Find the product using short methods.

(i) $25 \times 62 \times 4$ (ii) $5 \times 125 \times 2 \times 2$ (iii) $(75 \times 5) + (30 \times 5) + (25 \times 5)$

2.2 DIVISORS

Manoj has 6 cricket balls. He is trying to arrange them in a rectangular form.



Any natural number other than one can be written as a product of 2 or more numbers.

Can we arrange 6 balls in any other rectangular form?
The answer would be obtained by dividing 6 by a number smaller than 6.

1) 6 (6 6 ----------- 0 -----------	2) 6 (3 6 ----------- 0 -----------
3) 6 (2 6 ----------- 0 -----------	4) 6 (1 4 ----------- 2 -----------
5) 6 (1 5 ----------- 1 -----------	6) 6 (1 6 ----------- 0 -----------

It is found that by dividing 6 by some numbers the remainder is '0' and by some numbers the remainder is not zero.

Divisors of 6 are 1, 2, 3, 6.

All the numbers which divide a given number leaving 0 as remainder are called divisors of the given number.

Observe the following table

Number	Divisors	Expressing in different rectangular form
12	1, 2, 3, 4, 6, 12	1 x 12 ; 2 x 6 ; 3 x 4
17	1,17	1 x 17
25	1, 5, 25	1 x 25 ; 5 x 5
28	1, 2, 4, 7, 14, 28	1 x 28 ; 2 x 14 ; 4 x 7
31	1,31	1 x 31
35	1, 5, 7, 35	1 x 35 ; 5 x 7
42	1, 2, 3, 6, 7, 14, 21, 42	1 x 42 ; 2 x 21 ; 3 x 14 ; 6 x 7

The following are observed from the above table

★ 1 and number itself are divisors of any number.

★ Is there any number which has no divisor? No, because 1 is a divisor of all numbers.

★ Some numbers have many divisors. 42 has 8 divisors.

★ All the numbers from 1 to 10 except 7 are the divisors of 720.
Try to find more divisors.

★ Some numbers have only 2 divisors.

★ For Example : divisors of 7 are 1 and 7. Likewise prime numbers 11,13,17,19 have only two divisors.

Prime numbers are numbers which are divisible by 1 and itself.

2.2.1 FACTORS

In the previous section we have observed that 1 and the number itself were the divisor of any number along with other divisors. For Example : divisors of 45 are 1, 3, 5, 9, 15, 45. Here other than 1 and 45 the remaining numbers are called factors.

The divisors of a number other than 1 and the number itself are called the factors of that number.

Think Over:

"All factors are divisors". Are all divisors factors?

A prime number does not have any factors.

Can you factorise 7?

Numbers having more than two divisors are called composite numbers

2.2.2 METHODS OF FINDING PRIME NUMBERS

All even numbers are divisible by 2. Two is the only even prime number.
How to find whether a given number is a prime number? It is difficult. Why?
Is 200 divisible by 4? Yes by division.
Is 200 divisible by 9? No by division.
Is 131 divisible by 11? No.
Is 1137 divisible by 11?
Is 1234567 divisible by 133? Try to get the answer.

It is possible to find out whether a number is divisible by any number. But this is not enough to find out whether it is a prime number.

Prime numbers are only divisible by 1 and itself.

There is no other divisor for prime numbers proving the above is difficult for bigger numbers.

How many prime numbers are there from 1 to 100? Find them.

1. Form a tabular column for numbers 1 to 100.
2. Except 2, cross all the even numbers.
3. Next except 3, cross all the multiples of 3.
4. Next 5, because 4 and even number has already been crossed. Now cross all the multiples of 5.
5. Follow the same; the left out numbers are prime numbers.

A Greek mathematician Eratosthenes (BC 276 – BC 175) suggested this method for finding the prime numbers.

1	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Totally there are 25 prime numbers.

1 has only one divisor, so 1 is neither a prime nor a composite number.

2.2.3 MULTIPLES

Observe the given table

Multiples

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200

Example : 1

Write 4 multiples of 7 above 100
Answer: 105, 112, 119, 126.

105 is a multiple of 7, At the same time 7 is a divisor of 105. So a number is a multiple of its divisor.

Example : 2

Write 4 multiples of 5 just before 80 and 4 multiples of 5 just after 80 including 80.
Answer: 60, 65, 70, 75, 80, 85, 90, 95, 100.

Exercise 2.2

1. State true or false for the following

i. 4 is one of the divisors of 7.
ii. One of the factors of 21 is 3.
iii. 1 is one of the divisors of 24.
iv. 9 is one of the factors of 45.
v. One of the multiples of 5 is 105.

2. Choose the correct answer

i. Which of the following has all the divisors of 10?

(a) 1,2,5 (b) 2,5 (c) 1,2,5,10 (d) 2,10

ii. Which of the following has all the divisors of 4?

(a) 2,4 (b) 1,2 (c) 1,2,4 (d) 2

iii. 3 is the divisor of _____

(a) 18 (b) 19 (c) 20 (d) 29

iv. 4 is the multiple of _____

(a) 5 (b) 2 (c) 3 (d) 8

v. 15 is the multiple of _____

(a) 3 (b) 45 (c) 7 (d) 11

3. Find the divisors of the following

(i) 8 (ii) 15 (iii) 45 (iv) 121 (v) 14

4. Write all the multiples of 3 between 80 and 100.

5. Write all the multiples of 5 and 10 between 21 and 51. What do you understand from this?

6. Say true or false for the following

(i) 1 is the smallest prime number.

(ii) There are two even prime numbers.

(iii) 6 is a prime number.

(iv) 13 is a composite number.

(v) 61 is a prime number.

7. Choose the correct answer

(i) The prime factor of 24 is

(a) 3 (b) 4 (c) 6 (d) 12

(ii) The prime number between 5 and 11 is

(a) 6 (b) 7 (c) 8 (d) 10

(iii) Number of one digit prime numbers are

(a) 1 (b) 2 (c) 3 (d) 4

(iv) Number of prime numbers between 20 and 30 are

(a) 1 (b) 2 (c) 3 (d) 4

(v) The smallest two digit prime number is

(a) 37 (b) 7 (c) 11 (d) 10

8. Write all the prime numbers between 30 and 60

9. Are the addition, subtraction, multiplication and division of two prime numbers, is also a prime number? check with examples.

2.3 DIVISIBILITY

To find all the divisors of a natural number we should divide the number by all the numbers smaller than it, which is time consuming. Moreover the quotient or remainder is not important.

Our aim is to find whether it could be divided leaving 0 as the remainder. This could be found by simple methods.

Test of divisibility by 2

If we keep subtracting 2 from the odd numbers like 37, 453 we get a remainder. But, we get 0 as a remainder for even numbers such as 48, 376. So, all even numbers are divisible by 2.

Numbers ending with 0, 2, 4, 6 or 8 are divisible by 2.

Test of divisibility by 5

If we keep subtracting 5 from 1005 we get numbers such as 1000, 995, 900 whose last digit is 5 and 0 alternatively and finally it ends with 0.

If 5 is subtracted from a number ending with 7 (for e.g 237) we get numbers ending with 2, 7, 2, ... At last it ends with 2. So 237 is not divisible by 5.

Numbers ending with 0 or 5 are divisible by 5.

Test of divisibility by 10

If we keep subtracting 10 from 3010 we get numbers ending with 0 such as 3000, 2990, 2980.

Numbers ending with 0 are divisible by 10.

It is enough to see the last digit to know if a number is divisible by 2, 5, 10.

Test of divisibility by 4

Is 138 divisible by 4?

138 = 100 + 38; If we keep subtracting 4 from 100 we get 0 as the remainder. Therefore to know if 138 is divisible by 4 it is enough to find out if 38 is divisible by 4. Likewise 1792 = 1700 + 92. 92 is divisible by 4. So 1792 is divisible by 4. 2129 is not divisible by 4 (check), because 29 is not divisible by 4.

If the number formed by last two digits (unit and tenth digit) of a given number is divisible by 4, it will be divisible by 4.

Test of divisibility by 8

Is 1248 divisible by 8? 1248 = 1000 + 248. 1000 = 125 x 8. So it is enough to see if 248 is divisible by 8. 248 = 31 x 8. So 1248 is divisible by 8.

If the number formed by the last three digits of a given number is divisible by 8, the given number will be divisible by 8.

Are all numbers that are divisible by 2 are divisible by 4 also? For Example : 26 is divisible by 2 but not divisible by 4. Likewise all the numbers that are divisible by 4 need not be divisible by 8.

To test (i) if a number is divisible by 4, check only the last two digits.

(ii) If a number is divisible by 8 check only the last three digits.

Test of divisibility by 9

Is 45 divisible by 9?

45 = 10 + 10 + 10 + 10 + 5

= 9 + 1 + 9 + 1 + 9 + 1 + 9 + 1 + 5

If we keep subtracting 9 we get

= 1 + 1 + 1 + 1 + 5

= 4 + 5 = 9

If the last 9 is subtracted the remainder is 0. So, 45 is divisible by 9.

Is 123 divisible by 9?

123 = 100 + 10 + 10 + 3

= (99+1) + (9+1) + (9+1) + 3

= (99+1) + (9+9+2) + 3

If 9 or multiples of 9 are subtracted we get 1 + 2 + 3 = 6. So, 123 is not divisible by 9.

Note that after subtracting 9 the remainder is the sum of the digits of the given number.

If the sum of the digits of a number is divisible by 9, the number is divisible by 9.

Given Number	Sum of the digits	Is it divisible by 9?	Verify by multiplication
61	6 + 1 = 7	No	61 = 6 x 9 + 7
558	5 + 5 + 8 = 18; 1 + 8 = 9	Yes	558 = 62 x 9
971	9 + 7 + 1 = 17; 1 + 7 = 8	No	971 = 107 x 9 + 8
54000	5 + 4 + 0 + 0 + 0 = 9	Yes	54000 = 6000 x 9

Test of divisibility by 3

If we keep subtracting 3 from 42 we get 0 as remainder (ie. 42, 39, 36, … 0). This can also be checked by another method.

$$42 = 10 + 10 + 10 + 10 + 2$$
$$= 9+1 + 9+1 + 9+1 + 9+1 + 2$$

Instead of subtracting 3, 9 can be subtracted (because $9 = 3 \times 3$). Finally we get,

$$= 1 + 1 + 1 + 1 + 2$$
$$= 4 + 2$$
$$= 6$$

Note that after subtracting 9 the remainder is the sum of the digits of the given number.

6 is divisible by 3. So, 42 is also divisible by 3.

If the sum of the digits of a number is divisible by 3, the number is divisible by 3.

Note: Numbers that are divisible by 2 and 3 are also divisible by 6.

Test of divisibility by 11

	Digits						Sum of the digits in the odd places	Sum of the digits in the even places	Difference
	6	5	4	3	2	1			
3 x 11					3	3	3	3	0
71 x 11				7	8	1	8 (7+1)	8	0
948 x 11		1	0	4	2	8	13(1+4+8)	2 (0+2)	11
5102 x 11		5	6	1	2	2	8	8	0
73241 x 11	8	0	5	6	5	1	7	18	11

From the above table we know that the difference between the sum of the digits in the odd places and sum of the digits in the even places is a multiple of 11.

If the difference between the sum of the digits in the odd places and sum of the digits in the even places is either 0 or multiples of 11, the number is divisible by 11.

Generally it is difficult to find if a number is divisible by 11. But, if the numbers are in a particular pattern, we know that they are divisible by 11. For Example : 121, 1331, 4994, 56265, 1234321, 4754574 are divisible by 11. How?

Exercise 2.3

1. State true or false for the following
   - (i) 120 is divisible by 3.
   - (ii) All the numbers that are divisible by 8 are also divisible by 2.
   - (iii) All the numbers that are divisible by 10 are also divisible by 5.

2. Tabulate if the numbers given below are divisible by 2, 3, 4, 5, 6, 8, 9 , 10, 11

Numbers	DIVISIBILITY								
	2	3	4	5	6	8	9	10	11
77	No	No	No	No	No	No	No	No	Yes
896	Yes	No	Yes	No	No	Yes	No	No	No
918									
1,453									
8,712									
11,408									
51,200									
732,005									
12,34,321									

3. Fill the following tabular column with a suitable number.

The smallest number divisible by 2	7	6	0	4	3	1	2	
The biggest number divisible by 3						7	3	2
The smallest number divisible by 4				9	8	2	6	
The biggest number divisible by 5			4	3	1	9	6	
The smallest number divisible by 6		1		9	0	1	8	4
The biggest number divisible by 8	3	1	7	9	5		7	2
The smallest number divisible by 9				3	2	0		7
Any number divisible by 10	1	2	3	4	5	6	7	
Any number divisible by 11			8	6	9	4		4
The smallest number divisible by 3				5	6		1	0
Any number divisible by 11			9	2	3		9	3

4. Circle the numbers divisible by8.

   22, 35, 70, 64, 8, 107, 112, 175, 156

5. Check if the numbers divisible by 3 and 5 are also divisible by 15 with a suitable examples.

2.4 PRIME FACTORISATION

The method of expressing a number as a product of prime numbers is called prime factorization.

(i) Division method (ii) Factor tree method are the two methods to find the prime factors of the given numbers.

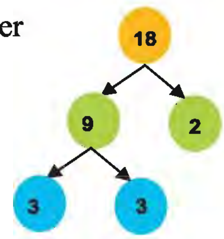
Factorise 18, 120 by division method and factor tree method.

Given number is 18

Division method

		Remainder
2	18	
3	9	-0
3	3	-0
	1	-0

Factor tree method



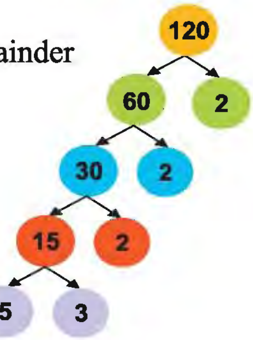
Prime factors of 18 are
$18 = 2 \times 3 \times 3$

Given number is 120

Division method

		Remainder
5	120	
3	24	-0
2	8	-0
2	4	-0
2	2	-0
	1	-0

Factor tree method



Prime factors of 120 are
$120 = 2 \times 2 \times 2 \times 3 \times 5$

Exercise 2.4

1. Express the following numbers as a product of prime factors.

(i)	6	(ii)	15	(iii)	21	(iv)	30	(v)	121
(vi)	145	(vii)	162	(viii)	170	(ix)	180	(x)	200

2. Which has more factors: 21 or 8? Find using a factor tree.

2.5 Greatest Common Divisor (G.C.D.) Least Common Multiple (L.C.M.)

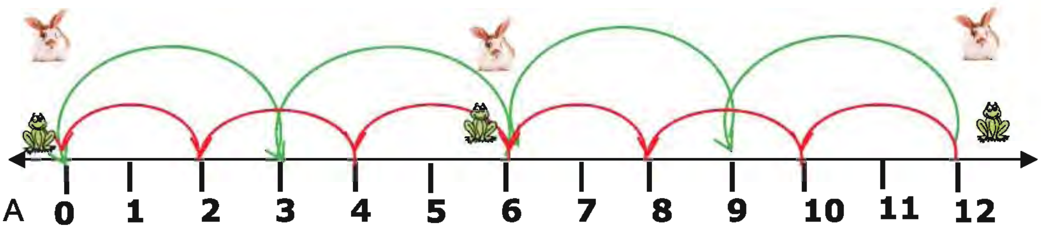
2.5.1 Least Common Multiple (L.C.M.)

A rabbit covers 3 feet in one jump. But a frog can cover only 2 feet in one jump.

Both of them start from the same point A.

From point A, the points of landing of the rabbit is at 3, 6, 9, 12,...

From point A, the points of landing of the frog is at 2, 4, 6 , 8,...



Both of them land at common points 6, 12 …. Here 6 is the LCM of 2 and 3.

> The smallest among the common multiples of two numbers is called their least common multiple (LCM).

We can find the LCM of given numbers by 2 methods

Common Multiple method

Step 1: List the multiples of the given numbers.

Step 2: Circle and write the common multiples

Step 3: The smallest common multiple is the LCM.

Given numbers: 16, 24

Multiples of 16: = 16, 32, 48, 64, 80, 96, 112, 128, 144, 160,.....

Multiples of 24: = 24, 48, 72, 96, 120, 144, 168,....

Common multiples of 16 and 24 = 48, 96, 144, ...

(The smallest multiple among the common multiples is the LCM)

∴ The LCM of 16 and 24 = 48.

Factorisation method

Step1: Find the prime factors of the given numbers.

Step2: Circle the common prime factors

Step3: Find the product of the common factors. Multiply this product with independent factors.

Given numbers: 16, 24

Factors of 16

		Remainder
2	16	
2	8	-0
2	4	-0
2	2	-0
	1	-0

Factors of 24

		Remainder
2	24	
2	12	-0
2	6	-0
3	3	-0
	1	-0

Factors of 16 = 2 x 2 x 2 x 2

Factors of 24 = 2 x 2 x 2 x 3

LCM is the product of the common factors and independent factors.

LCM of 16 &24 = 2 x 2 x 2 x 2 x 3 = 48

2.5.2 Greatest Common Divisor (G.C.D.)

We know that different numbers have common divisors. Among the common divisors the greatest divisor is the G.C.D.

There are 2 methods to find the G.C.D. of the given numbers.

Common divisor method

Step 1: Find the divisors of the given numbers.

Step 2: Circle and write the common divisors

Step 3: Among the common divisors the greatest divisor is the G.C.D.

Given numbers: 30, 42

Divisors of 30 : 1, 2, 3, 5, 6, 10, 15, 30

Divisors of 42 : 1, 2, 3, 6, 7, 14, 21, 42

Common divisors : 1, 2, 3, 6

G.C.D. = 6

Given numbers = 35, 45, 60

Divisors of 35 : 1, 5, 7, 35

Divisors of 45 : 1, 3, 5, 9, 15, 45

Divisors of 60 : 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Common divisors : 1, 5

G.C.D. : 5

Factorisation method

Step 1: Find the prime factors of the given numbers.

Step 2: Circle the common prime factors.

Step 3: Product of the common factors is the G.C.D. of the given numbers.

Given numbers: 30, 42

Prime factors of 30

		Remainder
2	30	
3	15	– 0
5	5	– 0
	1	– 0

Prime factors of 42

		Remainder
2	42	
3	21	–0
7	7	–0
	1	–0

Factors of 30: $= 2 \times 3 \times 5$

Factors of 42: $- 2 \times 3 \times 7$

(circle the common factors)

G.C.D = $2 \times 3 = 6$

Example : 3

Find the G.C.D. of 85, 45, 60 by factorization method.

Factors of 85

		Remainder
5	85	
17	17	-0
	1	-0

Factors of 45

		Remainder
3	45	
3	15	-0
5	5	-0
	1	-0

Factors of 60

		Remainder
2	60	
2	30	-0
3	15	-0
5	5	-0
	1	-0

Factors of 85 = 5 x 17

Factors of 45 = 3 x 3 x 5

Factors of 60 = 2 x 2 x 3 x 5

(Circle the common factors of all the three numbers)

G.C.D. of the given numbers = 5.

2.5.3 Relatively Prime Numbers

Let us form ordered pairs of any two numbers.

For Example : (5, 12), (9, 17), (11, 121), ...

G.C.D. of the ordered pair (3, 5) is 1. G.C.D. of the ordered pair (5, 15) is 5.

If the G.C.D. of any ordered pair is 1, then they are said to be relatively prime numbers.

(3, 5) are relatively prime numbers. (5, 15) are not relatively prime numbers.

Example : 4

Are the given ordered pairs relatively prime numbers?

(13, 17), (7, 21), (101, 201), (12, 13)

ANSWERS

1. (13, 17) – Relatively prime numbers as G.C.D. of (13, 17) is 1
2. (7, 21) – Not relatively prime numbers as G.C.D. of (7, 21) is 7
3. (101, 201) – Relatively prime numbers as G.C.D. of (101, 201) is 1
4. (12, 13) – Relatively prime numbers as G.C.D. of (12, 13) is 1.

Note: G.C.D. of any pair of consecutive numbers is 1.
So, they are said to be relatively prime numbers.

Exercise 2.5

1. State true or false for the following:
   (I) G.C.D. of 2, 3 is 1
   (ii) LCM of 4, 6 is 24
   (iii) (5, 15) are relatively prime numbers.
   (iv) G.C.D. of any two number is less than their L.C.M.

2. Choose the correct answer
   (i) The G.C. D. of 3, 6 is
   (a) 1 (b) 2 (c) 3 (d) 6
   (ii) The L.C.M. of 5, 15 is
   (a) 5 (b) 10 (c) 15 (d) none of these
   (iii) The G.C.D. of two prime numbers is
   (a) 1 (b) a prime number (c) a composite number (d) 0
   (iv) The G.C.D. and L.C.M. of the relatively prime numbers (3, 5) are
   (a) 1, 3 (b) 1, 5 (c) 1, 15 (d) 1, 8

3. Find the G.C.D. and L.C.M. of the following
   (i) 30, 42 (ii) 34, 102 (iii) 12, 42, 75 (iv) 48,72, 108.

4. Puspha bought 2 rice bags each weighing 75 kg and 60 kg. The rice must be completely filled in smaller bags of equal weight. What is the maximum weight of each bag?.

2.6. Relation between G.C.D. and L.C.M.

Observe the following table and fill in the blanks.

First number	Second number	Product	L.C.M.	G.C.D.	G.C.D. x L.C.M.
8	12	96	24	4	96
18	36	648	36	18	648
5	?	75	15	5	75
3	9	27	?	3	27

From the above table

Product of two numbers = G.C.D. x L.C.M.

Example : 5

The G.C.D. of 36, 156 is 12.

Find their L.C.M.

First number $= 36$

Second number $= 156$

G.C.D. $= 12$

$$\text{L.C.M.} = \frac{\text{Product of the two numbers}}{\text{G.C.D.}}$$

$$= \frac{36 \times 156}{12}$$

$$= 468$$

Example : 6

The G.C.D. and L.C.M. of two number are 3 and 72 respectively. If one number is 24. Find the other.

One number $= 24$.

G.C.D. $= 3$

L.C.M. $= 72$

$$\text{Other number} = \frac{\text{G.C.D. x L.C.M.}}{\text{One number}}$$

$$= \frac{3 \times 72}{24}$$

$$= 9$$

Exercise 2.6

1. Find the correct relationship between G.C.D. and L.C.M.
   (i) G.C.D. = L.C.M. (ii) G.C.D. $\leq$ L.C.M.
   (ii) L.C.M. $\leq$ G.C.d. (iv) L.C.M. $>$ G.C.D.
2. The L.C.M. of 78, 39 is 78. Find their G.C.D.
3. The G.C.D. and L.C.M. of two numbers are 2 and 28 respectively. One number is 4. Find the other number.

To think

1. What is the G.C.D. of any two consecutive even numbers?
2. What is the G.C.D. of any two consecutive odd numbers?
3. What is the G.C.D. of any two consecutive numbers?
4. Is the sum of any two consecutive odd numbers divisible by 4? Verify with examples.
5. Is the product of any three consecutive numbers divisible by 6? Verify with examples

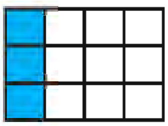
Points to remember

- Numbers can be added and multiplied in any order. (This is not applicable for subtraction and division)
- A number which divides a given number leaving '0' as remainder is called a divisor of the given number.
- 1 is a divisor for all numbers. A number is a divisor for itself.
- Numbers which are divisible by 1 and itself are called prime numbers. The remaining numbers are composite numbers.
- Divisibility of a number by 2, 3, 5, 6, 8, 9, 10, 11 can be easily found.
- The method of expressing a number as a product of prime numbers is called prime factorization.
- Among the common divisors of given numbers, the greatest divisor is the G.C.D.
- If the G.C.D. of any two numbers is 1 they are said to be relatively prime number.
- Among the common multiples of given numbers, the least is the L.C.M.
- The product of any two numbers is equal to the product of their G.C.D. and L.C.M.

3. FRACTIONS AND DECIMAL NUMBERS

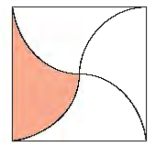
3.1 FRACTIONS – REVISION

A fraction is a part or parts of a whole which is divided into equal parts.

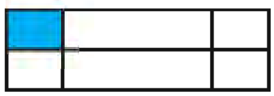


$\frac{3}{12}$

$\frac{2}{6}$



$\frac{1}{4}$



This is not $\frac{1}{6}$
(All the parts are
are not equal)

This is not $\frac{1}{2}$
(Both the parts
are not equal)

This is $\frac{2}{8}$
(All the parts
are equal)

In a fraction, the number above the line is called the numerator and the number below the line is called the denominator.

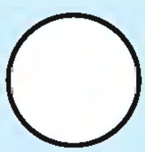
$$\text{FRACTION} = \frac{\text{NUMERATOR}}{\text{DENOMINATOR}}$$

We know to divide the whole into quarter, half and three quarters
We denote them as $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.
We call these numbers as fractions.

Do it yourself :

Shade the given shapes for the corresponding fraction in the following shapes.

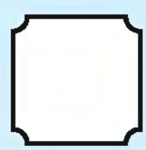
$\frac{2}{7}$



$\frac{3}{8}$

$\frac{1}{3}$

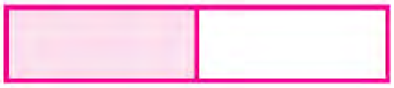
$\frac{3}{4}$



$\frac{1}{4}$

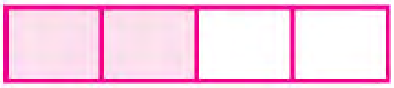
3.1.1 Equivalent Fractions (Revision)

Let us divide a rectangle into two equal parts and shade one part.



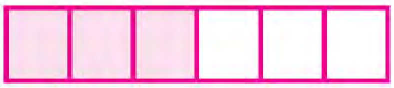
Shaded part = $\frac{1}{2}$

Let us divide the same rectangle into four equal parts and shade 2 parts.



Shaded part = $\frac{2}{4}$

Divide the same rectangle into 6 equal parts and shade 3 parts.



Shaded part = $\frac{3}{6}$

In all the above figures the shaded portions are equal but they can be represented by different fractions.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$$

When two or more fractions represent the same part of a whole, the fractions are called equivalent fractions.

Activity-Equivalent Fractions:

In a card write the multiples of 1 to 10. Cut as strips as shown below.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

3 6 9 12 15 18 21 24 27 30 33 36 39 42 45

5 10 15 20 25 30 35 40 45 50 55 60 65 70 75

8 16 24 32 40 48 56 64 72 80 88 96 104 112 120

These cards are called "multiple cards".

Let us now find the equivalent fractions of $\frac{2}{3}$

Solution:

Keep the multiple cards of the numerator and the denominator as shown in the figure.

2^{nd} multiple card.	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3^{rd} multiple card.	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45

$$\frac{2}{3}=\frac{2 \times 2}{3 \times 2}=\frac{4}{6}$$

$$\frac{2}{3}=\frac{2 \times 3}{3 \times 3}=\frac{6}{9}$$

$$\frac{2}{3}=\frac{2 \times 5}{3 \times 5}=\frac{10}{15}$$

$$\frac{2}{3}=\frac{2 \times 9}{3 \times 9}=\frac{18}{27}$$

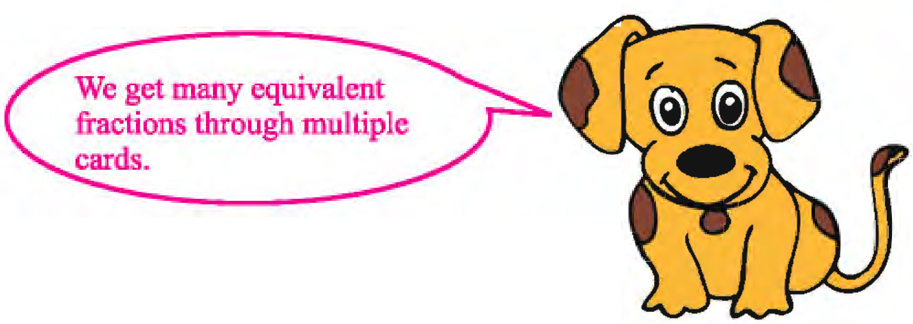
$$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}$$

These fractions are called equivalent fractions.

When the numerator and the denominator are multiplied by the same number, we get equivalent fractions.

Therefore $\frac{2}{3}=\frac{2 \times 2}{3 \times 2}=\frac{2 \times 3}{3 \times 3}=\frac{2 \times 5}{3 \times 5}=\frac{2 \times 9}{3 \times 9}=\frac{2 \times 10}{3 \times 10}$

$$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{10}{15}=\frac{18}{27}=\frac{20}{30}$$



Example : 1

Find the missing numbers in the following equivalent fractions using the multiple cards.

$$\frac{4}{9} = \frac{8}{18} = \frac{\square}{45} = \frac{32}{\square}$$

4th multiple card.	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
9th multiple card.	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135

From the above figure

1. If the denominator is 45, the numerator is 20.
2. Similarly if the numerator is 32, the denominator is 72.

$$\therefore \frac{4}{9} = \frac{8}{18} = \frac{\boxed{20}}{45} = \frac{32}{\boxed{72}}$$

Example : 2

Write any 5 equivalent fractions to $\frac{3}{7}$

To get equivalent fractions multiply the numerator and the denominator by the same number.

$$\frac{3}{7} = \frac{3x2}{7x2} = \frac{3x4}{7x4} = \frac{3x5}{7x5} = \frac{3x9}{7x9} = \frac{3x10}{7x10}$$

$$\frac{3}{7} = \frac{6}{14} = \frac{12}{28} = \frac{15}{35} = \frac{27}{63} = \frac{30}{70}$$

3.1.2 Expressing the fractions in its lowest form (simplest form)

Now consider $\frac{15}{18}$

Divisors of 15 are 1, 3, 5, 15

Divisors of 18 are 1, 2, 3, 6, 9, 18

$$\frac{15}{18} = \frac{3x5}{3x6}$$

$$\frac{15}{18} = \frac{\cancel{3}x5}{\cancel{3}x6} = \frac{5}{6}$$ (We cancel 3 which is common.)

Divisors of 5 are 1, 5
Divisors of 6 are 1, 2, 3, 6

As there is no common divisor for 5 and 6 (except 1)

$\frac{5}{6}$ is the lowest form of $\frac{15}{18}$

Equivalent fractions have the same value. So they can be represented by a single number. So we express it in the lowest form where there is no common factor for the numerator and denominator.

Example : 3

Reduce $\frac{12}{16}$ into lowest terms

Factors of 12 are 2, 3, 4, 6
Factors of 16 are 2, 4, 8

There are two common factors 2,4.

Considering 2 as a factor,

$$\frac{12}{16} = \frac{2x6}{2x8} = \frac{6}{8}$$

Factors of 6 are 2, 3
Factors of 8 are 2, 4

$$\frac{6}{8} = \frac{\cancel{2}x3}{\cancel{2}x4} = \frac{3}{4}$$

Considering 4 as a factor,

$$\frac{12}{16} = \frac{4x3}{4x4} = \frac{3}{4}$$

There is no common factor for 3 and 4.

$\therefore$ The Lowest form of $\frac{12}{16} = \frac{3}{4}$

So, when there are more than one common factor, use the greatest common factor, to get the lowest term easily.

Example : 4

Write the lowest form of $\frac{24}{40}$

Factors of 24 are 2, 3, 4, 6, 8, 12

Factors of 40 are 2, 4, 5, 8, 10, 20

8 is the greatest common factor. $\therefore \frac{24}{40} = \frac{8x3}{8x5} = \frac{3}{5}$

Exercise : 3.1

1. Write 4 equivalent fractions for each of the following : (i) $\frac{5}{6}$ (ii) $\frac{3}{8}$ (iii) $\frac{2}{7}$ (iv) $\frac{3}{10}$
2. Pick out the equivalent fractions: $\frac{2}{5}, \frac{12}{16}, \frac{1}{3}, \frac{5}{15}, \frac{16}{40}, \frac{3}{4}, \frac{9}{12}$
3. Express the following in its lowest form:

   (i) $\frac{12}{14}$ (ii) $\frac{35}{60}$ (iii) $\frac{48}{64}$ (iv) $\frac{27}{81}$ (v) $\frac{50}{90}$
4. Find the missing number.

   (i) $\frac{1}{4} = \frac{?}{20} = \frac{3}{?}$ (ii) $\frac{3}{5} = \frac{21}{?} = \frac{?}{20}$ (iii) $\frac{5}{9} = \frac{35}{?} = \frac{?}{72}$

3.1.3 Comparison of fractions, addition, subtraction – Revision

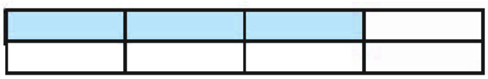
Like fractions have the same denominators.
$\left(\text{Eg: } \frac{2}{7}\ \frac{5}{7} \right)$

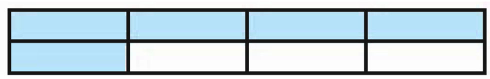
We have learnt to add, subtract and to compare the numbers. Similarly we can do it for the fractions also.

Comparison of fractions:

Which is greater? $\frac{3}{8}$ or $\frac{5}{8}$

Let us take a rectangle


$\frac{3}{8}$


$\frac{5}{8}$

From the figure, we observe that $\frac{3}{8} < \frac{5}{8}$. When fractions have the same denominator we can compare only the numerators and decide which fraction is greater.

$$\therefore \frac{3}{8} < \frac{5}{8}$$

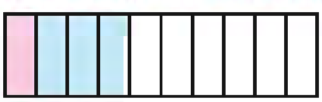
Example : 5

Which is greater ? $\frac{9}{11}$ or $\frac{7}{11}$

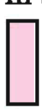
The denominators of $\frac{9}{11}$ and $\frac{7}{11}$ are same. So compare the numerators.

As 9>7, $\frac{9}{11} > \frac{7}{11}$

Addition of like fractions



In this figure


Represents $\frac{1}{10}$


Represents $\frac{3}{10}$

From the figure we can see that the total coloured part = $\frac{4}{10}$

$$\therefore \boxed{\frac{1}{10} + \frac{3}{10} = \frac{4}{10}}$$

We can see that both the fractions have same denominator.

Do it Yourself :

1. $\frac{3}{11} + \frac{1}{11} = ?$
2. $\frac{3}{8} + \frac{4}{8} + \frac{2}{8} = ?$
3. $\frac{1}{31} + \frac{15}{31} + \frac{7}{31} = ?$

For addition of fractional numbers with the same denominator, all the numerators are added and the sum is written as numerator in the result, keeping the denominator same.

Subtraction of like fractions

In subtraction of like fractions, find out which is greater and subtract the smaller from the greater.

1. $\frac{2}{4} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}$ 2. $\frac{6}{7} - \frac{4}{7} = \frac{6-4}{7} = \frac{2}{7}$

Can we subtract greater fraction from smaller fraction?

Exercise 3.2

1. Find which is greater fraction:

(i) $\frac{3}{7}, \frac{5}{7}$ (ii) $\frac{2}{12}, \frac{7}{12}$ (iii) $\frac{6}{19}, \frac{16}{19}$ (iv) $\frac{13}{34}, \frac{31}{34}$ (v) $\frac{37}{137}, \frac{33}{137}$

2. Add the following like fractions:

(i) $\frac{1}{4} + \frac{2}{4} = ?$ (ii) $\frac{3}{7} + \frac{4}{7} = ?$ (iii) $\frac{3}{13} + \frac{9}{13} = ?$ (iv) $\frac{5}{7} + \frac{3}{7} + \frac{4}{7} = ?$

(v) $\frac{5}{124} + \frac{43}{124} + \frac{33}{124} = ?$ (vi) $\frac{23}{432} + \frac{23}{432} + \frac{32}{432} = ?$

3. Simplify the following:

(i) $\frac{12}{13} - \frac{4}{13} = ?$ (ii) $\frac{9}{17} - \frac{6}{17} = ?$ (iii) $\frac{34}{39} - \frac{33}{39} = ?$ (iv) $\left\{\frac{75}{47} + \frac{3}{47}\right\} - \frac{14}{47} = ?$

(v) $\left\{\frac{125}{214} - \frac{25}{214}\right\} + \frac{50}{214} = ?$ (vi) $\left\{\frac{24}{122} + \frac{2}{122}\right\} - \frac{13}{122} = ?$

3.1.4 Unlike Fractions: Comparison, addition, subtraction.

Which is greater ? $\frac{1}{4}$ or $\frac{2}{5}$

Observe that the denominators are different.
Fractions having different denominators are called unlike fractions.
Convert the unlike fractions to like fractions to add, subtract and to compare.

How to convert the unlike fractions into like fractions ?

Consider the unlike fractions $\frac{1}{4}$ and $\frac{2}{5}$

Convert them into like fractions, without changing their values.
How to covert them in to like fractions without changing their values?
Convert unlike fractions into like fractions, by finding their equivalent fractions.

Equivalent fractions of $\frac{1}{4} \rightarrow \frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{4}{16}=\frac{5}{20}=\frac{6}{24}=\frac{7}{28}$

Equivalent fractions of $\frac{2}{5} \rightarrow \frac{2}{5}=\frac{4}{10}=\frac{6}{15}=\frac{8}{20}=\frac{10}{25}=\frac{12}{30}=\frac{14}{35}$

It is important to see, whether the denominators of the two fractions are same.

We can write $\frac{1}{4}$ as $\frac{5}{20}$ and $\frac{2}{5}$ as $\frac{8}{20}$ without changing their values.

Now $\frac{5}{20}$ and $\frac{8}{20}$ are like fractions.

As $\frac{8}{20} > \frac{5}{20}$, $\frac{2}{5} > \frac{1}{4}$

Example : 6

Which is greater? $\frac{1}{2}$ or $\frac{3}{5}$

$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{7}{14}=\frac{8}{16}=\frac{9}{18}=\frac{10}{20}$

$\frac{3}{5}=\frac{6}{10}=\frac{9}{15}=\frac{12}{20}=\frac{15}{25}=\frac{18}{30}=\frac{21}{35}=\frac{24}{40}=\frac{27}{45}=\frac{30}{50}$

Unlike fractions can be converted into like fractions by finding many sets of equivalent fractions. We can use any one pair of equivalent fractions to find which is greater.

$\frac{6}{10} > \frac{5}{10}$ Therefore $\frac{3}{5} > \frac{1}{2}$ or $\frac{12}{20} > \frac{10}{20}$ Therefore $\frac{3}{5} > \frac{1}{2}$

Activity

Prepare multiple cards for 1 to 10 as shown below

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45

Let us take $\frac{3}{4}$ and $\frac{2}{5}$ and convert them into like fractions

$\frac{3}{4}$	3	6	9	12	15	18	21	24	27	30
	4	8	12	16	20	24	28	32	36	40

$\frac{2}{5}$	2	4	6	8	10	12	14	16	18	20
	5	10	15	20	25	30	35	40	45	50

Keep the multiple cards of $\frac{3}{4}$ and $\frac{2}{5}$ as shown above.

Observe the denominators of multiple cards and find where they are equal.
20 and 40 are found in both the multiple cards.

Therefore we can write, $\frac{3}{4}=\frac{15}{20}$ and $\frac{2}{5}=\frac{8}{20}$

Using this activity, we can compare, add and subtract fractions.

3.1.5 Addition of unlike fractions

$\frac{1}{4}+\frac{2}{5}=?$

To add, convert the given fractions into like fractions.

Equivalent fractions of $\frac{1}{4}$ $\rightarrow \frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{4}{16}=\frac{5}{20}=\frac{6}{24}=\frac{7}{28}$

Equivalent fractions of $\frac{2}{5}$ $\rightarrow \frac{2}{5}=\frac{4}{10}=\frac{6}{15}=\frac{8}{20}=\frac{10}{25}=\frac{12}{30}=\frac{14}{35}$

$\frac{1}{4}=\frac{5}{20}$, $\frac{2}{5}=\frac{8}{20}$ $\quad \therefore \frac{1}{4}+\frac{2}{5}=\frac{5}{20}+\frac{8}{20}=\frac{13}{20}$

Example : 7

$\frac{2}{5}+\frac{5}{6}=?$ $\quad \frac{2}{5}=\frac{4}{10}=\frac{6}{15}=\frac{8}{20}=\frac{10}{25}=\frac{12}{30}=\frac{14}{35}$ $\quad \frac{5}{6}=\frac{10}{12}=\frac{15}{18}=\frac{20}{24}=\frac{25}{30}=\frac{30}{36}=\frac{35}{42}$

$\frac{2}{5}=\frac{12}{30}$, $\frac{5}{6}=\frac{25}{30}$ $\quad \therefore \frac{2}{5}+\frac{5}{6}=\frac{12}{30}+\frac{25}{30}=\frac{37}{30}$

Let us take the examples given above,

$$\frac{1}{4}+\frac{2}{5}=\frac{5}{20}+\frac{8}{20}$$

(i.e) $\frac{1}{4}$ is equivalent to $\frac{5}{20}$

$\frac{2}{5}$ is equivalent to $\frac{8}{20}$

(i.e) $\boxed{\frac{1x5}{4x5}+\frac{2x4}{5x4}=\frac{5}{20}+\frac{8}{20}=\frac{13}{20}}$

Similarly, $\frac{2}{5}+\frac{5}{6}=\frac{2x6}{5x6}+\frac{5x5}{6x5}$

$=\frac{12}{30}+\frac{25}{30}=\frac{37}{30}$

To add unlike fractions, we can use the following steps.

$$\frac{1}{4}+\frac{2}{5}$$

Step: 1 Multiply the two denominators $\frac{1}{4}+\frac{2}{5}=\frac{\quad}{4x5}$

Step: 2 Multiply the numerator of each fraction by the denominator of other fraction. $\frac{1x5}{4x5}+\frac{2x4}{5x4}=\frac{(1x5)+(2x4)}{4x5}$

Step: 3 $\frac{1}{4}+\frac{2}{5}=\frac{5+8}{4x5}=\frac{13}{20}$

Example : 8

$\frac{3}{8}+\frac{5}{7}=\frac{(3x7)+(5x8)}{8x7}$

$=\frac{21+40}{56}$

$=\frac{61}{56}$

Example : 9

$\frac{11}{10}+\frac{4}{9}=\frac{(11x9)+(4x10)}{10 x 9}$

$=\frac{99+40}{90}$

$=\frac{139}{90}$

3.1.6 Subtraction

Subtraction is similar to addition

To subtract,

(i) Convert the given fractions into like fractions.

(ii) Subtract the numerators

Example : $\frac{4}{5} - \frac{1}{3} = ?$

Step 1: Convert into like fractions

$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}, \quad \frac{1}{3} = \frac{5 \times 1}{5 \times 3} = \frac{5}{15},$$

$\frac{12}{15}$, $\frac{5}{15}$ are like fractions of $\frac{4}{5}$ and $\frac{1}{3}$ respectively

Step 2 : Subtraction

$$\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$$

$$\therefore \frac{4}{5} - \frac{1}{3} = \frac{7}{15}$$

Exercise:3.3

1. Which is greater?

(i) $\frac{5}{7}, \frac{3}{8}$ (ii) $\frac{2}{10}, \frac{7}{12}$ (iii) $\frac{6}{5}, \frac{2}{4}$ (iv) $\frac{6}{9}, \frac{4}{3}$ (v) $\frac{3}{2}, \frac{3}{7}$

2. Simplify the following:

(i) $\frac{3}{4} + \frac{2}{3} = ?$ (ii) $\frac{3}{8} + \frac{2}{4} = ?$ (iii) $\frac{3}{5} + \frac{9}{9} = ?$ (iv) $\frac{5}{3} + \frac{3}{8} + \frac{4}{3} = ?$

(v) $\frac{3}{10} + \frac{4}{100} = ?$ (vi) $\frac{3}{4} + \frac{2}{5} + \frac{4}{8} = ?$

3. Simplify the following:

(i) $\frac{2}{3} - \frac{1}{4} = ?$ (ii) $\frac{9}{10} - \frac{3}{5} = ?$ (iii) $\frac{3}{4} - \frac{3}{8} = ?$ (iv) $\frac{6}{7} - \frac{1}{4} = ?$ (v) $\left\{\frac{8}{9} - \frac{1}{9}\right\} - \frac{2}{9} = ?$

3.1.7 Improper fractions and mixed fractions

$\frac{3}{4}, \frac{1}{2}, \frac{9}{10}, \frac{5}{6}$ { In these fractions, denominator is greater than the numerator. They are called proper fractions.

If the numerator is greater than the denominator, the fraction is called an improper fraction. } (e.g) $\frac{5}{4}, \frac{6}{5}, \frac{41}{30}$

What is meant by $\frac{5}{4}$?

Velu, Appu, Vasu and Kala had 5 dosas with them. How to divide them equally ?

First we can give 1 dosa each to all the four. Then the remaining 5^{th} dosa can be divided into 4 equal parts, and each one can be given 1 part.

The total quantity of dosa received by Velu, Appu, Vasu and Kala= 1 whole dosa + $\frac{1}{4}$ dosa

$= 1 + \frac{1}{4}$ dosa

This can be written as $1\frac{1}{4}$

How else can you divide the dosa among them?

Each dosa can be divided into 4 equal parts and each one would receive five $\frac{1}{4}$ parts.

Each one would have got $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ = five $\frac{1}{4} = \frac{5}{4}$

But the dosa received by each in each method must be same.

$$\therefore \frac{5}{4} = 1\frac{1}{4}$$

$1\frac{1}{4}$ is called mixed fraction.

A mixed fraction has one natural number and one proper fraction.

Any improper fraction can be converted into mixed fraction.

Note: Mixed fraction = Natural number + proper fraction.

$4\frac{1}{2} = 4 + \frac{1}{2}$ and $22\frac{1}{3} = 22 + \frac{1}{3}$

3.1.8 Conversion of improper fractions into mixed fractions.

Example : 10

$$\frac{7}{3} = \frac{3}{3}+\frac{3}{3}+\frac{1}{3}$$
$$= \frac{6}{3}+\frac{1}{3}$$
$$= 2+\frac{1}{3} = 2\frac{1}{3}$$

Divide 7 by 3

```
3) 7 (2
   6
-----------
   1
-----------
```

Divisor = 3
Quotient = 2
Remainder = 1

$$\text{Mixed fraction} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

Think over!
There are two groups of people. In the first group, 4 apples are shared equally among 3 people and in the second group 3 apples are shared equally among 4 people. Which group you will join, if you want to get more apples?

Do it yourself : Convert the following improper fractions into mixed fractions.

(i) $\frac{11}{3}$ (ii) $\frac{23}{7}$ (iii) $\frac{22}{5}$

3.1.9: Conversion of mixed fractions into improper fractions.

Example : 11

Convert $3\frac{2}{7}$ into improper fraction.

$$3\frac{2}{7} = 3+\frac{2}{7} = 1+1+1+\frac{2}{7}$$
$$= \frac{7}{7}+\frac{7}{7}+\frac{7}{7}+\frac{2}{7}$$
$$= \frac{7+7+7+2}{7} = \frac{23}{7} \qquad \boxed{3\frac{2}{7} = \frac{23}{7}}$$

$$\text{Improper fraction} = \frac{\text{(Natural number x denominator) + Numerator}}{\text{Denaominator}}$$

$$3\frac{2}{7} = \frac{(3\times7)+2}{7}$$
$$= \frac{21+2}{7} = \frac{23}{7}$$

∴ The improper fraction of $3\frac{2}{7}$ is $\frac{23}{7}$

All non negative numbers can be considered as fractions. In these numbers, denominator can be considered as 1

Discuss: What kind of fractions are these?

$\frac{7}{7}$, $\frac{0}{7}$ and $\frac{1}{7}$

Do it yourself

Convert the following mixed fractions into improper fractions

$1\frac{1}{3}$, $2\frac{3}{5}$, $3\frac{5}{7}$, $1\frac{4}{10}$

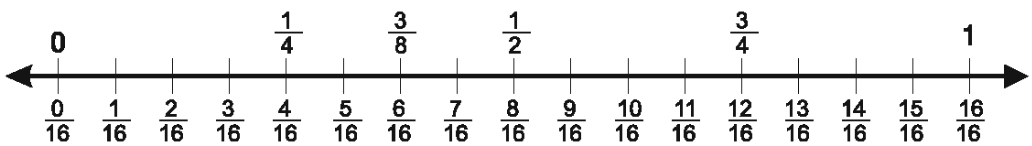
3.1.10 Fractions on number line

We have $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ between 0 and 1

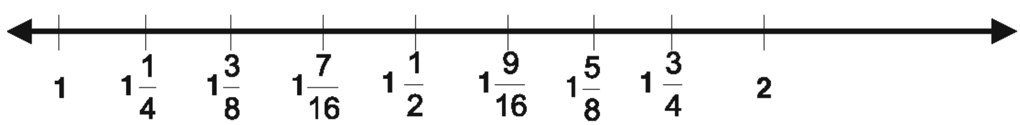
$\frac{3}{8}$ lies between $\frac{1}{4}$ and $\frac{1}{2}$

$\frac{7}{16}$ lies between $\frac{3}{8}$ and $\frac{1}{2}$

$\frac{9}{16}$ lies between $\frac{1}{2}$ and $\frac{3}{4}$



Similarly, there are many fractions between 1 and 2



Similarly, we can draw number lines between 101 and 102 , 134 and 135, 2009 and 2010.

On a number line, there are plenty of fractions. Moreover, when you add or subtract two fractions, we get a number or a fraction on the number line.

Between any two whole numbers we get many fractions.

Actually, between any two fractions, we can find a fraction! Thus many new fractions can be obtained! If each one of you find hundred fractions, there will be always a few more new fractions.

3.1.11 Miscellaneous Problems

Example : 12

There are 20 balls in a box. How many balls should be taken from the box, if you want to take three quarters of them?

Solution:

Total No.of balls = 20

Balls to be taken = $\frac{3}{4} \times 20$

$= 3 \times 5$

$= 15$ balls

Example : 13

There are 60 students in a class. $\frac{2}{5}$ of them are boys. Find the number of boys

Solution

Total number of students = 60

$$\text{No. of boys} = \frac{2}{5} \times 60$$
$$= 2 \times 12$$
$$= 24 \text{ boys}$$

Exercise: 3.4

1. Find any ten fractions between 0 and $\frac{1}{4}$.
2. There are 50 goats in a village. $\frac{2}{5}$ of them were lost. How many goats were lost?
3. The population of a village is 1000. One fourth of them are children. Find the number of adults.

Points to remember:

- When a whole is divided into a number of equal parts, we get fractions.
- When we multiply numerators and denominators of fractions by the same number, we get equivalent fractions.
- To compare, add or subtract the like fractions, we can take only the numerators and perform the operation.
- To compare, add or subtract unlike fractions, convert them into equivalent fractions.
- We can find a fraction between any two fractions on the number line.

3.2 DECIMAL NUMBERS

Introduction:

We have learnt about very big numbers (a number with more number of digits) and fractions which are less than 1. We often use fractions like $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$. By addition or subtraction of fractions we got fractions like $\frac{3}{8}, \frac{5}{8}, \frac{7}{16}$. Very small numbers also can be written as fractions.

Why can't we use fractions to represent all small numbers? It is because of the difficulties in using fractions.

$\frac{2}{3} + \frac{3}{4} = ?$

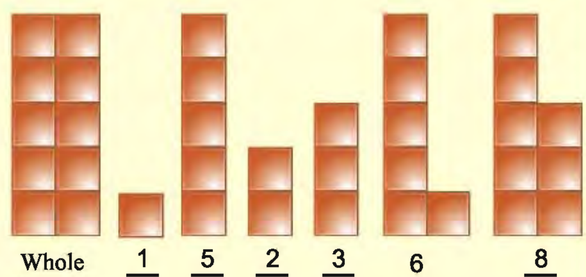
We convert them into like fractions by finding equivalent fractions and then add.
It is easy if all the fractions are in the form of $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$ ....

$\frac{15}{100} + \frac{235}{1000}$ can be easily added as $\frac{150}{1000} + \frac{235}{1000} = \frac{385}{1000}$

It was easy to use multiples of 10 in measurements. It will be easy if small numbers can be written as fractions with multiples of ten as denominators.

3.2.1 One Tenths ($\frac{1}{10}$)

Kannan has 6 chocolate bars,each with 10 connected pieces.
He gave some pieces to his friends.
He finds that
1 piece out of 10 from the first chocolate
5 pieces out of 10 from the second chocolate
2 pieces out of ten from the third
3 pieces out of ten from the fourth
6 pieces out of ten from the fifth
8 pieces out of ten from the sixth remaining.



We can write them as
$\frac{1}{10}, \frac{5}{10}, \frac{2}{10}, \frac{3}{10}, \frac{6}{10}, \frac{8}{10}$ in fractions
This can be written as **0.1**, 0.5 , 0.2 , 0.3 , 0.6 , 0.8 in decimals

0.1 is read as zero point one. The point between the numbers is called the decimal point.

Fractions with powers of ten as denominators are called decimal fractions.

3.2.2 Decimal numbers - Definition

A decimal number has two parts namely an integral part and a decimal part.

Example:

A. Decimal Number = 0.6 = 0 + 0.6 Integral part = 0 : Decimal Part = 6
B. Decimal Number = 7.2 = 7 + 0.2 Integral part = 7 : Decimal Part = 2

In a decimal number the digits to the left of the decimal point is the integral part. The digits to the right of the decimal point is the decimal part.

The value of all the decimal parts is less than 1.

Example : 13

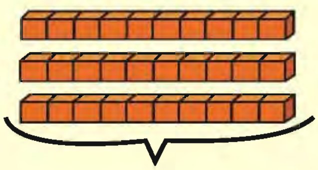


Fig.1

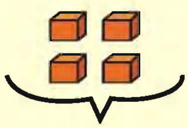


Fig.2

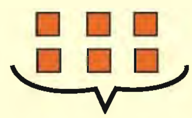


Fig.3

In figure 1 each wooden bar represents ten units.
In figure 2 each wooden bar represents one unit.
In figure 3 each wooden bar represents $\frac{1}{10}$ units.

Tens (10)	ones (1)	one tenths ($\frac{1}{10}$)
3	4	6

(i.e) $30 + 4 + \frac{6}{10} = 34 + 0.6 = 34.6$

It is read as thirty four point six

Example : 14

How to read decimal numbers?

S.No	Decimal Number	Integral Part	Decimal Part	Methods of reading numbers
1	6.5	6	5	Six point five
2	12.6	12	6	Twelve point six
3	91.8	91	8	Ninety one point eight

Do you know?
In olden days we used Ana, Chakkram, Kasu, Panam to denote money. Only from 1957 the decimal method of Rupees and paisa was introduced.

All whole numbers can be considered as decimals. 5 can be written as 5.0.
The zero to the extreme right of the decimal point has no value.

3.2.3 Place value of decimal numbers.

In decimal system, The place value of the integral part increases in powers of ten from right to left. The place value of the decimal part decreases in powers of ten from left to right.

Example : 15

Find the place value of the digits in the decimal number 67.8

Solution

Tens (10)	ones (1)	one tenths($\frac{1}{10}$)
6	7	8

Do it yourself: Find the place value of 32.7 , 78.6 , 201.0

Example : 16

Write the decimal numbers for the following:

1) Four ones and 3 tenths
2) Seventy two and 6 tenths.

Solution: i) Four ones and 3 tenths

$$4 + \frac{3}{10} = 4 + 0.3 = 4.3$$

ii) Seventy two and 6 tenths

$$72 + \frac{6}{10} = 72 + 0.6 = 72.6$$

Example : 17

Change the following fractions into decimal fractions.

(i) $30 + 8 + \frac{4}{10}$ (ii) $400 + 80 + \frac{6}{10}$

Solution:

(i) $30 + 8 + \frac{4}{10}$ (ii) $400 + 80 + \frac{6}{10}$

$= 38 + 0.4 = 38.4$ $= 480 + 0.6 = 480.6$

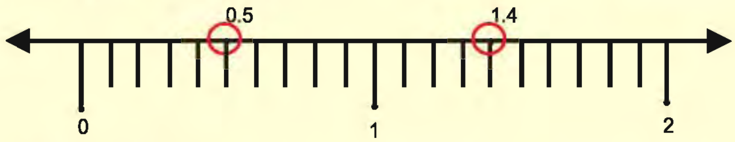
3.2.4 Representation of decimal numbers on the number line.

We know how to represent numbers and fractions on the number line. In a similar way we can represent decimal numbers on the number line.

Example : 18

Represent 0.5 , 1.4 on the number line

Solution

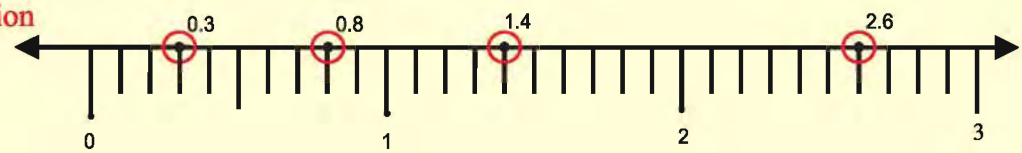


There are 10 equal parts between 2 whole numbers. The length of each equal part is $\frac{1}{10}$ part. So five tenths is the 5^{th} part from zero.

Example : 19

Represent 0.3 , 0.8 , 1.4 , 2.6 on the number line.

Solution



Do it yourself

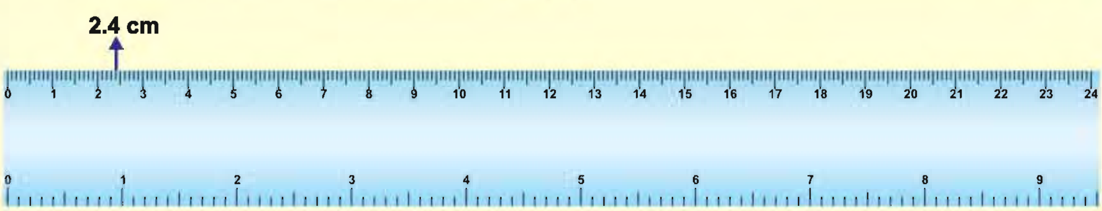
Represent 0.9 , 1.2 on the number line

Do you know ?

In cricket we denote 4 overs and 2 balls as 4.2 overs. This 4.2 is not a decimal number.

Example : 20

You can mark 2.4 cm on a scale like this



Exercise 3.5

1) Fill in the blanks

i) The decimal fraction of 0.7 is----------

ii) The integral part of 12.8 is------------------

iii) The digit in the one's place of 60.1 is---------------

iv) The place value of 4 in 9.4 is--------------

v) The point between the integral part and the decimal part of the decimal number is called------------

2) Complete the following table

Tens (10)	Ones (1)	One-tenths ($\frac{1}{10}$)	Decimal Nos
2	3	4	
6	9	2	
8	2	8	

3) Complete the following table

Decimal Nos.	Integral part	Decimal part	Value of the decimal part	Number name
7.6				
28.5				
24.0				
5.06				

4) Write the decimal for each of the following

i) One hundred and twenty four and six tenths

ii) Eighteen and three tenths

iii) Seven and four tenths

5) Represent the following decimal numbers on a number line

(i) 0.7 (ii) 1.9 (iii) 2.1

6) Convert the following fractions into decimal numbers.

(i) $\frac{2}{10}$ (ii) $3 + \frac{7}{10}$ (iii) $700 + 80 + 6 + \frac{3}{10}$

Activity

1. Divide the students of a class into groups. Ask them to visit a hotel, grocery shop, ration shop etc. Collect the price list and discuss.

2. Let them measure the length and breadth of different objects at home. Prepare a table using decimal numbers.

3.2.5 One-hundredths – Introduction

Mahesh measured the length of a black board in his class using a ruler. The length is 345 cm. Shall we help him to write the length of the blackboard in metres?

You know that 100cm = 1m

$\therefore$ 1cm = $\frac{1}{100}$ m so 345 cm = 300 cm + 45 cm

$= 3 \text{ m} + \frac{45}{100} \text{ m}$

$= \mathbf{3}\text{m} + \mathbf{0.45} \text{ m} = \mathbf{3.45} \text{ m}$

Therefore 345cm is converted into a decimal number as 3.45m.

We know what is one-tenths. Can we find one-tenths of one-tenths?
Let us see that in the following figure

Fig 1

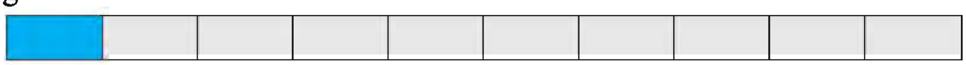
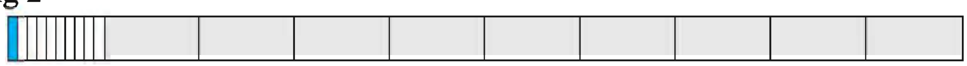


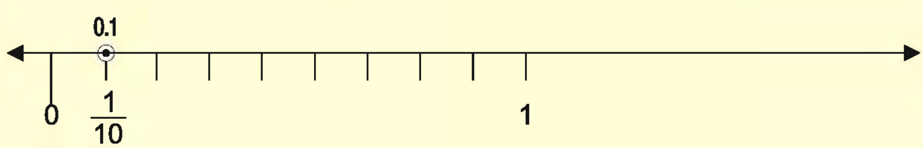
Fig 2

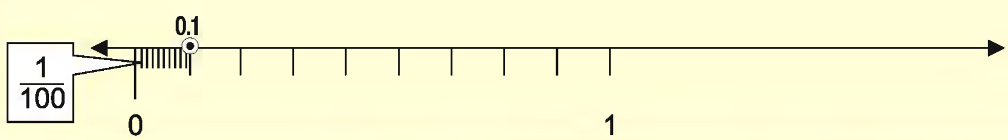


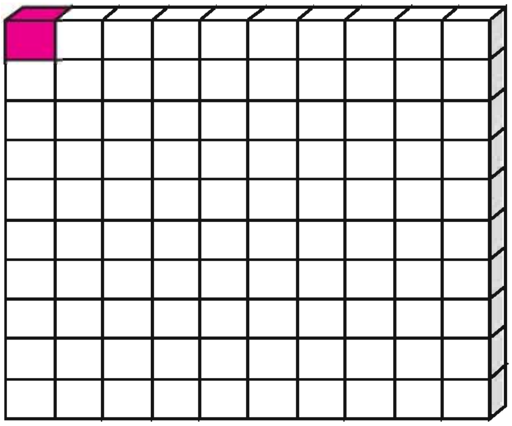
The shaded portion in fig 1 is $\frac{1}{10}$ and the shaded portion in fig 2 is $\frac{1}{100}$.

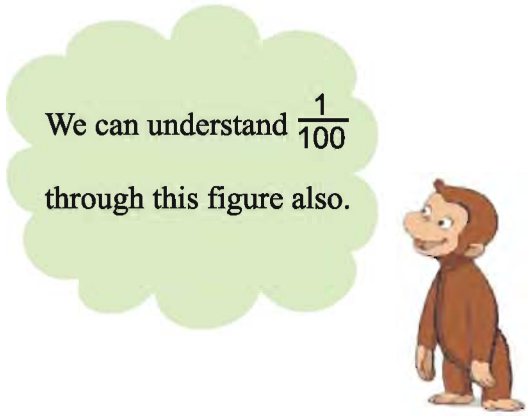
Example : 21

Represent $\frac{1}{10}$ and $\frac{1}{100}$ on the number line









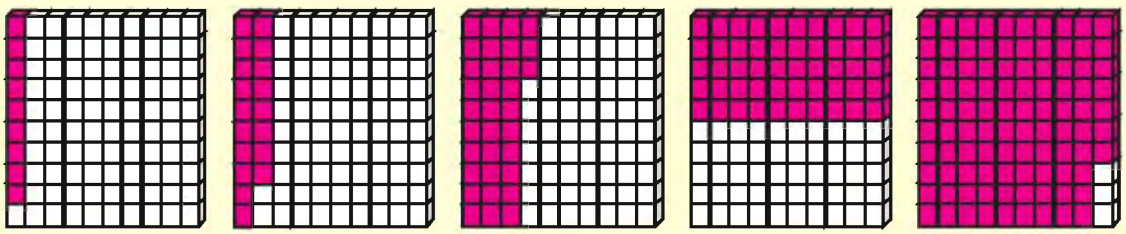
The shaded region in the figure is one-hundredths.

Fractional form = $\frac{1}{100}$

Decimal form = 0.01

Example : 22

Using the following figures, Convert into Fractional and Decimal forms.



S.No	Shaded Portions	Fractional form	Decimal form
1	9 squares	$\frac{9}{100}$	0.09
2	18 squares	$\frac{18}{100}$	0.18
3	33 squares	$\frac{33}{100}$	0.33
4	50 squares	$\frac{50}{100}$	0.50
5	97 squares	$\frac{97}{100}$	0.97

Example : **23**

Convert into decimals (i) $\frac{4}{100}$ (ii) $\frac{36}{100}$ (iii) $6 + \frac{7}{10} + \frac{8}{100}$

Solution

(i) $\frac{4}{100} = 0.04$ (ii) $\frac{36}{100} = 0.36$ (iii) $6 + \frac{7}{10} + \frac{8}{100} = 6 + \frac{70}{100} + \frac{8}{100}$

$= 6 + \frac{78}{100}$

$= 6 + 0.78 = 6.78$

Do it Yourself

Convert into decimal numbers

(i) $\frac{6}{100}$ (ii) $\frac{36}{100}$ (iii) $200 + 80 + 9 + \frac{3}{100}$

Example : **24**

Write the decimal number: Eighteen and forty five hundredths.

Solution

Eighteen and forty five hundredths$= 18 + \frac{45}{100} = 18 + 0.45 = 18.45$

Example : **25**

Convert the following decimal numbers into fractions

(i) 0.09 (ii) 0.83

Solution

(i) $0.09 = \frac{9}{100}$ (ii) $0.83 = \frac{83}{100}$

Do you know?

While reading decimal numbers, the digits to the Right of decimal point must be read individually. For Example : 8.29 must be read as "Eight point two nine"

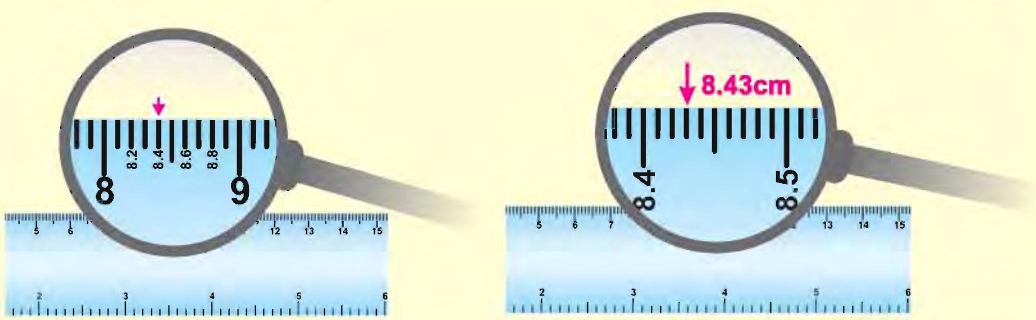
Do it yourself

Convert into fractions

a) 1.45 b) 0.13

Example : **26**

Let us mark 8.43 cm on a scale



Exercise 3.6

1) State whether the following are true or false:

   i) Non negative integers can be considered as decimals

   ii) Fractional form of 3.76 is $3 + \frac{76}{10}$

   iii) The place value of 3 in 82.03 is $\frac{3}{100}$

   iv) The place value of 0 in 70.12 is 70

2) Write as decimal numerals

   i) Twenty three and eighteen-hundredths

   ii) Nine and five-hundredths

3) Find the place value of the underlined digits in the following decimal numbers.

   i) $\underline{9}227.42$ ii) $208.0\underline{6}$ iii) $34\underline{3}.17$ iv) $166.\underline{2}4$

4) Convert the following fractions into decimals

   i) $20 + 3 + \frac{4}{10} + \frac{7}{100}$ ii) $137 + \frac{5}{100}$ iii) $\frac{3}{10} + \frac{9}{100}$

5) Convert the following decimals into fractions

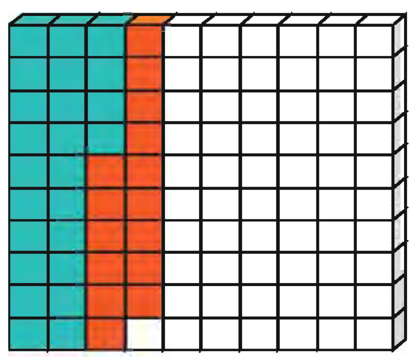
   i) 106.86 ii) 1.20 iii) 76.45 iv) 0.02

3.2.6 Addition and subtraction of decimals

The process of adding decimals is similar to that of adding whole numbers. The place value is important. It is important to arrange the digits one below the other according to place values.

While adding 7235 with 47 we arrange the numbers as $\begin{array}{r} 7235 \\ +\ 47 \\ \hline \\ \hline \end{array}$ Not as $\begin{array}{l} 7235 \\ +47 \\ \hline \\ \hline \end{array}$

Observe the figure



0.24 and 0.15 are shaded in two different colours.
Now the sum of 0.24 and 0.15 is 0.39
(i.e.) 3 tenths and 9 hundredths

Method: 1

	Ones	decimal point ●	one-tenths	one-hundredths
	0	●	2	4
	0	●	1	5
Sum	0	●	3	9

Method:
Arrange the decimal numbers according to place values as we arrange in whole numbers. Then add or subtract.

∴ 0.24 + 0.15 = 0.39

Method :2

$$\begin{array}{r} 0.24 \\ +\ 0.15 \\ \hline 0.39 \\ \hline \end{array}$$

∴ 0.24 + 0.15 = 0.39

Example : 27

(i) $\begin{array}{r} 0.5 \\ +\ 0.5 \\ \hline 1.0 \\ \hline \end{array}$ (ii) $\begin{array}{r} 0.75 \\ +\ 0.25 \\ \hline 1.00 \\ \hline \end{array}$ (iii) $\begin{array}{r} 0.75 \\ +\ 0.50 \\ \hline 1.25 \\ \hline \end{array}$

Note that in the problem (iii), while adding 0.75 and 0.5, 0.5 is written 0.50

Example : 28

Simplify (i) 7.3 + 11.46 (ii) 6.07 + 29

Solution (i) $\begin{array}{r} 7.30 \\ +\ 11.46 \\ \hline 18.76 \\ \hline \end{array}$ (ii) $\begin{array}{r} 6.07 \\ +\ 29.00 \\ \hline 35.07 \\ \hline \end{array}$

∴ 7.3 + 11.46 = 18.76 ∴ 6.07+29 = 35.07

Example : 29

(i) Subtract 1.52 from 3.29

Solution: $\begin{array}{r} 3.29 \\ -\ 1.52 \\ \hline 1.77 \\ \hline \end{array}$

(ii) Subtract 120-12.02

Solution: $\begin{array}{r} 120.00 \\ -\ 12.02 \\ \hline 107.98 \\ \hline \end{array}$

Exercise 3.7

1) Fill in the blanks

(i) 7.25 + 3.50 = ______ (ii) 8.18 − 5.00 = ______

(iii) 9.69 − 1.11 = ______ (iv) 5.83 − 3.14 = ______

2) Add: (i) 9.005 + 300 (ii) 142.36 + 158.25

3) Subtract: (i) 9.756 − 6.79 (ii) 250 − 202.54

Action plan

A student has made mistakes in all his sums in the home work. Discuss in groups to find the method of correcting his mistakes.

(i) 6.7 + 2.5

$$\begin{array}{r} 6.7 \\ +\,2.5 \\ \hline 8.12 \\ \hline \end{array}\ \text{X}$$

(ii) 8.9 + 4.3

$$\begin{array}{r} 8.9 \\ +\,4.3 \\ \hline 12.12 \\ \hline \end{array}\ \text{X}$$

(iii) 48.3 + 17.6

$$\begin{array}{r} 48.3 \\ +\,17.6 \\ \hline 515.9 \\ \hline \end{array}\ \text{X}$$

(iv) 38.3 − 17.9

$$\begin{array}{r} 38.3 \\ -\,17.9 \\ \hline 21.6 \\ \hline \end{array}\ \text{X}$$

(v) 28.4 − 4

$$\begin{array}{r} 28.9 \\ -\;\;4 \\ \hline 28.5 \\ \hline \end{array}\ \text{X}$$

(vi) 9.4 − 6.7

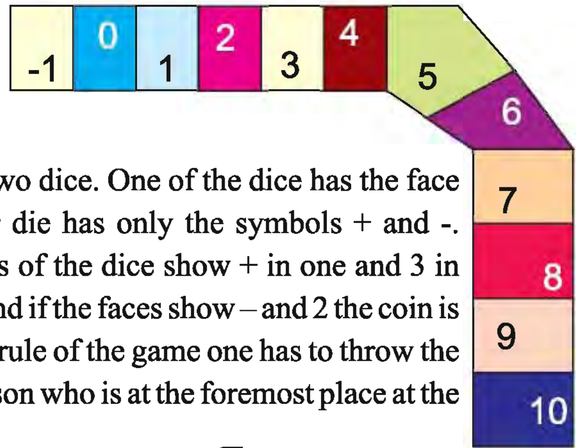
$$\begin{array}{r} 9.4 \\ -\,6.7 \\ \hline 3.3 \\ \hline \end{array}\ \text{X}$$

Points to remember:

- Decimal fractions are fractions having ten or powers of ten as denominators
- A decimal number has two parts namely (i) integral part (ii) decimal part
- They are separated by a decimal point
- All non negative integers can be considered as decimal numbers.
- In a decimal number, the zeros to the extreme right of decimal point has no value
- While adding or subtracting decimals, arrange the decimal numbers according to the place values as we do in whole numbers.

4. Integers

Activity



4.1 Problem in a Number Game.

Malliga and Victor play a game with two dice. One of the dice has the face numbers from 1 to 6 as usual. But the other die has only the symbols + and -. According to the rule of the game, if the faces of the dice show + in one and 3 in other, the coin is to be moved 3 steps forward and if the faces show – and 2 the coin is to be moved 2 steps backward. Also as per the rule of the game one has to throw the dice twice in each round. The winner is the person who is at the foremost place at the end of 5^{th} round.

Malliga played first. He got + and 3 in his first throw and – and 2 in the second throw. So, he moved the coin 3 steps forward and 2 steps backward then he placed the coin in the box 1. Then Victor played and he got + and 5 in first throw, - and 3 in second throw. So he placed the coin in box 2.

	Initial Position of the coin	Numbers in first throw	Numbers in second throw	Final Position of the coin
Malliga	0	+, 3	–, 2	1
Victor	0	+, 5	–, 3	2

They continued to play. At the end of the 5^{th} round the position is as follows.

	Initial Position of the coin	Numbers in first throw	Numbers in second throw	Final Position of the coin
Malliga	7	–, 3	–, 2	2
Victor	4	–, 6	+, 3	?

Victor had a problem to continue. He tried to go 6 steps backward from 4. But after 0 it is not possible. So, he went 3 steps forward and he kept coin in box 3. He declared that he was the winner.

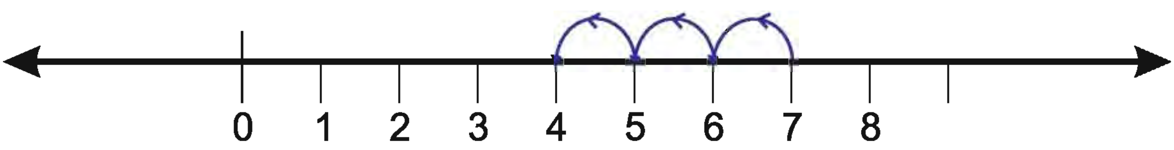
But, Malliga did not accept it. She said, "you are wrong. If you are not able to move – 6 from 4, you must move +3 first and then -6. Your coin will be in box 1. So, I am only the winner."

Can you guess the winner?

How to solve this problem?

Note: The solution is given in the last page of this unit:

What is the real problem in this game? We shall understand it by the number line. To find 7-3, we should move 3 units left from 7. The answer is 4.

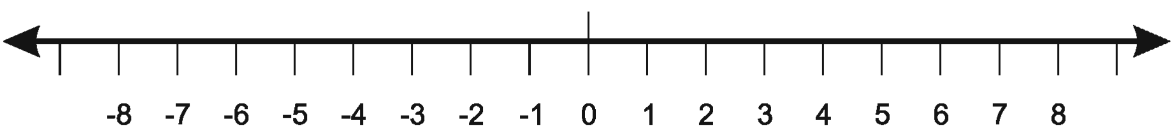


But, to find 4-6, we cannot move 6 units left from 4. Because there are no numbers before 0. Can we find the answer if we move beyond 0 leftwards?

4.2 Integers – Introduction

The numbers on left of 0 are decreasing as they are increasing on right of 0. We represent the numbers on left of zero with the symbol "-". The numbers can be written on left side of the number line as we write the Natural numbers on the right side.

Since the numbers on left of 0 are less than 0 they are called negative integers. The numbers on right of 0 are called positive integers.



Usually, we don't write + for positive integers. The numbers +5 and 5 are one and the same. But negative integers are preceeded with – sign.

We use many such numbers in our daily life.

A shopkeeper sells an article and gains Rs.500. It is represented as +500 Rupees gain. If an article is sold with loss of Rs.200, It is represented as -200 rupees loss.

The average temperature of Tamilnadu = +30°C

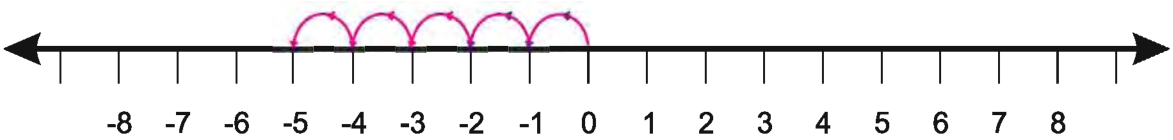
The average temperature of Antartica = -25°C

Positive integers, zero and Negative integers altogether constitute Integers.

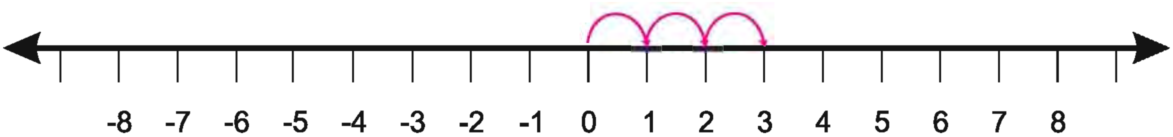
4.3 The position of integers on Number line.

First let us learn the method of marking numbers on the Number line.

-5 is marked on the number line after moving 5 units left of 0.

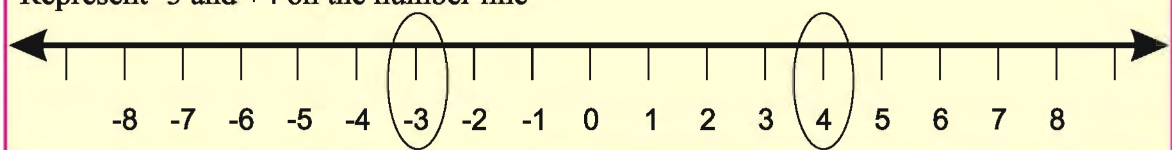


Similarly +3 is marked on the numbers after moving 3 units right of 0.



Example : 1

Represent -3 and +4 on the number line



Do it Yourself

Represent on the Number line:
+7,-2,-6,-1,8, -10

Here, smaller numbers alone are considered on the number line. But the number line extends on both the sides.

We have learnt that 5 > 1 in integers.

5 > 1 and 5 lies to the right of 1

3 > 0 and 3 lies to the right of 0

∴ 0 > -2 as 0 lies to the right of -2

-3 > -5 as -3 lies to the right of -5.

In other words,

Since -6 lies right of -8, we write -6 > -8

Since -2 lies right of -5, we write -2 > -5

So,

On number line, from right to left the numbers are getting decreased.

Every positive number is greater than a negative number.
Zero is less than a positive number.
Zero is greater than a negative number.

Is '0' negative? or Is '0' positive?
If not ' 0' is ……………..

Do it Yourself

Fill with proper symbols using < and >

1) 6 $\square$ 4   2) 5 $\square$ 0   3) 4 $\square$ -6   4) -3 $\square$ -1   5) -1 $\square$ 4

Example : 2

Find the predecessor and successor of the following.

−7, −3, 0, 4, 7

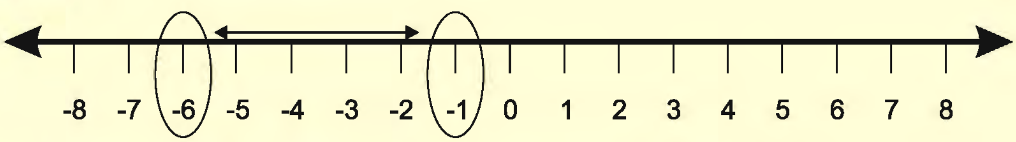
Solution

Predecessor	Integer	Successor
−8	−7	−6
−4	−3	−2
−1	0	1
3	4	5
6	7	8

Example : 3

Using the number line, write the integer between -6 and -1. Which of them is the largest? Which of them is the smallest?

Solution



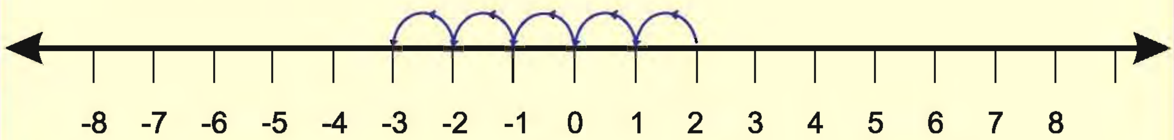
From the number line, the integers between -6 and -1 are -5, -4, -3, -2.
Since -2 lies right of -5, -2 > -5.
Largest integer = -2
Smallest integer = -5.

Example : 4

On the number line, (i) How many units are to be moved from 2 to reach -3? (ii) How many units are to be moved from -5 to reach 4?

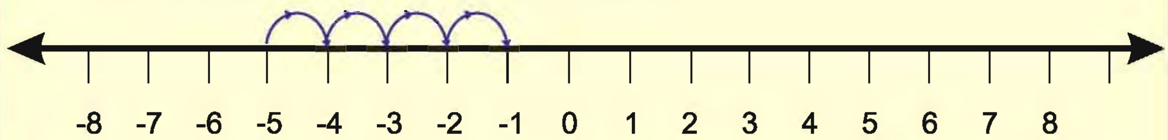
Solution:

(i) Represent the given number on a number line.



So, move five units left of 2 to reach -3.

(ii) Represent the given number on a number line



So, move four units right of -5 to reach -1.

Exercise 4.1

1. Say whether True or False.
   - (i) Zero is less than every positive number.
   - (ii) Towards the left side of 0, the numbers are getting decreased.
   - (iii) -5 is on the right side of -4 on the number line.
   - (iv) -1 is the least negative number.
   - (v) Every positive number is greater than the negative numbers

2. Identify the larger and smaller integer from the following using number line.
   (i) 7,3 (ii) -5, -3 (iii) -3, 2 (iv) 7, - 3 (v) 1, -4 (vi) -4, -7
3. List the integers between the given number using number line
   (i) 3, -3, (ii) -4, 2 (iii) -1, 1 (iv) -5, -2 (v) -4, 3 (vi) -2, 2
4. Answer the following using number line.
   - (i) What is the number when we move 3 units right of -2?
   - (ii) What is the number when we move 7 units leftward from 3?
   - (iii) How many units are to be moved from 5 to reach -3?
   - (iv) How many units are to be moved from -6 to reach -1?

4.4 Addition and subtraction of integers

We can add integers as we do in Natural numbers. But in integers we have already + and – signs. So, we should differenciate the addition and subtraction operation signs from the sign of the number. For Example : in (+5) + (+3) the second + sign represents the operation addition. But first and third + signs represent the sign of the number

Addition of two positive numbers is easy. (+5) + (+3) and 5 + 3 are one and the same. Since the answer for 5 + 3 is 8, we understand (+5) + (+3) = 8.

How to add two negative integers ? On a number line, when 1 is added to any number we get a number which lies in the immediate right of it. We know if 1 is added to a number 3 we get a new number 4, which lies to the right side of 3. What happens if (+1) is added to (-1)? Is it not 0 (zero)! That is the required number. So, (-1) + (+1) = 0. Using this concept we shall easily learn the addition and subtraction of positive and negative integers.

4.4.1. Addition using colour balls

We can easily understand the addition and subtraction of integers using balls of two different colours. Let us assume that green ball represents (+1) and red ball represents (-1). The integers are represented using colour balls in the following table.

Colour balls	Integers
⊕⊕⊕⊕⊕⊕⊕	+7
⊕⊕⊕⊕	+4
⊖⊖⊖	–3
⊖⊖⊖⊖⊖	–5
⊕⊕⊕	+3

We can understand that addition is nothing but union.

(a) Add +7 and +4.

⊕⊕⊕⊕⊕⊕⊕ + ⊕⊕⊕⊕ = ⊕⊕⊕⊕⊕⊕⊕⊕⊕⊕⊕

i.e, (+7) + (+4) = (+11)

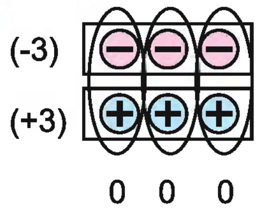
(b) Add -3 and -5

⊖⊖⊖ + ⊖⊖⊖⊖⊖ = ⊖⊖⊖⊖⊖⊖⊖⊖

i.e, (-3) + (-5) = (-8)

As we did earlier, we use the concept (-1) + (+1) = 0. That is, a green ball and a red ball are coupled and can be removed.

⊕ + ⊖ = 0



(-3) + (+3) = 0

Do it Yourself

(-2) + (+2) = ☐

(-1) + (+1) = ☐

(-5) + (+5) = ☐

(-8) + (+8) = ☐

> Sum of a positive number and its negative is zero Hence they are called additive inverse of each other.

Here, 3 and -3 are additive inverse of each other.

Now let us consider red balls and green balls of different numbers.

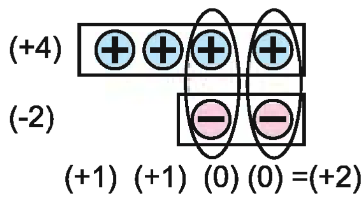
(a) Add: (+4) , (-2)

(+4) + (-2) = (+2) + (+2) + (-2)

= (+2) + 0

= +2

∴ (+4) + (- 2) = + 2



Do it Yourself

(-5) + (+2) = ☐

(+4) + (-3) = ☐

(-2) + (+7) = ☐

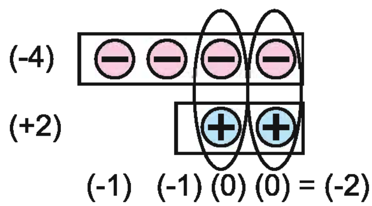
(-3) + (-5) = ☐

(b) Add: (-4) + (+2)

(-4) + (+2) = (-2) + (-2) + (+2)

= (-2) + 0

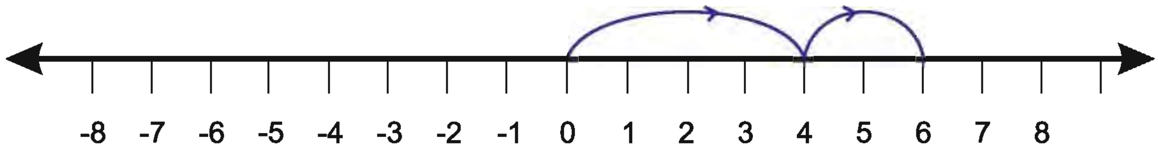
= -2



We have added the numbers using colour balls. Now, we shall do addition using number line.

4.4.2. Addition of integers using number line

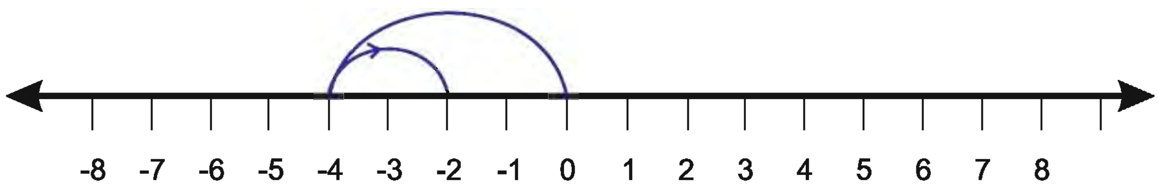
Now we shall learn to add +4 and +2 using a number line.



Since to add (+4) and (+2) starting from 4 we should move 2 units towards right, and we get +6.

$\therefore \quad (+4)+(+2)=(+6)$

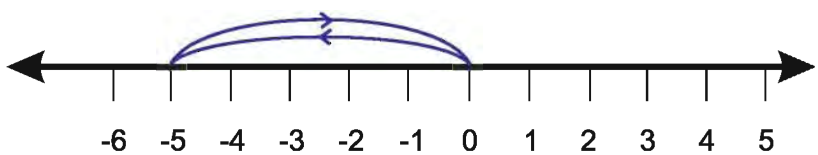
Now we shall add -4 and +2.



Since to add (-4) and (+2), starting from -4 we should move 2 units towards right and we get -2.

$\therefore \quad (-4)+(+2)=(-2)$

Now, we shall add -5 and +5.



Do it Yourself

(-5) + (+2) = $\boxed{}$

(-3) + (+6) = $\boxed{}$

(+1) + (+4)= $\boxed{}$

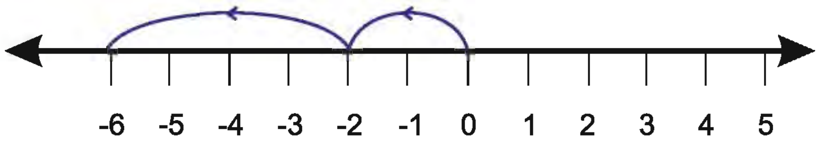
(-3) + (+5) = $\boxed{}$

Since to add (-5) and (+5), starting from -5 we should move 5 units towards right and we get 0. So, (-5) + (+5) = 0

Note: Move towards right for positive numbers and towards left for negative numbers.

We have already learnt using colour balls, when we add a negative and a positive of the same number (that is, additive inverse) we get 0. Just now we confirmed the same using number line. Here 5 and -5 are additive inverse of each other.

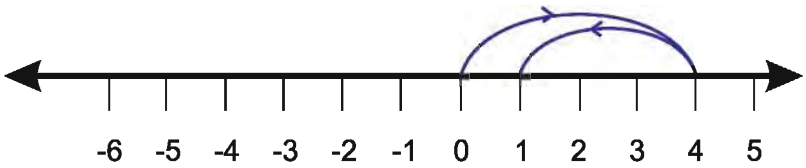
Now, we shall add -2 and -4. That is, (-2) + (-4). Now we should start from -2. The number to be added is -4. So, we should move towards left.



Since (-2) and (-4) are to be added, we should start from (-2) and move 4 units towards left. We reach -6.

$\therefore$ (-2) + (-4) = -6

Now, we shall add (+4) and (-3) using a number line,



Since (+4) and (-3) are to be added,
we should start from 4 and move 3 units towards left.
We reach (+1).

$\therefore$ (+4) + (-3) = (+1)

Do it Yourself

(-5) + (-2) = $\boxed{}$

(-3) + (+6) = $\boxed{}$

(+1) + (+4) = $\boxed{}$

(+3) + (-5) = $\boxed{}$

4.4.3 Subtraction using colour balls

We have already learnt addition of integers. Similarly subtraction also can be done. We should find the additive inverse of the number to be subtracted and then it should be added with the number.

Example : 5

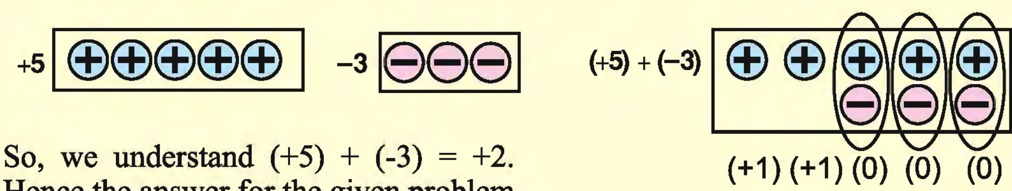
Find (+5) – (+3)

+3 is to be subtracted. Additive inverse of +3 is -3.

Given: (+5) – (+3)

> **Note**
> As (-3) +3=0
> Additive inverse of +3 is -3.

The answer will not be changed if we change it as (+5) + (-3). But we know, how to do (+5) + (-3).



The value is +2,

So, we understand (+5) + (-3) = +2. Hence the answer for the given problem is the same.

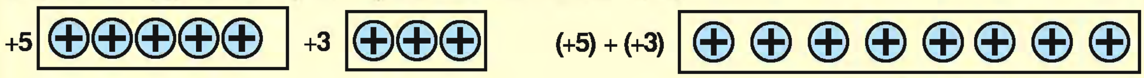
(ie) (+5) – (+3) = +2

Example : 6

Find (+5) – (-3).

Additive inverse of -3 is +3.

So, it is enough to find (+5) + (+3) instead of (+5) – (-3).



Its value is +8

(+5) + (+3) = +8

So, (+5)– (–3) = +8

Do it Yourself

(i) (–4) – (–3), (ii) (+7) – (+2), (iii) (–7) – (+3), (iv) (–5) – (+4)

4.4.4 Subtraction of integers using number line

To subtract an integer from another integer it is enough to add the additive inverse of the second number.

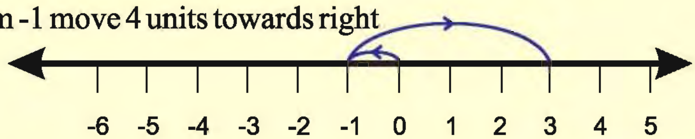
Example : 7

Solve using number line: (-1) – (-4).

Additive inverse of -4 = +4.

Instead of subtracting as (-1) - (-4) we can add it as (-1) + (+4).

Starting from -1 move 4 units towards right



Now we reach +3. So (-1) – (-4) = +3

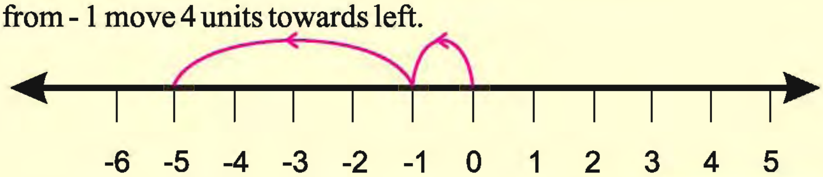
Example : 8

Solve using number line: (-1) – (+4)

Additive inverse of +4 = -4

Instead of subtracting as (-1) - (+4) we can add it as (-1) + (-4).

Starting from - 1 move 4 units towards left.



Now we reach -5. So (-1)-(+4) = -5

Exercise 4.2

1. Add using number line

   (i) $8 + (-4)$ (ii) $(-1) + (-9)$ (iii) $(-5) + (7)$ (iv) $3 + (-6)$ (v) $(+4) + (-7)$.

2. Find using number line.

   (i) What is the number 4 more than -3?

   (ii) What is the number 3 less than -7?

3. Add

   (i) $(-10) + (+17)$ (ii) $(+20) + (-13)$ (iii) $(-50) + (-20)$

   (iv) $(+40)+(+70)$ (v) $(+18) + (-75)$ (vi) $(+75) + (-75)$

   (vii) $(-30) + (12)$ (viii) $(-30) + (-22)$

4. Simplify

   (i) $5 + (-7) + (8) + (-9)$ (ii) $(-13) + (12) + (-7) + (18)$

5. Find the answer

   (i) $(+7) - (-3)$ (ii) $(-12) - (+5)$ (iii) $(-52) - (-52)$ (iv) $(+40)-(+70)$

Solution to the problem in first page of this unit.

If the number line is extended and the negative numbers are to be added, then Malliga will win the game. In the last round, Victor has to move the coin from 6 steps from 4 towards left and reaches -2 then move 3 steps towards right and reaches 1. But Malliga's coin is at box 2. So, she is only the winner.

Points to remember

1. Positive numbers, negative numbers and zero altogether constitute the integers.
2. In the number line, the numbers on the right of 0 are increasing and the numbers on the left of 0 are decreasing.
3. If the sum of two numbers is zero, then they are additive inverse of each other.
4. Sum of two positive numbers is positive. The sum of two negative numbers is negative.
5. The sum of a positive number and a negative number is either positive or negative or zero.
6. Subtracting an integer from another integer is same as adding the additive inverse of the second to the first number.

5. Constants, Variables, Expressions and Equations.

5.1 Introduction

You would have played many games eagerly and enthusiastically. Now, shall we play with numbers?

Divide the students in the class into small groups. Each group should think of a two digit number. Then ask them to do the following calculations.

Step 1: Multiply the two digit number by 2.

Step 2: Add 4 to the result.

Step 3: Multiply the result by 5.

Step 4: Finally subtract 20

From the final answer the number selected by a group can be found. The result obtained by dividing the final answer by 10 is the original number. This is applicable for all the groups.

For Example :

If the final answer is 380. Now divide 380 by 10.

Therefore, the selected number is 38.

How do we find this? Let us list the answers for the different numbers taken by the group. Observe the pattern formed.

Check

1. $38\times2 = 76$
2. $76 + 4 = 80$
3. $80 \times 5 = 400$
4. $400-20=380$

For Example:

Selected number = 23; **23x2=46; 46+4=50; 50x5=250; 250−20=230**

If the selected number is 23 the result is 230.

Let us verify this with a few more examples.

Selected number = 25, Result obtained = 250

Selected number = 40, Result obtained = 400

Selected number = 37, Result obtained = 370

Now we are able to see the relation between the selected number and its result.

Note: The algebraic explanation for the above is given at the end of the chapter.

Do it Yourself

Try the above game with three and four digit numbers. Create and solve a few more mathematical games.

Exercise 5.1

1. Find the missing number in the sequence. 5, 10, 15, _ , 25, 30.

(i) 20 (ii) 2 (iii) 22 (iv) 23

2. Choose the next three shapes from the pattern

○ △ □ ○ △ — — —

(i) ○ △ □ (ii) □ ○ △ (iii) △ ○ □

(iv) ○ □ △

3.

First number	1	2	3	4	5	6
Second number	10	20	30	40	50	60

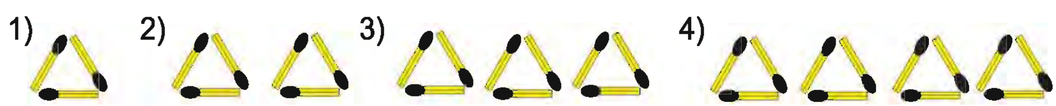
What is the pattern obtained from the table?

(i) Second number = 10 + first number. **(ii)** Second number = 10 – first number

(iii) Second number = 10 ÷ first number. **(iv)** Second number = 10 x first number

5.2 Introduction of constants and variables through patterns

Latha made the following triangular patterns with the match sticks she had.



To find out the total match sticks used for the above formation she prepared the following table.

Number of triangles	1	2	3	4	....	
Number of match sticks used.	3	6	9	12	....	
	3×1	3×2	3×3	3×4	....	

From the above table she found a relation between the number of triangles and the number of match sticks used. That is

Number of match sticks used = 3 x number of triangles

Here according to the number of triangles formed there is a change in the number of match sticks used. We find that the number of match sticks used to form a triangle is always the same. Likewise a quantity which takes a fixed numerical value is called a constant. But number of triangles keep changing. Therefore we denote the number of triangles by the letter x.

Therefore number of match sticks used = 3 x x = $3x$

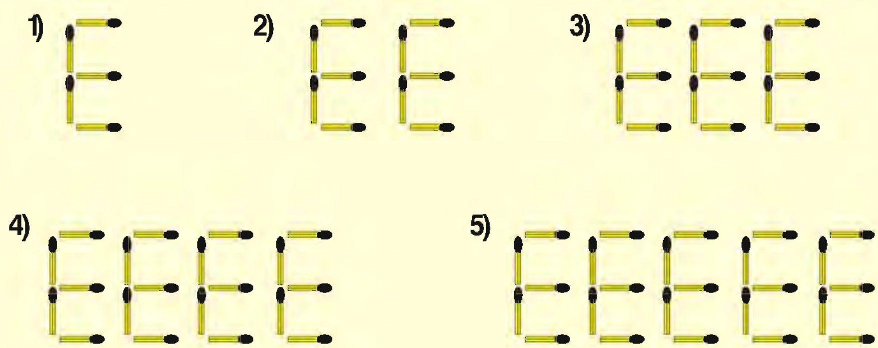
The above reduced law can be taken as "Laws of Patterns"

A quantity which takes different numerical values is called a variable. Usually variables are denoted by small letters.

a, b, c, ... x, y, z

Example : 1

Let us see the formation of letter E with the help of match sticks. We need 5 match sticks to form letter E



Number of E formation	1	2	3	4	5	....	
Number of match sticks used	5	10	15	20	25	....	
	5×1	5×2	5×3	5×4	5×5	....	

Law obtained from the above table.

Number of match sticks used = 5 x (Number of E formation)

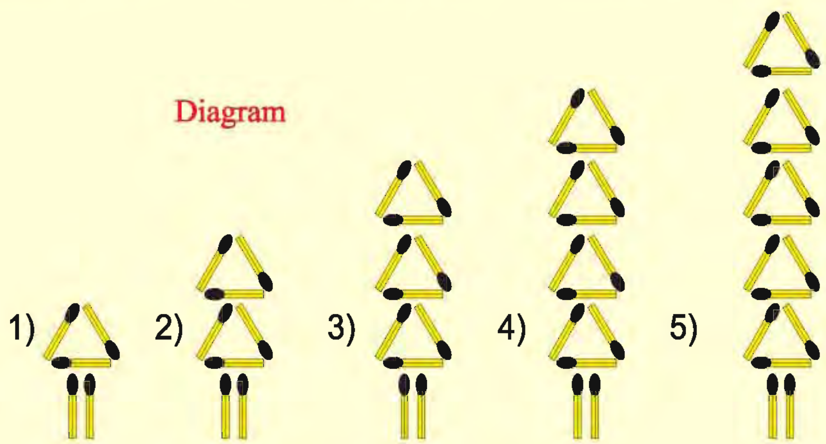
Number of E formation is denoted as the variable x.

Therefore, number of match sticks used = 5 x x = $5x$

Example : 2

Look at the pattern of the Asoka tree given. The base is always formed with two match sticks. The top portion of the tree differs in multiples of 3.

Diagram



Number of top portions	1	2	3	4	5	....	
Number of match sticks needed for the top portion	3	6	9	12	15	....	
	3×1	3×2	3×3	3×4	3×5	....	
Number of match sticks needed for the base	2	2	2	2	2	....	
Total number of match sticks used	3×1+2	3×2+2	3×3+2	3×4+2	3×5+2	....	

Law obtained from the above table,

Number of match sticks used = (3 x Number of top portions) + (Number of match sticks used for the base)

If the number of triangular formations is denoted as the variable x,

Number of match sticks used $= 3 \times x + 2 = 3x + 2$

Exercise 5.2

1. Choose the correct answer:

a)

First number	16	26	36	46	56	66
Second number	10	20	30	40	50	60

Choose the law in which the above pairs are based on?

(i) Second number = first number +6

(ii) Second number = first number -6

(iii) Second number = first number ÷ 6

(iv) Second number = first number x 6

b)

First number	1	2	3	4	5
Second number	9	10	11	12	13

Choose the law in which the above pairs are based on?

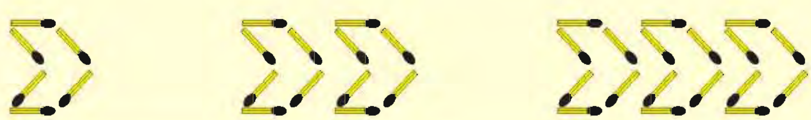
(I) Second number = first number x 5. (ii) Second number = first number - 8

(iii) Second number = first number + 8 (iv) Second number = first number x 8

2. If a box contains 40 apples, the total number of apples depends on the number of boxes given. Form an algebraic term (Consider the number of boxes as 'x').

3. If there are 12 pencils in a bundle, the total number of pencils depends on the number of bundles given. Form an algebraic term (Consider the number of bundles as 'b').

4. From the following pattern given, form an algebraic term.



5.3 Role of variables in the number system

Commutative property of addition of two numbers.

$1 + 2 = 2 + 1 = 3$
$4 + 3 = 3 + 4 = 7$
$4 + 5 = 5 + 4 = 9 \ldots.$

When the numbers are added in any order the value remains the same. So, this can be denoted using variables $a+b = b+a$ where a and b are any two whole numbers.

Do it Yourself

If a, b, c are variables in the set of whole numbers, verify the following laws

1. $a \times b = b \times a$
2. $a \times (b + c) = (a \times b) + (a \times c)$

5.4 Expressions

We have studied the following in the previous classes.

$11 = (1 \times 10) + 1,$
$12 = (1 \times 10) + 2$
$20 = (2 \times 10) + 0$
....

In the above numerical expressions we have used only numbers 1, 2, 3 . . . .

To form numerical expressions we use addition, subtraction, multiplication and division signs.

For Example :, in the numerical expression $(4 \times 10) + 5$ we have multiplied 10 with 4 and added 5 to the result.

Few more numerical expressions are : $(2 \times 10) - 7$, $3 + (7 \times 6)$, $(-5 \times 40) + 8$, $(6 \times 2) + 4$

A variable can take any numerical value.

All operation's +, -, x, ÷ used for numbers are also applicable for variable.

Example : 3

Write the algebraic expression for the following statements:

Situation	Introduction of variables	Algebraic expression
1. Length of a rectangle is 3 more than its breadth.	Let the breadth of the rectangle b 'x' units	Length of the rectangle is $(x+3)$ units
2. Raghu is 10 years younger than sedhu.	Let the age of Sedhu be 'x' years	Raghu's age is (x-10) years
3. Ramkumar is 2 times as old as Nandhagopal	Let the age of Nandhagopal be 'x' year	Ramkumar's age is (2x) years
4. Cost of one pen is Rs.9 less than the cost of one note book	Let the cost of one note book be Rs. 'y'.	Cost of one pen is Rs.(y-9)
5. The diameter of a circle is twice its radius	Let the radius of the circle be 'r' units.	Diameter of the circle is 2 r units

Example : 4

Write the algebraic expression for the following statements

Mathematical operations	Statements	Algebraic expression
Addition	Add 10 to a number	$x + 10$
Subtraction	Subtract 9 from a number	$x - 9$
Multiplication	5 times a number	$5x$
Division	One fourth of a person's monthly income	$\frac{x}{4}$
Less than	10 less than a given number	$x - 10$
Greater than	15 more than a given number	$y + 15$
Multiples	3 times Raghu's age	$3z$

Example : 5

Write the following expression in words

$3m+4$, $3m-4$, $\frac{3m}{4}$, $\frac{4m}{3}$.

Solution:

I. $3m+4$ Add 4 to 3 times a number

II. $3m-4$ Subtract 4 from 3 times a number

III. $\frac{3m}{4}$ One fourth of 3 times a number

IV. $\frac{4m}{3}$ One third of 4 times a number

Exercise 5.3

1. Write an expression for the following statements
   (i) Add 7 to x
   (ii) Subtract 10 from y
   (iii) Subtract 8 from $3y$
   (iv) Multiply x with (-7) and add (-5) to it.

2. Write the following expression in statement form
   (i) $2y + 5$ (ii) $2y - 5$ (iii) $\frac{2y}{5}$ (iv) $\frac{5y}{2}$

3. Write an expression containing y, 7 and a numerical operation.
4. If Mangai is 'z' years old, answer the following (form algebraic expressions)
   (i) What will be the age of Mangai after 5 years ?
   (ii) How old is Mangai's grand father, if he is 7 times as old as Mangai?
   (iii) How old is Mangai's father if he is 5 more than 3 times as old as Mangai?
5. A rabbit covers a distance of 30 feet by walk and then runs with the speed of 2 feet per second for 't' seconds. Frame an algebraic expressions for the total distance covered by the rabbit.
6. The cost of 1 pen is Rs.10. What is the cost of 'y' pens?
7. Sachin saves Rs.x every day. How much does he save in one week?

5.5 Formation and solving Equations

We can identify whether two numerical expressions are equal or not from the following: $7 + (30 + 7) = (40 - 2) + 6$
Is it true? Ans: Yes
Other than = sign, we can utilize the symbols like >, <, ≠. also,

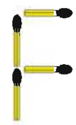
1) $135 \times (74 + 32) > 134 \times (72 + 34)$
2) $(20 - 10) \times 8 < (10 + 20) \times 8$
3) $(5 + 7) \times 6 \neq 5 + (7 \times 6)$

Check the above

When we use 'equal to' sign between two expressions we get an equation. (Both the expressions should not be numerical expressions).
Instead if we use signs like >, <, ≠ it is an inequation. For example,
(1) $3x - 7 = 10$ (equation) (2) $4x + 8 > 23$ (inequation) (3) $2x - 1 < 11$ (inequation)

Example:

Number of 'F' Formation	1	2	3	4	5 ....
Number of match sticks used	4	8	12	16	20 ......
	4×1	4×2	4×3	4×4	4×5 ....



If variable 'x' represent the number of sticks used in the formation of 'F', then, we get the following equation from the above table.

$x = 4$, $2x = 8$, $3x = 12$, $4x = 16$, $5x = 20$
$6x = 24$, $7x = 28$, $8x = 32$ ....

From the above table the value of 'x' which satisfies the equation $3x = 12$ is 4.

Now, let us solve the equation $3x = 12$ by substitution method.

Equation	Value of the variable	Substituting the value of the variable	Solution / Not a solution
$3x = 12$	$x = 1$	$3 \times 1 = 3$ (False)	Not a solution
	$x = 2$	$3 \times 2 = 6$ (False)	Not a solution
	$x = 3$	$3 \times 3 = 9$ (False)	Not a solution
	$x = 4$	$3 \times 4 = 12$ (True)	Solution
	$x = 5$	$3 \times 5 = 15$ (False)	Not a solution
	$x = 6$	$3 \times 6 = 18$ (False)	Not a solution

Result for the equation $3x = 12$ is 4.

Example : 6

Write an algebraic expression for the following statement:

Statement	Algebraic expression
1) 10 added to a number gives 20	$y + 10 = 20$
2) Two times a number is 40	$2x = 40$
3) 5 subtracted from a number gives 20	$x - 5 = 20$
4) A number divided by 6 gives the quotient 5 leaving no remainder.	$\frac{x}{6} = 5$
5) 8 subtracted from twice a number gives 10	$2y - 8 = 10$
6) 6 added to twice a number is 42	$42 = 2x + 6$

Example : 7

Complete the following table

Equation	Value of the variable	Substituting the value of the variable	Solution / Not a solution
(i) $x + 3 = 8$	$x = 4$	$4 + 3 = 7 \neq 8$ (False)	Not a solution
(ii) $x - 4 = 7$	$x = 11$	$11 - 4 = 7$ (True)	Solution
(iii) $3x = 12$	$x = 3$	$3 \times 3 = 9 \neq 12$ (False)	Not a solution
(iv) $\frac{x}{7} = 6$	$x = 42$	$\frac{42}{7} = 6$; (True)	Solution

Example : 8

Using the table find the value of the variable which satisfies the equation $x + 7 = 12$.

x	1	2	3	4	5	6	7	8	9	10	11
$x + 7$	8	9	10	11	12	13	14	15	16	17	18

From the table, solution for $x + 7 = 12$ is $x = 5$.

Exercise 5.4

1. Choose the correct answer:

   a) Which of the following is an equation?

   (i) $3 + 7 = 8 + 2$ (ii) $x < \frac{4}{3}$ (iii) $3x + 1 = 10$ (iv) $4 \times 7 = 28$

   b) Which equation has $y = 4$ as solution?

   (i) $2y + 3 = 0$ (ii) $y - 7 = 2$ (iii) $y + 3 = 7$ (iv) $y + 4 = 0$

   c) Which is the variable in the equation $2s - 4 = 10$?

   (i) 2 (ii) 10 (iii) -4 (iv) s

2. Match

	Equation	solution
a)	$y - 2 = 0$	(i) $y = 0$
b)	$2y = 6$	(ii) $y = 2$
c)	$2 = y + 2$	(iii) $y = 3$

3. Complete the table

Equation	Value of the variable	Substituting the value of the variable	Solution / Not a solution
$x - 8 = 12$	$x = 4$		
$x - 8 = 12$	$x = 6$		
$x - 8 = 12$	$x = 20$		
$x - 8 = 12$	$x = 15$		

4. Complete the table

Equation	Value of the variable	Substituting the value of the variable	Solution / Not a solution
$y + 7 = 15$	$y = 6$		
$y + 7 = 15$	$y = 7$		
$y + 7 = 15$	$y = 8$		
$y + 7 = 15$	$y = 9$		

5. Complete the table

S.No	Equation	Value of the variable	Substituting the value of the variable	Solution / Not a solution
(i)	$x - 3 = 0$	$x = 2$		
(ii)	$y + 7 = 2$	$y = -2$		
(iii)	n + 8 = -18	n= 28		
(iv)	3 – p = 10	$p = -7$		

6. Using the numbers given in the brackets find the value of the variable which satisfies the given equation.

(i) $x + 7 = 12$ (3, 4, 5, 6)
(ii) $x - 10 = 0$ (7, 8, 9, 10)
(iii) $3x = 27$ (6, 12, 9, 8)
(iv) $\frac{p}{7} = 5$ (21,14, 7, 35)
(v) $\frac{r}{10} = 2$ (18, 19, 20, 21)

7. Find the value of 'y' which satisfies the equation $y - 3 = 9$.

8. Complete the following table and find the value of the variable that satisfies 3z = 30

z	5	6	7	8	9	10	11	12	13	14	15
3z			21					36			

9. Complete the following table and find the value of the variable that

satisfies $\frac{P}{4} = 3$

P	4	8	12	16	20	24
$\frac{P}{4}$		2			5	

Point to remember

- Variable has no constant value. It take various values according to the given situation.
- Variables are denoted by small letters a, b, c …. x, y, z….
- Expressions can be related using variables.
- In arithmetic and geometry formulae are obtained using variables.
- If we equate one expression with another expression we get an equation. (One expression must be a non numerical expression).
- Value of the variable that satisfies the equation is the solution for the equation.

Note: Algebraic Explanation for the group game

Algebraic explanation for the group game given in the beginning of the chapter.

Let the number selected by the friend be 'x' multiply the selected number by 2, 2x; and 4 (2x +4); Multiply by 5 ($5 \times (2x + 4) = 10x + 20$)

Subtract 20 ($10x + 20 - 20 = 10x$)

Now the number selected can be found by dividing 10x by 10. ($\frac{10x}{10} = x$)

Finally we get the number selected.

Mathematical puzzles

1. I am a number. Go round the corners of the given figure 4 times. When you add my value with the number of corners you have crossed you get 46. Find my value.

2. I am a number. After crossing all the boxes given in the figure, the total of my value and the number of boxes crossed is 60. Find my value.

3. I am a two digit number. Moreover I am a multiple of 11. When I am divided by 7, I leave no remainder. When 4 is added to the quotient 15 is obtained. What is my value ?