Galaxies Problem Set 2

Emily Ramey

30 September, 2019

1. Stellar Population Synthesis Model

    (a) Assume that stars are formed within the stellar mass range 0.1 to 100 $M_\odot$ following the Salpeter initial mass function. We will use mass bins with a step size of $\Delta(\log M) = 0.2$ with 15 bins in total and an initial stellar population of 10^6 stars. Determine the average mass and the number of stars for each of these bins. Plot $\log_{10} N(\Delta \log M)$ vs. $\log_{10} M$. What is the fraction of stars within the population that is less massive than the sun?

    Calculating the mass bin endpoints:

$$\begin{aligned}
\log M_2 - \log M_1 &= 0.2 \\
10^{\log M_2 - \log M_1} &= 10^{0.2} \\
\frac{M_2}{M_1} &= 10^{0.2} \\
M_2 &= M_1 * 10^{0.2}
\end{aligned}$$

    Where M_1 is the lower bound of a bin and M_2 is its corresponding upper bound. According to the Salpeter IMF:

$$\begin{aligned}
N(\Delta M) &= \int_{M_1}^{M_2} \xi(M) dM \\
&= \int_{M_1}^{M_2} \xi_0 M^{-2.35} dM \\
&= \frac{\xi_0}{1.35}(M_1^{-1.35} - M_2^{-1.35})
\end{aligned}$$

    The average mass of each bin would then be:

$$\begin{aligned}
\langle M \rangle &= \int_{M_1}^{M_2} M\xi(M) dM \\
&= \frac{\xi_0}{0.35}(M_1^{-0.35} - M_2^{-0.35})
\end{aligned}$$

    Plotting these two quantities gives us Figure 1.
    Normalizing $N(M)$ to 10^6 stars gives us $\xi_0 \approx 60,307$ for our population. We can also normalize $N(M)$ to 1 to get the percentage of stars less massive than the sun:

$$N(M < 1 M_\odot) = 0.955$$

    Meaning that our sun is more massive than 95% of all stars.

    (b) For each of the mass bins, determine the average luminosity and explain your methods. Plot the M/L ratio in terms of $(M_\odot/L_\odot)$ as a function of stellar mass

    The average luminosity can be found from the mass using the luminosity-mass relations defined in the problem set. A more rigorous approach would use the expectation value of the luminosity, but graphing each of these methods shows that they are roughly equivalent. The final results for the

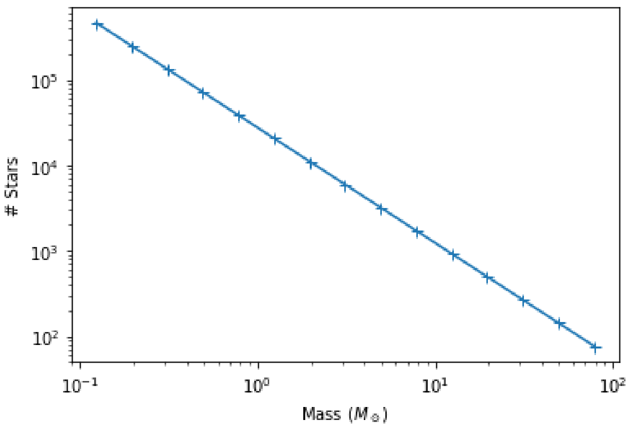

Figure 1: Number of stars vs. average mass for each bin.

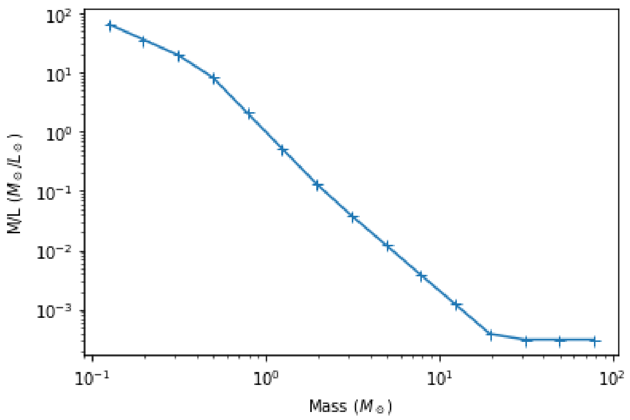

Figure 2: Mass-to-luminosity ratio for each mass bin.

mass-to-light ratio are shown in Figure 2 and the calculations for the true $\langle L \rangle$ are shown below:

$$\begin{aligned}\langle L \rangle &= \frac{1}{N_{bin}} \int_{M_1}^{M_2} L(M)\xi(M)dM \\ &= \frac{\xi_0}{N_{bin}} \int_{M_1}^{M_2} c * M^a * M^{-2.35} dM \\ &= -\frac{c\xi_0}{N(a-1.35)} \int_{M_1}^{M_2} (M_2^{a-1.35} - M_1^{a-1.35})\end{aligned}$$

c and a are the constants of the mass-to-luminosity relationship.

(c) Determine the approximate average temperatures for the 15 mass bins and plot the temperature as a function of stellar mass.

Using the provided luminosity-temperature data and the average luminosity values found in (b) for interpolation (via numpy's `interp` function), we can approximate the temperatures as shown in Figure 3.

(d) Assume that all main sequence stars burn a similar fraction of their total mass and that the lifetime of the sun is 10 Gyr. Derive the minimum, average, and maximum lifetime for each stellar mass bin. Plot the average lifetime as a function of the stellar mass.

Since the time on the main sequence is proportional to the mass-to-luminosity ratio for a given star,

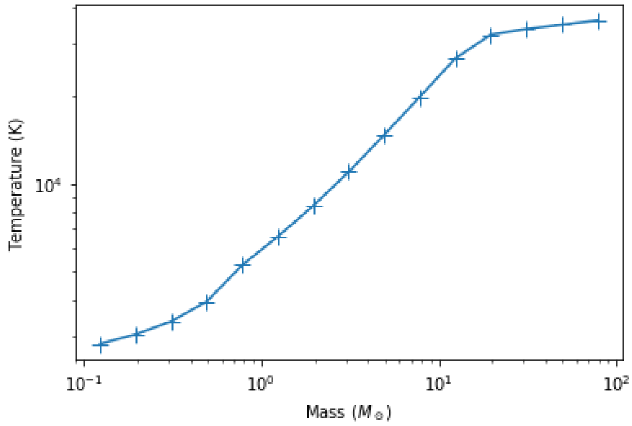

Figure 3: Average temperature per mass bin.

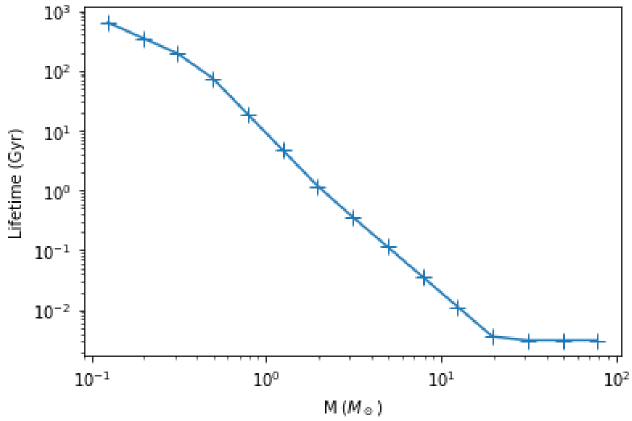

Figure 4: Average stellar lifetime per mass bin.

we can derive:

$$t_{ms} = \tau * \frac{M}{L}$$
$$t_\odot = \tau * \frac{M_\odot}{L_\odot}$$
$$\tau = (10\ Gyr)\frac{L_\odot}{M_\odot}$$
$$t_{ms} = (10\ Gyr)\frac{M/M_\odot}{L/L_\odot}$$

The minimum and maximum lifetimes would then be just the bounds of the mass bins plugged into this equation. The average lifetimes can be approximated using the average mass-to-luminosity ratios found in part (c). The graph of these is shown in Figure 4.

(e) Use the number of stars, the average mass and luminosity of the stars in each bin, as well as the lifetimes to derive the integrated mass-to-light ratio of the stellar population as a function of time. Assume that stars do not experience any mass loss throughout their lives and thus the remnant mass is the same as the mass on the main sequence.

To get the integrated mass-to-light ratio, we must simply add up all the stars that have not died off yet for each time interval. Thus, we start by summing the full set of luminosities for each bin and gradually remove the highest mass stars until finally we are only summing the low mass bins. The total mass remains the same for the entire sequence, so M_{total}/L_{total} only increases as total luminosity decreases, as shown in Figure 5.

(f) Assume that stars are perfect blackbody radiators. Determine and plot the average spectral energy

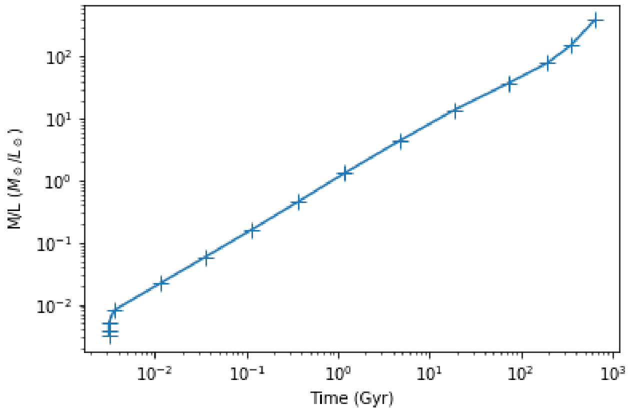

Figure 5: Evolution of the mass-to-luminosity ratio over time for a given star population.

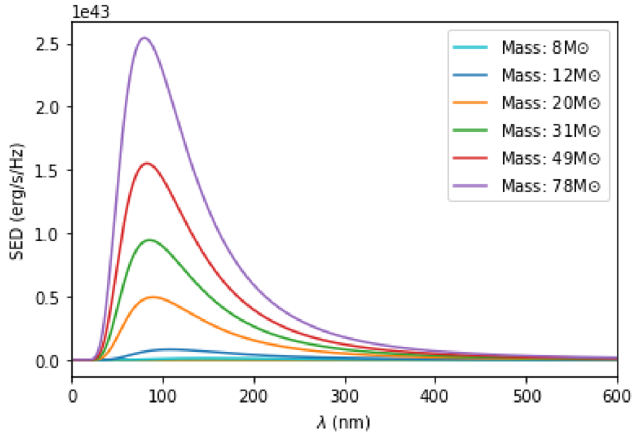

Figure 6: Average intensity for each mass bin as a function of wavelength.

distribution for each mass bin (the average L_ν as a function of wavelength).

We know the luminosity of a blackbody spectrum is proportional to the intensity of the source multiplied by its area. Therefore,

$$L_\lambda = A * B_\lambda = A * \frac{2hc^2/\lambda^5}{ehc/\lambda kT - 1}$$

Since $A = \frac{L_{tot}}{\sigma T^4}$, this gives us:

$$L_\lambda = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} * \frac{L}{\sigma T^4}$$

The average spectral energy distribution for each mass bin is shown in Figure 6, with the average mass of each bin displayed in the legend.

(g) Use the number of stars in each bin as well as the lifetimes and average spectral energy distributions to derive the integrated spectra for each bin. Plot the integrated spectra as a function of time in the range 0.1-13 Gyr.

We can sum the average spectral energy distributions in each bin to get the average spectra for each time period, shown in Figure 7. The spectra get redder and less luminous over time, as expected for an aging stellar population.

(h) Plot the evolution of the B-V color of the stellar population with time. Assume that the filter curves are delta functions at 445 and 551 nm for B and V, respectively. Also plot B-V vs. the integrated

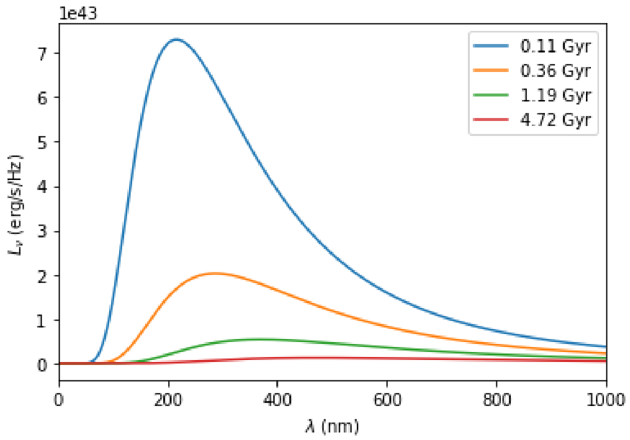

Figure 7: Integrated spectrum of the star population at different times in its evolution.

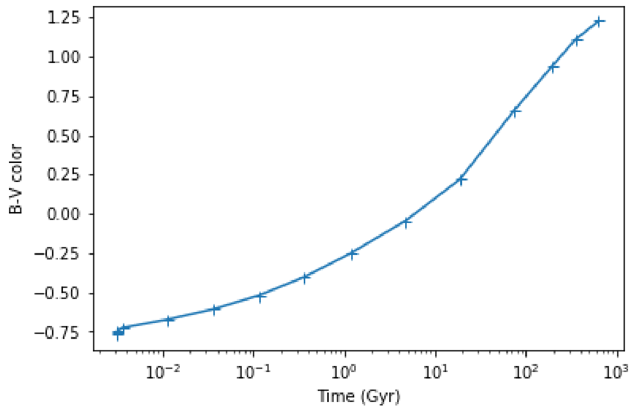

Figure 8: B-V color of the integrated spectrum as a function of time.

M/L ratio as derived in part e.

The B-V color is defined as:

$$BV = -2.5 \log_{10} \frac{f_{\lambda,B}}{f_{\lambda,V}}$$

We calculate this for each of the integrated spectra found in part (g) to get Figure 8. As the V color goes up, we would expect the log of B/V colors to decrease, and since the B-V color is related to the negative of the log, we would expect the B-V color to increase as the spectrum gets redder as we see in the figure.

2. Star Formation Rates

   (a) Consider an exponentially declining and a constant SFH. Assume that both form $10^{11} M_\odot$ in 5 Gyr, and plot their SFRs as a function of time.

   Calculating the SFRs from the SFHs:
   Constant SFR = total # stars formed / total time
   Exponential SFR = $S_0 e^{-\lambda t}$

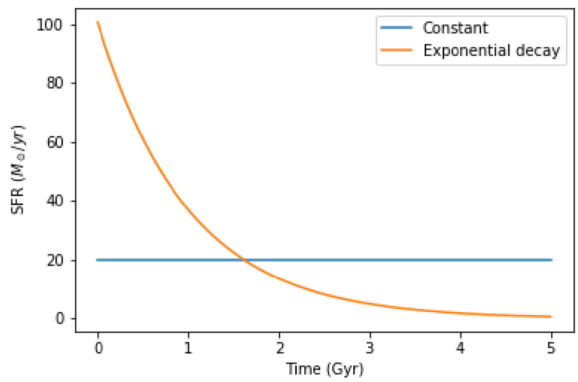

Figure 9: Star-formation rate of the stellar population vs. time for both a constant and an exponentially-decaying star-formation history.

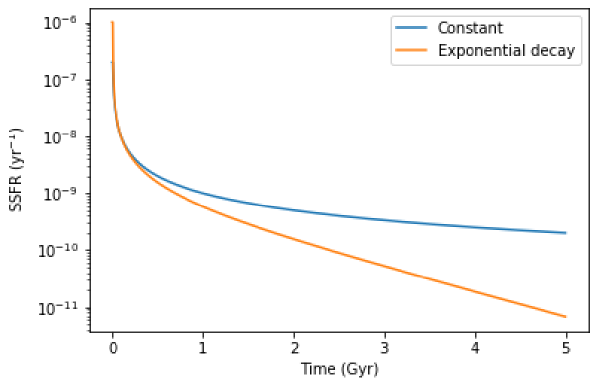

Figure 10: Specific star-formation rate vs. time for each star-formation history.

$$
\begin{aligned}
10^{11} &= \int_0^{5\ Gyr} S_0 e^{-\lambda t} dt \\
&= \frac{S_0}{\lambda}[1 - e^{-\lambda(5\ Gyr)}] \\
\lambda &= 1\ Gyr^{-1}\ \text{(arbitrary number)} \\
S_0 &= \frac{10^{11} * 10^{-9}\ yr^{-1}}{1 - e^{-5}} \\
&= 100.67\ M_\odot/yr
\end{aligned}
$$

The graph of each SFR is shown in Figure 9, and they look as we would expect for a constant vs. an exponentially decreasing function.

(b) Plot the evolution of the specific SFR as a function of time for both SFHs, assuming that no stellar mass is lost. Do you expect the U-B color to change with time for both SFHs?

The mass of each stellar population can be calculated from the integral of its star formation rate. Dividing each SFR by the mass of the stellar population at each point results in Figure 10. The mass term in the SSFR is monotonically increasing, so even for a constant SFR we would expect the plot of the SSFR to decrease as a function of time. For the exponentially decaying SFR, the mass term is increasing while the SFR itself is decreasing, so the slope of the SSFR is even lower than that of the constant rate.

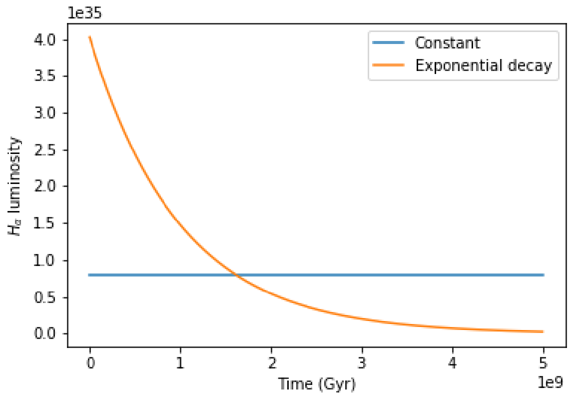

Figure 11: H_α luminosity as a function of time.

(c) What kind of SFH would produce a constant specific SFR? How would the color evolve for this SFH?

To get a constant SSFR, the star-formation rate would have to be the same as the mass at all points in time:

$$\begin{aligned} \text{SFR/Mass} &= const. \\ \text{SFR} &= c * \text{Mass} \\ &= c \int_0^t \text{SFR}\, dt \end{aligned}$$

Since the SFR must be equal to a constant times its own integral, one function that could satisfy this equation is an exponentially increasing one: SFR $= S_0 e^{\lambda t}$. In intuitive terms, if the mass term in the denominator of the SSFR formula is always increasing then the SFR must increase to keep up in order for the specific SFR to remain constant.

(d) Assuming no dust extinction, plot the expected H_α luminosity as a function of time for the two SFHs according to the Kennicutt relation.

The expected H_α luminosity is given by the Kennicutt relation:

$$\begin{aligned} SFR &= \frac{L(H_\alpha)}{1.26x10^{41}\ erg/s} \\ L(H_\alpha) &= SFR * (1.26x10^{41}\ erg/s) \end{aligned}$$

The plot of the H_α luminosity values is shown in Figure 11. As expected, it is the same graph as Figure 9 with a different scale.

(e) How would the Kennicutt relation look for an IMF with a slope of $\alpha = 1.3$ at $M < 0.5M_\odot$? Assume both IMFs have the same slope ($\alpha = 2.35$) at $M > 0.5M_\odot$ and that stars are formed in the range 0.8-100$M_\odot$.

A slope of $\alpha = 1.3$ at $M < 0.5M_\odot$ would correspond to a top-heavy IMF, meaning that fewer low-mass stars would be formed than in the usual Kennicutt relation. This means that using the traditional Kennicutt constant we would overestimate the star formation rate using the H_α luminosity. Thus the constant in the Kennicutt relation would have to be greater than $1.26x10^{41}$ erg/s to make up for fewer low-mass stars being created.

(f) The equivalent width of H_α, which is the luminosity divided by the continuum level, is also known as the birth parameter. Qualitatively explain how the equivalent width of H_α would evolve for the two different SFHs.

... sorry :/