Forward Kinematics

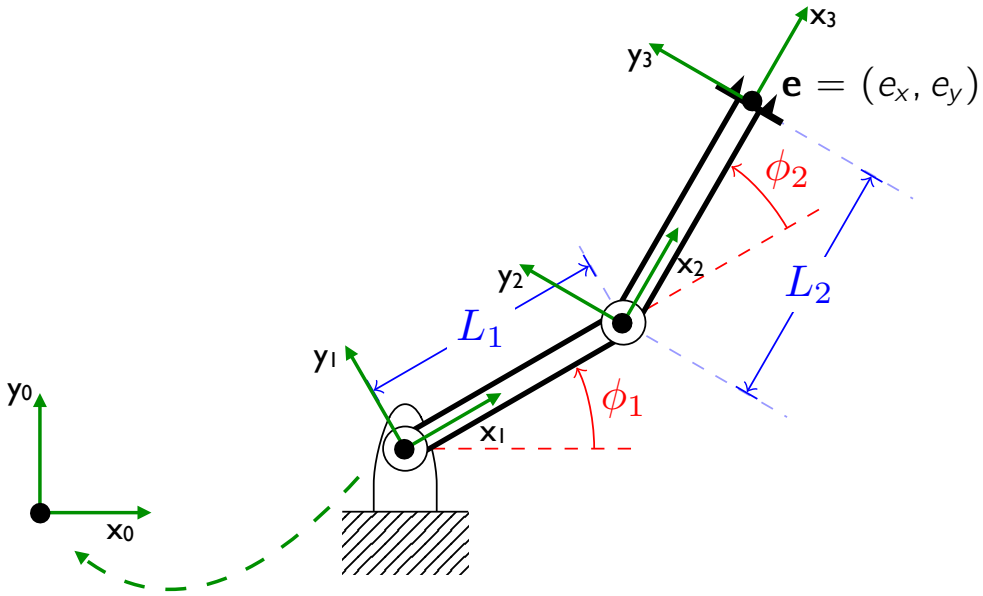

Figure 1: A two-dimensional articulated arm.

Consider the arm structure shown in Figure 1. Assume the following values for the arm configuration: the location of the first joint (i.e., the one attached to the ground support) is $\mathbf{p}_1 = (3, 2)^\mathsf{T}$, the lengths of the parts are $L_1 = 5$ and $L_2 = 8$.

1. Write the matrices that represent the local coordinate frames {1}, {2}, and {3}. These frames are indicated in green in Figure 1. The transformations you need to write are $T_{0,1}$, $T_{1,2}$, and $T_{2,3}$.

2. Write the matrices that represent each local frame w.r.t. the global frame {0}. The transformations you need to write are $T_{0,1}$, $T_{0,2}$, and $T_{0,3}$.

3. Use the transformation matrices to obtain the global coordinates (i.e., w.r.t. frame {0}) of the following points under the given joint-angle configurations:

   - The middle point of each part, for $\phi_1 = pi/8$ and $\phi_2 = pi/4$. Draw the configuration of the arm under these parameters.

- All the joint points and the end effector, for $\phi_1 = pi/4$ and $\phi_2 = pi/8$. Draw the configuration of the arm under these parameters.
- Write the matrix that represents the coordinate frame of the end effector w.r.t. frame $\{1\}$, i.e., $T_{1,3}$
- Write the matrix that represents the coordinate frame $\{1\}$ w.r.t. to the frame of the end effector.