The application of mathematical models to measure collateral concentration risk

Martin Seagroatt* and Ed Cockram

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*4sight Financial Software Ltd, Exchange Place 25 Semple Street, Edinburgh, EH3 8BL, UK;
E-mail: martin.seagroatt@4sight.com

Martin Seagroatt

Ed Cockram

Martin Seagroatt *is Director of Marketing and Product Innovation at 4sight Financial Software, a provider of securities finance, collateral management and collateral optimisation technology. He joined 4sight in 2005 after previously working as a consultant in technology systems for risk management in the energy industry.*

Ed Cockram *is Product Consulting Director at 4sight Financial Software. In previous roles he was Vice President of Securities Finance at ABN AMRO and worked as a collateral trader at Fortis Bank.*

ABSTRACT

The financial system is shifting towards greater use of collateral to mitigate counterparty credit risk. On a systemic basis, this is reducing credit risk; however, it is creating new market and liquidity risk on the collateral held, potentially resulting in weak points in the resilience of the financial system. Use of 'cheapest to deliver' collateral optimisation algorithms also may lead to firms allocating larger quantities of lower quality collateral to counterparties. Managing the concentration risk of the collateral portfolio is therefore becoming more important. This is particularly the case as high-quality liquid assets become scarcer due to rising demand, leading to a move to non-cash collateral or acceptance of lower-rated assets. In addition, many firms are using basic methods to monitor concentration at the trade or counterparty level; they have no clear view of collateral concentration at the firm level. This paper looks at some of the trends in monitoring collateral portfolio concentration. It also reviews the mathematical models available to measure build ups of concentration risk. This includes measures such as diversity scores, the Herfindahl-Hirschman index (HHI) and Gini coefficients. The paper concludes that market participants and particularly central clearing counterparties (CCPs) have incentives to move down the liquidity spectrum in terms of the collateral they will accept. More advanced models and technology solutions to manage concentration limits are therefore important tools in preventing future defaults and crises.

Keywords: collateral concentration, collateral management, collateral risk, concentration risk, non-cash collateral, exposure management, OTC collateral management

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INTRODUCTION

The global financial system is currently moving *en-masse* towards a more collateralised world. This is mostly a result of:

- market moves towards secured funding to manage counterparty risk in the wake of the Lehman default;

- regulatory mandates to trade derivatives via central clearing counterparties (CCPs);
- rules promoting bilateral exchange of two-way initial margin.

But collateral is not a catch-all solution. It mitigates credit risk but can never truly eliminate it and introduces other risks in its place. A widespread market move towards exchange of margin sees counterparty credit risk shift to market and liquidity risk on the collateral held, on a systemic level. As collateral becomes a scarcer resource and the sell-side seeks high-quality liquid assets (HQLA) to meet liquidity buffers, higher fees (securities lending) or lower cash interest rates (repo) are available for those prepared to accept lower rated/less liquid/cheaper to deliver collateral in exchange. This could provide an incentive for lenders in the securities finance markets to move towards more risky collateral.

Use of collateral optimisation tools by a wider range of market participants is also increasing, which could result in firms delivering the absolute 'cheapest to deliver' collateral a counterparty will accept in the greatest quantities possible. A recent Bank for International Settlements (BIS) report highlighted the dangers of this trend:

> 'The increased use of optimisation tools that focus on a cheapest-to-deliver model could result in a higher proportion of lower-quality collateral being provided to those willing to take it than was perhaps the case when the pledging of collateral was a less automated process. For example, if a Financial Market Infrastructure (FMI) or bilateral counterparty accepts corporate bonds for margining purposes, an automated cheapest-to-deliver model will always seek to provide the maximum amount of corporate bonds as collateral to that entity before considering the pledging of higher-quality collateral.'
>
> 'FMIs and bilateral counterparties accepting collateral should understand that they are likely to receive more of the lower-quality collateral on their collateral eligibility schedules and that they need to be able to appropriately manage the collateral they receive. FMIs can apply several principles in the [CPSS-IOSCO Principles for Financial Market Infrastructures] PFMI, including appropriate margining and concentration limits. Consistent application of the PFMI by market infrastructures can help mitigate potential concentration risks resulting from a more efficient allocation of collateral by market participants.'[1]

The quality and availability of collateral exchanged between market participants therefore could become a significant source of risk and result in future systemic crises. In this new environment, it is important for financial firms (and regulators) to measure, monitor and risk manage the concentration of collateral across their entire portfolio with all counterparties. A firm should have the ability to track the concentration risk it has to certain asset classes, currencies, countries and individual issuers, along with other criteria. Furthermore, there is an argument that it is prudent to maintain more diversified collateral portfolios and even accept limited amounts of less liquid assets in order to reduce concentration risk.

Crowded and increasingly procyclical collateral markets could see widespread liquidation of particular asset classes in a market-wide shock, leading to fire sales.[2] This would result in severe drops in collateral valuations. Particular asset types also could become completely illiquid. The

caveat to this is that, when taking less liquid assets, pricing must be transparent and collateral haircuts must be appropriate. Collateral concentration monitoring therefore allows a firm to accept a broader range of collateral, in the knowledge that it has properly risk assessed the diversification of these assets. This enables the firm to expand trading opportunities, while maintaining a lower risk profile, helping it to avoid exposure to macroeconomic shocks in a particular asset class, geographical region or industry sector. Financial firms also can use concentration rules to ensure they bring in the correct types of collateral to satisfy Basel III liquidity coverage ratios and CCP margin requirements, in the right quantities. In addition, concentration monitoring provides an important quantitative measure of how diverse the collateral portfolio is, compared with using more subjective, qualitative methods.

Setting concentration limits can help to avoid wrong-way risk. This can see a high correlation between the value of the collateral and the credit quality/default probability of the counterparty. For example, in a worst-case scenario, as the counterparty becomes more likely to default, the value of the collateral held to mitigate this risk is also decreasing, which results in a widening of the exposure as the counterparty nears default. This situation can be partially mitigated by haircuts and then enhanced by increased diversification of assets away from those most closely correlated to the credit-worthiness of the counterpart. Finally, from an efficiency point of view, consider an example where a trader must decide whether to execute a given trade with one of two otherwise identical counterparts. Counterpart 1 is offering to pledge collateral in which the trader's firm is already heavily concentrated. Counterpart 2 offers collateral to which the firm has less overall exposure. All other trade parameters being equal, it makes sense to maximise profit and loss (P&L) on a risk-adjusted basis by taking the more diversified collateral.

IMPROVED TOOLS TO MONITOR CONCENTRATION

The trend for firms to centralise the collateral management function across all business lines and desks has led to a better ability to assess concentration risk, which sees all collateral inventory and exposures managed in one system. Without this, it is more difficult to identify the true, real-time concentration risk of the collateral portfolio. Like portfolio approaches to investing, the risk of the entire portfolio across the firm often can be far greater than for individual trades or desks. Managing collateral in siloes for different business lines (derivatives, securities lending, repo etc) can result in one desk taking in collateral from a specific issuer, while another desk also accepts large quantities of the same collateral, which can lead to a build-up of firm-wide risk that is hard to track and measure effectively. In the past, it appears that many firms monitored collateral concentration using a basic common-sense approach. The market did not use advanced methods for measuring concentration risk, which probably was due to lower perceived counterparty credit risk and low market volatility. For example, at a given time, if there was a heightened sensitivity to commercial paper in the market, then risk managers would request that collateral managers push back on that. As another example of a basic approach to concentration, a firm's guidelines could dictate that, for securities lending transactions, at least five different stocks should cover the net asset value (NAV) of each trade. But these simple rules, while helpful, do not fully measure concentration risk, particularly on a firm-wide basis. As collat-

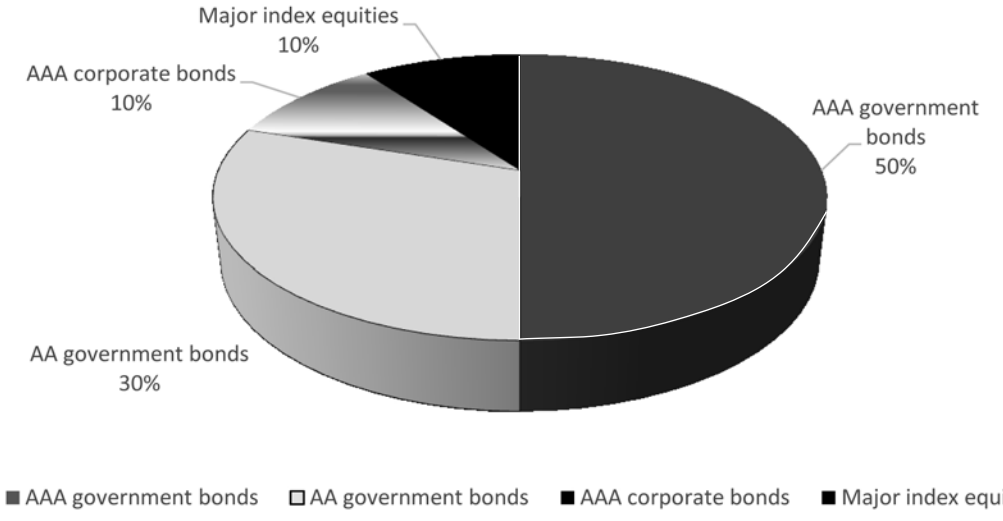


Figure 1 Example concentration schedule with maximum limits for received collateral

eral becomes more central to the functioning of financial markets in the post-crisis environment and a major source of systemic risk, it now could be useful to monitor collateral concentration more closely.

WHAT ARE CONCENTRATION RULES?

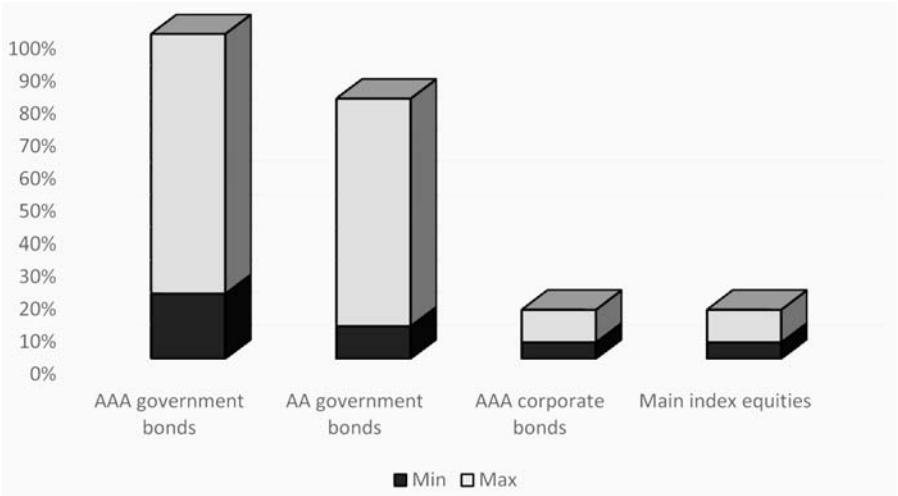
The function of collateral is to guarantee the performance of a loan or derivative in case of default by the counterparty. Collateral schedules state which types of collateral the counterparts to the trade consider eligible under a given contract. Concentration rules then limit the ratios between these types of collateral. A collateral concentration rule is a limit applied as either a percentage or a value. It represents the maximum value/percentage of securities that the collateral receiver will accept in a specific category. Figure 1 shows a basic example of a concentration schedule with maximum limits. In addition to maximum levels, concentration rules also can set minimum levels the firm requires of a given type of collateral, or even a range of values that a collateral receiver will accept, as shown in Figure 2. Collateral technology solutions can assist in this process by allowing users to specify concentration limits on factors such as:

- instruments;
- issuers;
- countries;
- regions;
- rating bands (both issuer and issue);
- industry sectors;
- market indices;
- market capitalisation (for equities);
- average daily traded volume.

It is possible to set limits at the individual trade/counterparty level and at the firm-wide level. More advanced systems can run 'what-if' scenarios to simulate the incremental effect on concentration of accepting a given asset into the pool. Limits are also sometimes combined to create more complex rule-sets. For example, a concentration schedule could stipulate that the firm will accept:

- a maximum of 10 per cent sub-investment grade bonds with a maturity greater than 10 years, combined with;

Figure 2 Example collateral concentration schedule with maximum/minimum rules



- a limit on securities issued by financial institutions making up a maximum of 20 per cent of the collateral set, combined with;
- an overall limit of 30 per cent sub-investment grade bonds across all tenors.

But more complex rules make it harder for counterparties to optimise collateral allocations to the receiver and are more difficult to fill. This situation can skew the trade economics, and make the trade more expensive for the collateral giver, which can then result in reduced trading opportunities for the receiver.

CONCENTRATION RULES MINIMISE CORRELATION RISKS BETWEEN ASSET DEFAULT PROBABILITIES

Concentration limits provide a useful tool as, theoretically, much of the risk in a collateralised loan can arise from the correlation between all of the assets and between each asset and the probability of default of the counterparty.[3] But these correlation coefficients are rarely available, and the calculations are often onerous, which makes them impractical for measuring risk on a daily basis. Schedules and concentration limits are then a way of minimising these underlying correlation coefficients without having to know what they actually are. Measures of concentration then can address areas where there could be a risk of high correlations that are hard to identify. It is beyond the scope of this paper to determine suitable concentrations of different asset types. This depends on many factors for each individual firm, including its risk tolerance, the counterparties it trades with, geographies, currencies, instrument types, wrong-way risks etc. Concentration rules must be flexible in order to respond to changing market conditions. The availability of technology systems to monitor concentration is also a key consideration. Furthermore, a robust stress-testing framework plays an important role in setting concentration limits. This sees the firm running 'what-if' scenarios to test how easy it is to liquidate different asset classes and instruments in a period of market disruption. Instead, this paper will examine the different mathematical models for measuring concentration

that firms can apply to the collateral portfolio.

MATHEMATICAL MODELS FOR MEASURING CONCENTRATION

There has been little discussion in the literature of applying mathematical measures of concentration to collateral. Most of the initial research focused on concentration of market competition.[4,5] Work also has been done on investment portfolio composition or counterparty concentration,[6] which is probably because exchange of collateral has been less widespread in the past. It was also harder to monitor at the firm level than in the present day, due to the factors discussed at the beginning of this paper. Furthermore, the global financial crisis has highlighted the need for improved control over collateral risk. Some example models to measure concentration include:

- simple NAV-based limits;
- concentration ratios;
- diversity scores;
- Herfindahl-Hirschman index (HHI);
- Gini coefficients;
- correlations of default probabilities between instruments in the pool.

It is possible to apply each of these concentration measures to different parameters of the collateral pool; for example, to measure the concentration of assets from countries, issuers, industry sectors etc. Each of these models now will be discussed.

Simple NAV-based limits

Taking a basic approach to concentration, a firm's guidelines could dictate that at least x different securities should cover the NAV of the exposure. This is easy to monitor, but only looks at concentration on the trade level. A more advanced calculation could look at the NAV of exposures at the firm level and then assign a percentage of the collateral held for various parameters (country, issuer, rating, industry etc). For example, no more than 20 per cent of total exposures collateralised by major index shares.

Concentration ratios

Concentration ratios can measure the share of the four/eight etc largest collateral asset holdings against the total collateral portfolio. The ratio can be expressed as:

$$CRa = \Sigma ai = 1\ Si$$

$CRm = s1 + s2 + \ldots + sm$ where s_i is the asset's share of the portfolio and m defines the ith asset.

Concentration ratios vary between 0 and 100 per cent. Lower scores signify no or low levels of concentration, while higher scores indicate greater or total concentration.

No concentration

A ratio of 0 per cent means perfect diversification or, at the very least, minimal concentration. If, for example, $CR4 = 0$ per cent, the four largest collateral asset holdings in the portfolio would not have any significant share of the total.

Low concentration

This category is in the range of 0–50 per cent.

Medium concentration

This category is in the range of 50–80 per cent.

High concentration

This category is in the range of 80–100 per cent.

Total concentration

A ratio of 100 per cent indicates a highly

concentrated portfolio. If, for example, CR1 = 100 per cent, there is zero diversity.

Diversity index

A diversity index is a measure of how many types of entities (for example, asset classes) there are in a dataset (the collateral pool). It also factors in how evenly distributed those entities (such as individual collateral assets) are among those types. While diversity scores are a useful measure, there is limited benefit in promoting equality of asset classes in the collateral portfolio. It is not necessarily desirable to have equal proportions of AAA-rated government debt and lower-rated corporate bonds, for example. A more simple measure of the diversity of the collateral pool (number of different asset classes etc) is sufficient; however, when applied to different aspects of the collateral portfolio (industry sectors, countries, currencies etc), it may have more value.

Herfindahl-Hirschman index (HHI)

The HHI originated as a way for antitrust regulators to measure the size of firms relative to an industry. The HHI is the sum of the squares of the percentage of the total portfolio share held by each respective collateral asset class. For example, a collateral portfolio made up of 60 per cent five-year US Treasuries and 40 per cent shares in IBM has an HHI of $60^2 + 40^2$, or 5,200. A collateral portfolio made up of four different asset types, making up 25 per cent of the portfolio each, would have an HHI score of $25^2 + 25^2 + 25^2 + 25^2$, or 2,500. An HHI score below 1,500 is 'unconcentrated', between 1,500 and 2,500 is 'moderately concentrated', and above 2,500 is 'highly concentrated'.[7] The HHI is a useful measure of concentration; however, it does not measure correlations between the different assets held in the portfolio, nor does it consider wrong-way risk in the collateral taken against the counterparty's likelihood of default, or right-way risk when there is an inverse correlation.

Gini coefficients

The Gini coefficient measures statistical dispersion.[8] The coefficient ranges between 0, which reflects complete equality, and 100, where the portfolio has full concentration in one asset, for example. It is defined as the ratio of the area between the Lorenz curve of the distribution and the Lorenz curve of the uniform distribution (which is a straight line) to the area under the Lorenz curve of the uniform distribution (see Figure 3).[9] The Gini coefficient is more of a measure of inequality than concentration; however, it can be useful when looking at whether the firm is holding unequal quantities of collateral issued by particular governments, for example, leading to a higher level of exposure to that country or region than is desirable.

APPLICATION OF THE MODELS

The models discussed above all measure different aspects of concentration. None of the measures gives a complete picture of the degree of concentration in a collateral pool; however, when applied together they can offer a powerful tool for monitoring concentration levels. In addition, it may be worthwhile to apply each measure to a different subset of concentration in turn. For example, the firm can measure concentration related to instrument types, industry sectors, countries, rating bands etc. It also could be useful to compare concentration risk among the firm's counterparties and then cross-reference against concentration risk in the firm-wide collateral portfolio, which could offer a method of identifying build-ups of wrong-way risk. For example, 30 per cent of the firm's counterparts may be based in a certain country while a simi-

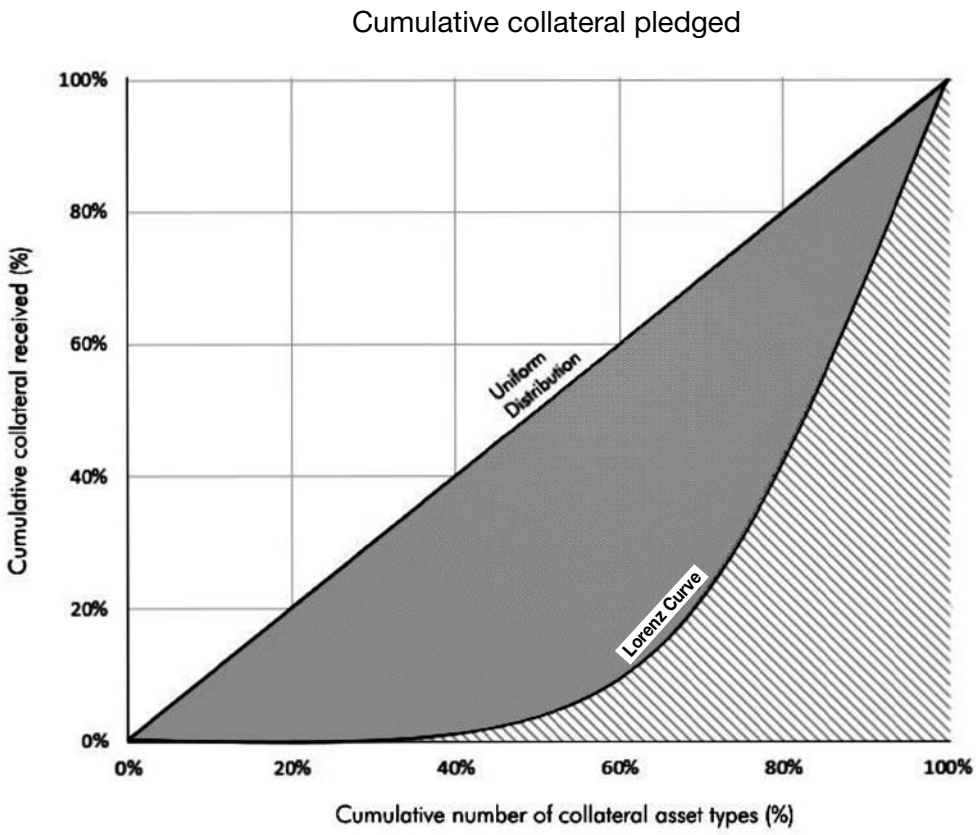


Figure 3 Example Lorenz curve showing received collateral asset types against the total amount of collateral submitted

Source: Adapted from Bindseil, U., Gonzalez, F. and Tabakis, E. (eds) (2009) 'Risk Management for Central Banks and Other Public Investors', Cambridge University Press, p. 372.

lar proportion of the total collateral pool consists of debt issued by that country.

A final, more advanced, option could involve:

- measuring concentration risk against each criteria (sector, issuer etc);
- summing each score to produce a single overall score of concentration across all criteria, which would require some normalisation between concentration numbers before summing them to avoid overweighting certain concentration types;
- multiplying by book notional, which is optional depending on whether one wants to measure concentration by counterparty or by largest potential loss;
- analysing/ranking each counterparty in terms of the concentration risks to which they are exposing one's firm from the collateral they are pledging to it; the technology system would then show graphical representations of where concentration risks arose for each specific counterparty across each criteria measured.

The methodologies above provide powerful tools to measure build-ups of concentration risk. When combined with careful consideration of a firm's risk appetite and a robust approach to stress testing, market participants can gain a greater insight into the risks they are facing and engage in

trading activities on a more informed and profitable basis.

CONCLUSION

It is acceptable and even desirable to take in a diverse range of collateral assets across the liquidity spectrum. Diversity of the collateral pool on a portfolio-wide basis can reduce a firm's overall risk exposure and open up new trading opportunities; however, this is based on the condition that:

- the correct haircuts are applied to collateral taken in;
- assets have transparent pricing;
- assets can be liquidated under stressed conditions;
- concentration rules are monitored according to the firm's risk appetite;
- concentration rules are flexible and adaptable to market conditions;
- the firm has the appropriate technology systems to accurately model its concentration profile.

It also requires the individual firm to have the ability to see its entire collateral portfolio in one place on a real-time basis. This allows the firm to model its overall collateral pool and concentration levels for various parameters, which requires an investment in technology and an alignment of previously siloed business units that many market participants (including many tier-one institutions) have not yet fully achieved.

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